OPTIMISATION

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COURSE-WORK – FOR M.SC. STUDENTS ONLY

The final report should be submitted electronically on Blackboard

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Part I [70 marks]

To compare the performance of optimisation methods it is customary to construct *test* functions and then compare the behavior of various optimization algorithms for such functions.
One, often used, test function is the so-called Rosenbrock function, *i.e.*

$$v(x, y) = 100(y - x^2)^2 + (1 - x)^2.$$

A1) Compute analytically all stationary points of the function $v(x, y)$ and verify if they are minimizers/maximizers/saddle points. [4 marks]

A2) Plot (using Matlab or a similar SW) the level sets of the function $v(x, y)$. [4 marks]

A3) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the gradient method (see Section 2.5) with Armijo line search (see Section 2.4.2). [8 marks]

A4) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using Newton method (see Section 2.6) with and without Armijo line search. [8 marks]

A5) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the Polak-Ribiere algorithm (see Section 2.7.3) with Armijo line search. [10 marks]

A6) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the Broyden-Fletcher-Goldfarb-Shanno algorithm (see Section 2.8) with Armijo line search. [10 marks]

A7) Implement (in Matlab or a similar SW) procedures for the minimization of the function $v(x, y)$ using the simplex method (see Section 2.9). Consider modifying the method in Section 2.9 to improve convergence properties of the algorithm. [12 marks]

A8) *Run* the minimization procedures written in points A3) to A7) with initial point $(x_0, y_0) = (-3/4, 1)$.

A8a) Plot, on the (x, y)-plane, the sequences of points generated by each algorithm. Are these sequences converging to a stationary point of $v(x, y)$? [4 marks]

A8b) For a sequence $\{x_k, y_k\}$ consider the *cost*

$$J_k = \log\left((x_k - 1)^2 + (y_k - 1)^2\right).$$

Plot, for each of the sequences generated by the above algorithms, the cost J_k as a function of k. Use such a plot to assess the speed of convergence of each of the considered algorithms. [10 marks]

Part II [30 marks]

Consider the constrained minimization problem

$$\min_{x_1,x_2} = -x_1 x_2$$
$$x_1 + x_2 - 2 = 0.$$

B1) Write the necessary conditions of optimality and find all points satisfying such conditions. [4 marks]

B2) Check if the candidate optimal points obtained in B1) are constrained local minimizers or otherwise. [4 marks]

B3) By solving the equation of the constraint transform the considered constrained optimization problem into an unconstrained problem. [4 marks]

B4) Construct the exact penalty function $G(x)$ associated to the considered problem (see Section 3.4.3). Minimize analytically the function $G(x)$ and show that the unconstrained minimizer of $G(x)$ is a solution of the considered constrained optimization problem. [8 marks]

B5) Construct the exact augmented Lagrangian function $S(x, \lambda)$ associated to the considered problem (see Section 3.4.4). Minimize analytically the function $S(x, \lambda)$ and show that the unconstrained minimizer of $S(x, \lambda)$ yields a solution of the considered constrained optimization problem and the corresponding optimal multiplier. [10 marks]