Primer on Fixed Income Performance Attribution

While many of us are still struggling with the details of implementing systems for performance attribution of equity portfolios, the author alerts us to a whole range of differences we'll encounter when addressing fixed income attribution analysis. Lucky for us he has arranged his information in highly digestible form, as a primer, and even supplies formulae for doing our own calculations and a case history illustration.

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Many performance attribution systems use the same process for stocks as they do for bonds. As with equities, the relevant benchmark and the manager's investment strategy are integral parts of the fixed income attribution process. But the process was developed for equity portfolios, and so it uses two equity-oriented factors to attribute performance: Sector Allocation and Issue Selection. The familiarity and simplicity of this model make it an appealing tool for analyzing bond performance. However, the characteristics of bonds, and the markets in which they are traded, are fundamentally different from stock markets. Consequently, bond performance is driven by factors unique to the bond market, and which differ from the drivers of stock performance. Accordingly, systems that analyze bond performance using the equity model produce results that are misleading and incorrect. This article outlines a simple, yet robust process for attributing bond performance to its unique risk factors.[1]

PERFORMANCE OVERVIEW

Linking Performance Attribution to the Portfolio Management Process

Portfolio management is primarily about risk management, and secondarily about return enhancement. The portfolio manager's first responsibility is to identify the types and levels of risk that are appropriate for the client, and then to develop an asset allocation that delivers these risks. The manager's second responsibility is to implement the agreed-upon asset allocation by purchasing and maintaining appropriate securities. Finally, the manager undertakes the ongoing responsibility of monitoring the portfolio and its investments to ensure that the investment strategies are working, and that the client is adequately compensated for the risk being borne. This is the stage at which a performance analysis system is essential. A performance system measures the level of return delivered to the client, and assists in determining whether the manager has added value by delivering a risk-adjusted return at least equal to, or perhaps even higher than the passive alternative, or benchmark. Performance attribution is an objective measure of whether the manager is both consistent and effective in the implementation of the asset allocation agreed to by the client.

Characteristics of a Good Performance Attribution System

A good performance attribution system has four characteristics:

- It is consistent with the investment process and the manager's decision-making process;
- It uses a benchmark that reflects the manager's strategic (long-term) asset allocation;
- Measures the effect of the manager's tactical (short-term) allocation shifts;

- And, it adjusts attribution of return for systematic risk(s).

Measuring Up

Many attribution systems fail at least one of these requirements, and some fail all of them. The first requirement is often violated by applying the standard "Sector/ Selection" model to all portfolios, without ensuring that the attribution model "fits" the manager's investment strategy. Comparing the manager's performance with a familiar benchmark that is inconsistent with the asset allocation is a common cause for failing the second requirement. The familiar equity attribution model emphasizes the third requirement: evaluating the effect of short-term sector bets by the manager. However, the model fails to explain why a certain sector underperformed or outperformed, relative to the benchmark's overall return. This is because it fails the last requirement of adjusting returns for both systematic risk and style (high beta vs. low beta, large vs. small, value vs. growth, etc). Simply overlaying the "Sector/Selection" model often results in attribution results that are not necessarily meaningful, since they do not reflect the investment process or the risks that drove performance over the evaluation period. This is often true for equities; it is always true for bonds.

FUNDAMENTALS OF BONDS

Why Bonds Need Their Own Attribution Model

An analysis of the unique characteristics of bonds results in a performance model that meets the requirements of a good attribution system; however, the bond attribution model will differ markedly from the familiar equity model. Given these differences, and the relative unfamiliarity of the drivers of bond performance, a fundamental review of bonds and the bond market is warranted. A complete explanation is beyond the scope of this article, but there are many texts that provide a thorough analysis of bond investments.

There are five critical differences between stocks and bonds. These differences concern the term of the investments, the return potential, the markets in which buyers and sellers participate, the risk factors that affect performance, and the degree to which a selection effect is significant.

1. Bonds are temporary lending agreements, with a stated maturity, whereas stocks are permanent investments.

2. Bonds promise a fixed return, with a limited upside; stocks promise the uncertain returns and unlimited upside associated with ownership.

3. Bonds are typically purchased by institutions, which hold them until maturity. This results in a limited secondary market for bonds. Stock investing takes place in an active secondary market. As a result, bonds are illiquid, when compared to stocks.

4. Bond performance is driven by promised income and by changes in market yields. When market yields rise above a bond's stated yield, the price of that bond falls. As a result, the relevant risk for bonds is the sensitivity to changes in yields. Stock performance is driven by the market's economic sectors, and the relevant risk for stocks is the sensitivity to the overall market, or beta.

