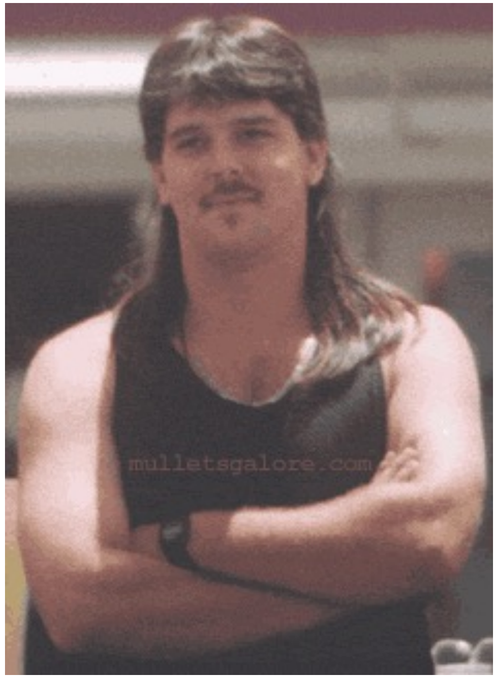
ECEN 758
Data Mining and Analysis

Recommendation Systems

J. Leskovec, A. Rajaraman, J. Ullman:

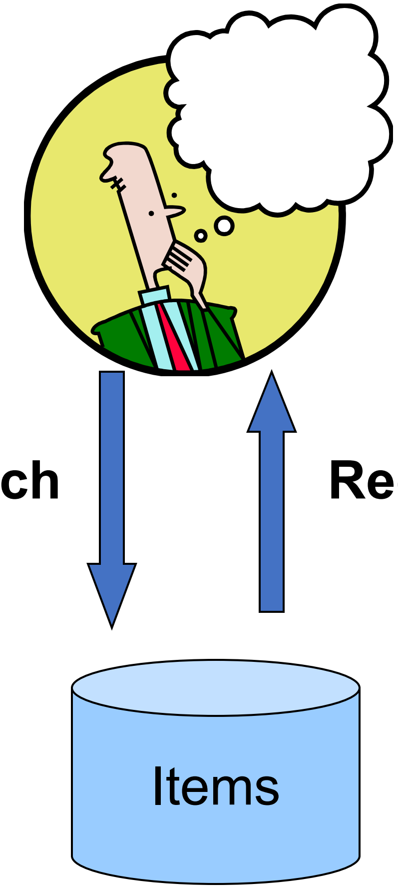
Mining of Massive Datasets, http://www.mmds.org

Example: Recommender Systems



- **Customer X**
  - Buys Metallica CD
  - Buys Megadeth CD
- **Customer Y**
  - Does search on Metallica
  - Recommender system suggests Megadeth from data collected about customer **X**

Recommendations



Search

Recommendations

Items

Products, web sites, blogs, news items, …

Examples:

amazon.com

PANDORA

StumbleUpon

del.icio.us

NETFLIX

movielens

helping you find the *right* movies

last.fm

the social music revolution

Google News

YouTube

XBOX LIVE

From Scarcity to Abundance

- **Shelf space is a scarce commodity for traditional retailers**
  - Also: TV networks, movie theaters,…

- **Web enables near-zero-cost dissemination of information about products**
  - From scarcity to abundance

- **More choice necessitates better filters**
  - Recommendation engines
  - How **Into Thin Air** made **Touching the Void** a bestseller: http://www.wired.com/wired/archive/12.10/tail.html

Types of Recommendations

- **Editorial and hand curated**
  - List of favorites
  - Lists of “essential” items

- **Simple aggregates**
  - Top 10, Most Popular, Recent Uploads

- **Tailored to individual users**
  - Amazon, Netflix, …

Formal Model

- ***X*** **= set of Customers**
- ***S*** **= set of Items**

- **Utility function *u*: $X \times S \rightarrow R$**
  - ***R*** = set of ratings
  - ***R*** is a totally ordered set
  - e.g., **0-5** stars, real number in **[0,1]**

Utility Matrix

	Avatar	**LOTR**	**Matrix**	**Pirates**
Alice	1		0.2	
Bob		0.5		0.3
Carol	0.2		1	
David				0.4

