Quantum Annealing and QAOA: connection and contrast

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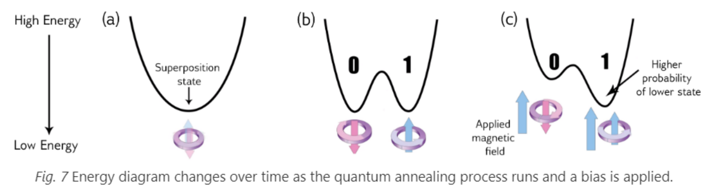
Introduction to Quantum Annealing

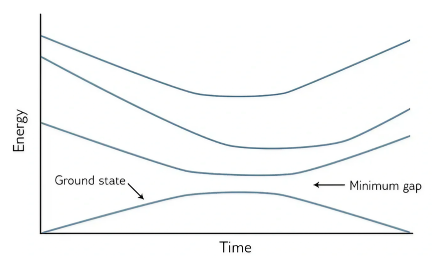
- A heuristic algorithm - applied to solve Optimization and Sampling problems.
- Quantum Mechanical properties to be followed:
  - Quantum Tunneling, Entanglement, Superposition
- Operates on the principles of Adiabatic Quantum Computation
  - Features of AQC
    - ∗ Continuous Time process
    - ∗ Analog in nature
    - ∗ Driven by the applications of time dependent external Magnetic field.



- The problem Hamiltonian can be represented as:

$$\mathcal{H}_{ising} = \underbrace{-\frac{A(s)}{2}\left(\sum_i \hat{\sigma}_x^{(i)}\right)}_{\text{Initial Hamiltonian}} + \underbrace{\frac{B(s)}{2}\left(\sum_i h_i \hat{\sigma}_z^{(i)} + \sum_{i>j} J_{i,j}\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)}\right)}_{\text{Final Hamiltonian}}$$

- Annealing in low energy states:



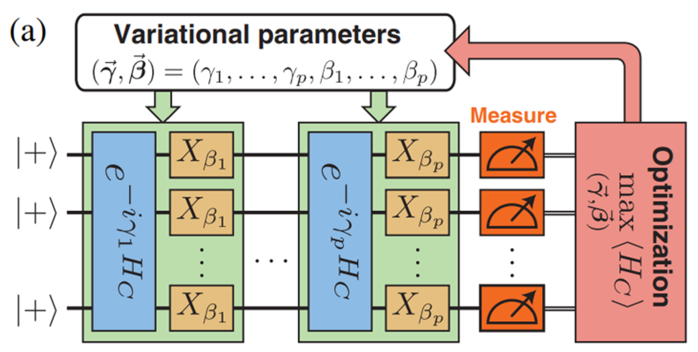
Eigenspectrum

Introduction to QAOA

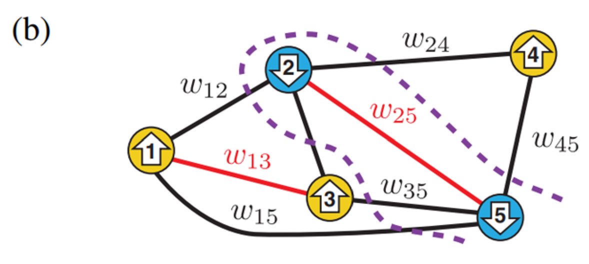
- MaxCut and other NP-complete problems can be mapped to Ising model.
- Efficient classical verification.

