Tutorial 5

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Intended Learning Outcomes

After this tutorial, you may know how to

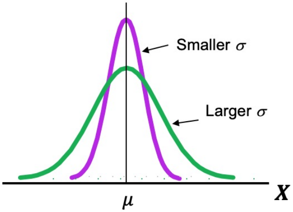
- compute probabilities of some random variables(Normal distribution) that falling in specific ranges.
  - transform a general distribution into the standard form.
  - identify specific events on a normal distribution graph.
- compute μ and σ from probabilities of specific events.

Recap

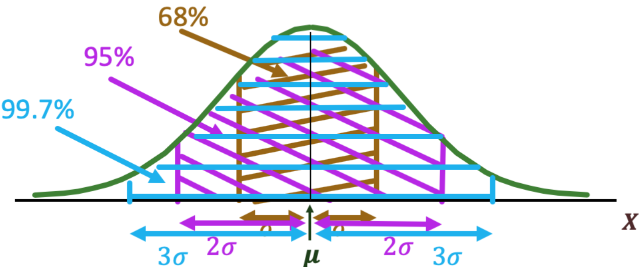
- Continuous distribution
- Definition of PDF(Probability density function):A probability density function, or density function of a continuous random variable is a function that describes the relative likelihood for this random variable to take on a given value
- Normal distribution
- PDF :$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{(-\frac{1}{2}[\frac{x-\mu}{\sigma}]^2)}$
- Notation: $X \sim N(\mu, \sigma^2)$
- KEY: If $X \sim N(\mu, \sigma^2)$, then $\frac{X-\mu}{\sigma} \sim N(0, 1^2)$

Recap

- For $X \sim N(\mu, \sigma^2)$
  - Has an infinite theoretical range, i.e. $-\infty$ to $+\infty$
  - Bell shaped
  - Symmetrical about $X = \mu$
  - Mean, median and mode are identical
  - The spread is determined by σ
    - For smaller σ, the X values are clustered more closely around μ
    - For larger σ, the X values are more spread out and away from μ
  - Follows the Empirical Rule

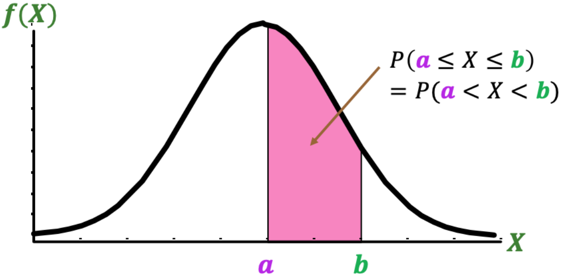


- The Empirical Rule said that
  - Area within $\mu \pm \sigma$ equals 68% approximately
  - Area within $\mu \pm 2\sigma$ equals 95% approximately
  - Area within $\mu \pm 3\sigma$ equals 99.7% approximately



Recap

- The total area under the curve is 1
- Probability is measured by the area under the curve
- Note that the probability of any individual value is zero by definition, i.e. $P(X = a) = 0$



Question 11

Q11

Given a normal distribution with $\mu = 100$ and $\sigma = 10$, what is the probability that

a) $X > 85$?

b) $X < 80$?

c) $X < 80$ or $X > 110$?

d) 80% of the values are between what two X values (symmetrically distributed around the mean)?

Question 11

Steps:

- 1.Use transformation based on $X \sim N(\mu, \sigma^2) \Longleftrightarrow \frac{X-\mu}{\sigma} \sim N(0, 1^2)$
- 2. Obtain the related range for $Z = \frac{x-\mu}{\sigma}$
- 3. Mark it on a standard normal distribution graph.
- 4. Derive the probability from the Z table.

Question 11

Q11

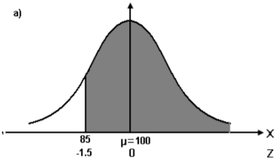
$X \sim N(100, 10^2)$

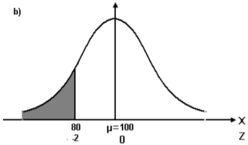
a) $P(X > 85)$

$$= P\left(Z > \frac{85-100}{10}\right)$$
$$= P(Z > -1.5)$$
$$= 1 - P(Z <= -1.5)$$
$$= 1 - 0.0668$$
$$= 0.9332$$

b) $P(X < 80)$

$$= P\left(Z < \frac{80-100}{10}\right)$$
$$= P(Z < -2)$$
$$= 0.0228$$

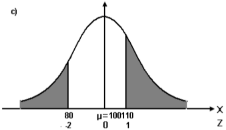




Question 11

c) $P(X < 80 \text{ or } X > 110) = P\left(Z < \frac{80-100}{10}\right) + P\left(Z > \frac{110-100}{10}\right)$

