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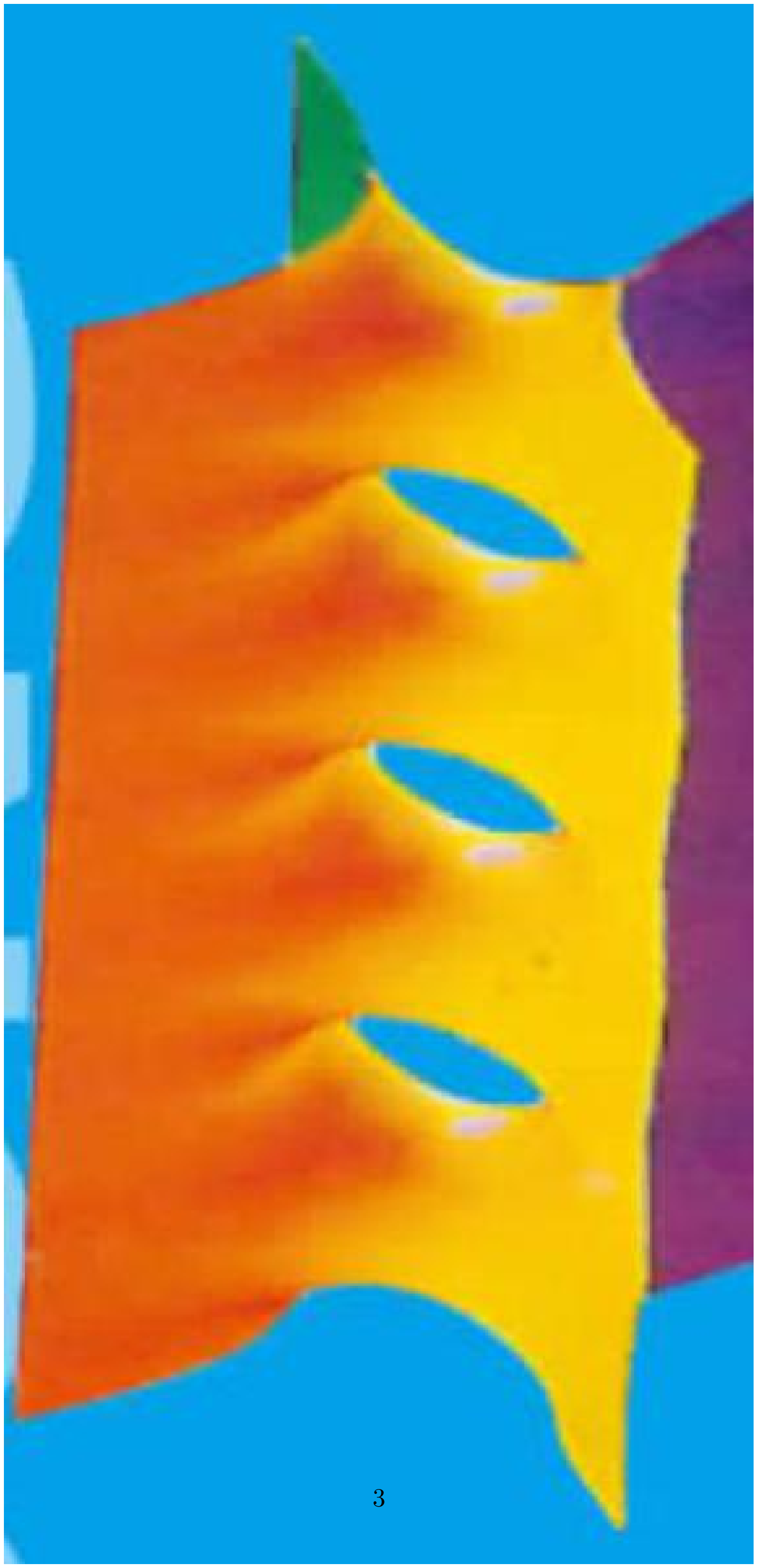

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第二版

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A. ⊠⊠⊠⊠⊠⊠⊠⊠95 ⊠-so easy!

B. ⊠⊠⊠⊠⊠⊠⊠80 ⊠⊠so difficult!

C. ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠Attitude first!

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☒☒☒☒☒☒ r, ☒☒☒☒☒ $C = 2\pi r$, ☒☒ $S = \pi r^2$.
☒☒☒☒☒☒☒ r, ☒☒☒☒☒☒☒ n°, ☒☒☒☒☒ $l = \frac{n\pi r}{180}$, ☒☒☒☒ $C = \frac{n}{180}\pi r + 2r$ ☒☒ ☒ $S = \frac{n\pi r^2}{360}$.

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1 - 1,弓形 ADB 的面积 $S = S_{扇形ADB} + S_{\triangle AOB}$,弓形 ACB ☒ ☒☒ $S = S_{☒☒ACB} - S_{\triangle AOB}$ ☒☒☒☒☒☒ $C =$ ☒☒+ ☒☒☒

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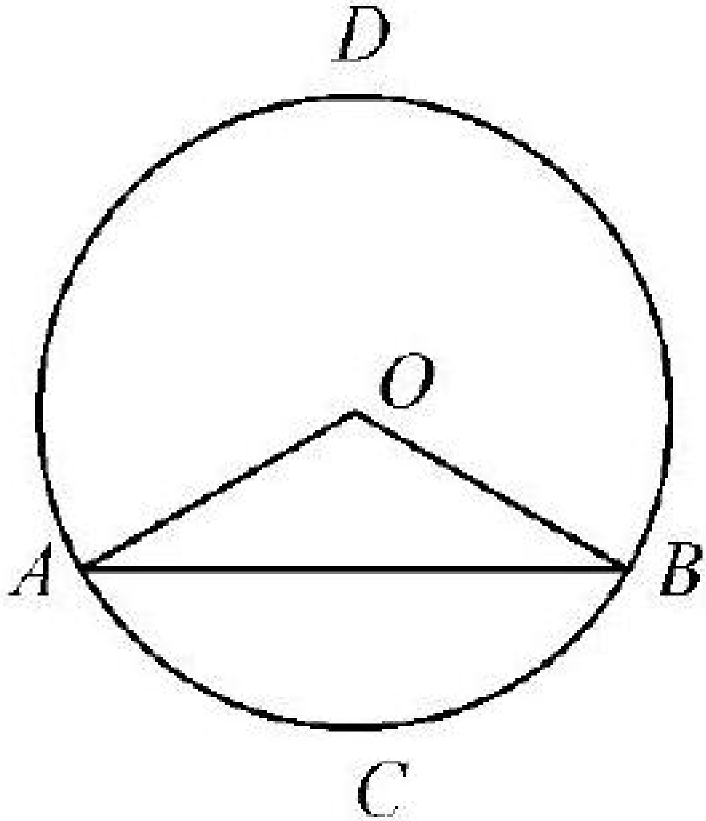

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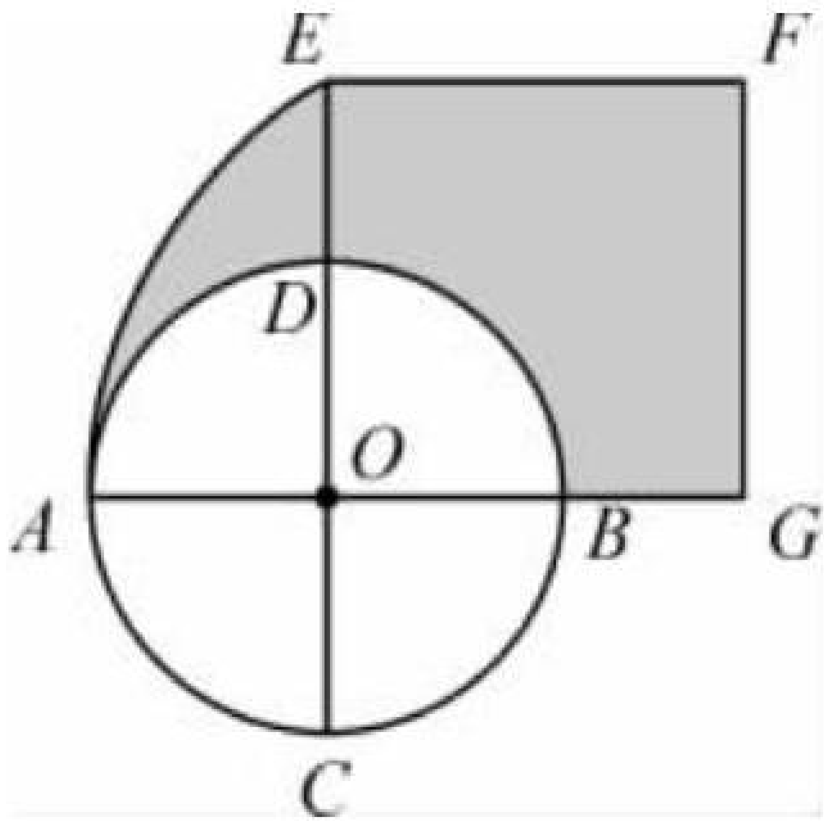
☒1 ☒☒1-2, AB CD ☒ $\odot O$ ☒☒☒☒☒☒☒☒☒☒☒☒☒ $AB = 2$ ☒☒☒ B ☒☒☒☒ BA ☒☒☒☒☒ AE ☒ CD ☒☒☒☒☒ E, ☒☒☒☒ $EFGO$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒☒☒☒☒☒☒☒ AED ☒☒☒ $DEFGB$ ☒☒☒☒☒ AED ☒☒☒☒☒☒☒☒☒ ABE ☒☒☒☒☒☒☒ AOD ☒☒☒☒ $\triangle OBE$ ☒☒☒☒☒☒

☒☒☒☒ $BA = BE$ ☒☒ EO ☒☒☒☒ AB ☒☒



⊠1-2

$\triangle ABE$ ⊠⊠⊠⊠⊠, $\angle ABE = 60^\circ, OE = \frac{\sqrt{3}}{2}AB = \sqrt{3}$, ⊠⊠

$$S_{\boxtimes\text{ADE}} = S_{\boxtimes\boxtimes ABE} - S_{\boxtimes\boxtimes AOD} - S_{\triangle OBE}$$

1 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠ $\triangle OFM \cong \triangle ION$ ⊠⊠⊠ $S_{QNMF} = S_{IQO}$ ⊠⊠⊠⊠ $IHGFQ$ ⊠⊠⊠⊠⊠⊠⊠ $S_{\boxtimes\boxtimes 1OF} = S_{FMNI}$ ⊠

⊠ $\triangle GJO \cong \triangle OKH$ ⊠⊠⊠ $S_{GPKJ} = S_{OHP}$ ⊠⊠⊠ GHP ⊠⊠⊠⊠⊠⊠⊠ $S_{GHKJ} = S_{\boxtimes\boxtimes\text{HOG}}$.

