Quality Analysis of a Homemade Pendulum

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Introduction

In this lab, I tested the relationship between the drop angle, period of oscillation and length of pendulum for a homemade pendulum. The experiment explored how the amplitude of the pendulum's oscillation changed over time. In the second part of the lab, I experimented on the relationship between the length of the pendulum and its period and quality (Q) factor.

Drop Angle vs Period (Experiment 1)

The experiment was predicted to follow a damped harmonic motion model described by the equation $\theta(t) = \theta_0 e^{-\frac{t}{\tau}} cos(2\pi\frac{t}{T} + \phi_0)$ [1] (equation 1), where θ_0 is the initial angle [rad], t is time [s], φ_0 is the phase constant, and τ and T are setup dependent constants. This predicted the drop angle would have no effect on the period of oscillation, meaning a constant period would exist. However, the experiment revealed the period had a quadratic dependence on the drop angle for large angles, but small angles between $\pm\ 0.319$ rad were consistent with the theoretical model where $T = T_0$. The results were best fitted to the equation $T = T_0(1 + B\theta_0 + C\theta_0^2)$ (equation 2), where $T_0 = 1.1026 \pm 0.0004$, θ_0 is the initial angle in radians, $B = -\ 0.0020 \pm 0.0005$, and $C = 0.0700 \pm 0.0007$ (B and C are best fit parameters).

The Q factor of the pendulum was measured using $Q = \pi\frac{\tau}{T}$ [1] (equation 3), where τ (time constant) and T (period [s]) are setup dependent constants, and counting how many oscillations it took for the amplitude to decay to $e^{-\pi/2}$ ~ 20%, giving $\frac{Q}{2}$ [1]. The Q factor was calculated to be $Q = \pi\frac{\tau}{T} = \pi\frac{194.1\pm0.8}{1.133\pm0.008} = 538 \pm 4$ where the T value was the average of all the periods.

Q Factor Dependence on Length (Experiment 2)

The period was predicted to be dependent on the length of the pendulum with a relationship defined by the equation $T = 2L^{0.5}$ [1], where L (length) is the pendulum length [m] and T is the period [s[. The length and period data for 8 lengths between 0.297m and 0.212m were fitted to the power law function $T = kL^n$ [1] (equation 4) where k and n are best fit parameters. The predicted parameters of $k = 2$ and $n = 0.5$ were close to the experimentally determined values of $k = 2.39 \pm 0.02$ and $n = 0.605 \pm\ 0.006$.

The Q factor was found to experimentally have a quartic relationship with the length factor with a fitted R^2 value of 0.8003. The relationship was defined by the equation $y = ax^4 + bx^3 + cx^2 + dx + e$ (equation 5) where $a = 4 \times 10^8 \pm 1 \times 10^8$, $b = -\ 4 \times 10^8 \pm 1 \times 10^8$, $c = 1.4 \times 10^8 \pm 4 \times 10^7$, $d = -\ 2.3 \times 10^7 \pm 7 \times 10^6$, and $e = 1.5 \times 10^6 \pm 5 \times 10^5$.

Method

The pendulum was constructed using a 3.6cm x 61.0cm x 1.1cm wood slab with a hole drilled 1.8 cm from each edge to feed the 1 mm diameter nylon string through. A screw was positioned 4.6 cm away from the hole, allowing the string to wrap around and enable the pendulum to have an adjustable length (see Figure 1). The pendulum's mass was made using 10 washers tied tightly together to ensure a consistent centre of mass throughout the experiment and reduce any uncertainty caused by a moving centre of mass (see Figure 2). The mass of the 10 washers was 29 g (Appendix A) which meant the lightweight nylon string had a negligible mass in comparison to the washers.

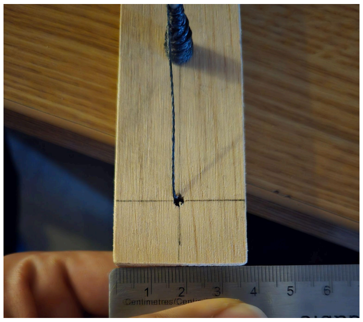


Figure 1. A birds eye view of the pendulum frame with the hole, screw and coiled nylon string visible. Ruler for scale.

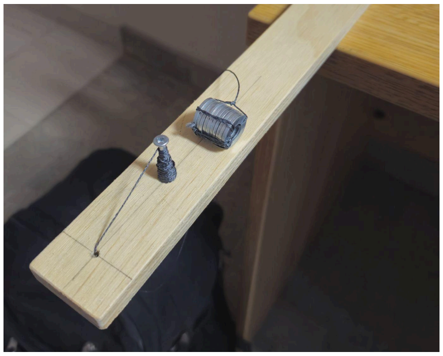


Figure 2. A photo of the 10 washers (pendulum's mass) and the setup of the pendulum. The nylon string is visible and secured around the washers.

To stabilize the pendulum setup and reduce uncertainty caused by movement in the pendulum stand, I put 2 fully filled 40 oz water bottles (see Figure 3 for placement) weighing approximately 4.0 kg (Appendix B) on the wood to provide a large enough friction force between the pendulum and the table to ensure the pendulum stand didn't move with the pendulum mass, and a large enough moment to ensure the pendulum didn't fall off the table. The desk was measured to be level using the built-in levelling function on a Samsung Galaxy S24 phone camera. This minimized the potential for asymmetry in the pendulum's motion.

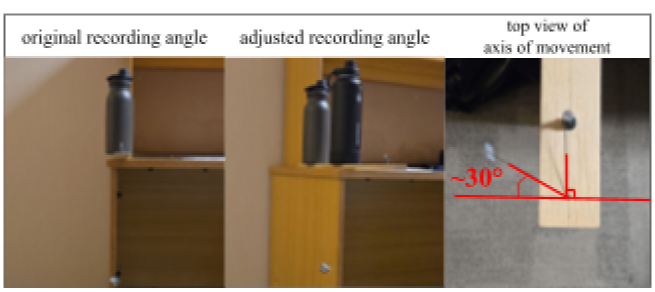


Figure 3. The pendulum was not recorded straight on, but rather at an angle ~30° to the left, aligning the camera perpendicular to the natural axis of motion.