Key Problems

- **(1) Gathering "known" ratings for matrix**
  - How to collect the data in the utility matrix
- **(2) Extrapolate unknown ratings from the known ones**
  - Mainly interested in high unknown ratings
    - ❑ We are not interested in knowing what you don't like but what you like
- **(3) Evaluating extrapolation methods**
  - How to measure success/performance of recommendation methods

(1) Gathering Ratings

- **Explicit**
  - Ask people to rate items
  - Doesn't work well in practice – people can't be bothered
- **Implicit**
  - Learn ratings from user actions
    - E.g., purchase implies high rating
  - What about low ratings?

(2) Extrapolating Utilities

- **Key problem: Utility matrix *U* is sparse**
  - Most people have not rated most items
  - **Cold start:**
    - ☐ New items have no ratings
    - ☐ New users have no history

- **Discuss Two approaches to recommender systems:**
  - **1)** Content-based
  - **2)** Collaborative

Content-based Recommender Systems

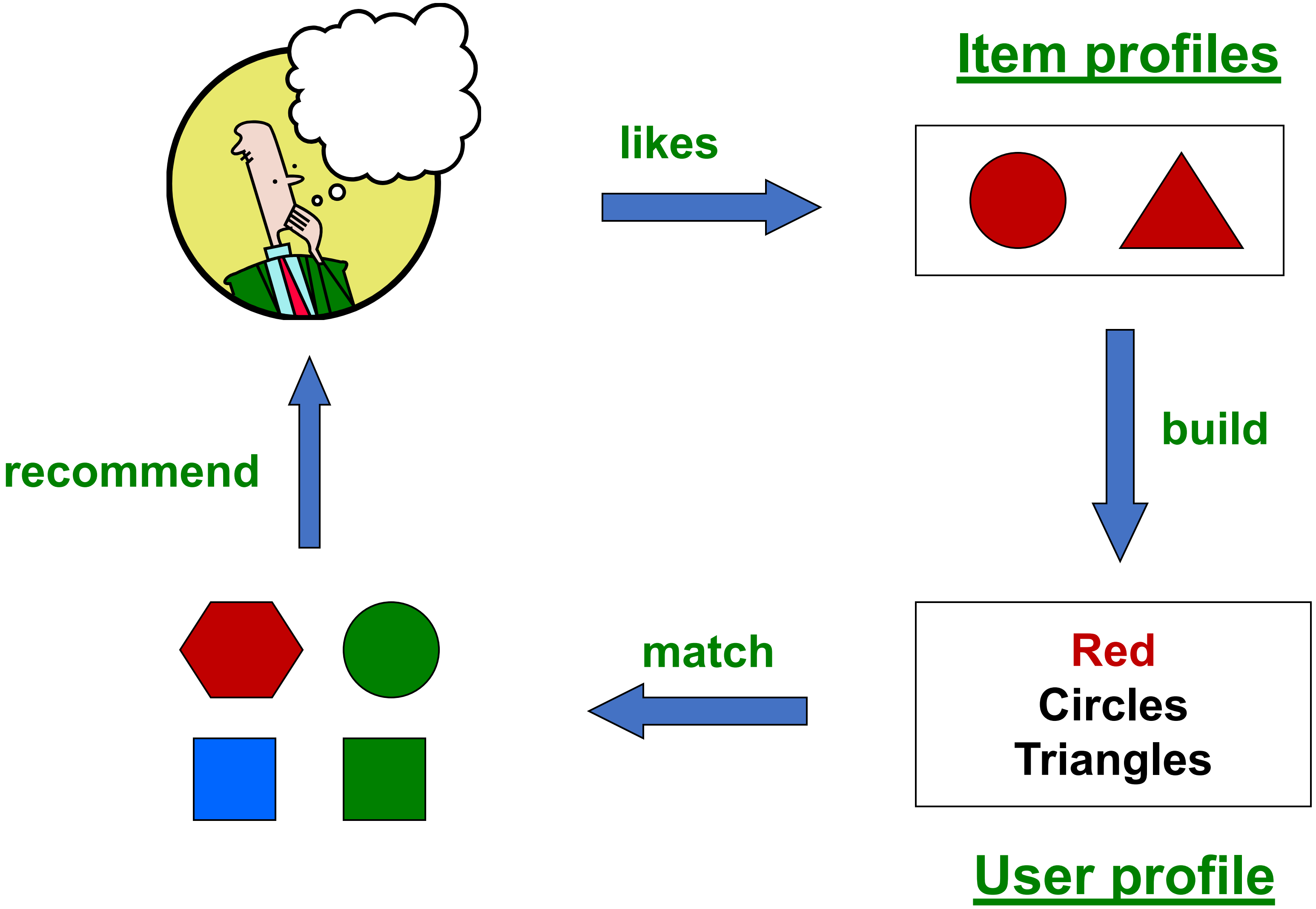
Content-based Recommendations

- **Main idea: Recommend items to customer *x* similar to previous items rated highly by *x***

Example:

- **Movie recommendations**
  - Recommend movies with same actor(s), director, genre, …
- **Websites, blogs, news**
  - Recommend other sites with "similar" content

Plan of Action



Item profiles

likes

build

Red
Circles
Triangles

User profile

match

recommend

Item Profiles

- **For each item, create an item profile**

- **Profile is a set (vector) of features**
