Quality Analysis of a Homemade Pendulum

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Introduction

In this lab, I tested the relationship between the drop angle and period of oscillation for a homemade pendulum. Additionally, the experiment explored how the amplitude of the pendulum's oscillation changed over time. The experiment was predicted to follow a damped harmonic motion model described by the equation $\theta(t) = \theta_0 e^{-\frac{t}{\tau}} cos(2\pi\frac{t}{T} + \phi_0)$ [1] (equation 1), where θ_0 is the initial angle, t is time, φ_0 is the phase constant, and τ and T are setup dependent constants. This predicted the drop angle would have no effect on the period of oscillation, meaning a constant period would exist. The Q factor of the pendulum was measured using $Q = \pi\frac{\tau}{T}$ [1] (equation 2), where τ (time constant) and T (period) are setup dependent constants, and counting how many oscillations it took for the amplitude to decay to $e^{-\pi/2}$ ~ 20%, giving $\frac{Q}{2}$ [1].

Method

The pendulum was constructed using a 3.6cm x 61.0cm x 1.1cm wood slab with a hole drilled 1.8 cm from each edge to feed the 1 mm diameter nylon string through. A screw was positioned 4.6 cm away from the hole, allowing the string to wrap around and enable the pendulum to have an adjustable length (see Figure 1). The pendulum's mass was made using 10 washers tied tightly together to ensure a consistent centre of mass throughout the experiment and reduce any uncertainty caused by a moving centre of mass (see Figure 2). The mass of the 10 washers was 29 g (Appendix A) which meant the lightweight nylon string had a negligible mass in comparison to the washers.

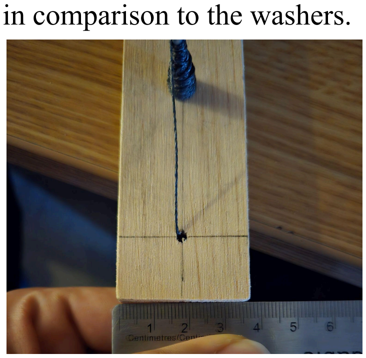

Figure 1. A birds eye view of the pendulum frame with the hole, screw and coiled nylon string visible. Ruler for scale.

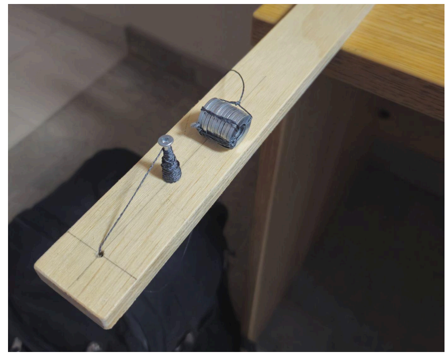

Figure 2. A photo of the 10 washers (pendulum's mass) and the setup of the pendulum. The nylon string is visible and secured around the washers.

To stabilize the pendulum setup and reduce uncertainty caused by movement in the pendulum stand, I put 2 fully filled 40oz water bottles (see Figure 3 for placement) weighing approximately 4.0 kg (Appendix B) on the wood to provide a large enough friction force between the pendulum and the table to ensure the pendulum stand didn't move with the pendulum mass, and a large enough moment to ensure the pendulum didn't fall off the table. The desk was measured to be level using the built-in levelling function on a Samsung Galaxy S24 phone camera. This minimized the potential for asymmetry in the pendulum's motion.

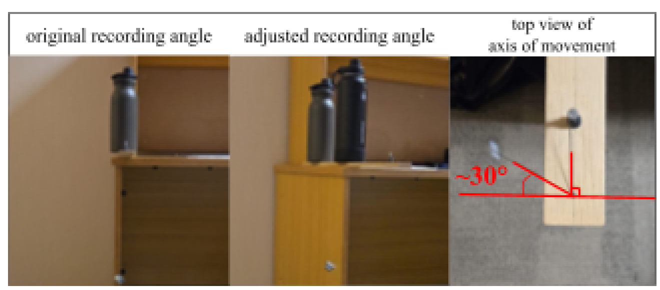

Figure 3. The pendulum was not recorded straight on, but rather at an angle ~30° to the left, aligning the camera perpendicular to the natural axis of motion.

The phone was stabilized on a tripod to the same height as the pivot point and the angle it was positioned at was adjusted to be perpendicular to the pendulum's natural axis of movement, which wasn't perpendicular to the length of the pendulum frame. This off-angle, noticeable in Figure 3, removed the possibility of uncertainty in the measured angles caused by parallax/viewing angle from the camera. The pendulum was released from an initial angle/amplitude of 1.50 radians. It was recorded in 1080p at 30 frames per second on a Samsung Galaxy S24 for 6 minutes to ensure the amplitude decreased a significant amount to calculate Q factor both ways. This recording was imported into Tracker [2] software, and the time and angle data tracked using the auto tracking feature was imported onto a spreadsheet (Appendix C).