$= P(Z < -2) + P(Z > 1)$
$= 0.0228 + 1 - P(Z <= 1)$
$= 0.0228 + 1 - 0.8413$
$= 0.1815$



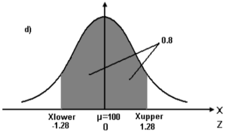
d) $P(X_{lower} < X < X_{upper}) = 0.8$

$P\left(\frac{X_{lower} - 100}{10} < Z < \frac{X_{upper} - 100}{10}\right) = 0.8$

$P(Z < -1.28) = 0.10$ and $P(Z < 1.28) = 0.90$

$Z = \frac{X_{lower} - 100}{10} = -1.28$ and $Z = \frac{X_{upper} - 100}{10} = 1.28$

$X_{lower} = -1.28(10) + 100 = 87.2$ and $X_{upper} = 1.28(10) + 100 = 112.8$



Q12

The breaking strength of plastic bags used for packaging produce is normally distributed, with a mean of 5 pounds per square inch and a standard deviation of 1.5 pounds per square inch. What proportion of the bags have a breaking strength of

a) Less than 3.11 pounds per square inch?
b) At least 3.8 pounds per square inch?
c) Between 5 and 5.5 pounds per square inch?
d) 95% of the breaking strength will be contained between what two values symmetrically distributed around the mean?

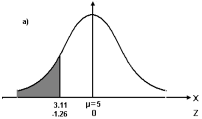
Question 12

Q12

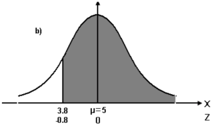
Let X be the breaking strength of plastic bags.

$X \sim N(5, 1.5^2)$

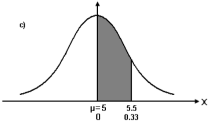
a) $P(X < 3.11) = P\left(Z < \frac{3.11-5}{1.5}\right)$
$= P(Z < -1.26)$
$= 0.1038$



b) $P(X >= 3.8) = P\left(Z >= \frac{3.8-5}{1.5}\right)$
$= P(Z >= -0.8)$
$= 1 - P(Z < -0.8)$
$= 1 - 0.2119$
$= 0.7881$

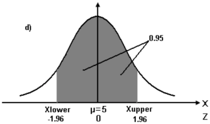


c) $P(5 < X < 5.5) = P\left(\frac{5-5}{1.5} < Z < \frac{5.5-5}{1.5}\right)$
$= P(0 < Z < 0.33)$
$= 0.6293 - 0.5$
$= 0.1293$



d) $P(X_{lower} < X < X_{upper}) = 0.95$

$$P\left(\frac{X_{lower}-5}{1.5} < Z < \frac{X_{upper}-5}{1.5}\right) = 0.95$$



$P(Z < -1.96) = 0.025$ and $P(Z < 1.96) = 0.975$

$Z = \frac{X_{lower}-5}{1.5} = -1.96$

$X_{lower} = -1.96(1.5) + 5 = 2.06$

$Z = \frac{X_{upper}-5}{1.5} = 1.96$

$X_{upper} = 1.96(1.5) + 5 = 7.94$

Q13

A statistical analysis of 1,000 long-distance telephone calls made from the headquarters of the Bricks and Clicks Computer Corporation indicates that the length of these calls is normally distributed with $\mu = 220$ seconds and $\sigma = 30$ seconds.

a) What is the probability that a call lasted less than 175 seconds?

b) What is the probability that a call lasted between 175 and 265 seconds?

c) What is the probability that a calls lasted between 115 and 175 seconds?

d) What is the length of a call if only 1% of all calls are shorter?

Question 13

Q13

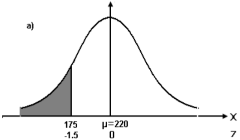
Let X be the length of long-distance telephone call.

$X \sim N(220, 30^2)$

a) $P(X < 175) = P\left(Z < \frac{175-220}{30}\right)$

$= P(Z < -1.5)$

$= 0.0668$

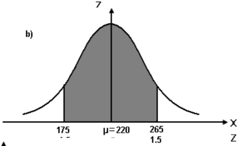


b) $P(175 < X < 265) = P\left(\frac{175-220}{30} < Z < \frac{265-220}{30}\right)$

$= P(-1.5 < Z < 1.5)$

$= 0.9332 - 0.0668$

$= 0.8664$

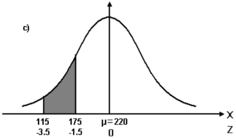


c) $P(115 < X < 175) = P\left(\frac{115-220}{30} < Z < \frac{175-220}{30}\right)$

$= P(-3.5 < Z < -1.5)$

$= 0.0668 - 0.00023$

$= 0.06657$



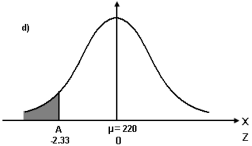
d) Let A be the length of a call if only 1 % of all calls are shorted

$P(X < A) = 0.01$

Since $P(Z<-2.33) = 0.01$

$$\frac{A-220}{30} = -2.33$$

$A = -2.33\,(30) + 220 = 150.1$



Q14

The exam marks of a large class of students follow a normal distribution with mean μ and standard deviation σ. 1% of the students got 90 or above. 10% of the students got 40 or below. The passing mark is 50.

a) Find the values of μ and σ.
b) Find the chance that a randomly selected student passes the exam.