The phone was stabilized on a tripod to the same height as the pivot point and the angle it was positioned at was adjusted to be perpendicular to the pendulum's natural axis of movement, which wasn't perpendicular to the length of the pendulum frame. This off-angle, noticeable in Figure 3, removed the possibility of uncertainty in the measured angles caused by parallax/viewing angle from the camera.

For the purpose of tracking, a solid white board was placed behind the pendulum as seen in Figure 4. This created a consistent background that provided contrast to the washers, making it easier for the Tracker [2] app to detect them during autotracking.

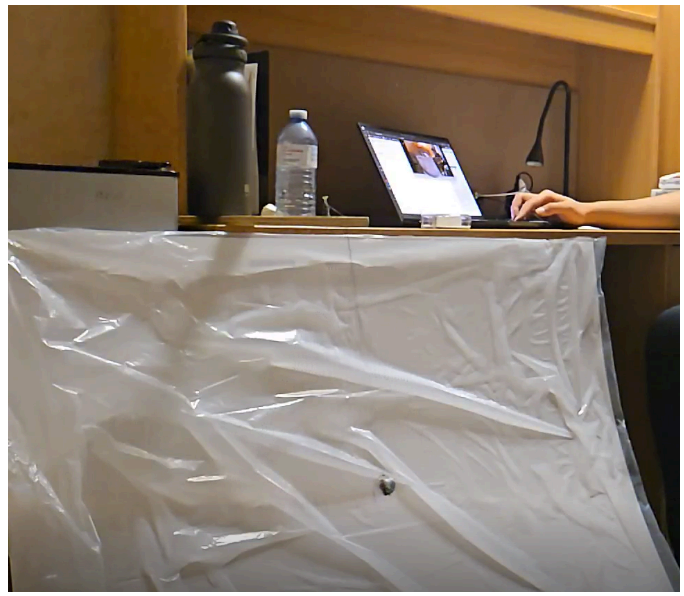


Figure 4. A white board was placed behind the pendulum to provide a consistent contrast between the pendulum and the background.

The pendulum was recorded in 1080p at 240 frames per second (fps) on a Samsung Galaxy S24 and converted to 30 fps using CapCut [3]. The data was originally recorded in 30fps, but was increased to 240fps in a second data collection to allow for more precise data points and reduce the uncertainties from motion blur while autotracking in the Tracker [2] software. However, I was constrained by the limitations of my laptop that couldn't handle tracking videos in 240fps, so the videos were converted from 240fps to 30fps. The recordings were imported into Tracker [2] software, and the time and angle data tracked using the auto tracking feature was imported onto a spreadsheet (Appendix C).

Drop Angle vs Period

The pendulum with a string length of 262mm was released from an initial angle/amplitude of 1.403 radians and recorded for 6 minutes. To analyze the relationship between drop angle and period, the data collected from Tracker [2] was analyzed using a Python program [4] that found the period and the angle by searching for the local maximums and minimums, This approach allowed for the measurement of both positive and negative amplitudes from a single dataset. The period and angle data, along with their uncertainties, were proceed using a Python program (Appendix D) that fitted to the power series $T = T_0(1 + B\theta_0 + C\theta_0^2)$ [1] (equation 2), where T_0 is the period for small angles, θ_0 is the initial angle in radians, and B and C are best fit parameters. The power series was used to generate a graph plotting the data and line of best fit. The pendulum was tested for symmetry using the fitted equation.

Time vs Amplitude (Q Factor)

To examine the relationship between time and amplitude, the data processed in Tracker [2] app for drop angle vs period was inputted into a Python program [5] that isolated the measured amplitude values and their respective timestamps. The processed data with its uncertainties was fitted using a Python program (Appendix D) to the exponential function $ae^{\frac{-t}{\tau}}$ [1] (equation 6), where t to create a graph with the line of best fit. The τ value from equation 6 was used to calculate the Q factor (see equation 3). The Q factor was also calculated by counting the number of oscillations it took to get to $e^{-\pi/2}$ ~ 20%, giving $\frac{Q}{2}$[1].

Q-Factor vs Length and Period vs Length

The pendulum was released at an angle θ at 8 different string lengths where $-0.319 < \theta < 0.319$, meaning small angle approximation was applicable (Appendix E) and period was independent of angle. The string lengths used were 297mm, 287mm, 273mm, 262mm, 251mm, 236mm, 226mm, and 212mm.

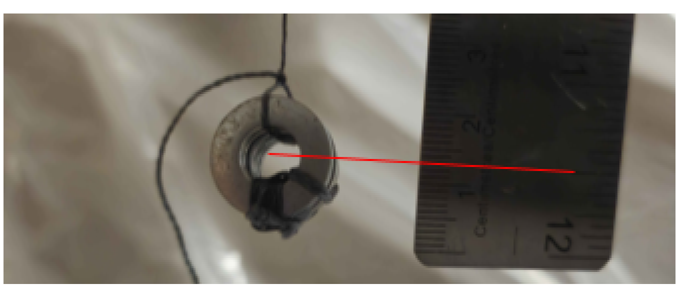


Figure 5. A depiction of how the length of the pendulum was measured. The measurement went from the centre of mass of the pendulum to the connection between the pendulum stand and string.

Each length's data was individually autotracked in Tracker [2] and was processed by a Python program [5] that outputted the amplitude vs time data. The period for each of the measured lengths was found by taking the average of all the periods for angle values where the period was independent of angle (Appendix E). This data was fit to the power law function $T = kL^n$ [1] (equation 4) using the best fit Python program (Appendix D).

Additionally, the Q factor for each trial was calculated using both methods; with the τ value from equation 6 and by counting how many oscillations it took to get from 0.319 rad (Appendix F) to $e^{-\frac{\pi}{16}} \simeq 82.2\%$ of the amplitude and multiplying by 16. The Q factor vs length values were plotted and fitted to linear, quadratic, cubic, quartic, power law, exponential and logarithmic functions using curve.fit [6]. Curve.fit [6] also found their respective R^2 values which I used to find which model best fit the Q factor vs length relationship.