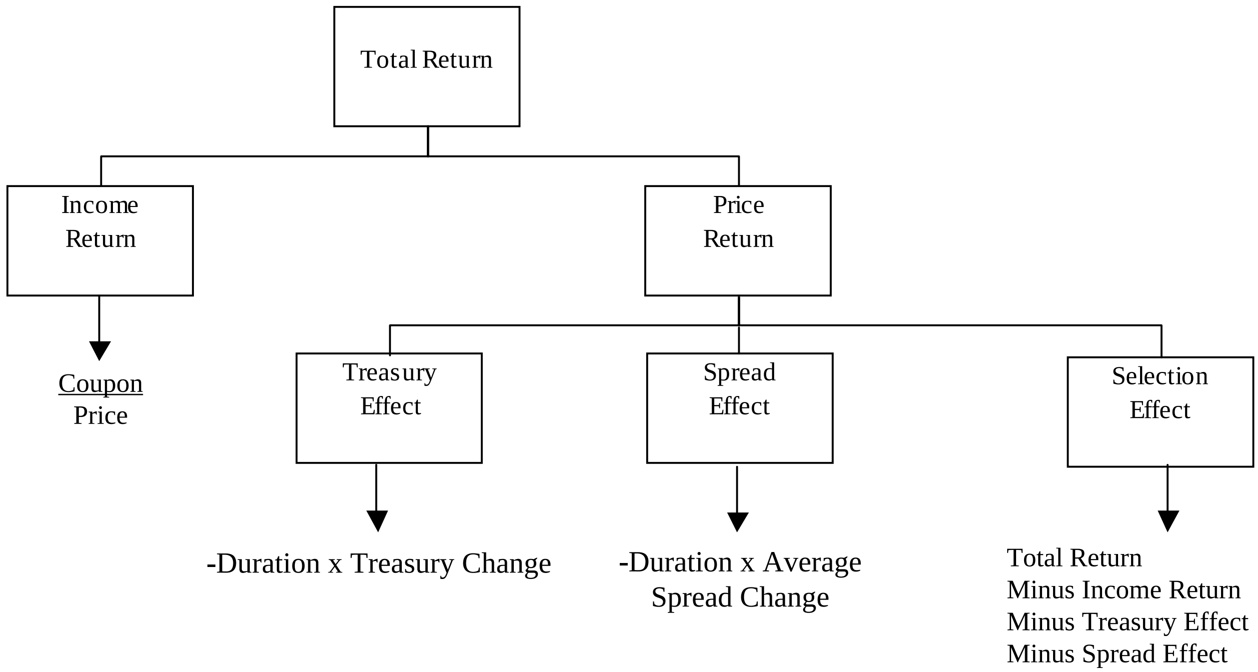
5. Bonds are simply the promise of a cashflow annuity, and so are relatively homogenous in their pricing. For example, bonds with the same maturity and default risk will generally sell at the same price. Because of this, bonds reflect very little selection effect. Stock prices respond dramatically to company-specific conditions, and reflect a very large selection effect.

As a result of these differences, two distinct attribution approaches emerge. Stock portfolio performance is driven by exposure to the market, and by the allocation to the major economic sectors that the manager has followed. Issue Selection provides the remaining explanation of stock performance. Bond portfolio performance is driven by promised income, and by the effect that changes in yields have on bond prices. The small, unexplained residual is the selection effect.

Decomposing the Return Process

As shown in Figure 1 (*see page 16*), the return for any investment is the sum of its income and its price change

Figure 1
Diagram of Return Process



over the measurement period. This can be expressed as:

Total Return = Income Return + Price Change

For a stock, this is the dividend yield plus the change in market value. We can adapt this familiar return decomposition for the unique risks of bonds, see Figure 1.

Where:

- Income Return = Annual Coupon Rate ÷ Beginning Market Price

- Price Change = Effect of Yield Changes

The income return is identical to the current yield on stocks, after substituting the bond coupon rate for the stock dividend rate. To measure the price change, an understanding of bond yields is required, specifically, the components of bond yields, and the relationship between bond yields and bond prices.

<u>Explanation of Bond Yields</u>

Bond yields have two components:

- An underlying Treasury Interest Rate, which generally increases with the maturity of the bond. (Investors require a higher return for giving up the liquidity of short-term investments.)

- A risk premium to compensate for uncertainties related to the payment of principal and interest. This premium is known as a "Spread" or "Spread over Treasuries."

Bonds exhibit four distinct types of risk, each of which provides a risk premium:

- *Interest Rate Risk* – The risk that bond prices will decline as Treasury interest rates rise. When current interest rates rise above the promised coupon yield for a bond, that bond becomes undesirable (it pays less than the going rate of interest). In response to lack of demand, the bond's price falls.

- *Credit Risk* – The risk that the borrower will not pay interest and repay the principal of the loan. This is also known as default risk. Bonds are rated by their creditworthiness, using a letter system. Investment grade bonds are rated AAA, AA, A and BBB. The less creditworthy the borrower, the higher the default risk premium required by investors.

- *Prepayment Risk* – The risk that borrowers will refinance their current bonds as interest rates decline, leaving lenders to reinvest at lower yields, and to suffer the loss of expected interest income. This risk is greatest in bonds backed by pools of home mortgages, which may be prepaid at any time without penalty (Mortgage Backed Securities, also know as "GNMAs").

