Multivariable Design Coursework

An Introduction to $\mathcal{H}_\infty$ Design

1 Introduction

The aim of the coursework is to introduce you to $\mathcal{H}_\infty$ control system design. All the background material you may need is found in the *Robust Control Toolbox*. You should read Section 3 in the tutorial section before attempting any part of the coursework. The theoretical work can be found in the lectures on multivariable control.

2 A lightly damped beam example

The purpose of the section is to introduce you to the siso mixed sensitivity problem (see the *Robust Control Toolbox* for a precise definition). The plant considered is contrived to illustrate various features of the $\mathcal{H}_\infty$ theory, but is vaguely reminiscent of a lightly damped beam with its sensors and actuators

$$g(s) = \frac{(s-2)(s^2+s+1.25)(s^2+0.6s+9.09)}{(s-1)(s^2+0.2s+1.01)(s^2+0.2s+25.01)}$$

The design will be carried out in a number of steps:

1. Use the *conv* command to input the plant model into the variables *num* and *den* in *matlab*; we want $g(s) = num/den$.

2. Find a state-space model for $g(s)$ using the command *tf2ss*.

3. Find the poles and zeros of $g(s)$ using the commands *eig* and *tzero* respectively.

4. Plot a frequency response of $g(s)$ using the command sequence:

   - $\omega =$ logspace(-2,3,150);
   - $mag =$ bode(num,den,w);
   - semilogx$(\omega, 20 * \log_{10}(mag))$

   and reconcile this plot with the poles and zeros of $g(s)$ (i.e. identifying the poles and zeros by looking at the shape of the bode plot such as the asymptotes).

5. We will now design a controller for the sensitivity alone; we want a good tracking property over the frequency range 0-5 rad/s. To this end we solve the problem

$$\left\| w_1(s)[1 + g(s)k(s)]^{-1} \right\|_\infty \leq 1,$$

   or equivalently,

$$|[1 + g(j\omega)k(j\omega)]^{-1}| \leq |w_1^{-1}(j\omega)|,$$

   where,

$$w_1(s) = \frac{\gamma}{(s+5)^2}, \qquad \gamma = 15.$$

6. Embed this design in a larger problem as follows. Let,

$$w_2(s) = w_3(s) = 0.0001,$$

   and form,

$$P(s) = \left[\begin{array}{c|c} w_1 & -w_1 g \\ 0 & w_2 \\ 0 & w_3 g \\ \hline 1 & -g \end{array}\right],$$

   using the *augment* command as follows: (create your own .m file containing this),

- $Gam = 15;$
- $dnw1i = 1; nuw1i = [1, 10, 25];$
- $dnw1 = [1, 10, 25]; nuw1 = 1;$
- $dnw2 = 1; nuw2 = 0.0001;$
- $dnw2i = nuw2; nuw2i = dnw2;$
- $dnw3 = 1; nuw3 = 0.0001;$
- $dnw3i = nuw3; nuw3i = dnw3;$
- $[aw1, bw1, cw1, dw1] = tf2ss(dnw1i * Gam, nuw1i);$
- $sysg = [ag, bg; cg, dg]; sysw1 = [aw1, bw1; cw1, dw1];$
- $[rdg, cdg] = size(dg);$
- $sysw2 = [0.0001]; sysw3 = [0.0001];$
- $dim = [5, 2, 0, 0];$
- $[A, B1, B2, C1, C2, D11, D12, D21, D22]$
  $= augment(sysg, sysw1, sysw2, sysw3, dim);$

7. Calculate an $\mathcal{H}_\infty$ controller using the command $hinfsyn$.

8. Plot $W_1^{-1}(s)$, $W_3^{-1}(s)$, the sensitivity and the complementary sensitivity by typing the command *pltopt* in the command window. The value of γ is user-defined and in this case $\gamma = 15$. Follow the steps shown in the command window. Alternatively, you can open the relevant m–file by typing in *open pltopt.m* in the command window. Be aware that the parameters $ag, bg, cg, dg, nuw1i, dnw1i, nuw3i, dnw3i$ are the inputs of this m–file and they need to be specified beforehand. Analyse the plots and comment on the loop tracking properties of the design.

9. Find the closed-loop poles and check they are stable. Find the controller poles and zeros and explain what the controller is doing to the stable and minimum phase factor of the plant.

10. Suppose the $(s-2)$ term in the numerator of $g(s)$ were replaced by $(s+2)$. How big could we make γ and why?

11. Interpret the complementary sensitivity as a robustness measure and comment on the robustness of the above design. Would you be happy with this design from a robustness point of view? (Ignore step (10)).

12. Suppose we want the complementary sensitivity smaller than

$$W_3^{-1}(s) = \frac{1 + 0.01s}{0.01s}$$

for robustness reasons. Introduce this W_3 and let $\gamma = 11$ in the weight $W_1(s)$ above.

13. Find the new augmented plant as above and repeat the design calculation using $hinfsyn$.

14. Plot $W_1^{-1}(s)$ and $W_3^{-1}(s)$. Compare the robust stability properties of the old and new designs. Are there any improvements? Explain the trade made between performance and robustness.

15. Use *Simulink* to validate the tracking abilities of both designs above by performing time responses (i.e. step or pulse responses). Explain your findings.

3 A 1990's aircraft example

In this experiment we consider the longitudinal dynamics of the HI-MAT experimental aircraft trimmed at an altitude of 25,000ft and a speed of 0.9 Mach. The control variables are the elevon and canard actuators, while the output variables are the angles of attack and attitude. See the tutorial section of the *Robust Control Toolbox* for further details. Obtain the aircraft model by running *hinfdemo* and using Option 2. Alternatively make a copy of the Matlab file *hmatdemo.m* which you can change to perform the remainder of the experiment. This can be done by typing in *open hmatdemo.m* in the command window.

Preliminary analysis

Calculate the open-loop poles and zeros, plot the frequency response and comment on the expected difficulties associated with synthesising a control system for such a plant. Explain your answer.

The design specifications

1. The robustness specification requires that the complementary sensitivity function rolls off at -40db per decade over frequencies beyond 100 rad/s and that it is no greater than -20db at 100 rad/s. Design a weighting function that meets these specifications without imposing any extra conditions at low frequencies.

2. With the above robustness specifications in mind, we require that the sensitivity is as small as possible between 0 and 1 rad/s.

The controller design and analysis phase

1. Choose weighting matrices $W_1(s)$ and $W_3(s)$ which reflect the design specifications on performance and robustness, respectively. Remember that $W_2(s)$ must be set equal to some small full rank constant matrix. Explain why this is necessary. Explain how you would synthesise the improper weighting function $W_3(s)$. Follow step (6) of the first design to augment the plant.

2. Calculate your own controller using the $hinfsyn$ command; all the standard plots may be obtained using the command *pltopt* as before. Interpret the robustness and sensitivity characteristics of this design.

3. Plot $W_1^{-1}(s)$, $W_3^{-1}(s)$, the sensitivity and the complementary sensitivity functions. Tune the value of γ to obtain the best possible design which meets the specifications.

4. Plot $\bar{\sigma}[K(I+GK)^{-1}]$. In the light of this plot, is this control system implementable? Carefully explain your answer and back up your comments with appropriate graphs. Indicate how this situation may be remedied and do so.

5. Interpret $\bar{\sigma}[K(I+GK)^{-1}]$ as a robustness indicator by comparing the sizes of the perturbations that might be accommodated in the two designs (i.e. step (3) and step (4)). Indicate how the two designs are changed and what trade–offs have been made.

6. Compare the time responses of the near optimal design of step (3) with those of the near optimal design of step (4).

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