Results and Analysis

Uncertainty Analysis

Before looking at the results of the experiment it is important to understand what uncertainties exist, where they come from and how they affected the experiment

Type B uncertainty in the time values arises from the frame rate of the recording, which was 30 frames per second (fps). This resulted in a type B uncertainty of $\pm \frac{1}{30} \times \frac{1}{4} = \pm \frac{1}{120} \simeq \pm 0.008s$. For time, there were no type A uncertainties because only 1 trial was performed.

Type B uncertainties for angle measurements came from the motion blur of the recording. The motion blur allowed for a larger area that the Tracker [2] auto tracker could identify as the mass (see Figure 5). This resulted in an uncertainty of $\pm 0.1°$.

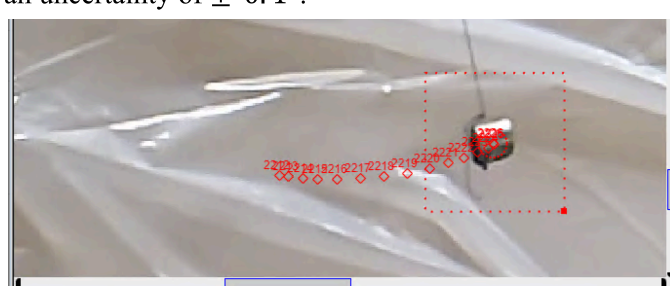


Figure 5. An example of the motion blur present in the video which was measured to cover 0.5°, leaving an uncertainty of $\pm \frac{0.5}{4} \simeq \pm\ 0.1°$.

Type B uncertainties for length measurements arose from inaccuracies from measuring with a ruler and were $\pm\ 0.0005$mm. Only 1 trial was done for each length so no type A uncertainties were available.

The largest source of uncertainty came from processing the video data in 30 fps. The motion blur uncertainty was already dealt with by recording at 240fps the second time data was collected, being $0.262 \div 0.002 = 131$ times smaller than the measured values. The length uncertainty was at least $212 \div 0.5 = 424$ times smaller than the measured values. However, using 30fps video data resulted in imprecise time data, accurate only to the nearest 1/30th of a second. This meant that it was only $0.933 \div \frac{1}{30} = 27.99$ times smaller than the measured values, making them significantly less accurate than the motion blur and length measurements. For example, only 8 period lengths in figure 6, when in reality there were many more in between those 8 that couldn't be precisely determined. The impreciseness impacted the best fit equations for period vs drop angle, amplitude vs time, period vs length and Q factor vs length. Inaccurate best fit equations meant that the calculated angles where small angle approximation was applicable and the period and drop angle were independent (Appendix E) were also imprecise and probably unreliable as a result.

To reduce these uncertainties I used 240fps to record the data which decreased the amount of motion blur and increased precision of the auto tracker. Unfortunately, I had to convert the 240fps videos to 30fps in order to run the auto tracker on my computer without it crashing. I tried running Tracker on the U of T Remote ECF workstations, but the online version of Tracker [2] would crash before it finished processing the data. If I wasn't limited by my computer's processing capacity, recording in 240fps would've allowed my data in every figure to look more fluid and provide a more accurate conclusion to my experiment.

Drop Angle vs Period

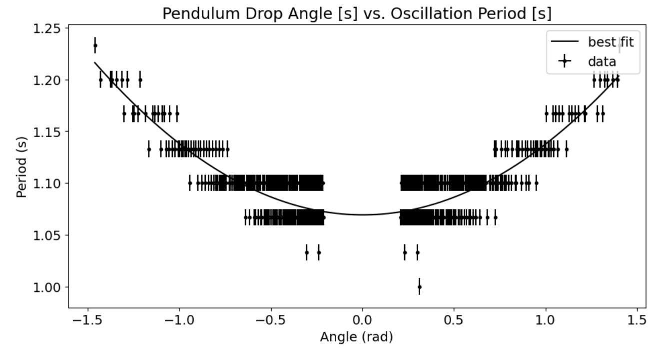


Figure 6. The relationship between the pendulum drop angle [rad] and the oscillation period [s] best fitted to $T = T_0(1 + B\theta_0 + C\theta_0^2)$ (equation 2), where $T_0 = 1.0692 \pm 0.0004$, $B = -\ 0.0003 \pm 0.0005$, $C = 0.0642 \pm 0.0008$, θ_0 is the drop angle and T is the period of oscillation.

It was predicted that the experiment would follow a damped harmonic motion model (see equation 1), where drop angle would have no effect on the period of oscillation, resulting in a consistent period across all drop angles. It was predicted that $B = C = 0$, meaning the fitted equation would've had a slope of 0. Figure 5 illustrates the relationship between the pendulum's drop angle and its period, displaying a nonlinear relationship which resembles a parabola.

This deviation from the prediction suggests that the pendulum's drop angle influences the oscillation period. This is evident from the best fit parameters for equation 2, specifically the C value. The C value of 0.0642, significantly larger than it's uncertainty ±0.0008, is a dominant and unignorable factor in the behaviour of the relationship when $|\theta|$ increases. The C value would have been even more accurately fitted if the experiment was processed at a higher frame rate, like 240fps. It can be concluded that higher magnitude drop angles result in longer periods.

However, the predicted constant relationship was found to be more accurate towards smaller angles $-\ 0.319 < \theta < 0.319$ where the change in period as a result of drop angle was experimentally zero, allowing for small angle approximation to be applicable (see Appendix E).

Asymmetry

In the equation of best fit from Figure 5, the B value from equation 2 is $-\ 0.0003 \pm 0.0005$. Given that the

B value is smaller than the uncertainty of B, the value of B is experimentally zero, meaning that the pendulum is symmetrical.

Q Factor (Experiment 1)

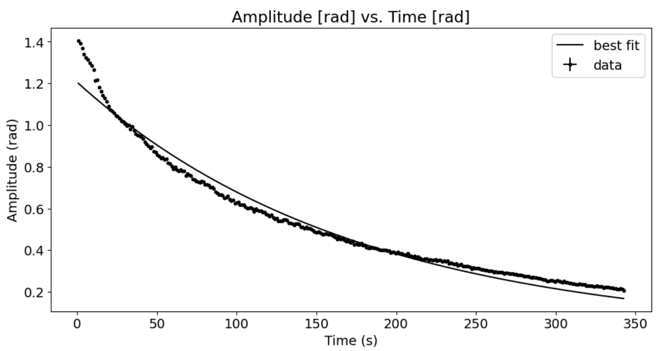


Figure 7. The relationship between the pendulum's amplitude [rad] versus time [s] and is approximated using the exponential equation (equation 6) $\theta = (1.2051 \pm 0.0003)e^{\frac{-t}{174.25 \pm 0.08}}$ where t is the time [s] and θ is the amplitude [rad].