- *"Equity-type" Risk* – The risk of speculative grade bonds ("junk bonds"), which have a greater probability of default than investment grade bonds. These bonds are rated BB, B, C and D (defaulted). They reflect a higher degree of default risk, and a much greater than average selection effect, compared with investment grade bonds.

A bond's yield is therefore the sum of:

- A short-term rate to compensate for expected inflation, plus a small real return.

- A maturity premium for unanticipated inflation over the bond's life.

- A risk premium to compensate for default or lost income from prepayments.

<u>Relationship Between Yield and Price:</u>
<u>Bond Duration</u>

A bond's price changes inversely with changes in yields: as yields rise, bond prices decline, and as yields decline, bond prices rise. The amount of price change depends on a bond's sensitivity to yield changes. This price sensitivity is known as "Duration." Duration (actually "Modified Duration") provides an estimate of the percentage change in a bond's price for a 1% change in yields.[2] This change in yield could be the result of changes in interest rates, risk premiums, or both.

The general model for bond price change is: Change in Bond Price = -Duration x Change in Yield

Note that the negative value of the bond's duration is used. This preserves the inverse relationship between yield change and bond price change. For example, the price of a 7-year bond with a modified duration of 5 years would decline by approximately 5% in response to a 1% (100 basis point) increase in yields. Or, the price of a 10-year bond with a modified duration of 6.5 years would increase by about 3.25% in response to a 50 bps decline in yields. Duration generally increases with longer bond maturities, and decreases with higher coupon rates.

Duration is therefore the systematic risk measure for bonds – it is analogous to beta for stocks. Just as beta measures the sensitivity of a stock to changes in the overall stock market, duration measures the sensitivity of a bond to changes in yields (both changes in Treasury Rates and Risk Premiums). And, just as market risk cannot be diversified away in a stock portfolio, the risk of price declines resulting from rising yields cannot be diversified away. By incorporating the systematic risk of bonds, a true risk-adjusted attribution process begins to emerge. Thus far, this attribution process identified the income component of return, along with the price change from systematic risk. The remaining component is a small selection effect. Calculating the selection effect involves comparing the portfolio's average spread change with the spread change for an identically allocated index, after controlling for any differences in price sensitivity. By adjusting for differences in duration, this process eliminates the error of assigning a selection effect to differences in systematic risk or style.

The next section addresses the practical aspects of performance attribution, including data collection, benchmark creation, and calculation of both contribution to return and the attribution effects.

THE PERFORMANCE ATTRIBUTION MODEL

<u>Data Requirements for Bond Performance Attribution</u>

The first step in analyzing bond performance is to gather and calculate the following summary data for the investment portfolio and the benchmark:

- Duration
- Coupon
- Beginning Market Price
- Allocation to sectors
- Total Return
- Interest Rate Change
- Spread Changes

This data is available at an aggregate level from the index providers for the various benchmark components. It should also be available from the manager's accounting and information systems at an issue level for investment portfolios. For all summary statistics, the portfolio's value is simply the market value weighted average of the issue data. For example, the portfolio's duration is the average of the individual issue durations.

Creating a Portfolio Benchmark

A risk-adjusted bond benchmark is a normal portfolio whose allocation reflects the long-term exposure to systematic risk factors: Interest Rate Risk, Credit Risk, Prepayment Risk and Equity-type Risk. Creating and maintaining a benchmark is relatively simple, and requires the periodic calculation of the weighted average returns and statistics of the indexes corresponding to each type of risk.

In this analysis, the benchmark is:

- 6% Interest Rate Risk, represented by US Treasuries.
- 66% investment grade Credit Risk, represented by an A-rated Corporate index.
- 18% Prepayment Risk, represented by a Mortgage-Backed Securities index.
- 10% Equity-type Risk, represented by 8% Domestic High Yield Bonds and 2% Emerging Markets Bond indexes.

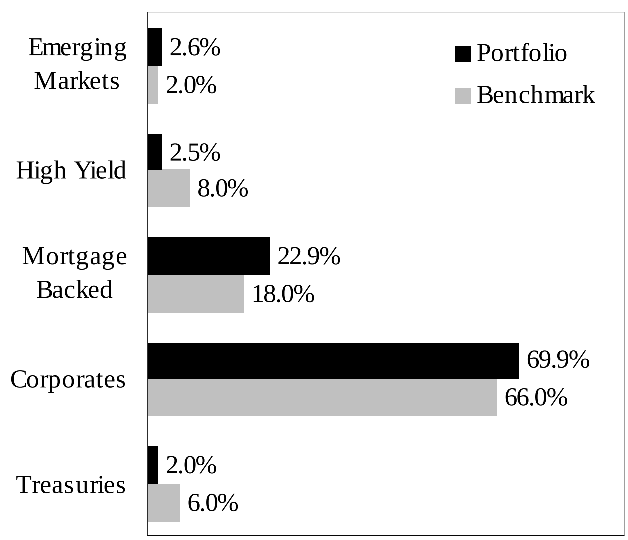
Figure 2 shows the broad sector allocations of the portfolio, compared with the benchmark.