⊠⊠⊠⊠⊠FMNI ⊠⊠

$$\begin{aligned} S_{\boxtimes} &= S_{\boxtimes\boxtimes\text{IOF}} - S_{\boxtimes\boxtimes HOG} \\ &= S_{\boxtimes\boxtimes\text{IOG}} \\ &= \frac{36}{360}\pi r^2 \\ &= 40\pi. \end{aligned}$$

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AIFD$ ⊠⊠$S_{⊠} = 80\pi$.
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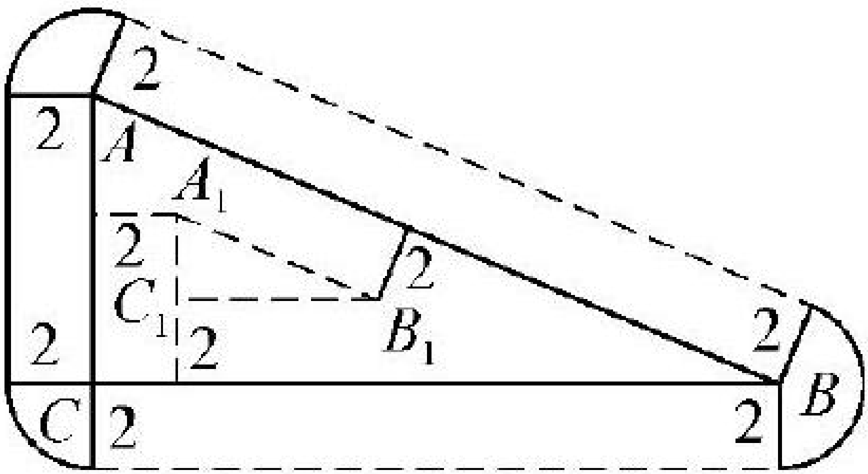
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠1 - 5 ⊠ $\triangle A_1B_1C_1$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠
$\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠2 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠2 ⊠⊠⊠⊠⊠⊠
$\triangle ACB$ ⊠⊠⊠⊠ $S_1 = \pi \times 2^2 + (50 + 120 + 130⊠\times 2 = 600 + 4\pi$.



⊠1-5

⊠⊠, ⊠⊠ $\triangle A_1C_1B_1 \backsim \triangle ACB$, ⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠2 . ⊠⊠ $\triangle ACB$ ⊠⊠⊠⊠
⊠⊠⊠ $\frac{50+120-130}{2} = 20$ ⊠⊠ $\triangle A_1B_1C_1$ ⊠⊠⊠⊠⊠⊠⊠18 ⊠

⊠⊠

$$\frac{S_{A_1B_1C_1}}{S_{ABC}} = \frac{81}{100}$$

⊠⊠

$$\begin{aligned} S_{A_1B_1C_1} &= \frac{81}{100} \times S_{ABC} \\ &= \frac{81}{100} \times \frac{1}{2} \times 50 \times 120 \\ &= 2430 \end{aligned}$$

⊠⊠⊠⊠ $\triangle A_1B_1C_1$ ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠ S_2 ⊠570 .
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $S_1 + S_2 = 600 + 4\pi + 570 = 1170 + 4\pi$.

1 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠5 ⊠⊠⊠⊠1 ⊠⊠⊠⊠⊠ $ABCDE$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠1 ⊠⊠, ⊠⊠⊠⊠⊠⊠
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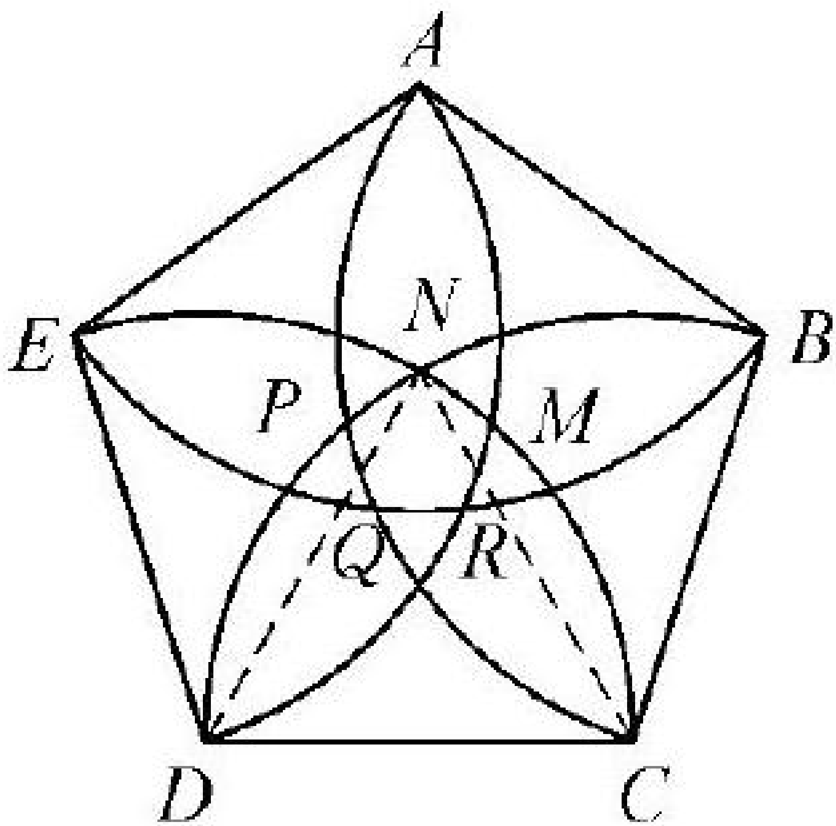

☒1-6

☒☒1 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒" ☒☒☒☒☒" NBC ☒☒☒☒☒☒

☒☒☒☒☒☒☒☒☒ NBC ☒☒☒ NDC ☒☒☒☒☒☒☒☒ $\triangle DNC$ ☒☒☒ NCB ☒☒☒☒ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

$$\begin{aligned} S &= 5\left(S_{\triangle DNC} + S_{☒☒NCB} - S_{☒☒NDC}\right) \\ &= 5\left(\frac{\sqrt{3}}{4} + \frac{108-60}{360}\pi - \frac{60}{360}\pi\right) \\ &= \frac{5\sqrt{3}}{4} - \frac{\pi}{6}. \end{aligned}$$

☒6 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒1 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒☒☒☒☒☒0.34 ☒

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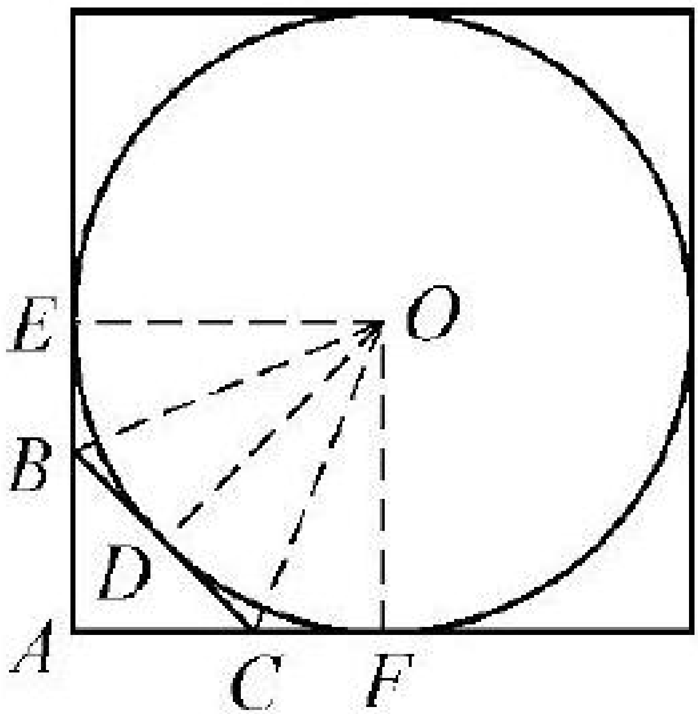
☒☒1-7, $\odot O$ ☒☒☒☒☒☒☒☒☒ E F ☒☒ $\triangle ABC$ ☒☒☒☒ D, ☒☒ OE OB OD OC OF ☒☒☒☒☒ $OEAF$ ☒☒☒☒☒☒

☒ $AB = x, AC = y$, ☒ $BD = EB = 1 - x, CD = CF = 1 - y$, ☒☒Rt $\triangle ABC$ ☒☒

$$(1 - x + 1 - y)^2 = x^2 + y^2,$$



☒1-7

☒☒☒☒☒

$$2 + xy = 2(x + y)$$

☒☒

$$2 + xy = 2(x + y) \geqslant 4\sqrt{xy},$$

☒☒

$$\sqrt{xy} \geqslant 2 + \sqrt{2}, \text{☒} \sqrt{xy} \leqslant 2 - \sqrt{2},$$

☒ $0 < x\ y < 1$, ☒☒ $\sqrt{xy} < 1$.