The Q factor is calculated by counting the number of oscillations to get from 1.403 rad to $1.403 \times e^{-\pi/2} \simeq 0.29$ was 245 (see Appendix C), which represents the value of $Q/2$. This means that Q is experimentally $245 \times 2 = 490$.

The Q factor is also calculated using equation 3. In order to do so, the τ and T value have to be identified. $\tau =$ 174.25 ± 0.08 was taken from $(1.2051 \pm 0.0003)e^{\frac{-t}{174.25 \pm 0.08}}$, the fitted equation of equation 6. T is not a constant, so to calculate the Q factor, the period values for positive amplitudes are averaged using the built in average function in Google Sheets. The T value calculated in this way is 1.09 ± 0.01. The uncertainty was calculated by taking the largest uncertainty% and multiplying that by the largest period value: $\frac{0.008}{1.033} \times 1.233 \simeq 0.01$. The Q factor is calculated to be $Q = \pi\frac{\tau}{T} = \pi\frac{174.25 \pm 0.08}{1.09 \pm 0.01} = 502 \pm 5$.

The values of the counted and calculated Q factors are very close and deviate by at most 17 according to the uncertainty. This only varies by around $\frac{12 \pm 5}{502} \simeq 2 \pm 1\%$ which is very little and can be considered negligible or consistent with zero.

Period Dependence on Length

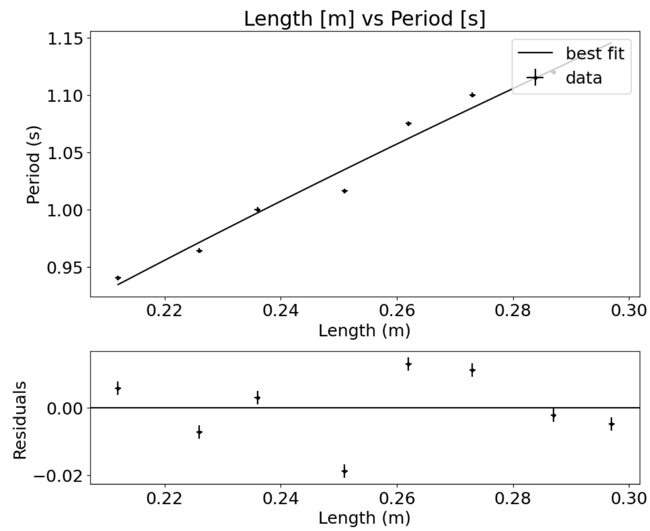


Figure 8. The relationship between the pendulum length [m] and period [s] fitted to the power law function (equation 4) $T = (2.39 \pm 0.02)L^{0.605 \pm 0.006}$ where T is the period [s] and L is the length [m].

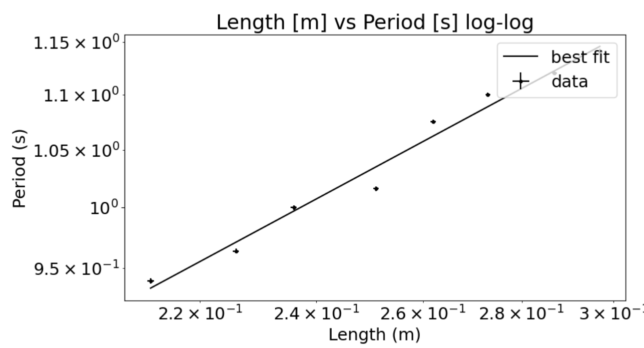


Figure 9. The same pendulum length [m] and period [s] data as Figure 8 in log-log form.

Figure 8 shows the relationship between the length of the pendulum and its period, with best fit parameters of k = 2.39 ± 0.02 and n = 0.605 ± 0.006. These results somewhat align with the predicted pendulum behaviour modeled by equation 4. Compared to the theoretical equation $T = 2L^{0.5}$ (equation 4), the experimental values are both roughly 20% larger (Appendix F), meaning they are consistent with the predicted values. The data is also plotted on log axes in Figure 9.

Q Factor Dependence on Length

The Q factor was found using both equation 3 and the counting method outlined in the method section. However, using the equation produced a smaller uncertainty (Appendix C), so the Q factor values were taken using that method. From the R^2 values calculated for each fit (Appendix H), the Q factor vs length data is best represented by a quartic equation (equation 5) with an R^2 value of 0.8003, significantly higher than the rest of the equations fit to the Q factor vs length data (Appendix H).

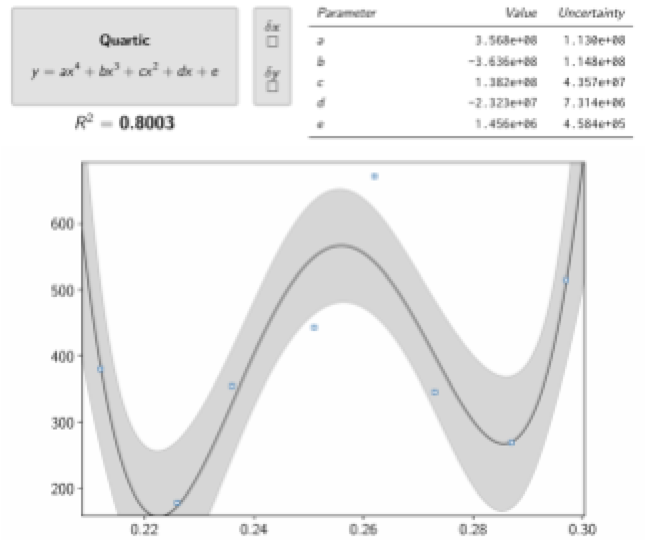


Figure 10. Q factor vs length [m] relationship best fitted to a quartic equation in the form $y = aL^4 + bL^3 + cL^2 + dL + e$ (equation 5) where $a = 4 \times 10^8 \pm 1 \times 10^8$, $b = -4 \times 10^8 \pm 1 \times 10^8$, $c = 1.4 \times 10^8 \pm 4 \times 10^7$, $d = -2.3 \times 10^7 \pm 7 \times 10^6$, $e = 1.5 \times 10^6 \pm 5 \times 10^5$ where *y* is the Q factor, *L* is the length of the pendulum [m], and *a, b, c, d, e* are best fit parameters. The R^2 value is 0.8003.