  - **Movies:** author, title, actor, director,…
  - **Text:** Set of "important" words in document

- **How to pick important features?**
  - Usual heuristic from text mining is **TF-IDF**
    - (Term frequency * Inverse Doc Frequency)
  - **Term … Feature**
  - **Document … Item**

Sidenote: TF-IDF

f_{ij} = frequency of term (feature) *i* in doc (item) *j*

$$TF_{ij} = \frac{f_{ij}}{\max_k f_{kj}}$$

Note: we normalize TF to discount for “longer” documents

n_i = number of docs that mention term *i*

N = total number of docs

$$IDF_i = \log \frac{N}{n_i}$$

TF-IDF score: $w_{ij} = TF_{ij} \times IDF_i$

Doc profile = set of words with highest TF-IDF scores, together with their scores

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User Profiles and Prediction

- **User profile possibilities:**
  - Weighted average of rated item profiles
  - **Variation:** weight by difference from average rating for item
- **Prediction heuristic:**
  - Given user profile ***x*** and item profile ***i***, estimate

$$u(\boldsymbol{x}, \boldsymbol{i}) = \cos(\boldsymbol{x}, \boldsymbol{i}) = \frac{\boldsymbol{x} \cdot \boldsymbol{i}}{||\boldsymbol{x}|| \cdot ||\boldsymbol{i}||}$$

Pros: Content-based Approach

- **+: No need for data on other users**
  - No cold-start or sparsity problems
- **+: Able to recommend to users with unique tastes**
- **+: Able to recommend new & unpopular items**
  - No first-rater problem
- **+: Able to provide explanations**
  - Can provide explanations of recommended items by listing content-features that caused an item to be recommended