Drop Angle vs Period

To analyze the relationship between drop angle and period, the data collected from Tracker [2] was analyzed using a Python program [3] that found the period and the angle by searching for the local maximums and minimums, This approach allowed for the measurement of both positive and negative amplitudes from a single dataset. The period and angle data, along with their uncertainties, were proceed using a Python program (Appendix D) that fitted to the power series $T = T_0(1 + B\theta_0 + C\theta_0^2)$ [1] (equation 3), where T_0 is the period for small angles, θ_0 is the initial angle in radians, and B and C are best fit parameters. The power series was used to generate a graph plotting the data and line of best fit. The pendulum was tested for symmetry using the fitted equation.

Time vs Amplitude (Q Factor)

To examine the relationship between time and amplitude, the data from the Tracker [2] app was inputted into a Python program [4] that isolated the measured amplitude values and their respective timestamps. The processed data with it's uncertainties was fitted using a Python program (Appendix D) to the exponential function $ae^{\frac{-t}{\tau}}$ [1] (equation 4), where to create a graph with the line of best fit. The τ value from equation 4 was used to calculate the Q factor (see equation 2).

Results and Analysis

Uncertainty Analysis

Before looking at the results of the experiment it is important to understand what uncertainties exist, where they come from and how they affected the experiment

Type B uncertainty in the time values arises from the frame rate of the recording, which was 30 frames per second. This resulted in a type B uncertainty of $\pm \frac{1}{30} \times \frac{1}{4} = \pm \frac{1}{120} \simeq \pm 0.008s$. For time, there was no type A uncertainties because only 1 trial was performed.

Type B uncertainties for angle measurements came from the motion blur of the recording. The motion blur allowed for a larger area that the Tracker [2] auto tracker could identify as the mass (see Figure 4). This resulted in an uncertainty of $\pm 1°$.

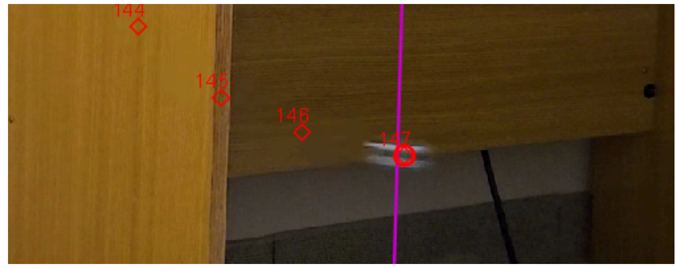

Figure 4. An example of the motion blur present in the video which was measured to cover 5°, leaving an uncertainty of $\pm \frac{5}{4} \simeq \pm 1°$.

To reduce these uncertainties, I could have used a higher frame rate, like 240 fps, to record the data. This would have reduced the time uncertainty by a factor of 8 and decreased the amount of motion blur in the video substantially, which would have made the precision of the auto tracker from Tracker [2] less uncertain. This would've allowed my data in Figure 5 to look more fluid and provide a more accurate conclusion of my experiment.

Drop Angle vs Period

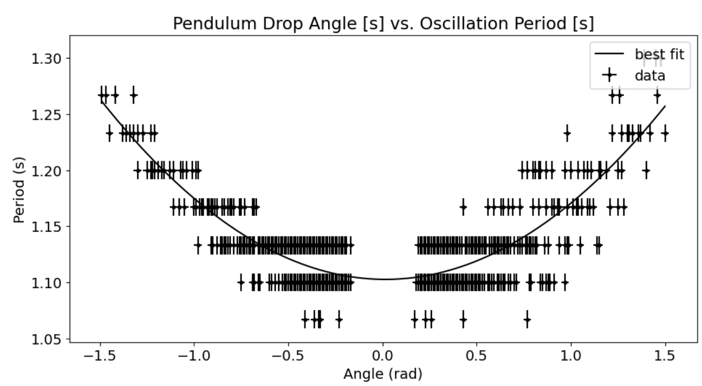

Figure 5. This figure shows the relationship between the pendulum drop angle [rad] and the oscillation period [s] It is best fitted to $T = T_0(1 + B\theta_0 + C\theta_0^2)$ (equation 3), where $T_0 = 1.1026 \pm 0.0004$, $B = -0.0020 \pm 0.0005$, $C = 0.0700 \pm 0.0007$, θ_0 is the drop angle and T is the period of oscillation.