Q14

a) Let X be the exam marks of the student

$X \sim N(\mu, \sigma^2)$

$\Pr(X \geq 90) = 0.01$

$\Pr(Z \geq \frac{90-\mu}{\sigma}) = 0.01$

$\frac{90-\mu}{\sigma} = 2.33$

$90 - \mu = 2.33\sigma ......(1)$

$\Pr(X \leq 40) = 0.1$

$\Pr(Z \leq \frac{40-\mu}{\sigma}) = 0.1$

$\frac{40-\mu}{\sigma} = -1.28$

$40 - \mu = -1.28\sigma ......(2)$

By solving equation (1) & (2)

$(1)-(2): 50 = 3.61\sigma$

$\sigma = 13.85$

Sub $\sigma = 13.85$ into (2)

$\mu = 57.73$

b) $\Pr(X \geq 50) = \Pr(Z \geq \frac{50-57.73}{13.85}) = \Pr(Z \geq -0.56) = 0.7123$

Question 14

Steps:

- 1. making equations
- 2. Making transformation based on $X \sim N(\mu, \sigma^2) \iff \frac{X-\mu}{\sigma} \sim N(0, 1^2)$
- 3. Find related probabilities on the normal distribution table.
- 4. Calculate the μ, σ

Question 15

Q15

The fill amount of bottles of soft drink has been found to be normally distributed with a mean amount of 2.0 liters and a standard deviation of 0.05 liter. Bottles that contain less than 95% of the listed net content (1.90 liters in this case) can make the manufacturer subject to penalty by the Consumer Council, whereas bottles that have a net content above 2.12 liters may cause excess spillage upon opening.

a) What proportion of the bottles is subject to penalty by the Consumer Council?

b) What proportion of the bottles is risking to excess spillage upon opening?

c) In an effort to reduce the possible penalty due to insufficient net content in the bottles, the manufacturer has set out the following quality control requirement: 99% of bottles should comply with the Consumer Council's standard. To achieve this, the bottler decides to set the filling machine to a new mean amount. Determine the mean amount to be set for the bottle filling machine such that the above requirement can be met.

Q15

a) $X \sim N(2, 0.05^2)$

$\Pr(x < 1.9) = \Pr(Z < \frac{1.9-2}{0.05}) = \Pr(Z < -2) = 0.0228\ (2.28\%)$

The proportion of the bottles is subject to penalty by the Customer Council is 0.0228.

b) $\Pr(x > 2.12) = \Pr(Z > \frac{2.12-2}{0.05}) = \Pr(Z > 2.4) = 0.0082\ (0.82\%)$

The proportion of the bottles is risking to excess spilling upon opening is 0.0082.

c) $P(x < 1.9) = 1 - 0.99$

$$\Pr\left(Z < \frac{1.9-\mu}{0.05}\right) = 0.01$$

$$\frac{1.9-\mu}{0.05} = -2.33$$

$$\mu = 2.0165$$

Q16

At the CityU Computer Service Centre, the loading time for e-Portal page on Internet Explorer is normally distributed with mean 3 seconds.

a) Without doing the calculations, for a randomly selected student, which of the following intervals of loading time (in second) is the most likely to be: 2.9-3.1, 3.1-3.3, 3.3-3.5, 3.5-3.7? Which interval of loading time is the least likely to be? Explain.

b) What is the chance that the loading time is exactly 2 seconds?

Q16

a) ∵ The loading time is normally distributed with mean of 3 seconds
Most likely: 2.9-3.1, since it lies in the central part of the normal distribution model, which has the largest area, thus the largest probability to occur.
Less likely: 3.5-3.7, since it is the farthest interval from the mean, thus has the least probability to occur under the normal distribution model.

b) P(exactly 2) = 0, since it is a line, not an area, this probability = 0

Q17

The volume of a randomly selected bottle of a new type of mineral water is known to have a normal distribution with a mean of 995ml and a standard deviation of 5ml. What is the volume that should be stamped on the bottle so that only 3% of bottles are underweight?

Question 17

Q17

Let x be the volume that should be stamped on the bottle:

$P(X<x) = 0.03$

$P(Z<\frac{x-995}{5})=0.03$

$\frac{x-995}{5}=-1.88$

$x= 985.6$