Figure 10 shows the fit of a quartic equation with the data collected in this experiment. The R^2 value of 0.8003 corresponds to a decent fit that still carries some variance, however, not an extremely strong fit. All of the best fit parameters are greater than their uncertainties, meaning that a relationship between Q factor and length does exist. However, 2 of the values between 0.24 and 0.28 don't fall close to the line of best fit which shows this isn't a perfect fit of the relationship between Q factor and length. Therefore, for the limited dataset I have recorded in this experiment, the Q factor has a quartic relation with the length of the pendulum, though not a strong one. However, if more data points were collected, a more accurate relationship (higher R^2 value) could be found.

Conclusion

This lab tested whether the behaviour of a homemade pendulum was accurately represented by theoretical pendulum models.

The drop angle of the pendulum was found to have a quadratic relationship, represented by $T = T_0(1 + B\theta_0 + C\theta_0^2)$ (equation 2), which differed from the theoretical prediction of the pendulum following the motion of a damped harmonic motion (equation 1). However, the angles between -0.319 rad and 0.319 rad showed a roughly independent relationship between period and angle that was consistent with the predicted model due to small angle approximation being applicable between those angles. The Q factors, 490 and 502 ± 5, derived from oscillation counting and equation 3 respectively, are in agreeance with each other with a percent error of only $2 \pm 1\%$.

The predicted relationship between the period and the pendulum length of $T = 2L^{0.5}$ (equation 4) was consistent with the experimental result $T = (2.39 \pm 0.02)L^{0.605 \pm 0.006}$, only differing by 20%. The length of the pendulum was found to have a weak quartic correlation ($R^2 = 0.8003$) with the Q factor represented by the equation $y = ax^4 + bx^3 + cx^2 + dx + e$ (equation 5).

The limitations of the results mainly stemmed from the type B uncertainty of processing the video in 30 fps that limited the precision of time measurements to increments of 1/30th of a second. This imprecision affected the reliability of the best fit equations for period vs drop angle, amplitude vs time, period vs length and q factor vs length. Consequently, the calculated angles at which small angle approximation was applicable (Appendix E) were also imprecise and likely unreliable. For Q factor dependence on pendulum length and period dependence on pendulum length, a more accurate correlation could have been found if more data points were acquired because of the likeliness of outliers being present in such a small dataset.

I tried resolving these limitations by recording in 240 frames per second, but this resulted in my laptop not being able to process the data in Tracker [2] app, ultimately leading me to lower the fps to 30 fps to complete this report. In future experiments I will look into building or borrowing a computer with more capable components to ensure my conclusions are based upon more precise experiment results. I will also look to record more data points where possible to increase the size of datasets so fitted models are a better representation of the relationships between the variables I'm testing.

Resources

[1] B. Wilson, “PHY180 Lab Project (2024),” PHY180-PendulumProject-2024.pdf, https://q.utoronto.ca/courses/363836/files/32826310?module_item_id=6082503 (Accessed Oct. 4, 2024).

[2] D. Brown, W. Christian, and R. M. Hanson, “Tracker Video Analysis and Modeling Tool for Physics Education”, https://physlets.org/tracker/ (Accessed Nov. 29, 2024).

[3] CapCut, "CapCut - Video Editor," CapCut, https://www.capcut.com. (Accessed: Nov 6, 2024).

[4] Y. Zhang, “angle_vs_period.py”, https://github.com/YYZ-CR/PHY180/blob/main/data_sorting/angle_vs_period.py (accessed Oct. 10, 2024)

[5] Y. Zhang, “angle_vs_time.py” https://github.com/YYZ-CR/PHY180/blob/main/data_sorting/angle%20vs%20time.py (accessed Oct. 10, 2024)

[6] Curve Fit, “Curve Fitting with X and Y Uncertainties”, https://curve.fit/ (accessed Dec 5, 2024)

Appendices

Appendix A (washer mass calculation)

The volume of the washers is the volume of the hole in the middle subtracted from the volume of everything including the hole.

Washers diameter = 1.8 cm
Hole diameter = 0.8 cm
Washers height = 1.8 cm

Volume of everything = $\pi \times (\frac{1.8\ cm}{2})^2 \times 1.8\ cm = 4.5804\ cm^3$

Volume of hole = $\pi \times (\frac{0.8\ cm}{2})^2 \times 1.8\ cm = 0.9148\ cm^3$

Volume of pendulum = $4.5804\ cm^3 - 0.9148\ cm^3 = 3.6757\ cm^3$
Density of steel = 7850 kg/m³ = 7.85 g/cm³
Mass of pendulum = $7.85\ g/cm^3 \times 3.6757\ cm^3 = 28.85\ g \approx 29\ g$

Additionally, the mass was weighed using a scale and found to be 29.0 g, confirming the calculation above.

Appendix B (water bottle weight calculation)

Per the thermoflask website https://mythermoflask.com/products/thermoflask-insulated-stainless-steel-4-lid-combo-pack-40oz-1-1l-2-pack#:~:text=Weight%3A%201.93,lbs.%20total , each 40oz stainless steel water bottle weighs 1.93lb without any liquid in it. Filled with 40 oz of water (2.5 lb) would be 1.93 lb + 2.5 lb = 4.43 lb ~ 2.01 kg. 2 of these bottles would be 2.01 kg x 2 = 4.01 kg ≈ 4.0 kg.

Appendix C (data collected from Tracker [2] and processed data)

The tracker data (100,000+ data points), the processed data and all other data used in this lab can be viewed here: https://docs.google.com/spreadsheets/d/1VSwNOmmxI4KrP7Po4nxmL91l822pYo9TJ0T3jLy3fb8/edit?usp=sharing.