A bond manager can implement this target allocation by using a number of "sub-sectors," which provide an opportunity to enhance investment returns. Examples of sub-sectors within each risk category are:

- *Interest Rate Risk* – Treasuries, US Agencies, AAA-rated Corporates.
- *Credit Risk* – AA, A, BBB-rated bonds, Asset-Backed Securities (ABS), Commercial Mortgage Backed Securities (CMBS), Private Placement bonds.
- *Prepayment Risk* – Mortgage-Backed securities, Collateralized Mortgage Obligations (CMOs), Callable Corporate bonds.
- *Equity-type Risk* – BB, B, C-rated Corporate Bonds, Convertible bonds.

Figure 3 (*see page 19*) shows the manager's allocation to the various bond sub-sectors.

Figure 2
Sector Allocation:
Bond Portfolio vs. Benchmark



A good confirmation of the validity of a portfolio's benchmark is the degree of correlation between the portfolio and the benchmark. If the portfolio's systematic risk exposures and style characteristics are consistent with the benchmark, then its correlation with the benchmark will be high. The square of the correlation coefficient, (the coefficient of determination, or "r-squared"), is an intuitively appealing measure of how well the benchmark fits the portfolio, since it shows the percentage of the portfolio's return pattern that is explained by the benchmark's systematic risks and style characteristics. As shown in Figure 4 (*see page 20*), the benchmark used in this analysis has an almost perfect correlation with the portfolio. As a result, the manager can have great confidence in the usefulness of the attribution results.

Figure 3
Sub-Sector Allocations of Bond Portfolio

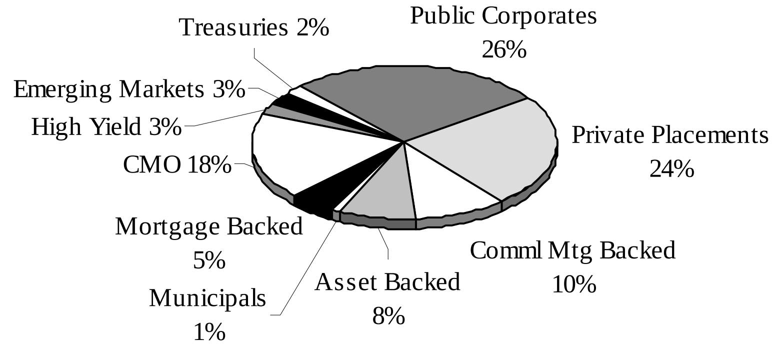


Table 1 shows the relevant benchmark and sub-sector data for 1999:

Table 1
Benchmarks and Sub Sector Data

	Total Return	Beginning Market	Coupon	Duration
Treasuries	-1.75	107.19	6.45	5.02
Mortgage-Backed	1.86	101.98	7.03	1.95
Asset-Backed	1.81	102.15	6.39	3.14
Comml Mtg-Bk	-1.27	102.62	6.67	5.98
A-rated Corp.	-1.96	105.32	7.18	6.03
High Yield	2.39	88.69	8.02	4.64
Private Placements	-0.17	100.84	7.12	6.66
Emerging Mkts	23.07	67.04	7.20	3.59
Municipals	-2.06	102.97	5.51	7.30

Using the data found in Table 1, we can calculate the following contribution to return factors for the broad benchmark (the strategic allocation) and also for the identically-allocated sub-index (the tactical implementation):

- *Income Effect* – Interest earned on the market value invested: (Coupon ÷ Price)
- *Treasury Effect* – Price change from Treasury Change: (-Modified Duration x Change in Treasury Interest Rate)
- *Spread Effect* – Price change from change in risk premium: (Total Return - Income - Duration Effect)
- *Spread Change* – Change in Risk Premium (in basis points): (Spread Effect ÷ -Modified Duration)

Since there is no selection effect in indexes, the index spread effect is the result of all performance that is not explained by income and changes in Treasury Rates.

The next section provides a step-by-step guide to the calculation of each return factor. For each return factor, the difference in contribution to return between the portfolio and the benchmark provides the risk-adjusted attribution of return.