It was predicted that the experiment would follow a damped harmonic motion model (see equation 1), where drop angle would have no affect on the period of oscillation, resulting in a consistent period across all drop angles. It was predicted that $B = C = 0$, meaning the fitted equation would've had a slope of 0. Figure 5 illustrates the relationship between the pendulum's drop angle and it's period, displaying a nonlinear relationship which resembles a parabola.

This deviation from the prediction suggests that the pendulum's drop angle influences the oscillation period. This is evident from the best fit parameters for equation 3, specifically the C value. The C value of 0.0700, significantly larger than it's uncertainty ±0.0007, is a dominant and unignorable factor in the behaviour of the relationship when |θ| increases. The C value would have been even more accurately fitted if the experiment was recorded at a higher frame rate, like 240fps. It can be concluded that higher magnitude drop angles result in longer periods. Possible explanations for this non-linear behaviour could be the slowing effect of resisting forces like air resistance or friction that decrease as the distance traveled (amplitude) decrease.

However, the predicted constant relationship was found to be more accurate towards smaller angles $-0.358 < \theta < 0.358$ where the change in period as a result of drop angle was experimentally zero, allowing for small angle approximation to be applicable (see Appendix E).

Asymmetry

In the equation of best fit from Figure 5, the B value from equation 3 is -0.0020 ± 0.0005. Given that the B value is 4 times larger than the uncertainty of B, the value of B is close to zero, meaning that the pendulum is slightly asymmetrical, favoring the positive angle values a small but unnegligible amount more.

Q Factor

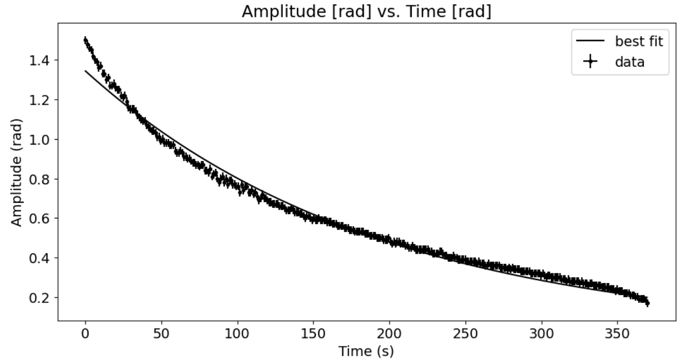

Figure 6. This figure shows the relationship between the pendulum's amplitude [rad] versus time [s] and is approximated using the exponential equation (equation 4) $\theta = (1.339 \pm 0.003)e^{\frac{-t}{194.1\pm0.8}}$ where t is the time [s] and θ is the amplitude [rad].

The Q factor is calculated by counting the number of oscillations to get from 1.50 rad to $1.50 \times e^{-\pi/2} \simeq 0.31$ was 265 (see Appendix C), which represents the value of $Q/2$. This means that Q is experimentally $265 \times 2 = 530$.

The Q factor is also calculated using using equation 1. In order to do so, the τ and T value have to be identified. $\tau = 194.1 \pm 0.8$ was taken from $\theta = (1.339 \pm 0.003)e^{\frac{-t}{194.1\pm0.8}}$, the fitted equation of equation 4. T is not a constant, so to calculate the Q factor, the period values for positive amplitudes are averaged using the built in average function in Google Sheets. The T value calculated in this way is 1.1329 ± 0.01. The uncertainty was calculated by taking the largest uncertainty% and multiplying that by the largest period value: $\frac{0.008}{1.067} \times 1.13 \simeq 0.008$. The Q factor is calculated to be

$$Q = \pi\frac{\tau}{T} = \pi\frac{194.1\pm0.8}{1.133\pm0.008} = 538 \pm 4.$$

The values of the counted and calculated Q factors are very close and deviate by at most 12 according to the uncertainty. This only vary by around $\frac{8\pm4}{530} \simeq 2 \pm 1\%$ which is very little and can be considered negligible or consistent with zero.
If the experiment were recorded at more than 30fps, the T and τ values would have been more precise to their actual values, making the Q factor more precise.

Next Steps

The results of this experiment have shown me that for future measurements in this lab, I should record in a higher frame rate to ensure more precise time data and less uncertain angle data caused by motion blur. I should find a different surface to support my pendulum that is more levelled to reduce any asymmetry I haven't already addressed. Because I will be measuring the length of the pendulum, I should ensure that the centre of mass of the pendulum mass doesn't move around, so I might need to tie the string tighter around it.

