Tutorial 7

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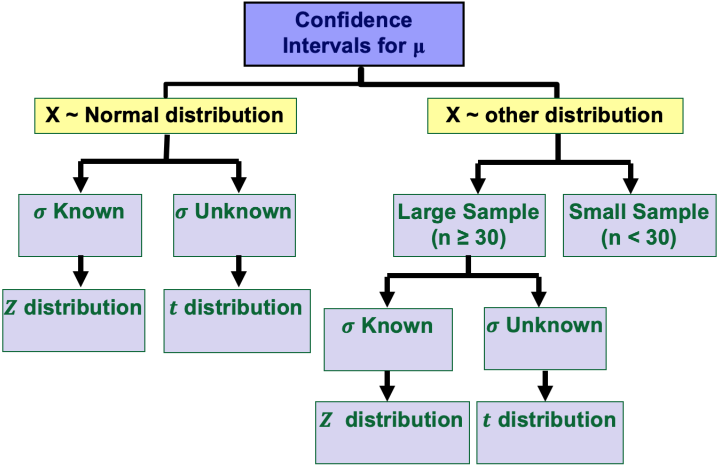
Intended Learning Outcomes

After this tutorial, you may know how to

- utilize Z table and t table
- calculate the C.I(Confidence interval) with respect to a confidence level.
- interpret the confidence interval


Confidence Intervals for μ
X ~ Normal distribution
X ~ other distribution
σ Known
σ Unknown
Large Sample (n ≥ 30)
Small Sample (n < 30)
Z distribution
t distribution
σ Known
σ Unknown
Z distribution
t distribution

Conditions of Utilizing Z table

- Conditions
  - Population standard deviation (σ) is known
  - Population is normally distributed → $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$
  - If population is not normal, but with a large sample ($n \geq 30$), by Central Limit Theorem → $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$

- $100(1-\alpha)\%$ Confidence interval estimate

$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Standard Error, $\sigma_{\bar{X}}$

Sampling Error, E
(Also called margin of error)

Critical Value

Conditions of Utilizing t table

- Conditions
  - Population standard deviation (σ) is unknown
  - Population is normally distributed → $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$
  - If population is not normal, but with a large sample ($n \geq 30$), by Central Limit Theorem → $\bar{X} \sim N(\mu_{\bar{X}}, (\frac{\sigma}{\sqrt{n}})^2)$
- $100(1-\alpha)\%$ Confidence interval estimate

$$\bar{X} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$$

  - With the use of t-distribution with $(n-1)$ degrees of freedom in this context

Question 1

Q1

If $\overline{X}$=120, σ=24 and n=36, construct a 99% confidence interval estimate of the population mean, μ

Q1

Since n = 36 > 30 from unknown population distribution, we may use the Central Limit Theorem to conclude that the sampling distribution of $\overline{X}$ is approximately normal. σ is known, so we use the Z-distribution.

$\overline{X} = 120, \quad \sigma = 24, \quad n = 36,$

$\alpha = 1 - 0.99 = 0.01, \quad Z_{0.01/2} = 2.575$

99% confidence interval for μ

$$= \overline{X} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$= 120 \pm 2.575\left(\frac{24}{\sqrt{36}}\right)$$

$$= [109.7, 130.3]$$

We are 99% confident that the population mean is between 109.7 and 130.33.

Question 2

Q2

A stationery store wants to estimate the mean retail value of greeting cards that it has in its inventory. A random sample of 100 greeting cards indicates a mean value of \$2.65 and a standard deviation of \$0.44.

Assuming a normal distribution, construct a 95% confidence interval estimate of the mean value of all greeting cards in the store's inventory.

Question 2

Q2

Since population of the retail value of greeting cards is normally distributed, we can conclude that the sampling distribution of $\overline{X}$ is also normally distributed.

σ is unknown, so we use the t-distribution.

$\overline{X} = 2.65, \quad s = 0.44, \quad n = 100,$

$\alpha = 1 - 0.95 = 0.05, \quad t_{0.05/2,\,100-1} = 1.9842$

95% confidence interval for μ

$$= \overline{X} \pm t_{0.05/2,\,100-1}\left(\frac{s}{\sqrt{n}}\right)$$

$$= 2.65 \pm 1.9842\left(\frac{0.44}{\sqrt{100}}\right)$$

$= [2.5627, 2.7373]$ in dollar

We are 95% confident that the true population mean value of all greeting cards in the store's inventory is between \$2.5627 and \$2.7373.