Calculating the Treasury Effect

The Treasury Effect is the component of price change resulting from the change in Treasury Interest Rates and the sensitivity of the investment to changes in rates. Figure 5 (*see page 21*) illustrates the increase in Treasury Rates that occurred in 1999, data in Table 2 (*see page 20*). The Treasury Effect is calculated using the change in Interest Rates at the point on the Treasury Curve corresponding to the duration of the investment (or portfolio).[3] For example, the Treasury Effect for Asset-Backed Securities (ABS) would use the Treasury Change at the 3-year point, since ABS had a 3-year duration. The duration effect for Mortgage-Backed Securities (MBS) would use the Treasury Change at the 2-year point, since MBS duration was about 2 years.[4]

Table 2
1999 Interest Rate Changes and Price Effects

Duration	Rate Change	Price Effect
0.50	1.15	-0.58
1.00	1.41	-1.41
2.00	1.70	-3.40
3.00	1.62	-4.86
3.50	1.67	-5.85
4.00	1.73	-6.90
4.50	1.78	-8.00
5.00	1.83	-9.15
5.25	1.81	-9.49
5.50	1.79	-9.82
5.75	1.76	-10.13
6.00	1.74	-10.44
6.50	1.70	-11.02
7.00	1.65	-11.55
7.50	1.68	-12.59
8.00	1.71	-13.65
9.00	1.76	-15.87
10.00	1.82	-18.20
20.00	0.98	-19.60
30.00	1.35	-40.50

*See Figure 5 *on page 21*.

The modified duration of most coupon-paying bonds is limited to about 10 years. Only zero coupon bonds[5] have durations in the 20 to 30 year range, since their durations are equal to their years until maturity.

Calculating the Income Effect

The Income Effect is simply the average annual coupon rate divided by the average beginning market price. Alternatively, this can be calculated in dollar terms:

Income Effect = Interest Income (in dollars) ÷ Beginning Market Value (in dollars)

= Beginning par value x Average Coupon Rate ÷ Beginning Market Value

For example, the Income Effect for Asset Backed Securities would be:

- 6.39% coupon ÷ \$102.15 beginning dollar price = 6.26% income

Calculating the Average Spread Change

Since there is no selection effect in indexes, the spread effect is the residual of the total return that is not explained by the Treasury and Income Effects:

Figure 4
Bond Portfolio vs Normal Portfolio Benchmark (1995 - 1999)

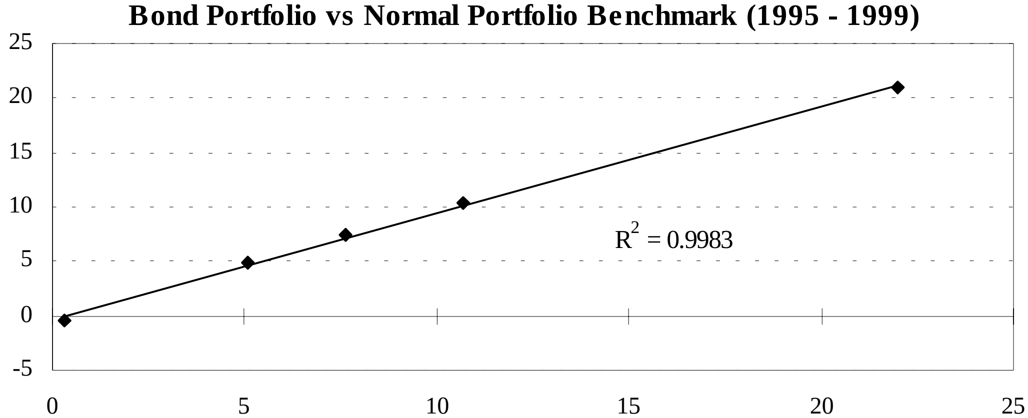
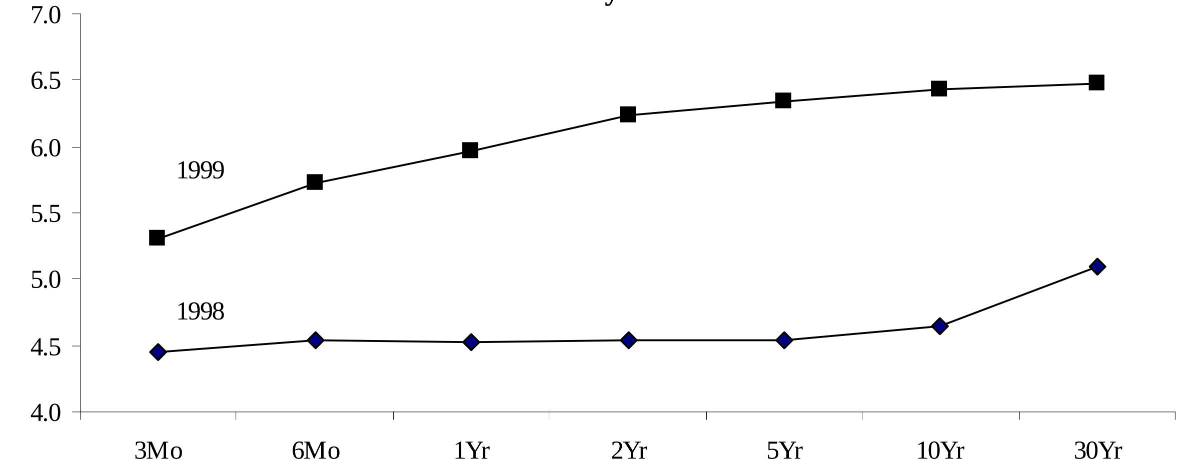


Figure 5
Treasury Rates



- Index Spread Effect = Total Return - Income Effect - Treasury Effect.