⊠⊠ $\sqrt{xy} \leqslant 2-\sqrt{2}, S_{\triangle BBC} = \frac{1}{2}xy \leqslant (\sqrt{2}-1)^2$.
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠" ⊠"⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $S_{⊠} - 3S_{\triangle ABC}$, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ S, ⊠

$$
\begin{aligned}
S &\geqslant S_{⊠} - 3S_{\triangle ABC} - S_{⊠} \\
&\geqslant 2^2 - 3\times(\sqrt{2}-1)^2 - \pi\times 1^2 \\
&= 6\sqrt{2} - 5 - \pi > 0.34.
\end{aligned}
$$

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1

1. Rt $\triangle ABC$ ⊠, $AC = BC, \angle C = 90°$, ⊠ D ⊠⊠ AB ⊠, ⊠ A ⊠⊠⊠, AD ⊠⊠⊠⊠⊠⊠ BC ⊠⊠ E, ⊠ AC ⊠⊠⊠⊠⊠ F. ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AD : DB$ ⊠
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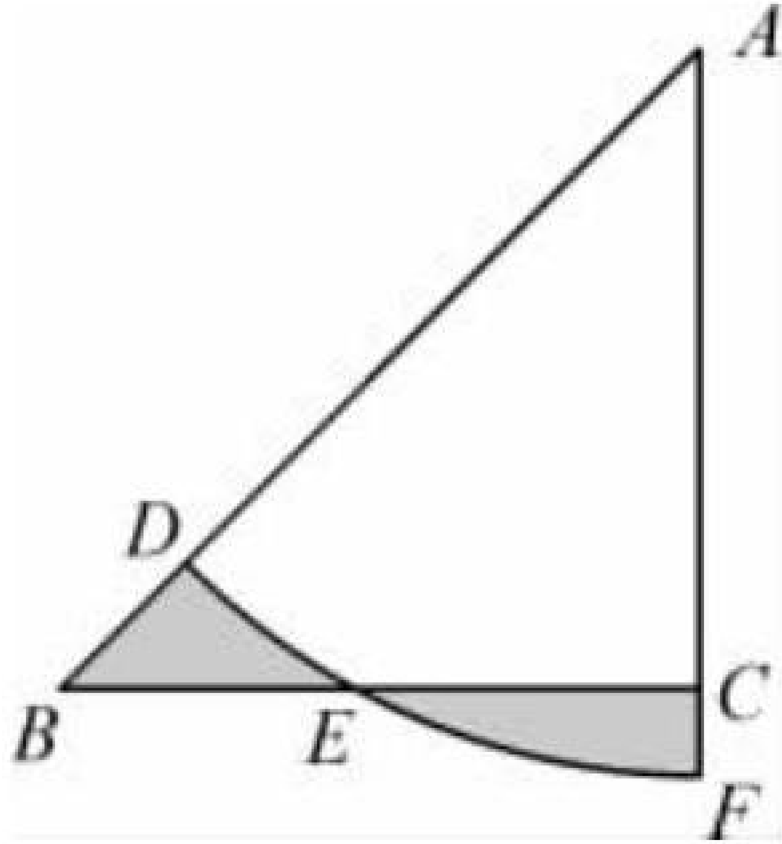

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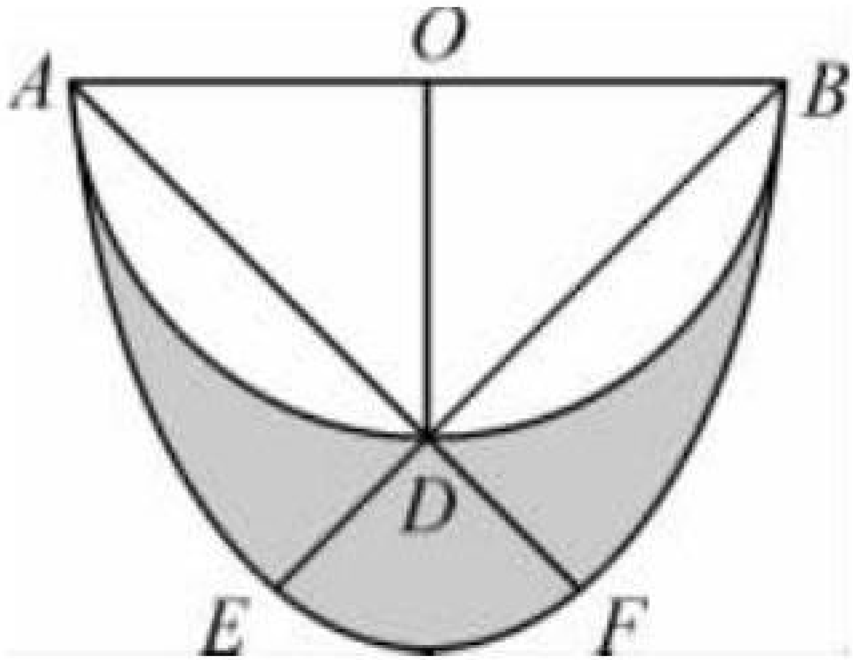

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3 . AB ⊠⊠⊠ O ⊠⊠⊠⊠⊠ $OD \perp AB$ ⊠⊠⊠ O ⊠⊠ D ⊠⊠⊠⊠⊠ A B ⊠⊠⊠⊠ AB ⊠⊠⊠⊠⊠⊠ AD BD ⊠⊠⊠⊠ F E ⊠⊠⊠⊠ D ⊠⊠⊠⊠DE ⊠⊠⊠⊠⊠⊠⊠⊠ E F ⊠⊠ $AB = 2$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

4 ⊠⊠⊠⊠⊠ ACB ⊠, $AC = CB = 2, \angle C = 90°$ ⊠⊠ $\triangle ACB$ ⊠ C ⊠⊠⊠⊠ $90°$ ⊠⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠⊠

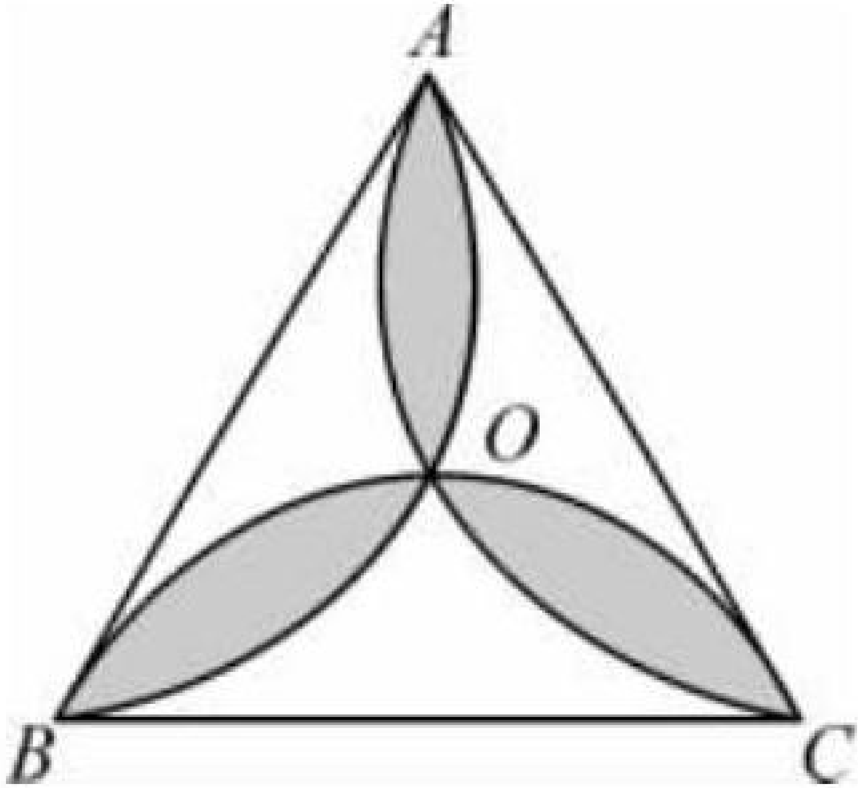
5 ⊠⊠⊠⊠⊠⊠1 cm ⊠⊠⊠⊠⊠⊠ n ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ n ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

1 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

6 ⊠⊠⊠⊠⊠⊠⊠⊠ a, ⊠⊠⊠⊠⊠⊠⊠⊠⊠ O ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $S_{⊠⊠}$ ⊠

7 ⊠⊠⊠⊠⊠⊠⊠⊠ $2a$, ⊠⊠⊠⊠⊠⊠⊠⊠$\sqrt{2}a$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

8$\triangle ABC$ ⊠⊠⊠⊠ $AB = c, BC = a, CA = b$ ⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ AB BC CA ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠



(☒6 ☒)

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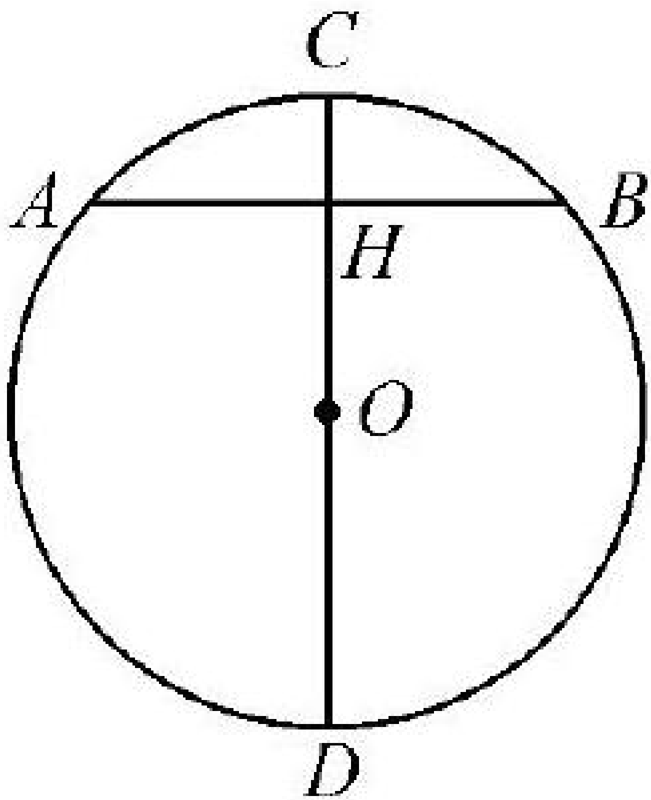
☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒. ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒. ☒☒☒☒☒ ☒☒☒☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒. ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒☒☒☒☒☒☒☒

☒☒☒☒: ☒☒☒☒☒☒☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒☒☒☒.

☒☒2-1 ☒☒, ☒☒ CD ☒☒☒☒ AB ☒☒ H, ☒☒☒☒☒☒: $AH = HB; AC = CB; AD = DB$ ☒

☒☒☒☒☒☒☒☒☒☒☒☒☒☒(1) ☒☒☒CD☒☒(2) ☒☒☒ AB, (3) ☒☒☒☒ AB, (4) ☒☒☒ ACB, ☒5☒☒☒☒ ADB ☒☒☒ (1)☒(2) # (3), (4), (5) ☒, ☒☒☒☒☒☒☒☒☒☒ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒☒ CD ☒☒☒ AB ☒☒ H ☒, ☒☒ O ☒☒☒ H ☒



⊠2-1

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\triangle OAH$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

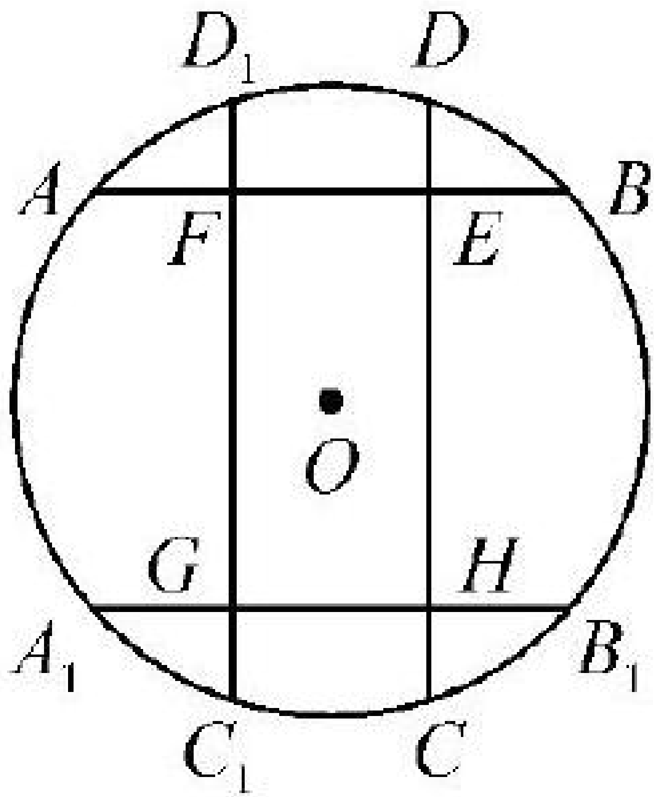
⊠⊠⊠ (⊠⊠⊠) ⊠, ⊠⊠⊠ $\Leftrightarrow$ ⊠⊠⊠⊠⊠⊠

⊠⊠⊠ (⊠⊠⊠) ⊠, ⊠⊠⊠ $\Leftrightarrow$ ⊠⊠⊠⊠⊠⊠

⊠1 ⊠⊠2-2 ⊠⊠, AB ⊠ CD ⊠ $\odot O$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ O ⊠⊠⊠⊠⊠⊠⊠⊠⊠ AB ⊠⊠⊠⊠⊠ a ⊠⊠ CD ⊠⊠⊠⊠⊠ b ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

解　如图 2－2 所示，作弦 $A_1B_1 \mathrel{\underline{\mathbin{/\!/}}}$ 弦 AB，弦 $C_1D_1 \mathrel{\underline{\mathbin{/\!/}}}$ ⊠ CD ⊠⊠ $\odot O$ ⊠⊠⊠⊠9 ⊠⊠⊠⊠⊠⊠" $\triangle$ " ⊠⊠⊠⊠⊠



⊠2-2

" " ⊠⊠, ⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $EFGH$ ⊠⊠⊠.
⊠⊠⊠⊠⊠⊠⊠ $EF = 2b, FG = 2a$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠

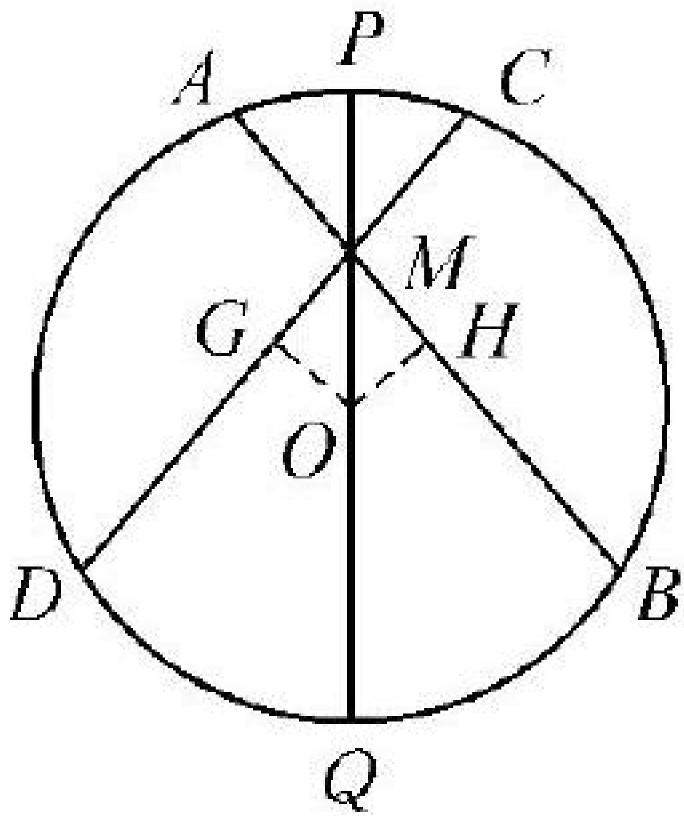
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $4ab$ ⊠

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠2 ⊠⊠2-3 ⊠⊠⊠⊠ $\odot O$ ⊠⊠⊠ PQ ⊠⊠⊠⊠ M ⊠⊠ M ⊠⊠⊠⊠ AB CD ⊠⊠ $\angle PMA = \angle PMC$ ⊠⊠⊠⊠$MD = MB$.

⊠⊠⊠⊠⊠⊠⊠ O ⊠⊠ AB CD ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\angle PMA = \angle PMC$ ⊠⊠⊠⊠⊠ $\triangle OHM$ ⊠ $\triangle OGM$ ⊠.

⊠⊠ $OG \perp CD$ ⊠⊠ $G, OH \perp AB$ ⊠⊠ H, ⊠⊠ $\triangle OGM$ ⊠ $\triangle OHM$ ⊠,



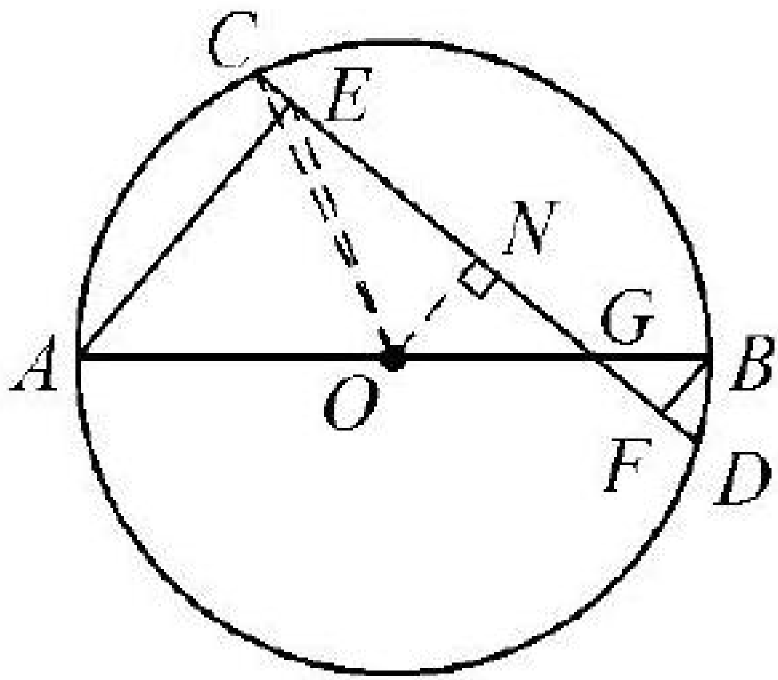
⊠2-3

$\angle OGM = \angle OHM, \angle OMG = \angle PMC = \angle PMA = \angle OMH$, $OM = OM$ ⊠⊠⊠ $\triangle OGM \cong \triangle OHM$ ⊠⊠ $GM = HM, OG = OH$ ⊠⊠⊠ $GD =$ HB.