Promising in NISQ:

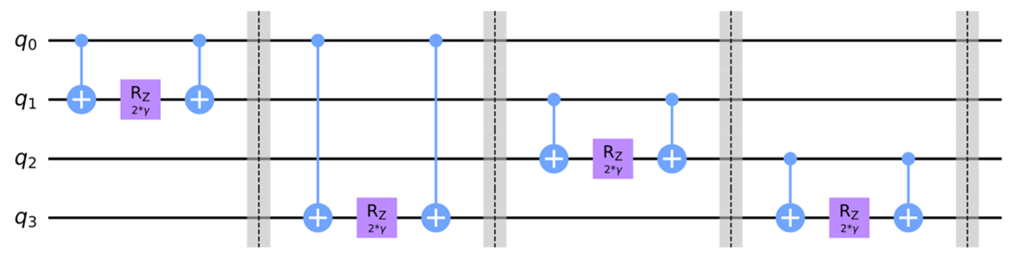
1. The depth of the algorithm is scalable
2. Somewhat resistant to noise
3. Flexibility of the ansatz



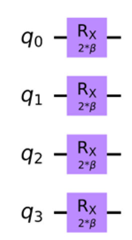
QAOA Procedure



MaxCut problem



$U(H_C, \gamma) = e^{-i\gamma H_C}$



$U(H_M, \beta) = e^{-i\beta H_M}$

Construct annealing path from QAOA[2]

- Schrödinger equation and solutions are:

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle \qquad |\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

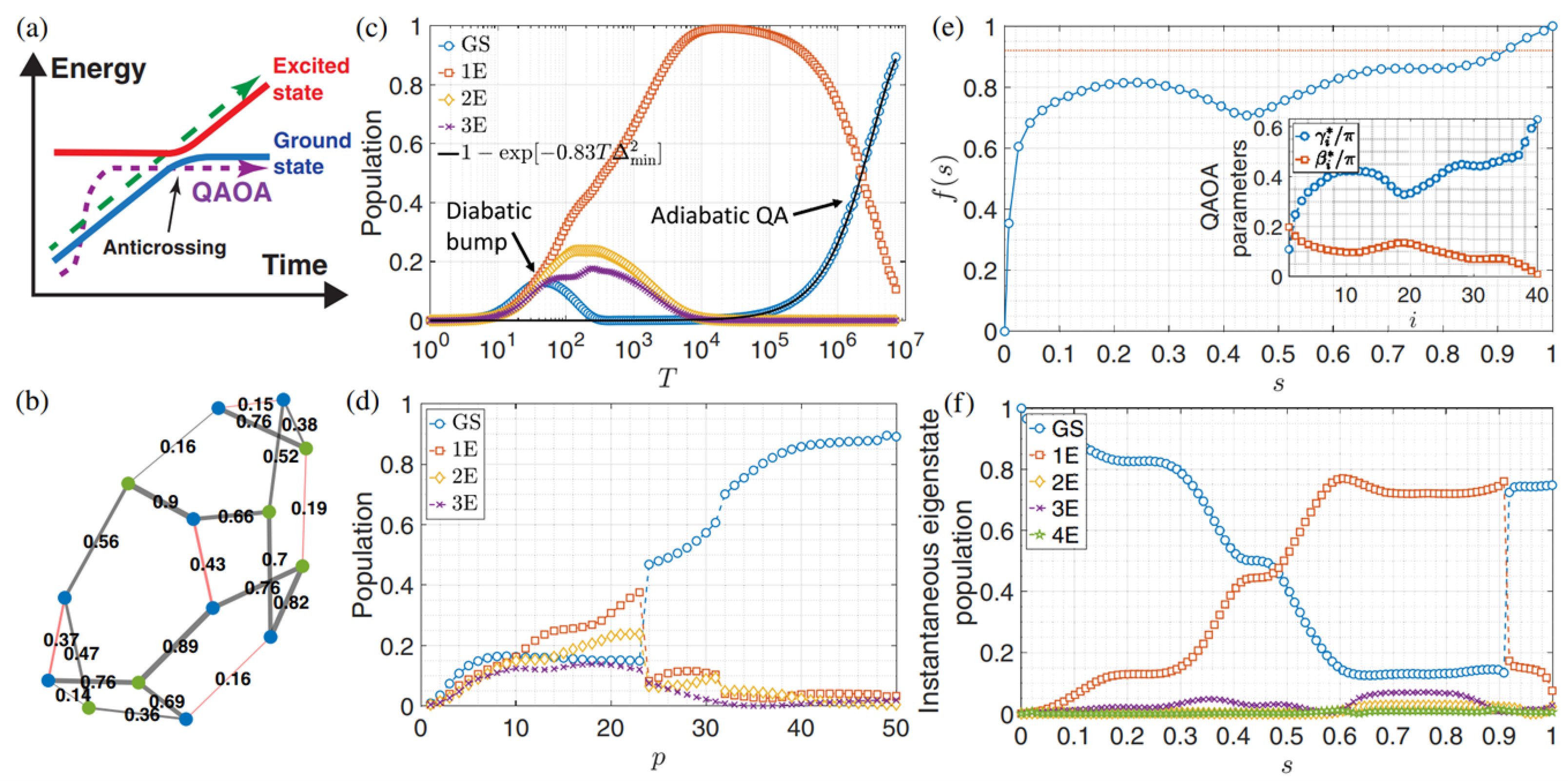
- Compare with the operators in QAOA, beta and gamma can be interpreted as time.
- An annealing path can be constructed:

$$H_{\text{QAOA}}(t) = -\{f(t)H_C + [1-f(t)]H_B\},$$

$$f\left(t_i = \sum_{j=1}^{i}(|\gamma_j^*| + |\beta_j^*|) - \frac{1}{2}(|\gamma_i^*| + |\beta_i^*|)\right) = \frac{\gamma_i^*}{|\gamma_i^*| + |\beta_i^*|},$$

- QA and QAOA are run on MaxCut problem in (b), and the problem Hamiltonian is:

$$H_C = \sum_{\langle i,j\rangle} \frac{w_{ij}}{2}(1 - \sigma_i^z\sigma_j^z)$$



Conclusion

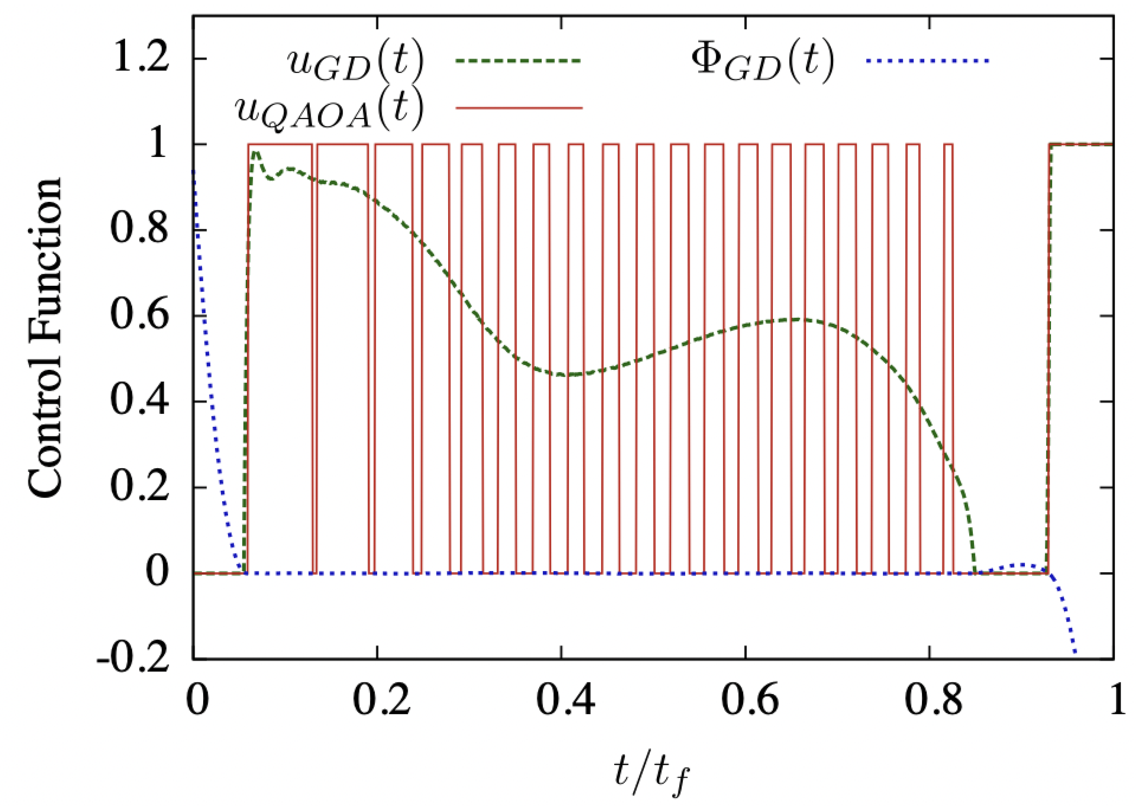
- QAOA itself is inspired by QA and it can be understood as a trotterized version of QA. The performance guarantee of adiabatic quantum annealing is given by Quantum Adiabatic Theorem. As p $\rightarrow \infty$, QAOA parameters corresponding to a trotterized adiabatic quantum computation can achieve a perfect approximation ratio. [3]
- If we construct an annealing path from optimized parameters in QAOA, the behavior of the QAOA annealing path is very different from QA.
- QAOA and QA can be understand as two cases of optimizing u(t). By optimizing energy with respect to u(t), the optimal protocol is a mixture of QAOA and QA. (QAOA-like at start and end, annealing like in middle.)
- In the finite p regime it is not fully understood whether or not the optimal parameters for QAOA should appear adiabatic.
- Some improved versions of QAOA are inspired by shortcuts to adiabaticity (STA). [4-6]