The Spread Change can be generalized by adjusting for the index duration, as follows:

- Index Spread Change (in bps) = Index Spread Effect ÷ -Index Modified Duration.

Again, using ABS as an example:

• Sector Spread <u>Effect</u>	=	<u>TR</u>	- Income <u>Effect</u>	-Treasury <u>Effect</u>
	=	1.81	- (6.39/102.35)	- (-3.14 x 1.62)
	=	1.81	- 6.26	- (-5.09)
	=	1.81	- 6.26	+ 5.09
	=	0.64%	or	+ 64 bps

- Sector Spread Change = 0.64 ÷ (-3.14) = -20 bps

This analysis indicates that Asset-Backed Securities earned 1.81% total return in 1999, resulting from:

- A gain of 6.26% from interest income.

- A loss of 5.09% from a 1.62% increase in interest rates.

- A gain of 0.64% from a 0.20% decrease in risk premium.

This can be further generalized as:

Income:	+6.26%
Price Change:	<u>- 4.45%</u>
Total Return:	+1.81%

<u>Calculating the Selection Effect</u>

The majority of a bond portfolio's performance can be explained by the effects of income and the price change resulting from changes in general interest rates and risk premiums. These are "Systematic Factors" that cannot be eliminated through diversification or issue selection. That is, these identical return effects will occur in all bonds with these common characteristics. All bonds with the same coupon and beginning market price will deliver the same income return, if held throughout the performance measurement period. All bonds with the same duration will reflect the same price effect as interest rates change. Finally, all bonds in a given sector will reflect a general change in risk premiums resulting from the market's change in required compensation for risk.

The Selection Effect in a bond portfolio can be calculated from the difference between the Spread Change in the bond portfolio and the Spread Change of an identically allocated sub-sector index. The sub-sector index must be used (instead of the normal portfolio allocation of broad risk sectors) because the sub-sectors may provide significant spread advantage over the broader risk sectors. Simply comparing the Spread Change Effects (SCE) of the portfolio and the benchmark would incorporate both the allocation effect of using sub-sectors instead of broad risk sectors, as well as the effect of any differences in systematic risk, measured by duration.

The method described here makes the important distinction between selecting superior sub-sectors within a broad risk sector, and selecting superior issues, after controlling for differences in risk. The process involves calculating the following components:

- *Index Spread Effect* – Return component resulting from changes in the index average spread.
- *Market Spread Change (MSC)* – Average spread change (in bps) for the index.
- *Portfolio Spread Effect* – Return component resulting from the MSC and the portfolio's duration.
- *Portfolio Selection Effect* – Residual return after adjusting for all systematic effects.

Calculating the Spread Change for the Sub-Sector Index:

1. Calculate the weighted average sub-index statistics (return, duration, coupon, price) and the first two components of total return (Income Return and Treasury Effect).

2. Using these statistics, calculate the Spread Change Effect (SCE):

   SCE = Total Return minus Income Return minus Treasury Effect.

3. Using the Spread Change Effect, calculate a Market Spread Change (MSC):

   MSC = SCE ÷ the duration of the sub-sector index.

	Index
Duration	4.90
Return	0.50
Income Return	6.90
Treasury Effect	-8.97
Spread Effect	2.56
Spread Change	-0.52

Calculating the Selection Effect for the Portfolio:

1. Calculate the average statistics for the portfolio (coupon, price, duration).

2. Using these statistics, calculate the Income Return and Treasury Change Effect.

3. Using the sub-sector Market Spread Change, calculate a Portfolio Spread Change Effect: Portfolio SCE = MSC x -Portfolio Duration.

4. Calculate the selection effect as the difference between the portfolio's total return and the contribution to return from the systematic factors (income, Treasury rate change, spread change):

   Selection Effect = Portfolio Total Return minus Income Effect minus Treasury Effect minus Spread Effect.

CASE STUDY IN ATTRIBUTION ANALYSIS

Bringing it all Together: Performance Attribution

The return components for the portfolio and the benchmark are now brought together to analyze contribution to return, Figure 6 (*see page 24*), and attribution of return, Figure 7 (*see page 24*).