⊠⊠ $MD = MG + GD = MH + HB = MB.$

⊠3 ⊠⊠2-4 ⊠⊠⊠$\odot O$ ⊠⊠⊠ AB ⊠ 20 cm, G ⊠⊠⊠ AB ⊠⊠⊠, CD ⊠⊠ G ⊠⊠⊠⊠, $CD = 16$ cm, ⊠ A B ⊠⊠⊠ $AE \perp CD$ ⊠⊠ $E, BF \perp CD$ ⊠⊠ F, ⊠ AE ⊠ BF ⊠⊠⊠⊠⊠⊠

⊠⊠⊠ $ON \perp CD$ ⊠⊠ N ⊠⊠⊠ CO ⊠⊠⊠⊠⊠⊠Rt $\triangle CON$ ⊠⊠⊠ AB CD ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\triangle GON$ $\triangle GAE, \triangle GBF$ ⊠⊠⊠⊠⊠⊠ AE BF ⊠⊠⊠⊠⊠⊠



☒2-4

☒☒ $ON \perp CD$ ☒☒ N, ☒☒ OC, ☒

$ON = \sqrt{OC^2 - CN^2} = \sqrt{\left(\frac{1}{2}AB\right)^2 - \left(\frac{1}{2}CD\right)^2} = 6(\text{ cm})$,

☒☒

$$\triangle GON \backsim \triangle GAE \backsim \triangle GBF,$$

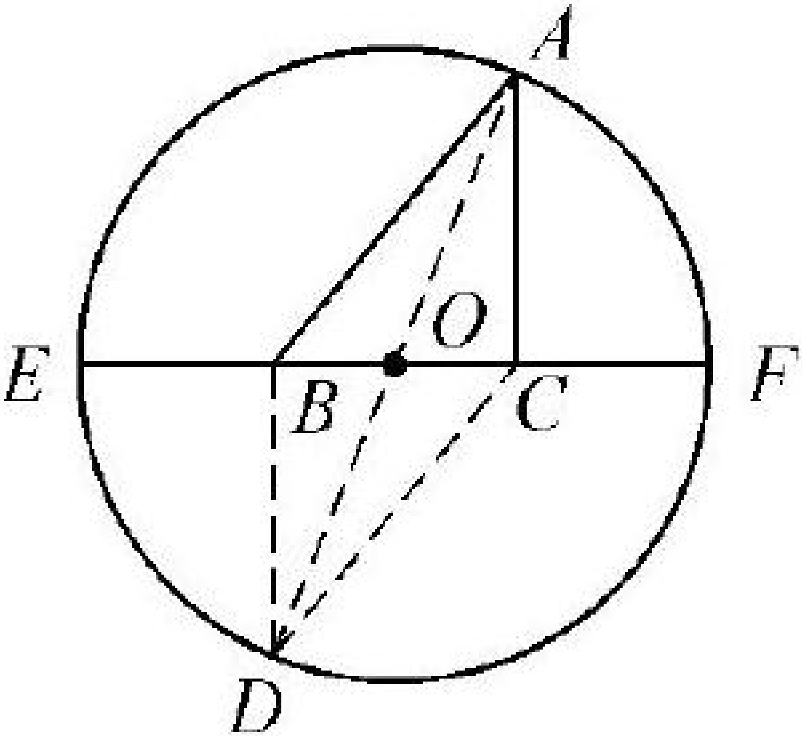
☒☒ $\frac{ON}{AE} = \frac{OG}{GA}$, $\frac{ON}{BF} = \frac{OG}{BG}$, ☒☒

$$\begin{aligned} AE - BF &= \frac{ON}{OG} \cdot GA - \frac{ON}{OG} \cdot BG \\ &= \frac{ON}{OG}[OA + OG - (OB - OG)] \\ &= \frac{ON}{OG} \times 2OG = 2ON = 12(\text{ cm}) \end{aligned}$$

☒4 ☒☒2-5 ☒☒, EF ☒ $\odot O$ ☒☒☒, B C ☒☒☒ EF ☒☒ $EB = BC = CF$, ☒ A ☒ $\odot O$ ☒. ☒☒: $AB + AC \leqslant \frac{\sqrt{10}}{3}EF$.

☒☒☒☒☒☒ O ☒ BC ☒☒☒☒☒☒☒☒☒☒☒, ☒☒ AO ☒ $\odot O$ ☒☒ D, ☒☒☒☒ $ABDC$ ☒☒☒☒☒☒, ☒☒☒ AB AC ☒☒☒☒☒☒☒☒.

☒☒☒ AO ☒ $\odot O$ ☒☒ D ☒☒☒ BD CD ☒☒☒☒ O ☒ BC ☒☒, ☒☒☒☒☒ $ABDC$ ☒☒☒☒☒☒,



⊠2-5

⊠⊠

$$2\left(AB^2+AC^2\right)=AD^2+BC^2=EF^2+\left(\frac{1}{3}EF\right)^2=\frac{10}{9}EF^2$$

⊠

$$\begin{aligned}&2\left(AB^2+AC^2\right)-(AB+AC)^2\\=&(AB-AC)^2\geqslant 0\end{aligned}$$

⊠⊠

$$\frac{10}{9}EF^2=2\left(AB^2+AC^2\right)\geqslant(AB+AC)^2$$

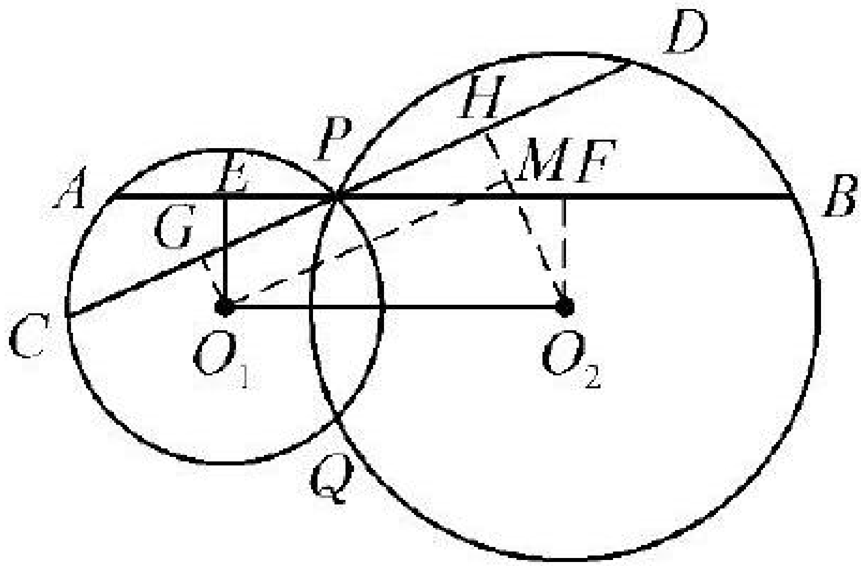
⊠

$$AB+AC\leqslant\frac{\sqrt{10}}{3}EF$$

⊠⊠⊠⊠⊠⊠⊠, ⊠⊠ AO, ⊠⊠⊠⊠⊠ AB AC ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠5 ⊠⊠2-6 ⊠⊠⊠⊠⊠ $\odot O_1$ ⊠ $\odot O_2$ ⊠⊠ P Q ⊠⊠⊠⊠⊠ P ⊠⊠⊠ APB ⊠ $\odot O_1$ ⊠⊠ A, ⊠ $\odot O_2$ ⊠⊠ B, ⊠ $AB//O_1O_2$; ⊠⊠ P ⊠⊠⊠⊠ CPD ⊠ $\odot O_1$ ⊠⊠ C, ⊠ $\odot O_2$ ⊠⊠ D. ⊠⊠⊠ AB ⊠ CD ⊠⊠⊠.

⊠⊠⊠ $O_1E \perp AB$ ⊠ $E, O_2F \perp AB$ ⊠



⊠2-6

F, ⊠ $O_1O_2 = \frac{1}{2}AB$, ⊠⊠ AB ⊠⊠⊠ O_1O_2 ⊠⊠⊠⊠⊠. ⊠⊠⊠, ⊠ $O_1G \perp CD$ ⊠ $G, O_2H \perp CD$ ⊠ H, ⊠⊠ $O_1M \perp O_2H$ ⊠ M, ⊠ $O_1M = \frac{1}{2}CD$. ⊠⊠⊠ AB CD ⊠⊠⊠⊠⊠⊠Rt $\triangle MO_1O_2$ ⊠, ⊠⊠⊠⊠.