The data from the first time I recorded can be found here (it wasn't used in this report): https://docs.google.com/spreadsheets/d/1zY-a8DOFjC4rEsYK8gz0ikJFwNO3PC4cED7r3G2fHok/edit?usp=sharing

Appendix D (python files)

The Python files that were used to generate the graphs and equations of best fit in the experiment were based off of the provided files from the PHY180 Quercus page. These files were altered and used to graph data points collected by Tracker App [2]. All the files used for this project can be found on this GitHub repository: https://github.com/YYZ-CR/PHY180.

Appendix E (calculations of angles where angles are small enough that $C\theta_0^2$ can be ignored and the data roughly follows the prediction)

The uncertainty of time values in this experiment was found to be ±0.008s, meaning that if the period was within ±0.008s of T_0, the slope would be experimentally zero. The values of θ that allow for the period to be ±0.008s of T_0 were calculated by finding the values of θ where $C\theta_0^2$ could be ignored. Note: $B\theta_0$ is ignored in this calculation since it is experimentally determined to be zero and has a significantly smaller impact on the behaviour of the pendulum.

$$|\theta| < \sqrt{(\frac{(1.0692 \pm 0.0004) \pm 0.008}{(1.0692 \pm 0.0004)} - 1) \div (0.0694 \pm 0.0004)}$$

Note: only the + from the ± are used because $\theta^2 > 0$ and we are trying to find the minimum value $|\theta|$ needs to be less than.

$$|\theta| < \sqrt{(\frac{(1.0692 + 0.0004) + 0.008}{(1.0692 + 0.0004)} - 1) \div (0.0694 + 0.0004)} = \sqrt{(\frac{1.0772}{1.0696} - 1) \div 0.0698} \simeq 0.319$$

$$|\theta| < 0.319$$

$$\therefore -0.319 < \theta < 0.319$$

Appendix F (calculations for how much larger the experimental values of equation 2 are than the theoretical values)

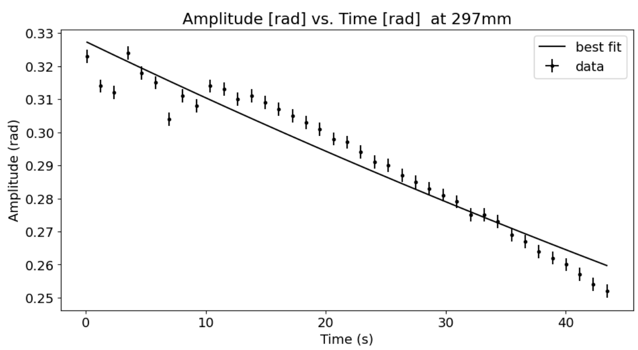
$2.39 \div 2 = 1.195 = 119.5\%$

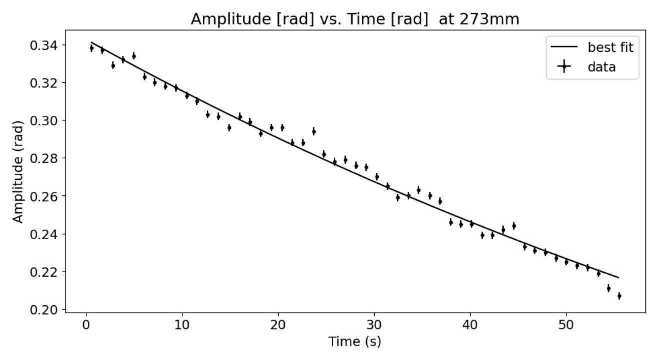
$0.605 \div 0.5 = 1.21 = 121\%$

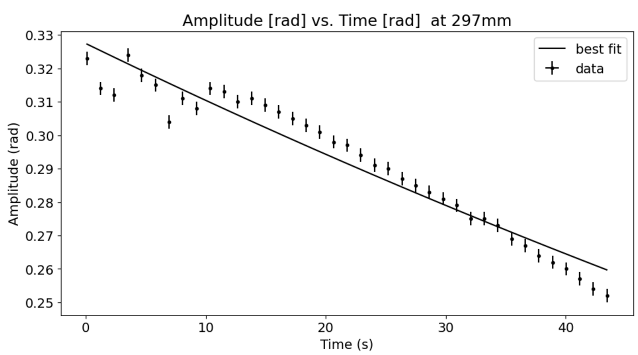
Both of the above values are roughly 120%, meaning the experimental values are roughly 20% larger than the theoretical values and can be considered consistent with the theoretical prediction.

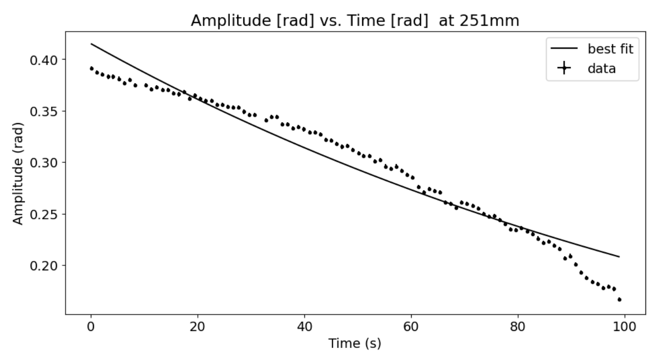
Appendix G (Amplitude [rad] vs Time [s] graphs for each length)

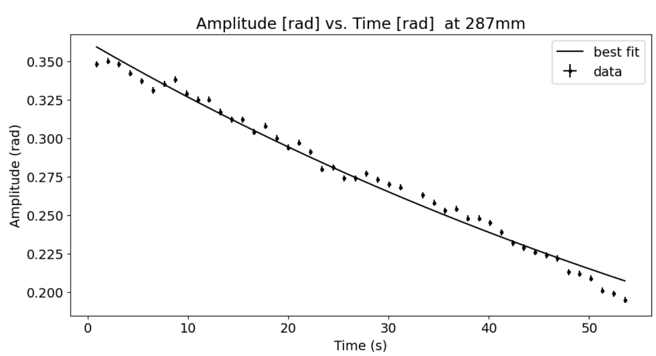
These graphs were created using the python files in Appendix D