The Market Spread Change (-52 bps) and the portfolio duration (5.58) produce a spread effect of +2.92 percent. The portfolio's actual SE (Total Return minus Income Return minus Treasury Effect) produces a Spread Effect of +3.01%, which exceeds the average Spread Effect by 0.09 percent. This excess return from greater spread tightening is the incremental return from Issue Selection:

Table 3
Equity Attribution Model

	Benchmark Weighting	Portfolio Weighting	Benchmark Return	Portfolio Return	Allocation Effect	Selection Effect	Total Alpha
Treasuries	6%	2%	-2.56	-6.22	0.08	-0.07	0.01
Corporates	66%	70%	-1.96	-1.07	-0.06	0.62	0.56
Mortgage Backed	18%	23%	1.86	2.60	0.11	0.17	0.28
High Yield	8%	3%	2.39	7.04	-0.16	0.12	-0.04
Emerging Markets	2%	3%	23.07	15.32	0.15	-0.20	-0.06
Total			-0.46	0.30	0.13	0.63	0.76

Statistics	Portfolio	
Total Return	0.30	
Income Return	7.25	
Treasury Effect	-9.96	(-5.58 x 1.79)
Net Effect	3.01	(0.30 - 7.25 - (-9.96))
Average Spread Effect	2.92	(-5.58 x -52bps)
Selection Effect	0.09	(3.01 - 2.92)

The attribution of excess return relative to the appropriate benchmark is simply the difference between the contributions to return for each factor:

Statistics	Benchmark	Portfolio	Alpha
Total Return	-0.46	0.30	0.76
Income Return	7.04	7.25	0.21
Treasury Effect	-9.29	-9.96	-0.67
Spread Effect	1.78	2.92	1.14
Selection Effect	N/A	0.09	0.09

Note how different the results would have been using the Equity Attribution Model shown in Table 3.

This discrepancy (63bps vs. 9 bps of selection effect) demonstrates that any model that ignores systematic risk and style factors will assign too much value to selection. In this case, even if the benchmark is appropriate, the alpha can be incorrectly assigned. Further proof of this concept is provided by the negative selection effect for Treasury bonds, caused by their apparent "underperformance" (-6.22% vs. -2.56%). Since all Treasury Bonds are identical, except for maturity, there can be no return differential from issue selection. The difference in performance is completely explained by differences in systematic risk: the portfolio held longer-maturity, higher-duration Treasuries than the benchmark at a time of rising interest rates.

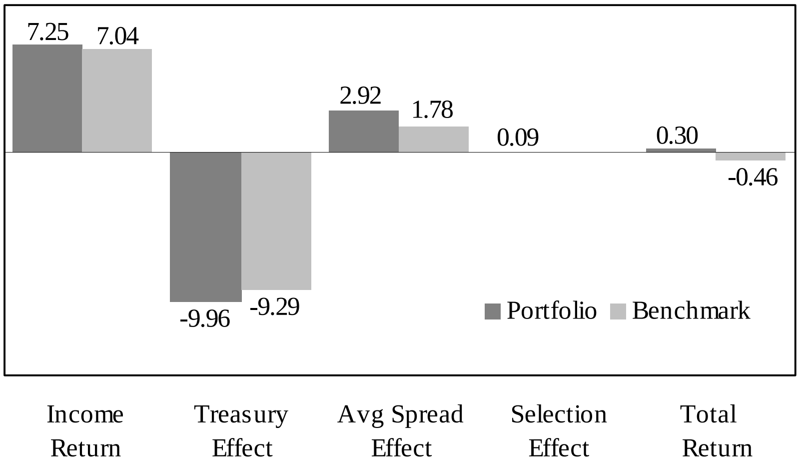
Explanation of Results

The portfolio outperformed its benchmark by 0.76 percent. The economy during this period was strong, producing an increase in interest rates and a decline in risk premiums. This is typical for any continued economic expansion. The portfolio's price declined by 7.04% as a result of an increase in yields. The yield increase was the combined effect of rising interest rates (+1.79%) and lower risk premiums (-0.52%), which produced price effects of -9.96% and +2.92. The portfolio delivered an income return of 7.25%, which was mainly the result of its long-term asset allocation, both to broad risk sectors, such as Corporate Bonds, as well as its use of finer sub-sectors, such as ABS and Private Placements. Finally, all active management effects produced a relatively minor selection effect of 0.09 percent.

SUMMARY AND CONCLUSIONS

Bonds are unique investments that require their own performance attribution model. The systematic risk of bonds is their sensitivity to changes in market yields. The bonds' price changes due to fluctuations in market yields, plus the income provided by the bond's coupon, explain most of a bond portfolio's total return.

Figure 6
Contribution to Return for Bond Portfolio and Benchmark (1999)



A small selection effect is to be expected, after accounting for these systematic factors, because bonds are a fairly homogenous investment class.

This performance attribution model provides a simple and straightforward method for identifying the systematic risk and stylistic factors of the investment manager's strategy. By incorporating these factors into the attribution model, the performance analysis can provide a meaningful explanation of results that are an integral part of the portfolio management process.