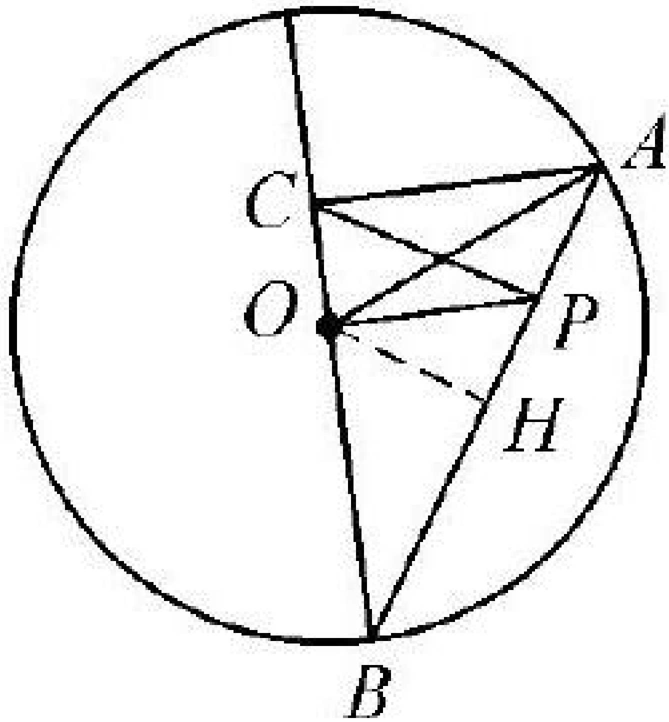
⊠⊠ $O_1E \perp AB$ ⊠ $E, O_2F \perp AB$ ⊠ F, ⊠ $O_1O_2 = \frac{1}{2}AB$, ⊠ $O_1G \perp CD$ ⊠ $G, O_2H \perp CD$ ⊠ H, ⊠ $O_1M \perp O_2H$ ⊠ M, ⊠ $O_1M = \frac{1}{2}CD$. ⊠Rt $\triangle O_1O_2M$ ⊠, $O_1M < O_1O_2$, ⊠⊠ $\frac{1}{2}CD < \frac{1}{2}AB$, ⊠ $CD < AB$.

⊠6 ⊠⊠2-7⊠⊠⊠⊠ $\odot O$ ⊠⊠⊠⊠⊠⊠⊠⊠ OA ⊠OB, ⊠⊠ A ⊠ $AC \perp OB$ ⊠⊠ C, ⊠⊠ C ⊠ $CP \perp AB$ ⊠⊠ P, ⊠⊠ OP ⊠⊠⊠⊠$OP^2 + CP^2 = R^2$ ⊠⊠⊠ R ⊠ $\odot O$ ⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $OH \perp AB$ ⊠⊠ H ⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠ $OH \perp AB$ ⊠⊠ H ⊠⊠

$$\begin{aligned} OP^2 &= OH^2 + HP^2 \\ &= OB^2 - HB^2 + HP^2 \\ &= R^2 - \frac{1}{4}AB^2 + HP^2 \end{aligned}$$



⊠2-7

⊠Rt $\triangle ACB$ ⊠, $CP \perp AB$ ⊠⊠ P, ⊠ $AC^2 = AP \cdot AB$ (⊠⊠⊠⊠), ⊠⊠,

(2)

$= AC^2 - AP^2$ $= AP \cdot AB - A$

(1) +(2) ⊠:

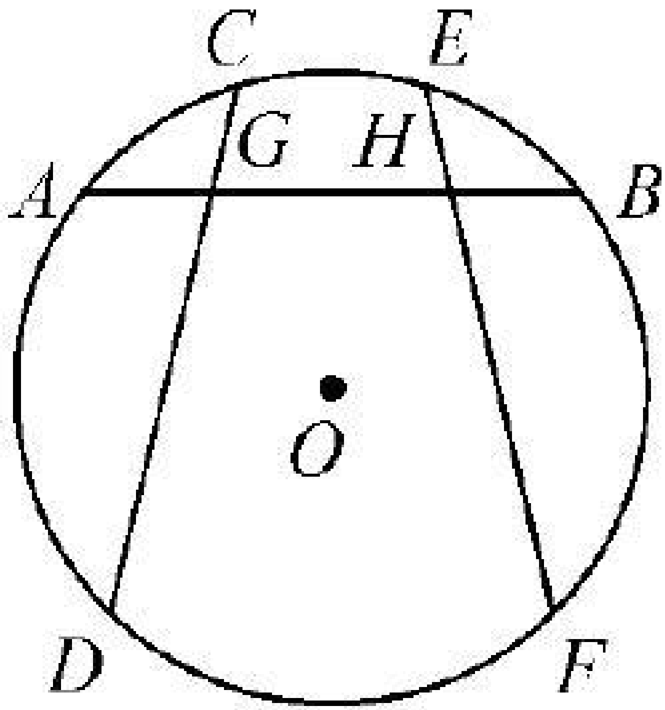
$$OP^2 + CP^2 = R^2$$

⊠⊠ $CP^2 = AP \cdot PB$ ⊠⊠⊠ $\triangle ACP \backsim \triangle CBP$ ⊠⊠⊠⊠.

2

1 ⊠⊠⊠⊠⊠⊠, $\angle C = 90^\circ, AC = 8, CB = 15$, ⊠⊠ C ⊠⊠⊠⊠⊠⊠⊠8⊠⊠⊠ AB ⊠⊠ D, ⊠ BD ⊠⊠.

2 ☒☒☒☒☒AB ☒ $\odot O$ ☒☒☒☒ G H ☒☒ AB ☒☒☒ $AG = BH$ ☒☒☒☒☒ G H ☒☒ CD EF ☒☒ $\angle DGB = \angle FHA$ ☒☒☒☒$CD = EF$ ☒



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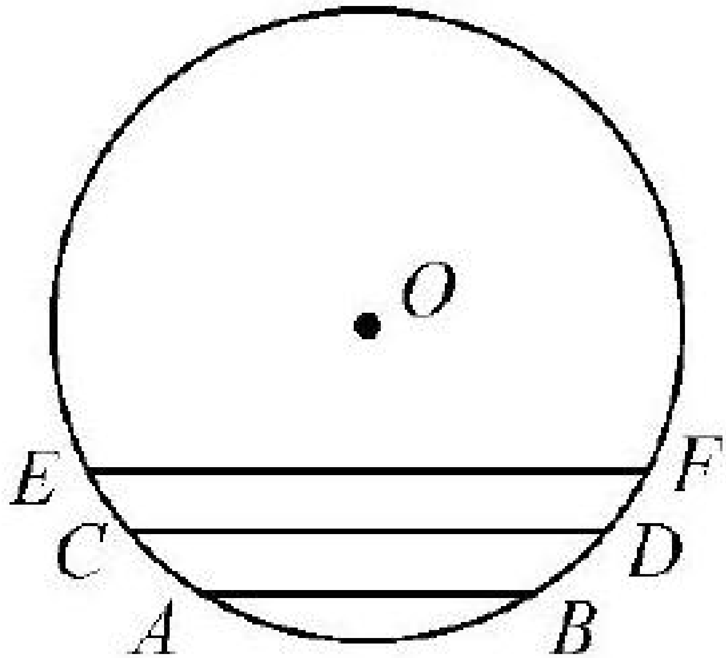
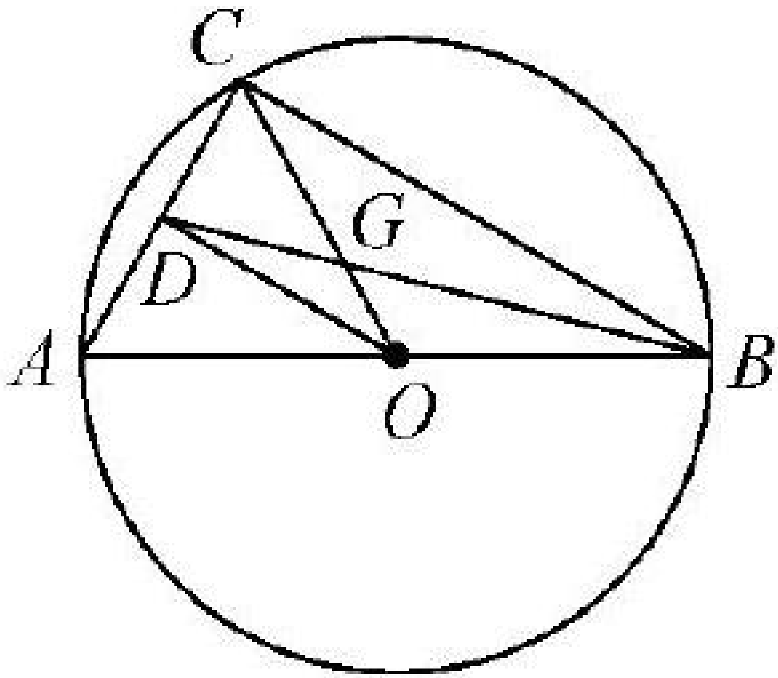


(⊠3⊠)

