Tutorial 8

CB2200 Business Statistics, YANG Yihang

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Intended Learning Outcomes

After this tutorial, you may know how to

- Construct a hypothesis t-test.
- Compute p-value and interpret it.
- analyze the relation between decision making and parameter changing.

Q3

A manufacturer of chocolate candies uses machines to package candies as they move along a filling line. Although the packages are labeled as 8 ounces, the company wants the packages to contain a mean of 8.17 ounces so that virtually none of the packages contain less than 8 ounces. A sample of 50 packages is selected periodically, and the packaging process is stopped if there is evidence that the mean amount packaged is different from 8.17 ounces. Suppose that in a particular sample of 50 packages, the mean amount dispensed is 8.159 ounces, with a sample standard deviation of 0.051 ounce.

a) Is there evidence that the population mean amount is different from 8.17 ounces? (Use a 0.05 level of significance.)

b) Compute the p-value and interpret its meaning.

Q3

a) H_0: $\mu = 8.17$ ounces

H_1: $\mu \neq 8.17$ ounces

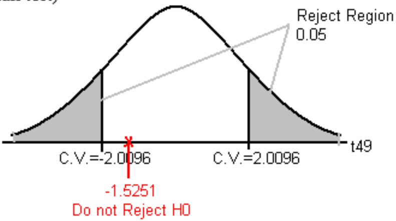
∵ n = 50 > 30 from unknown population distribution, by Central Limit Theorem, the sampling distribution of $\overline{X}$ is approximately normal

∵ σ unknown ∴ t test should be used (two-tail test)

$\alpha = 0.05$, Critical value $= \pm t_{\frac{0.05}{2},50-1} = \pm 2.0096$

Reject H_0 if t < -2.0096 or t > 2.0096

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{8.159 - 8.17}{0.051/\sqrt{50}} = -1.5251$$



Since t = -1.5251 >-2.0096 and <2.0096

Do not Reject H_0 at $\alpha = 0.05$

There is insufficient evidence that population mean amount is different from 8.17 ounces.

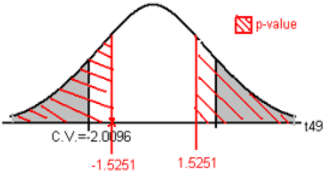
Question 3

b) p-value = $\Pr(t \le -1.5251) + \Pr(t \ge 1.5251) = 2 \times \Pr(t \ge 1.5251)$

$= 2 \text{ x } (0.05, 0.1) = (0.1, 0.2)$

Interpretation:

Probability of obtaining a test statistics 1.5251 or more or -1.5251 or less is between 0.1 and 0.2 exclusively, given H_0 is true.



Q4

The Glen Valley Steel Company manufactures steel bars. If the production process is working properly, it turns out that steel bars are normally distributed with mean length of at least 2.8 feet. Longer steel bars can be used or altered, but shorter bars must be scrapped. You select a sample of 25 bars, and the mean length is 2.73 feet and the sample standard deviation is 0.20 feet. Do you need to adjust the production equipment?

a) If you test the null hypothesis at the 0.05 level of significance, what decision do you make using the critical value approach to hypothesis testing?
b) If you test the null hypothesis at the 0.05 level of significance, what decision do you make using the p-value approach to hypothesis testing?
c) Interpret the meaning of the p-value in this problem.
d) Compare your conclusions in (a) and (b).

Question 4

Q4

a) H0: $\mu \geq 2.8$ feet

H1: $\mu < 2.8$feet

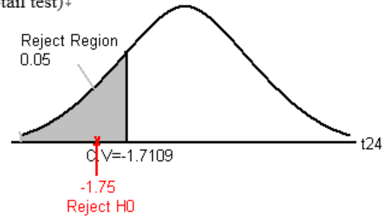
$\because$ the population is normal distribution, the sampling distribution of $\overline{X}$ is also normal distribution

$\because$ σ unknown $\therefore$ t test should be used (lower-tail test)

$\alpha = 0.05$, Critical value =- $t_{0.05,25-1} = -1.7109$

Reject H0 if t < -1.7109

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{2.73 - 2.8}{0.2/\sqrt{25}} = -1.75$$



Since t = -1.75 < -1.7109

Reject H0 at $\alpha = 0.05$

There is sufficient evidence that the production equipment needs adjustment.

Question 4

b) H0: $\mu \geq 2.8$ feet

H1: $\mu < 2.8$feet

∵ the population is normal distribution, the sampling distribution of $\overline{X}$ is also normal distribution

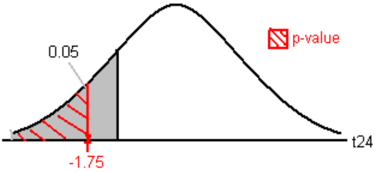
∵ σ unknown ∴ t test should be used (lower-tail test)

$\alpha = 0.05$

Reject H0 if p-value < 0.05

$$t = \frac{\overline{X} - \mu_0}{s/\sqrt{n}} = \frac{2.73 - 2.8}{0.2/\sqrt{25}} = -1.75$$

p -value = P(t ≤ -1.75) = (0.025, 0.05)



Since p-value = (0.025, 0.05) < 0.05

Reject H0 at $\alpha = 0.05$

There is sufficient evidence that the production equipment needs adjustment.

c) Probability of obtaining a test statistics -1.75 or less is between 0.025 and 0.05 exclusively, given H0 is true.

d) The conclusions are the same.