Optimal Protocols in QA/QAOA[1]

- Control Problem:

$$H(t) = u(t)B + (1-u(t))C$$
$$J = (t_f)|C|x(t_f)\rangle$$
$$\hat{B} = -\sum_{i=1}^{N}\hat{\sigma}_i^x, \quad \hat{C} = \sum_{ij} J_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z.$$

- Time Constraints: fixed parameter
  - ”Hard constraint”
  - ”Soft constraint”
- Numerical results:



Optimal control functions found through either gradient descent or constrained-time QAOA for a random instance of the MaxCut problem.

References

[1] L T. Brady, C L. Baldwin, A. Bapat, Y. Kharkov & A V. Gorshkov. Optimal Protocols in Quantum Annealing and Quantum Approximate Optimization Algorithm problems. Physical Review Letters 126, 070505 (2021).

[2] L. Zhou, S. Wang, S. Choi, H. Pichler & M. Lukin. Quantum Approximate Optimization Algorithm: performance, Mechanism, and Implementation on Near-Term Devices. Physical Review X 10, 021067 (2020).

[3] E. Farhi, J. Goldstone & S. Gutmann. A Quantum Approximate Optimization Algorithm. arXiv:1411.4028v1 (2014).

[4] P. Chandarana, N. N. Hegade, K. Paul, F. Albarran-Arriagada, E. Solano, A. del Campo, & X. Chen. Digitized-counterdibatic Quantum Approximate Optimization Algorithm. Physical Review Research 4, 013141 (2022).

[5] L. Zhu, H. L. Tang, G. S. Barron, F. A. Calderon-Vargas, N. J. Mayhall, E. Barnes & S. Economou. Adaptive Quantum Approximate Optimization Algorithm for solving Combinatorial Problems on a Quantum Computer. Physical Review Research 4, 033029 (2022).

[6] D. Guery-Odelin, A. Ruschhaupt, A. Kiely, E. Torrontegui, S. martinez-Garaot & J. G. Muga. Shortcuts to Adiabaticity: Concepts, Methods, and Applications. Reviews of Modern Physics 91, 045001 (2019).