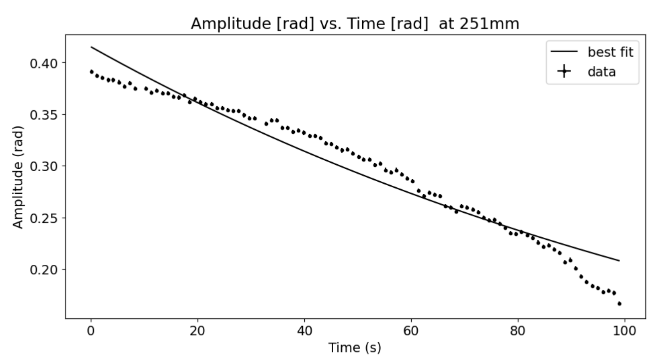


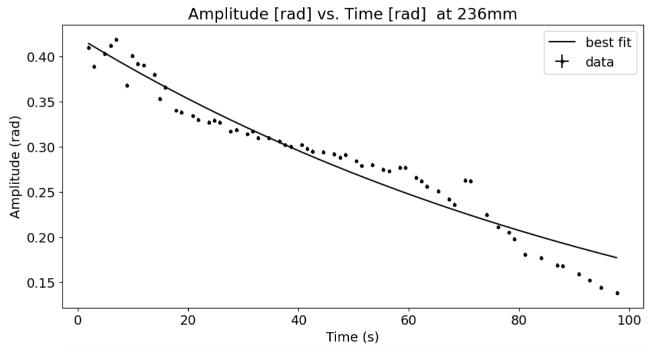


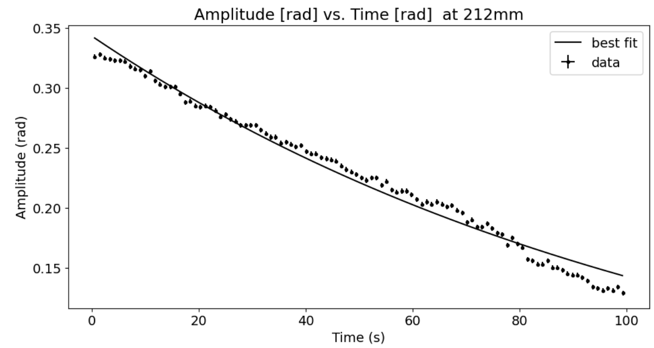












Appendix H (best fit graphs from curve.fit[6] and R^2 values for each fitting of Q factor vs length)

[Linear Fit]

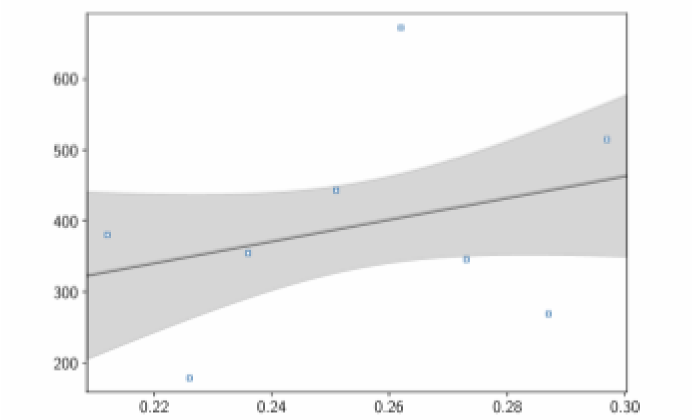
Linear

$y = mx + b$

δx δy

Parameter	*Value*	*Uncertainty*
m	1.532e+03	1.976e+03
b	3.232e+00	5.079e+02

$R^2 = \mathbf{0.0911}$



[Quadratic Fit]

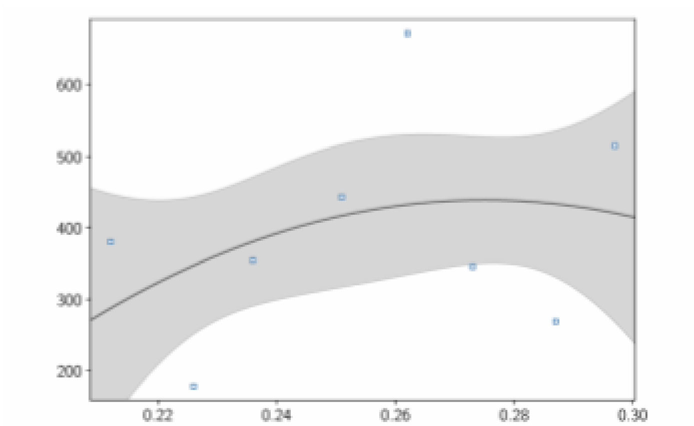
Quadratic

$y = ax^2 + bx + c$

δx δy

Parameter	*Value*	*Uncertainty*
a	-3.811e+04	8.745e+04
b	2.096e+04	4.464e+04
c	-2.444e+03	5.642e+03

$R^2 = \mathbf{0.1243}$



Cubic Fit

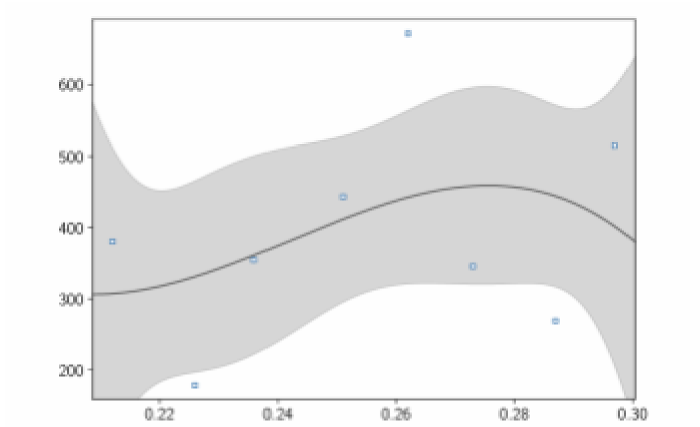
Cubic

$y = ax^3 + bx^2 + cx + d$

δx δy

$R^2 = \mathbf{0.1368}$

Parameter	Value	Uncertainty
a	-1.031e+06	4.285e+06
b	7.497e+05	3.274e+06
c	-1.782e+05	8.290e+05
d	1.422e+04	6.951e+04