Cons: Content-based Approach

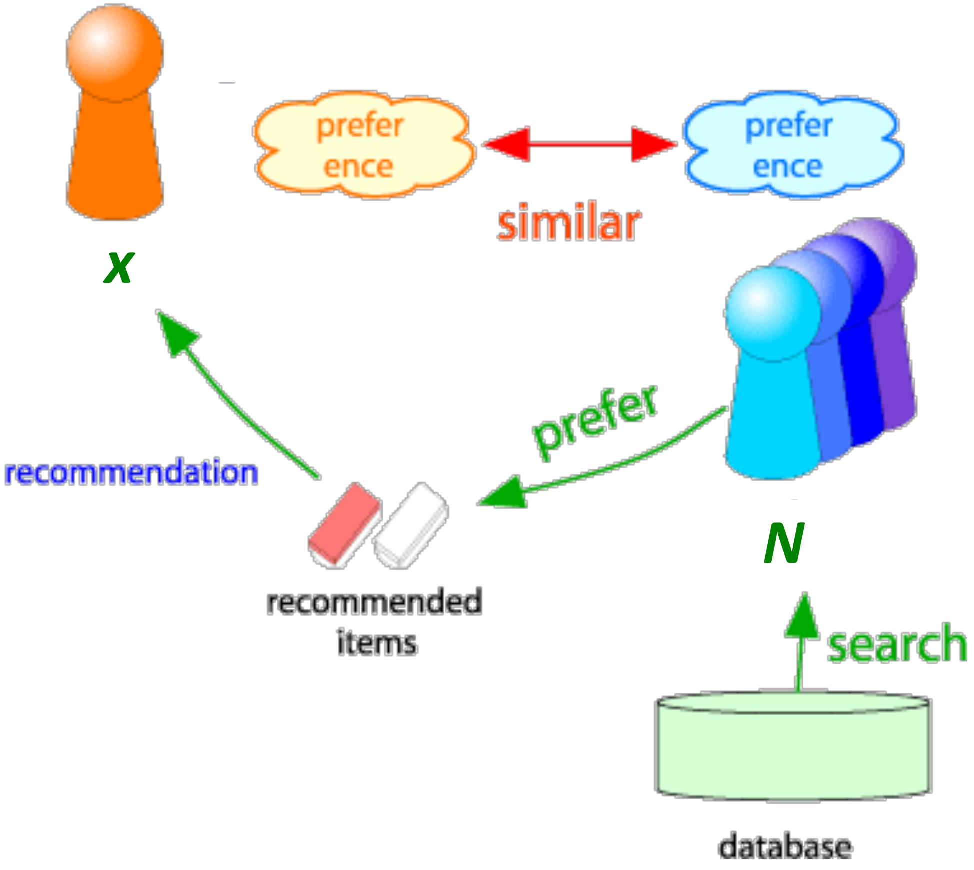
- **–: Finding the appropriate features is hard**
  - E.g., images, movies, music
- **–: Recommendations for new users**
  - **How to build a user profile?**
- **–: Overspecialization**
  - Never recommends items outside user's content profile
  - People might have multiple interests
  - **Unable to exploit quality judgments of other users**

Collaborative Filtering

Harnessing quality judgments of other users

Collaborative Filtering

- **Consider user *x***
- **Find set *N* of other users whose ratings are "similar" to *x*'s ratings**
- **Estimate *x*'s ratings based on ratings of users in *N***



Finding "Similar" Users

- **Let r_x be the vector of user x's ratings**
- **Jaccard similarity measure: J(A,B)=|A∩B|/|A∪B|**
  - **Problem:** Ignores the value of the rating
- **Cosine similarity measure**
  - sim(***x***, ***y***) = cos(r_x, r_y) = $\frac{r_x \cdot r_y}{||r_x|| \cdot ||r_y||}$
  - **Problem:** Treats missing ratings as "negative"
- **Pearson correlation coefficient**
  - S_{xy} = items rated by both users ***x*** and ***y***

$$sim(x,y) = \frac{\sum_{s \in S_{xy}} (r_{xs} - \overline{r_x})(r_{ys} - \overline{r_y})}{\sqrt{\sum_{s \in S_{xy}} (r_{xs} - \overline{r_x})^2} \sqrt{\sum_{s \in S_{xy}} (r_{ys} - \overline{r_y})^2}}$$

r_x, r_y as sets:
r_x = {1, 4, 5}
r_y = {1, 3, 4}

r_x, r_y as points:
r_x = {1, 0, 0, 1, 3}
r_y = {1, 0, 2, 2, 0}

$\overline{r_x}$, $\overline{r_y}$ … avg. rating of **x**, **y**

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Similarity Metric

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

- **Intuitively we want:** **sim(*A*, *B*) > sim(*A*, *C*)**
- **Jaccard similarity: 1/5 < 2/4**
- **Cosine similarity: 0.386 > 0.322**
  - Considers missing ratings as "negative"
  - **Solution: subtract the (row) mean**

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	$2/3$			$5/3$	$-7/3$		
B	$1/3$	$1/3$	$-2/3$				
C				$-5/3$	$1/3$	$4/3$	
D		0					0

sim A,B vs. A,C:
0.092 **>** -0.559

Notice cosine sim. is
correlation when
data is centered at 0

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Rating Predictions

From similarity metric to recommendations:

- **Let r_x be the vector of user *x*'s ratings**
- **Let *N* be the set of *k* users most similar to *x* who have rated item *i***
- **Prediction for item *i* of *user x*:**
  - $r_{xi} = \frac{1}{k} \sum_{y \in N} r_{yi}$
  - $r_{xi} = \frac{\sum_{y \in N} s_{xy} \cdot r_{yi}}{\sum_{y \in N} s_{xy}}$

Shorthand:
$\boldsymbol{s_{xy} = sim(x, y)}$

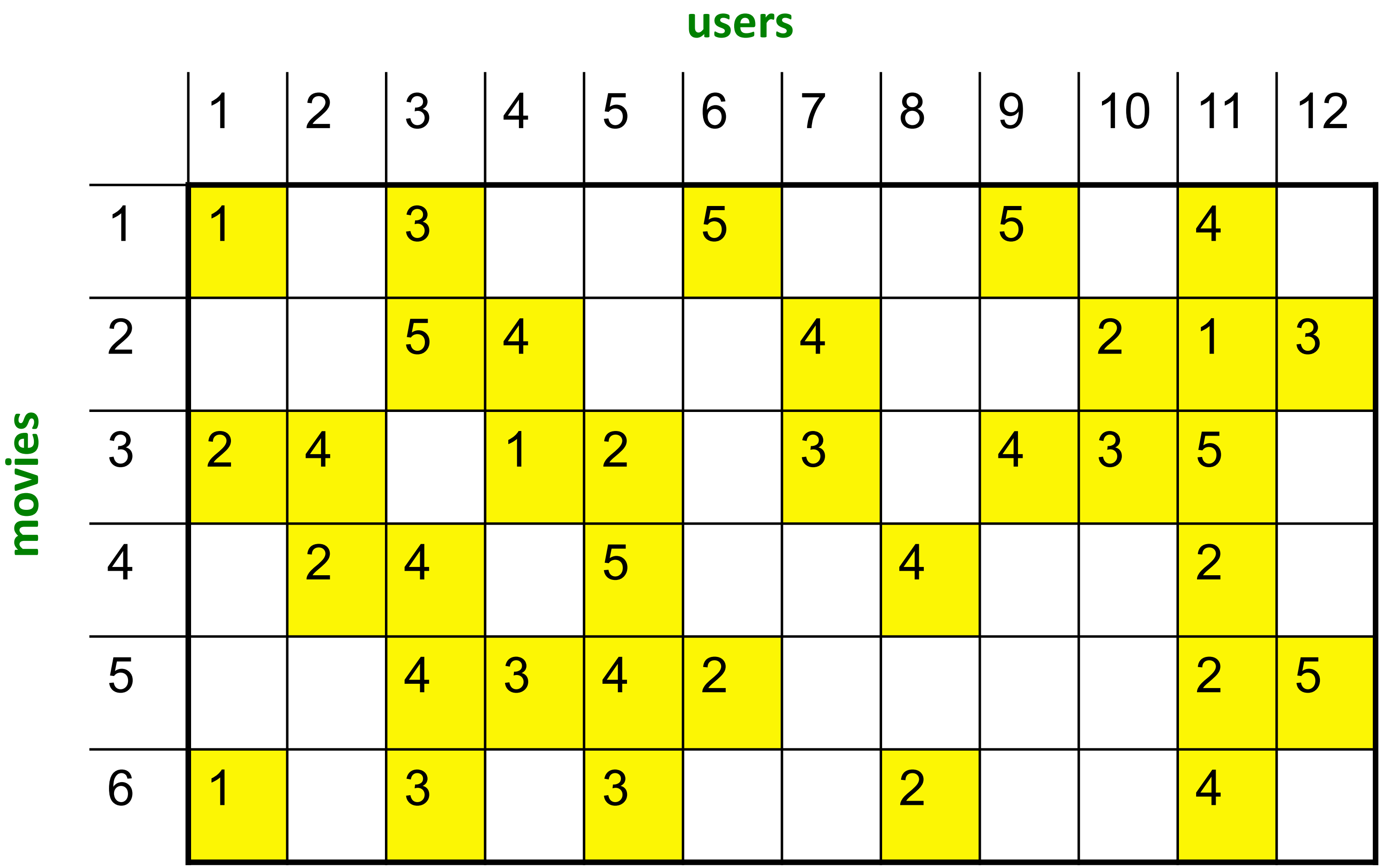
Item-Item Collaborative Filtering

- **So far: User-user collaborative filtering**
- **Another view: Item-item**
  - For item ***i***, find other similar items
  - Estimate rating for item ***i*** based on ratings for similar items