Q3

The U.S. Department of Transportation requires tire manufacturers to provide tire performance information on the sidewall of the tire to better inform prospective customers when making purchasing decisions. One very important measure of tire performance is the tread wear index, which indicates the tire's resistance to tread wear compared with a tire graded with a base of 100. This means that a tire with a grade of 200 should last twice as long, on average, as a tire graded with a base of 100. A consumer organization wants to estimate the actual tread wear index of a brand name of tires that claims "graded 200" on the sidewall of the tire. A random sample of n=18 indicates a sample mean tread wear index of 195.3 and a sample standard deviation of 21.4.

a) Assuming that the population of tread wear indexes is normally distributed, construct a 95% confidence interval estimate of the population mean tread wear index for tires produced by this manufacturer under this brand name.
b) Do you think that the consumer organization should accuse the manufacturer of producing tires that do not meet the performance information provided on the sidewall of the tire? Explain.
c) Explain why an observed tread wear index of 210 for a particular tire is not unusual, even though it is outside the confidence interval developed in (a).

Question 3

Q3

a) Since population of tread wear indexes is normally distributed, we can conclude that the sampling distribution of $\overline{X}$ is also normally distributed.

σ is unknown, so we use the t-distribution.

$\overline{X} = 195.3, \quad s = 21.4, \quad n = 18,$

$\alpha = 1 - 0.95 = 0.05, \quad t_{0.05/2, 18-1} = 2.1098$

95% confidence interval for μ

$$= \overline{X} \pm t_{\alpha/2, n-1}\left(\frac{s}{\sqrt{n}}\right)$$

$$= 195.3 \pm 2.1098\left(\frac{21.4}{\sqrt{18}}\right)$$

$$= [184.6581, 205.9419]$$

We are 95% confident that the population mean tread wear index for tires produced by this manufacturer under this brand name is between 184.6581 and 205.9419.

b) No, a grade of 200 is in the interval

c) By comparing this value to the sample mean and sample standard deviation: It is not unusual. A tread-wear index of 210 for a particular tire is only (210-195.3)/21.4 = 0.69 standard deviation above the sample mean of 195.3

Q6

A food inspector, examining 10 bottles of a certain brand of honey, obtained the following percentages of impurities:

23.5	19.8	21.3	22.6	19.4	18.2	24.7	21.9	20.0	21.1

a) What are the mean and standard deviation of this sample?

b) With 95% confidence, what is the sampling error if the inspector used the sample mean to estimate the mean percentage of impurities in this brand of honey?

Q6

a) $$\text{mean} = \frac{23.5+19.8+21.3+22.6+19.4+18.2+24.7+21.9+20.0+21.1}{10} = 21.25$$

$$\text{s.d.} = \sqrt{\frac{(23.5-21.25)^2+\ldots+(21.1-21.25)^2}{10-1}} = 1.9896$$

b) Assume the population distribution is normal

$\because$ σ is unknown, set $\sigma = s$ and n = 10 <30

$\therefore$ use t-distribution

$\alpha = 0.05$

$$\text{Sampling error, E} = t_{\alpha/2,10-1}\frac{s}{\sqrt{n}} = t_{0.05/2,10-1}\frac{1.9896}{\sqrt{10}} = 2.2622\left(\frac{1.9896}{\sqrt{10}}\right) = 1.4233$$

Question 7

Q7
A sample of 12 observations is obtained from an infinite population with normal distribution. Based on the sample data, the 95% confidence interval for the population mean is calculated to be [20, 30]. Form this 95% confidence interval, determine the mean and standard deviation of the sample.

Question 7

Q7

$\because$The population distribution is normal and σ is unknown, use t-distribution.

$\alpha = 0.05$, $t_{0.025,11} = 2.2010$

$$\because \text{CI} = [20,30] = \overline{X} \pm t_{\frac{\alpha}{2},n-1} \frac{s}{\sqrt{n}}$$

$$\therefore \overline{X} - (2.2010)\frac{s}{\sqrt{12}} = 20 \text{ and } \quad \overline{X} + (2.2010)\frac{s}{\sqrt{12}} = 30$$

Thus, sample mean, $\overline{X} = 25$ and sample s.d., s = 7.8694