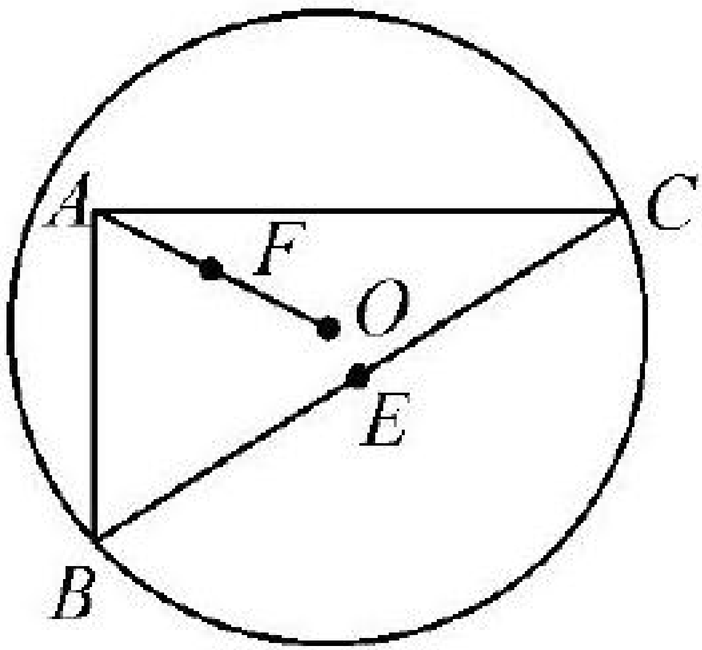
3 ⊠⊠⊠AB CD EF ⊠ $\odot O$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ EF ⊠ CD ⊠⊠⊠⊠⊠⊠⊠ CD ⊠ AB ⊠⊠⊠⊠⊠⊠⊠ $AB = 6, CD = 8, EF = 2\sqrt{21}$, ⊠ $\odot O$ ⊠⊠⊠⊠

4 ⊠⊠ AB ⊠ $\odot O$ ⊠⊠⊠, P ⊠ AB ⊠⊠⊠, ⊠⊠ P ⊠⊠ CD, ⊠ $\angle DPB = 45^\circ$ ⊠⊠⊠⊠ $PC^2 + PD^2 = 2OA^2$ ⊠

5 ⊠⊠⊠C ⊠ $\odot O$ ⊠⊠⊠⊠AB ⊠⊠⊠⊠⊠ $OD \perp AC$ ⊠ D ⊠⊠⊠ BD ⊠ OC ⊠⊠ G, ⊠ $BD = 9$, ⊠ DG ⊠⊠⊠



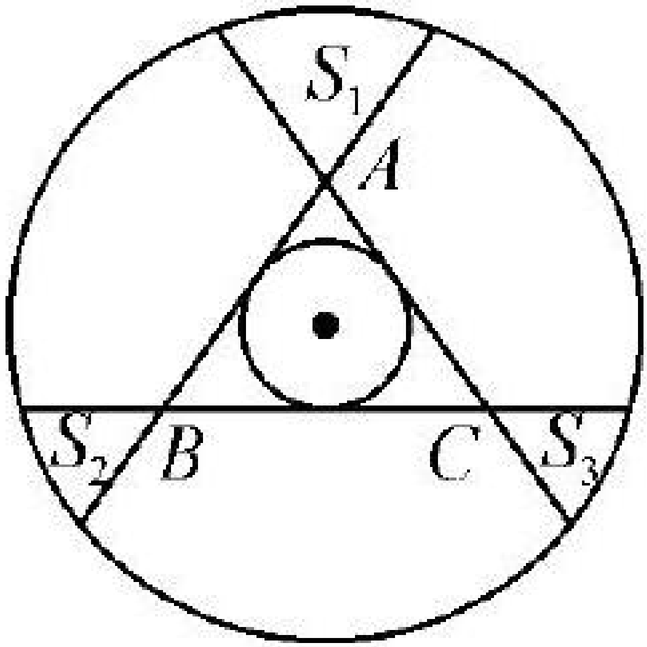
⊠⊠ 5 ⊠⊠



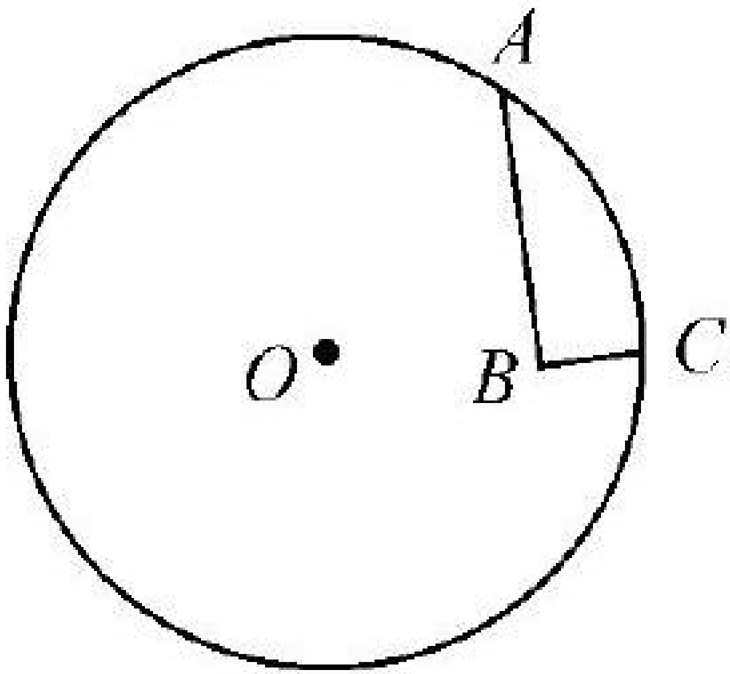
(⊠6 ⊠)

6 ⊠⊠, A ⊠⊠⊠⊠ R ⊠ $\odot O$ ⊠⊠⊠⊠, $OA = m$, ⊠⊠ A ⊠ AC ⊠ $\odot O$ ⊠⊠ C, AB ⊠ $\odot O$ ⊠⊠ B ⊠ $\angle BAC = 90^\circ$, ⊠⊠ E ⊠ BC ⊠⊠, F ⊠ AO ⊠⊠, ⊠ EF ⊠

7 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ A B C ⊠⊠, ⊠⊠ A B C ⊠⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ S_1 S_2 S_3, $\triangle ABC$ ⊠⊠⊠⊠ S ⊠⊠⊠: $S_1 + S_2 + S_3 - S$ ⊠ ⊠⊠⊠



(⊠7 ⊠)



(☒8 ☒)

8 ☒☒☒$\odot O$ ☒☒☒☒ A C, B ☒ $\odot O$ ☒☒☒ $AB = 6, BC = 2, AB \perp BC$, $\odot O$ ☒☒☒☒ $5\sqrt{2}$, ☒ OB.

☒☒

☒☒

☒☒☒☒☒☒☒☒☒☒

☒1☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒2☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒3☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒4☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

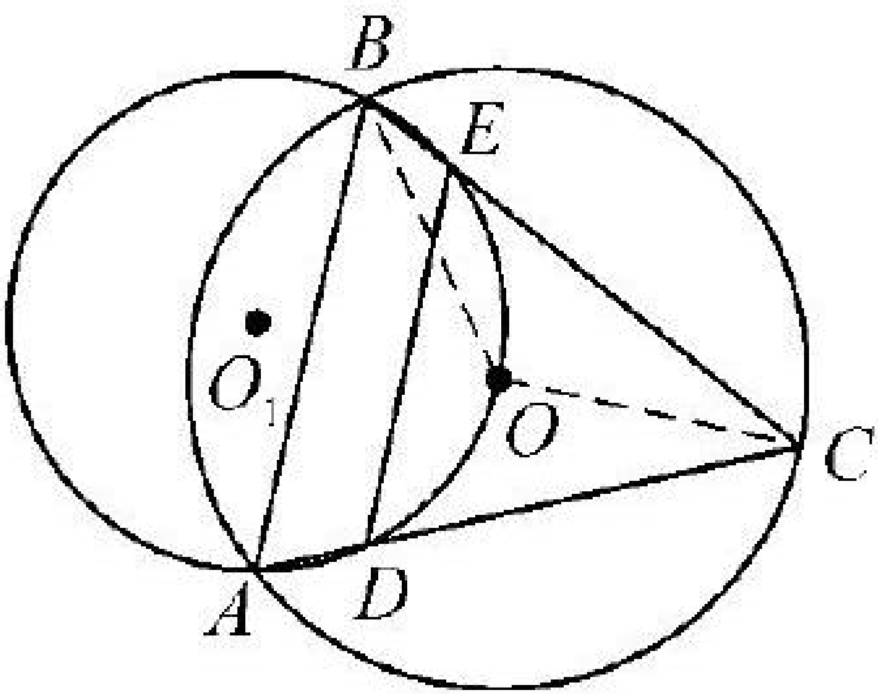
☒5☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒6☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒1 ☒☒ $\triangle ABC$ ☒☒☒☒☒ $\odot O, \odot O_1$ ☒☒ A B O ☒☒☒☒ BC AC ☒ E D (☒☒ B A ☒☒), ☒☒: $CO \perp DE$ ☒

☒☒☒☒ A B E D ☒☒ $\odot O_1$, ☒ $\angle DEC = \angle BAC$ (☒☒☒☒☒☒☒).

