Tutorial 3

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Intended Learning Outcomes

After this tutorial, you may know how to

- construct a contingency table.
- compute probabilities under different settings: (simple event, joint event) and apply general addition rule.
- understand the term 'mutually exclusive', 'collectively exhaustive' and 'independent'
- calculate joint probability and conditional probability

Recap

- Contingency table

	Ace	Not Ace	Total
Black	2	24	**26**
Red	2	24	**26**
Total	4	48	52

Sample Space

Q1

Each year, ratings are compiled concerning the performance of new cars during the first 90 days of use. Suppose that the cars have been categorized according to whether the car needs warranty-related repair (yes or no) and the country in which the company manufacturing the car is based (United States or not United States). Based on the data collected, the probability that the new car needs warranty repair is 0.04, the probability that the car was manufactured by a U.S.-based company is 0.60, and the probability that the new car needs a warranty repair and was manufactured by a U.S.-based company is 0.025. Construct a **contingency table** to evaluate the probabilities of a warranty-related repair. What is the probability that a new car selected at random

a) needs a warranty repair?
b) needs a warranty repair and was manufactured by a U.S.-based company?
c) needs a warranty repair or was manufactured by a U.S.-based company?
d) needs a warranty repair or was not manufactured by a U.S.-based company?

Question 1

Q1

Needs warranty-related repair	U.S.	Non-U.S.	Total
Yes	0.025	0.015	0.04
No	0.575	0.385	0.96
Total	0.600	0.400	1.00

a) *P*(needs warranty repair) = 0.04

b) *P*(needs warranty repair and manufacturer based in U.S.) = 0.025

c) *P*(needs warranty repair or manufacturer based in U.S.)
=*P*(needs warranty repair) + *P*(manufacturer based in U.S.)
- *P*(needs warranty repair and manufacturer based in U.S.)
= 0.04 + 0.6 – 0.025 = 0.615

d) *P*(needs warranty repair or manufacturer not based in U.S.)
=*P*(needs warranty repair) + *P*(manufacturer not based in U.S.)
- *P*(needs warranty repair and manufacturer not based in U.S.)
= 0.04 + 0.4 - 0.015 = 0.425

Conclusion from Question 1

- Construct a contingency table.
- $P(A \text{ or } B) = P(A) + P(B) - P(A \bigcap B)$
- Conclude probabilities of specific events from the contingency table.

Question 2

Q2

A sample of 500 respondents was selected in a large metropolitan area to study consumer behavior with the following results:

Enjoys Shopping for Clothing	Gender: Male	Gender: Female	Total
Yes	126	234	360
No	104	36	140
Total	230	270	500

a) Suppose the respondent chosen is a female. What is the probability that she does not enjoy shopping for clothing?

b) Suppose the respondent chosen enjoys shopping for clothing. What is the probability that the individual is a male?

c) Are enjoying shopping for clothing and the gender of the individual independent? Explain.

Q2

a) $P(not\ enjoy \mid female) = \dfrac{P(not\ enjoy\ and\ female)}{P(female)} = \dfrac{36/500}{270/500} = 0.1333$

b) $P(male \mid enjoy) = \dfrac{P(male\ and\ enjoy)}{P(enjoy)} = \dfrac{126/500}{360/500} = 0.35$

c) $P(male) = \dfrac{230}{500} = 0.46$

From part (b), $P(male \mid enjoy) = 0.35$

$P(male) \neq P(male \mid enjoy)$

=> enjoy shopping and gender are not independent

Conclusion from Question 2

- Conditional probability calculation.
- $P(A \mid B) = \frac{P(A \bigcap B)}{P(B)}$
- Identify the dependency by whether $P(A \mid B) = P(A)$

Q3

a) An advertising executive is studying television viewing habits of married men and women during prime time hours. On the basis of past viewing records, the executive has determined that during prime time, husbands are watching television 60% of the time. It has also been determined that when the husband is watching television, 40% of the time the wife is also watching. When the husband is not watching television, 30% of the time the wife is watching television.

(i) Find the probability that both the wife and the husband are watching television in prime time.

(ii) Find the probability that the wife is watching television in prime time.

Q3

a)

(i) Let H: husband watch TV, $\overline{H}$: husband do not watch TV

W: wife watch TV, $\overline{W}$ wife do not watch TV

$\because$ P(H) = 0.6 , P(W | H) = 0.4,

$$P(W/H) = \frac{P(W \cap H)}{P(H)}$$

$\therefore$ P(W $\cap$ H) = P(W | H)P(H) = (0.4)(0.6) = 0.24

(ii) $\because$ P(W | $\overline{H}$) = 0.3 , P($\overline{H}$) = 1-0.6 =0.4

P(W $\cap$ $\overline{H}$) = P(W | $\overline{H}$) P($\overline{H}$) = (0.3)(0.4) = 0.12

$\therefore$ P(W) = P(W $\cap$ H) + P(W $\cap$ $\overline{H}$) = 0.24 + 0.12 = 0.36

Conclusion from Question 3(a)

- Conditional probability.
- $P(A \mid B) = \frac{P(A \bigcap B)}{P(B)}$
- $P(A \bigcap B) + P(A \bigcap B') = P(A)$

Question 4

Q4

If there are 10 multiple-choice questions on an exam, each having three possible answers, how many different sequences of answers are there?

Question 4

- 10 tracks with 3 possible answers. Then 3^{10} is the answer.

Question 5

Q5

You would like to "build-your-own-burger" at a fast-food restaurant. There are five different breads, seven different chesses, four different cold toppings, and five different sauces on the menu. If you want to include one choice from each of these ingredient categories, how many different burgers can you build?

Question 5

- $5 \times 7 \times 4 \times 5 = 700$

Question 6

Four members of a group of 10 people are to be selected to a team. How many ways are there to select these four members?

Question 5

$$_{10}C_4 = \frac{10!}{(10-4)!4!}$$