Joint & Marginal Distributions
Linearity of Expectation

Joint and Marginal Distributions

- Let X_1 and X_2 be discrete r.v.'s with PMFs f_1 and f_2
- Let $f(x_1,x_2)$ be joint PMF of X_1 and X_2

$$P[X_1 = x_1, X_2 = x_2] = f(x_1,x_2)$$

- Can write f_1 and f_2 in terms of f:

$$f_1(x_1) = P[X_1 = x_1] = \sum_{x2} P[X_1 = x_1, X_2 = x_2] = \sum_{x2} f(x_1,x_2)$$
$$f_2(x_2) = P[X_2 = x_2] = \sum_{x1} P[X_1 = x_1, X_2 = x_2] = \sum_{x1} f(x_1,x_2)$$

- When dealing with joint distribution f of (X_1, X_2), f_1 and f_2 are sometimes referred to as the *marginal distributions* of X_1 and X_2

Linearity of Expectation

- Let X_1 and X_2 be discrete RVs and a_1 and a_2 constants
- Linearity of Expectation:

$$E[a_1X_1 + a_2X_2] = a_1 E[X_1] + a_2 E[X_2]$$

- Proof:

$$\begin{aligned}
E[a_1X_1 + a_2X_2] &= \textstyle\sum_{x1,x2} f(x_1, x_2)(a_1x_1+a_2x_2) \\
&= a_1\textstyle\sum_{x1,x2} f(x_1, x_2)\, x_1 + a_2\sum_{x1,x2} f(x_1, x_2)\, x_2 \\
&= a_1\textstyle\sum_{x1} f_1(x_1)\, x_1 + a_2\sum_{x2} f_2(x_2)x_2 \\
&= a_1 E[X_1] + a_2 E[X_2]
\end{aligned}$$