ENDNOTES

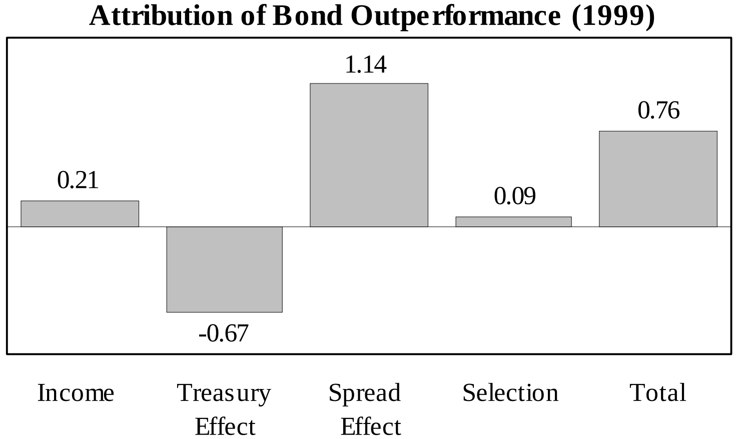
[1] This analysis uses annual returns and weightings for simplicity of presentation. The methodology can easily be adapted to quarterly or monthly frequency, with the periodic results linked to form an annual result.

[2] Duration is used as the single price sensitivity measure. However, the bond price/yield curve reflects a curvilinear or "convex" shape, rather than the linear relationship implied by duration. This means that for a given change in yield, the price movement is greater for gains than implied by duration, and less for losses. The degree to which the bond price/yield curve deviates from the linear duration approximation can be measured by the "convexity" statistic. By including the convexity term, a more precise estimate of bond price change results. The more precise model for bond price change follows a Taylor series, using both duration and convexity factors:

Price Change = -Duration x Yield Change + Convexity/2 x Yield Change^2.

The effect of using only duration to measure the effects of yield change is relatively minor. For example, spread changes for negatively convex sectors (mortgage backed and callable corporate bonds) will reflect a bias toward widening, since both widening of spreads and negative convexity both cause prices to decline. (In 1999, mortgage backed bonds actually exhibited a tightening of spreads, yet prices declined as a result of negative convexity. The effect on price change is the same as a spread widening.) Also, a diversified portfolio that contains both positively convex investments (Treasuries and noncallable Corporates) along with negatively convex Mortgage Backed Securities will often have an overall

Figure 7
Attribution of Bond Outperformance (1999)



convexity effectively equal to zero, making the convexity effect immaterial.

The basic attribution methodology outlined here can be enhanced to include a convexity measure. This requires the substitution of a quadratic equation for the simple linear duration expression:

$X = (-B \pm \sqrt{B^2 - 4AC}) / 2A$

Where:

- A = Convexity ÷ 2
- X = Change in spread
- B = - Duration
- C = Total return minus Income Return minus Treasury Effect

Solving for the implied change in spread can also be accomplished (without the math!) by trial and error, or using the optimizer that is embedded in most spreadsheet software.

[3] The Treasury curve may change by the same amount for each maturity (a "parallel shift"), or by a different amount for each maturity (a "non-parallel shift"). Non-parallel shifts are quite common, resulting from a "steepening" or "flattening" of the curve. A steepening occurs when long rates increase by more than short rates. A flattening occurs when long rates increase by less then short rates, or when short rates increase while long rates decrease. (This occurred in the first quarter of the year 2000.) A more robust measurement of the Treasury Effect would evaluate the price change for the portfolio's exposure at each point along the Treasury curve. However, this methodology requires a significantly higher level of resources and analytic capabilities. First, the analyst must be able to project the expected interest and principal cashflows for every bond, and then sum these cashflows for all bonds at each relevant point along the Treasury curve. Once these are determined, the price effect from Treasury rate changes can be calculated. Next, this process must also be replicated for the benchmark. This requires the analyst to know the comparable Treasury curve exposures for the benchmark. One way to accomplish this is to input every bond in the benchmark into an analytic system or data repository that can project each bond's cashflows. Since bond benchmarks contain many issues, this can be a data and resource intensive task (for example, the Lehman Aggregate Index contains about 6,000 bonds).

Whichever method is used to analyze price performance, it is essential that the same process be used for both the portfolio and the benchmark. The temptation is to use a more robust process for the portfolio, and a simpler process for the benchmark, but this would introduce a form of "model error" and thus compromise the integrity of the results.

[4] Any partial-year durations use Treasury changes calculated by interpolation between whole-year interest rate changes. For example, the Treasury change for Agencies (having a duration of 4.45 years) equals the average of the 4-year and 5-year rate changes:

(1.73% + 1.83%) ÷ 2 = 1.78%.

The duration effect then equals: -4.45 years x 1.78% = -7.92 percent. Obviously, partial year adjustments to interest rate changes are more important (for accuracy) with steeper yield curves, and less critical with flatter yield curves. Generally, half-year adjustments are adequate, and quarter-year adjustments are probably only needed when a significant portion of the portfolio is allocated to that point on the Treasury curve.

[5] Zero coupon bonds do not pay interest. Instead, the source of their total return is the difference between the bond's purchase price and the par value that is repaid at maturity.