Question 5

Q5

A bank branch located in a commercial district of a city has developed an improved process for serving customers during the 12:00 to 1 p.m. peak lunch period. The waiting time in minutes (operationally defined as the time the customer enters the line to the time he or she is served) of all customers during this hour is recorded over a period of a week. A random sample of 15 customers is selected, and the results are as follows:

4.21	5.55	3.02	5.13	4.77	2.34	3.54	3.20
4.50	6.10	0.38	5.12	6.46	6.19	3.79	

At the 0.05 level of significance, is there evidence that the average waiting time at a bank branch in a commercial district of the city is less than five minutes during the lunch period?

Question 5

Q5

Assume population distribution is normal

Let μ be the true average waiting time at back in commercial district.

H_0: $\mu \geq 5$

H_1: $\mu < 5$

$\because$ n < 30 and σ unknown

$\therefore$ Use *t*-test

$\alpha = 0.05$, $t_{critical} = -t_{0.05,15-1} = -1.7613$; Reject H_0 if t < -1.7613

$\bar{X} = 4.286667$, s = 1.637985

$t = \frac{4.286667-5}{\frac{1.687985}{\sqrt{15}}} = -1.6867 > -1.7613$

$\therefore$ We do not reject H_0 at $\alpha = 0.05$. There is insufficient evidence that the population average waiting time is less than 5 mins.

Question 6

Q6

A television documentary on over-eating claimed that Americans are about 10 pounds overweight on average. To test this claim, 18 randomly selected individuals were examined, and their average excess weight was found to be 12.4 pounds, with a sample standard deviation of 2.7 pounds.

a) What assumption(s) is(are) required for performing the hypothesis testing in (ii) below?
b) At a significance level of 0.01, is there any reason to doubt the validity of the claimed 10-pound value?
c) Define the probability of type I error α and that of type II error β according to the context of this part.

Q6

a) As $n = 18 < 30$, we need to assume the population distribution is normal.

b) Let μ be the population mean of overweight

$H_0 : \mu = 10$

$H_1 : \mu \neq 10$

$\because n = 18 < 30$, σ is unknown, assume population distribution is normal

$\therefore$ Use the t test, $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ (two-tail test)

$\alpha = 0.01$; Critical Value $= -t_{\frac{\alpha}{2}, n-1} = -t_{0.005, 17} = -2.8982$

$= t_{\frac{\alpha}{2}, n-1} = t_{0.005, 17} = 2.8982$

Reject H_0 if $t < -2.8982$ or $t > 2.8982$

$\bar{x} = 12.4 \quad \mu_0 = 10 \quad s = 2.7 \quad n = 18$

$t = \dfrac{12.4 - 10}{2.7/\sqrt{18}} = 3.7712$

$\because t = 3.7712 > 2.8982$

$\therefore$ We reject H_0.

There is sufficient evidence that the population mean overweight is not 10 pounds

Question 6

c) Type I error (α) = Pr (do not agree the claim of 10-pound overweight when in fact the claim is true)

Type II error (β)= Pr (Agree the claim of 10-pound overweight when in fact the claim is false)

Question 7

Q7

A management consultant has introduced new procedures to a reception office. He claims that the receptionist should not do more than 10 minutes of paperwork in each hour. A check is made on 40 random hours of operation. The sample mean and sample standard deviation of the time spent on paperwork are found. Based on these figures, the null hypothesis that the new procedures meet specifications is rejected at a 1% level of significance.

a) After the consultant has asked the data entry clerk to show him the original data, he finds that the sample size should be 41, instead of 40. Should the null hypothesis that the new procedures meet specifications be rejected? Why or why not?

b) Peter, the manager of the reception office, asks the consultant to test the same hypothesis with a new level of significance of 5%. Should the null hypothesis that the new procedures meet specifications be rejected? Why of why not?

Q7

a) $n = 40 > 30$ and σ is unknown, use t-distribution

H_0: $\mu \le 10$

H_1: $\mu > 10$

$\alpha = 0.01$; $t_{critical} = t_{0.01,39} = 2.4258$

$$t = \frac{\overline{X} - 10}{\frac{s}{\sqrt{40}}} > 2.4528, \text{ as } H_0 \text{ is rejected}$$

Now sample size, $n\uparrow$, $n' = 41$, hence

$t'_{critical} = t_{0.01,40} = 2.4233$, and

$\frac{\sigma}{\sqrt{n}} \downarrow$, as a result t' will be increased

i.e. $t' > t > 2.4258 > 2.4233$

$\therefore$ H_0 is still rejected

b) Now $\alpha'' = 0.05$

$t''_{critical} = 1.6839$, with 40 degrees of freedom

$\because$ $t' > 2.4233$ from above

$\therefore$ $t' > 2.4233 > 1.6839$

Therefore, H_0 is still rejected