Resources

[1] B. Wilson, “PHY180 Lab Project (2024),” PHY180-PendulumProject-2024.pdf, https://q.utoronto.ca/courses/363836/files/32826310?module_item_id=6082503 (accessed Oct. 4, 2024).

[2] D. Brown, W. Christian, and R. M. Hanson, “Tracker Video Analysis and Modeling Tool for Physics Education”, https://physlets.org/tracker/ (accessed Sep. 25, 2024).

[3] Y. Zhang, “angle_vs_period.py”, https://github.com/YYZ-CR/PHY180/blob/main/data_sorting/angle_vs_period.py (accessed Oct. 10, 2024)

[4] Y. Zhang, “angle_vs_time.py” https://github.com/YYZ-CR/PHY180/blob/main/data_sorting/angle%20vs%20time.py (accessed Oct. 10, 2024)

Appendices

Appendix A (washer mass calculation)

The volume of the washers is the volume of the hole in the middle subtracted from the volume of everything including the hole.

Washers diameter = 1.8 cm
Hole diameter = 0.8 cm
Washers height = 1.8 cm

Volume of everything = $\pi \times (\frac{1.8\,cm}{2})^2 \times 1.8\,cm = 4.5804\,cm^3$

Volume of hole = $\pi \times (\frac{0.8\,cm}{2})^2 \times 1.8\,cm = 0.9148\,cm^3$

Volume of pendulum = $4.5804\,cm^3 - 0.9148\,cm^3 = 3.6757\,cm^3$
Density of steel = 7850 kg/m³ = 7.85 g/cm³
Mass of pendulum = $7.85\,g/cm^3 \times 3.6757\,cm^3 = 28.85\,g \approx 29\,g$

Additionally, the mass was weighed using a scale and found to be 29.0 g, confirming the calculation above.

Appendix B (water bottle weight calculation)

Per the thermoflask website https://mythermoflask.com/products/thermoflask-insulated-stainless-steel-4-lid-combo-pack-40oz-1-1l-2-pack#:~:text=Weight%3A%201.93,lbs.%20total , each 40oz stainless steel water bottle weighs 1.93lb without any liquid in it. Filled with 40 oz of water (2.5 lb) would be 1.93 lb + 2.5 lb = 4.43 lb ~ 2.01 kg. 2 of these bottles would be 2.01 kg x 2 = 4.01 kg ≈ 4.0 kg.

Appendix C (data collected from Tracker [2] and processed data)

The tracker data (11,000+ data points) and the processed data can be viewed here: https://docs.google.com/spreadsheets/d/1zY-a8DOFjC4rEsYK8gz0ikJFwNO3PC4cED7r3G2fHok/edit?usp=sharing.
The number of oscillations to get to $e^{-\pi/2}$ was counted using the row numbers of the amplitude values (row 2 for 1.50 rad and row 266 for 0.31 rad) in the "time vs amplitude" tab. $266 - 2 + 1 = 265$ oscillations.

Appendix D (python files)

The Python files that were used to generate the graphs and equations of best fit in the experiment were based off of the provided files from the PHY180 Quercus page. These files were altered and used to graph data points collected by Tracker App [2].

Appendix E (calculations of angles where angles are small enough that $C\theta_0^2$ can be ignored and the data roughly follows the prediction)

The uncertainty of time values in this experiment was found to be ±0.01s, meaning that if the period was within ±0.01s of T_0, the slope would be experimentally zero. The values of θ that allow for the period to be ±0.01s of T_0 were calculated by finding the values of θ where $C\theta_0^2$ could be ignored. Note: $B\theta_0$ is ignored in this calculation since it has a significantly smaller impact on the behaviour of the pendulum at larger angles.

$|\theta| < \sqrt{(\frac{(1.1026 \pm 0.0004) \pm 0.01}{(1.1026 \pm 0.0004)} - 1) \div (0.0700 \pm 0.0007)}$

Note: only the + from the ± are used because $\theta^2 > 0$ and we are trying to find the minimum value $|\theta|$ needs to be less than.

$|\theta| < \sqrt{(\frac{(1.1026+0.0004)+0.01}{(1.1026+0.0004)} - 1) \div (0.0700 + 0.0007)} = \sqrt{(\frac{1.1130}{1.1030} - 1) \div 0.0707} \simeq 0.3581$

$|\theta| < 0.351$

$\therefore -0.358 < \theta < 0.358$

Appendix F (Python Files and Data Files)

The other files I used for this project can be found on this GitHub repository: https://github.com/YYZ-CR/PHY180.