  - Can use same similarity metrics and prediction functions as in user-user model

$$\boxed{r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}}$$

s_{ij}… similarity of items ***i*** and ***j***
r_{xj}…rating of user ***u*** on item ***j***
N(i;x)… set items rated by ***x*** similar to ***i***

Item-Item CF (|N|=2)



users

movies	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3			5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

□ - unknown rating 🟨 - rating between 1 to 5

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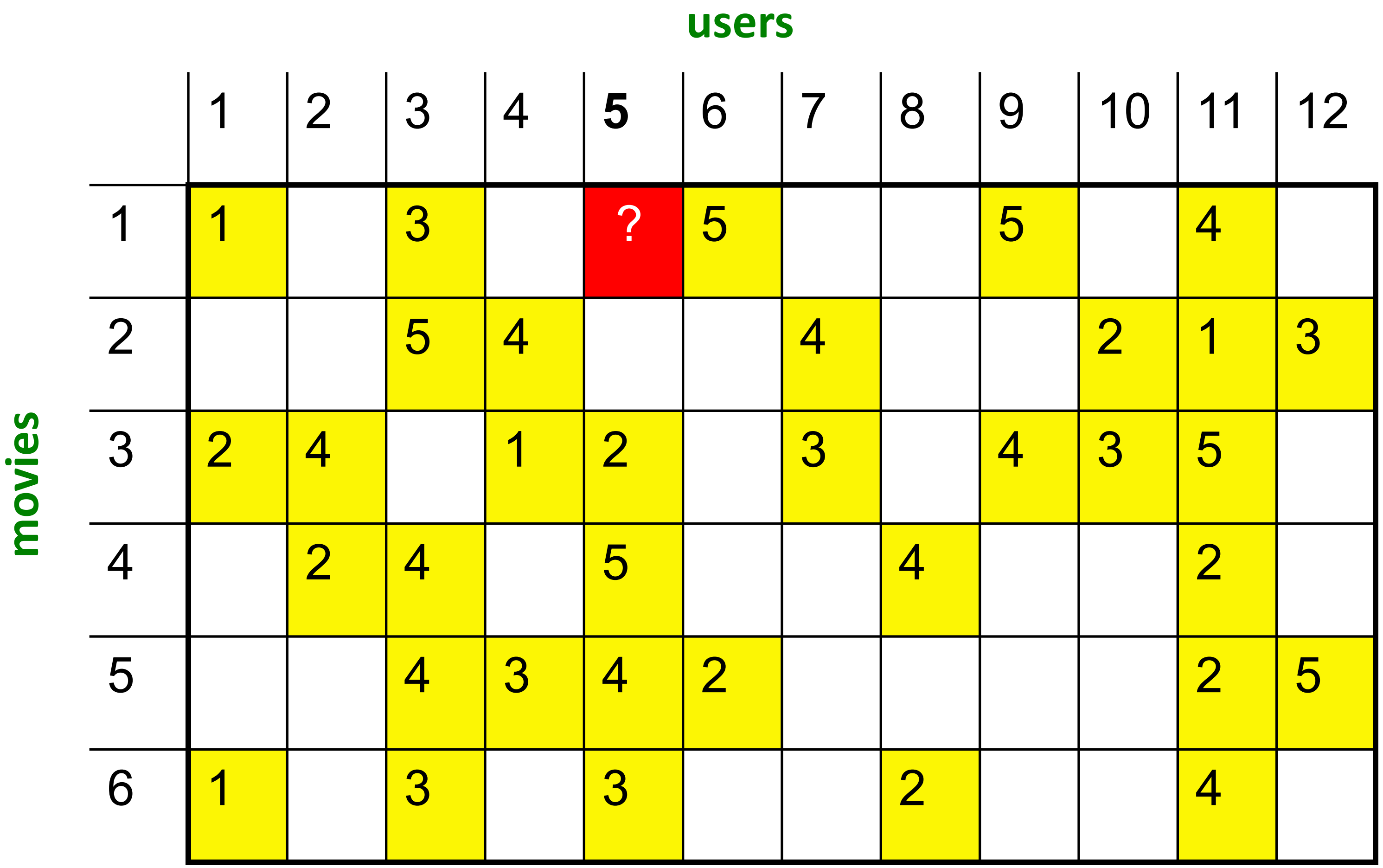
Item-Item CF (|N|=2)