Quartic Fit

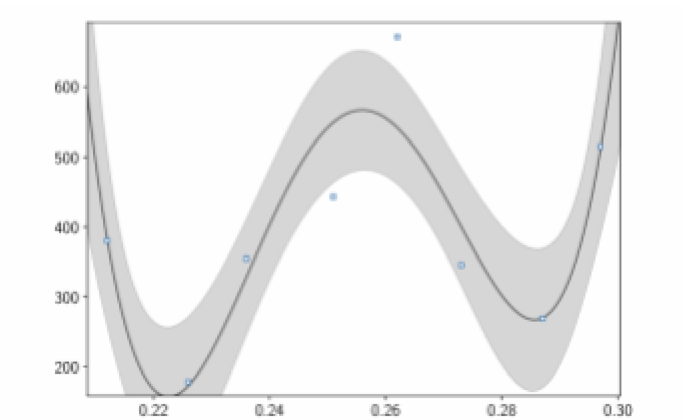
Quartic

$y = ax^4 + bx^3 + cx^2 + dx + e$

δx δy

$R^2 = \mathbf{0.8003}$

Parameter	Value	Uncertainty
a	3.568e+08	1.130e+08
b	-3.636e+08	1.148e+08
c	1.382e+08	4.357e+07
d	-2.323e+07	7.314e+06
e	1.456e+06	4.584e+05



Calculations for Power Law, Exponential and Natural Log Fit R^2 values. All values are based on the “q factor vs length” table in Appendix C

$$\bar{y} = \frac{\Sigma y_i}{n} = 394.75$$

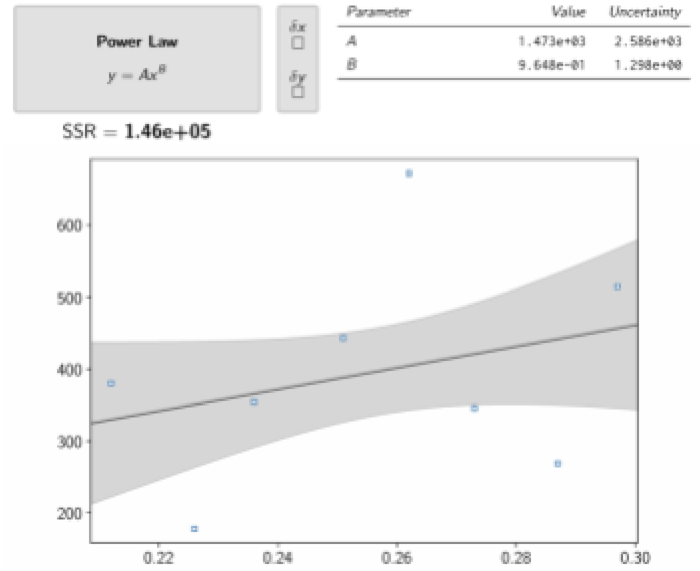
$$SST = \sum (y_i - \bar{y})^2 = 160517.89$$

Power Law: $R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{146000}{160517.89} = 0.0904$

Exponential: $R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{146000}{160517.89} = 0.0904$

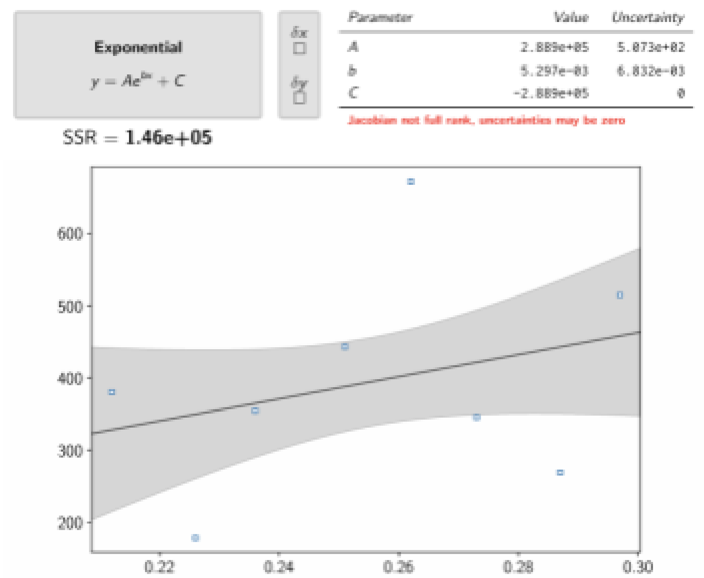
Natural Log: $R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{146000}{160517.89} = 0.0967$

Power Law Fit



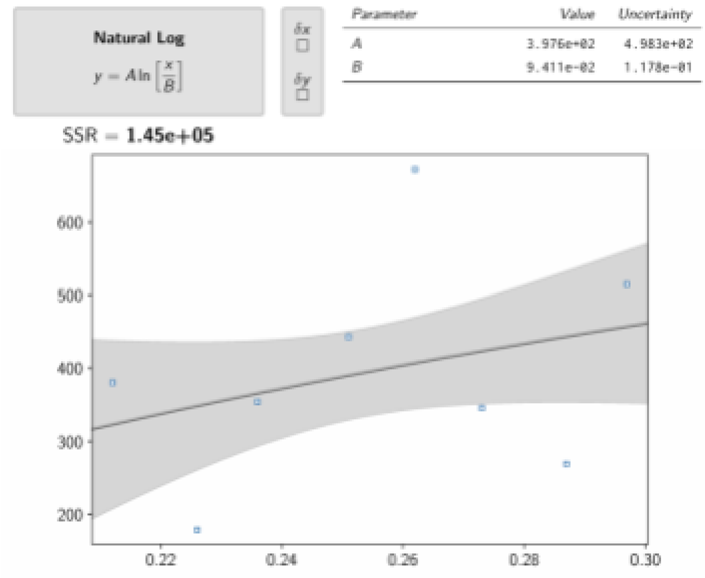
$R^2 = 0.0904$

Exponential Fit



$R^2 = 0.0904$

Natural Log Fit



$R^2 = 0.0967$