users

movies

	1	2	3	4	**5**	6	7	8	9	10	11	12
1	1		3		?	5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

🟥 - estimate rating of movie **1** by user **5**

Item-Item CF (|N|=2)

users

movies

	1	2	3	4	**5**	6	7	8	9	10	11	12	**sim(1,m)**
1	1		3		?	5			5		4		**1.00**
2			5	4			4			2	1	3	**-0.18**
3	2	4		1	2		3		4	3	5		**0.41**
4		2	4		5			4			2		**-0.10**
5			4	3	4	2					2	5	**-0.31**
6	1		3		3			2			4		**0.59**

Neighbor selection:
Identify movies similar to movie **1**, **rated by user 5**

Here we use Pearson correlation as similarity:

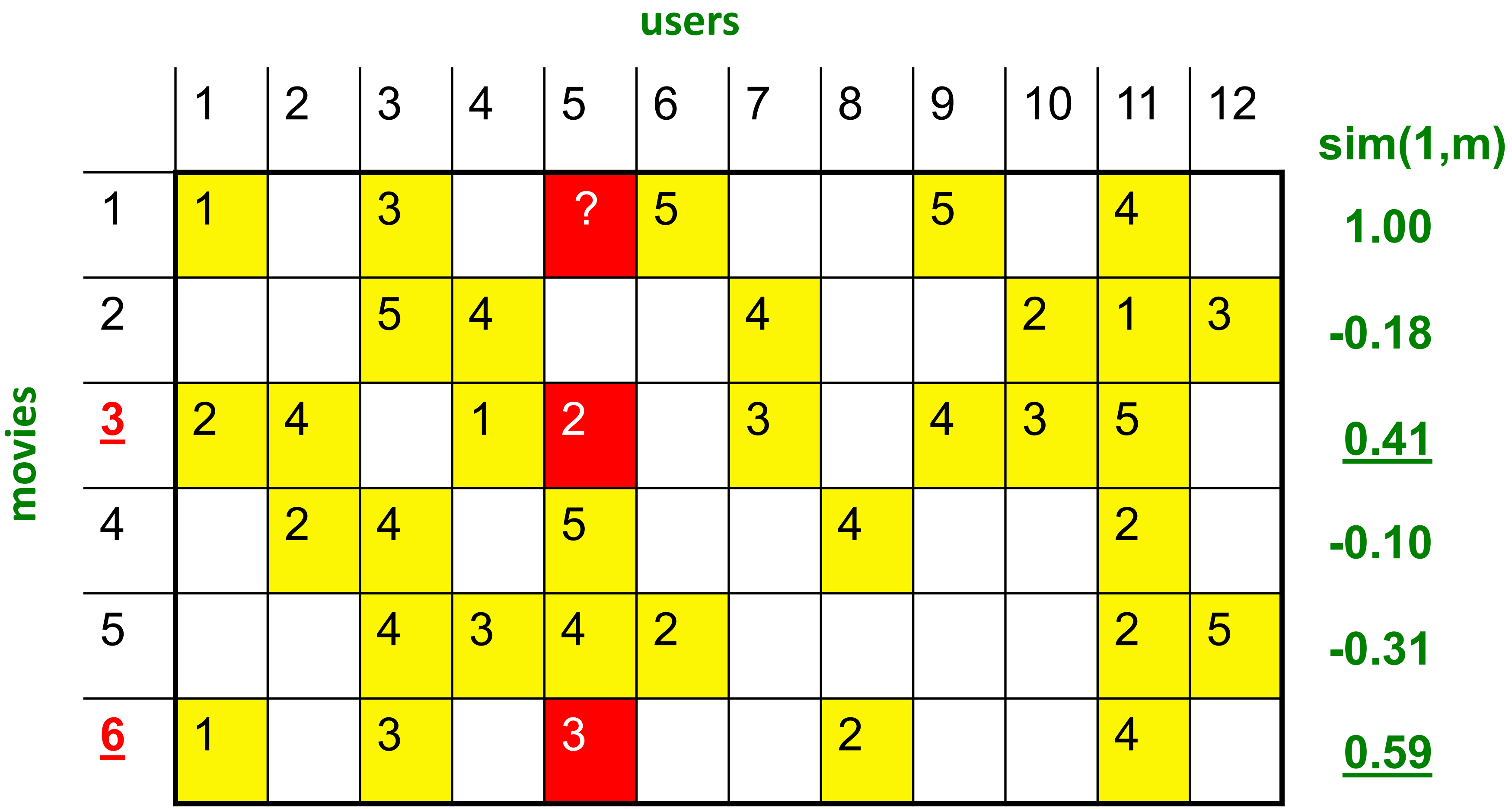
1) Subtract mean rating m_i from each movie i

$m_1 = (1+3+5+5+4)/5 = 3.6$

row 1: [-2.6, 0, -0.6, 0, 0, 1.4, 0, 0, 1.4, 0, 0.4, 0]

2) Compute cosine similarities between rows

Item-Item CF (|N|=2)



users

movies

	1	2	3	4	5	6	7	8	9	10	11	12	sim(1,m)
1	1		3		?	5			5		4		1.00
2			5	4			4			2	1	3	-0.18
3	2	4		1	2		3		4	3	5		**0.41**
4		2	4		5			4			2		-0.10
5			4	3	4	2					2	5	-0.31
6	1		3		3			2			4		**0.59**

Compute similarity weights: **$s_{1,3}=0.41$, $s_{1,6}=0.59$**

Item-Item CF (|N|=2)

users

movies	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3		**2.6**	5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

Predict by taking weighted average:

$r_{1.5}$ = (0.41*2 + 0.59*3) / (0.41+0.59) = 2.6

$$r_{ix} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{jx}}{\sum s_{ij}}$$

CF: Common Practice

Before:

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

- **Define similarity s_{ij} of items *i* and *j***
- **Select *k* nearest neighbors $N(i; x)$**
  - Items most similar to ***i***, that were rated by ***x***
- **Estimate rating r_{xi} as the weighted average:**

$$r_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

$$\boldsymbol{b_{xi} = \mu + b_x + b_i}$$

- μ = overall mean movie rating
- b_x = rating deviation of user ***x***
  = *(avg. rating of user **x**)* – μ
- b_i = rating deviation of movie ***i***

Item-Item vs. User-User

	Avatar	**LOTR**	**Matrix**	**Pirates**
Alice	1		0.8	
Bob		0.5		0.3
Carol	0.9		1	0.8
David			1	0.4

- **In practice, it has been observed that item-item often works better than user-user**
- **Why?** Items are simpler, users have multiple tastes

Pros/Cons of Collaborative Filtering

- **+ Works for any kind of item**
  - No feature selection needed

- **- Cold Start:**
  - Need enough users in the system to find a match
- **- Sparsity:**
  - The user/ratings matrix is sparse
  - Hard to find users that have rated the same items
- **- First rater:**
  - Cannot recommend an item that has not been
previously rated
  - New items, Esoteric items
- **- Popularity bias:**
  - Cannot recommend items to someone with unique taste
  - Tends to recommend popular items

Hybrid Methods

- **Implement two or more different recommenders and combine predictions**
    - Perhaps using a linear model

- **Add content-based methods to collaborative filtering**
    - Item profiles for new item problem
    - Demographics to deal with new user problem