☒☒ OB OC, ☒ $\angle BOC = 2\angle BAC = 2\angle DEC$ ☒☒



☒3-1

$$\begin{aligned}\angle OCB &= \frac{180^\circ - \angle BOC}{2} \\ &= 90^\circ - \frac{1}{2}\angle BOC \\ &= 90^\circ - \angle DEC\end{aligned}$$

☒

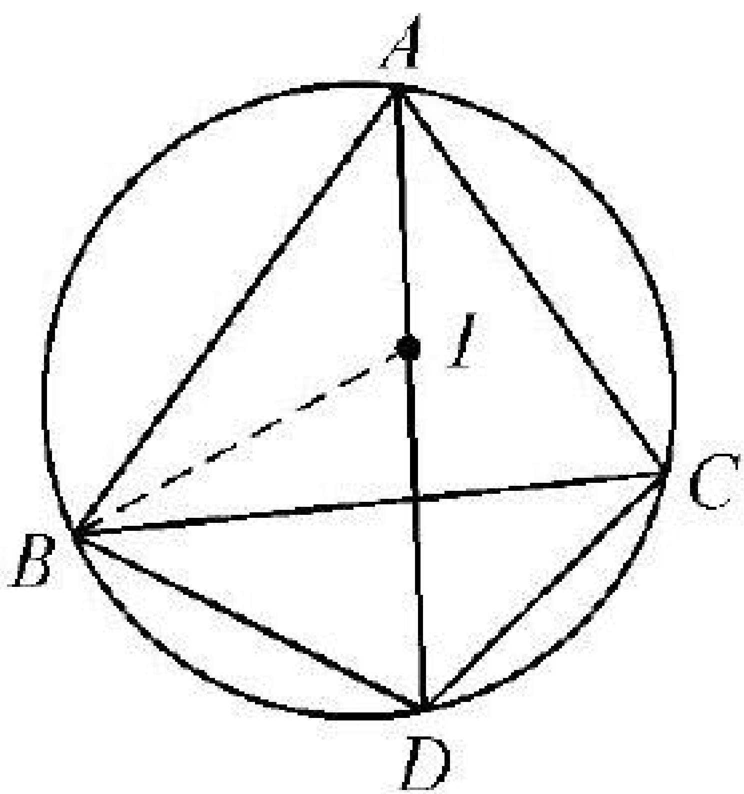
$\angle OCB + \angle DEC = 90^\circ$ ☒☒ $CO \perp DE$ ☒

☒2 ☒☒ I ☒ $\triangle ABC$ ☒☒☒☒☒☒ AI ☒ $\triangle ABC$ ☒☒☒☒ D ☒☒☒ BD DC ☒☒☒☒ $BD = DI = DC$ ☒

☒☒☒☒ BI ☒☒

☒

$$
\begin{aligned}
\angle BID &= \frac{1}{2}\angle BAC + \frac{1}{2}\angle ABC \\
\angle IBD &= \angle IBC + \angle CBD \\
&= \frac{1}{2}\angle ABC + \angle DAC \\
&= \frac{1}{2}\angle ABC + \frac{1}{2}\angle BAC
\end{aligned}
$$



⊠3-2

⊠⊠ $\angle BID = \angle IBD$, ⊠⊠ $BD = DI$.

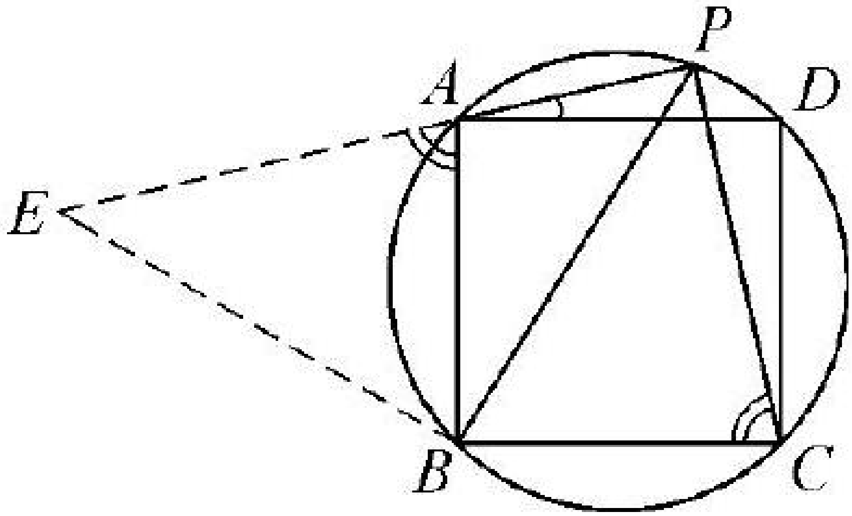
⊠⊠⊠ AI ⊠⊠ $\angle BAC$, ⊠⊠ $BD = DC$, ⊠⊠ $BD = DC$, ⊠⊠ $BD = DI = DC$.

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠" ⊠⊠⊠⊠⊠".

⊠3 ⊠ P ⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠ O ⊠⊠ AD ⊠⊠⊠, ⊠⊠ PA PB ⊠PC, ⊠ $\frac{PA+PC}{PB}$ ⊠⊠.

⊠⊠⊠⊠⊠ $\angle PAD + \angle PCB \overset{m}{=} \frac{1}{2}DP + \frac{1}{2}PB = 90^\circ, \angle DAB = 90^\circ$ ⊠⊠ $BC = BA$ ⊠⊠⊠ $\triangle PBC$ ⊠ B ⊠⊠⊠⊠ 90° ⊠⊠⊠ PA PC ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠ PA ⊠⊠⊠ B ⊠⊠⊠ PB ⊠⊠⊠



⊠3-3

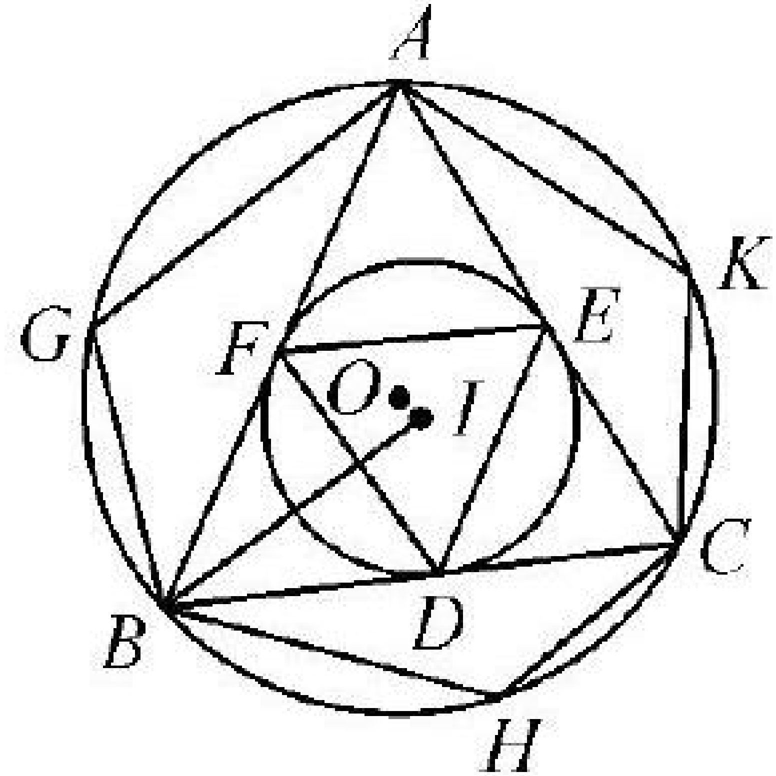
⊠⊠⊠ E ⊠

⊠ A B C P ⊠⊠, ⊠⊠ $\angle EAB = \angle PCB.$
⊠⊠ $\angle EBA$ $\angle PBC$ ⊠⊠ $\angle ABP$ ⊠⊠⊠, ⊠ $\angle EBA = \angle PBC$, ⊠⊠ $AB = CB$ ⊠⊠ ⊠⊠⊠⊠ $\triangle EAB \cong \triangle PCB$ ⊠⊠⊠ $EB = PB, EA = PC$ ⊠⊠⊠ $\triangle EBP$ ⊠⊠⊠⊠⊠ ⊠⊠⊠, ⊠⊠ $\frac{PA+PC}{PB} = \frac{PA+AE}{PB} = \frac{EP}{PB} = \sqrt{2}.$

⊠ 4 $\odot O$ ⊠⊠⊠⊠ $AGBHCK$ ⊠⊠⊠⊠⊠$\odot I$ ⊠⊠⊠ $\triangle ABC$, ⊠ D E F ⊠⊠⊠ (⊠⊠), ⊠ $\angle DEF = 55^\circ$, $\angle DFE = 60^\circ$, ⊠ $\angle G$ $\angle H$ $\angle K$ ⊠

⊠⊠⊠⊠ AB AC ⊠ $\odot I$ ⊠⊠ $\angle AFE = \angle AEF = \angle FDE = 180^\circ - \angle DEF - \angle DFE = 65^\circ$ ⊠⊠⊠ $\angle BAC$ ⊠⊠⊠⊠ $\angle BAC + \angle BHC = 180^\circ$ ⊠⊠ $\angle H$ ⊠⊠⊠

⊠⊠⊠ AB AC ⊠ $\odot I$ ⊠⊠ F E ⊠⊠ $\angle AFE =$



☒3-4

$\angle AEF = \angle EDF.$

☒☒ $\angle FDE = 180° - \angle DEF - \angle DFE = 180° - 55° - 60° = 65°,$

☒☒

$$\angle AFE = \angle AEF = 65°,$$

☒☒

$$\angle BAC = 180° - \angle AFE - \angle AEF = 50°.$$

☒☒

$$\angle BAC + \angle BHC \overset{m}{=} \frac{1}{2}BAC + \frac{1}{2}\widehat{BHC} = 180°$$

☒☒ $\angle BHC = 130°.$

☒☒ $\angle AGB = 120°, \angle AKC = 110°$ ☒

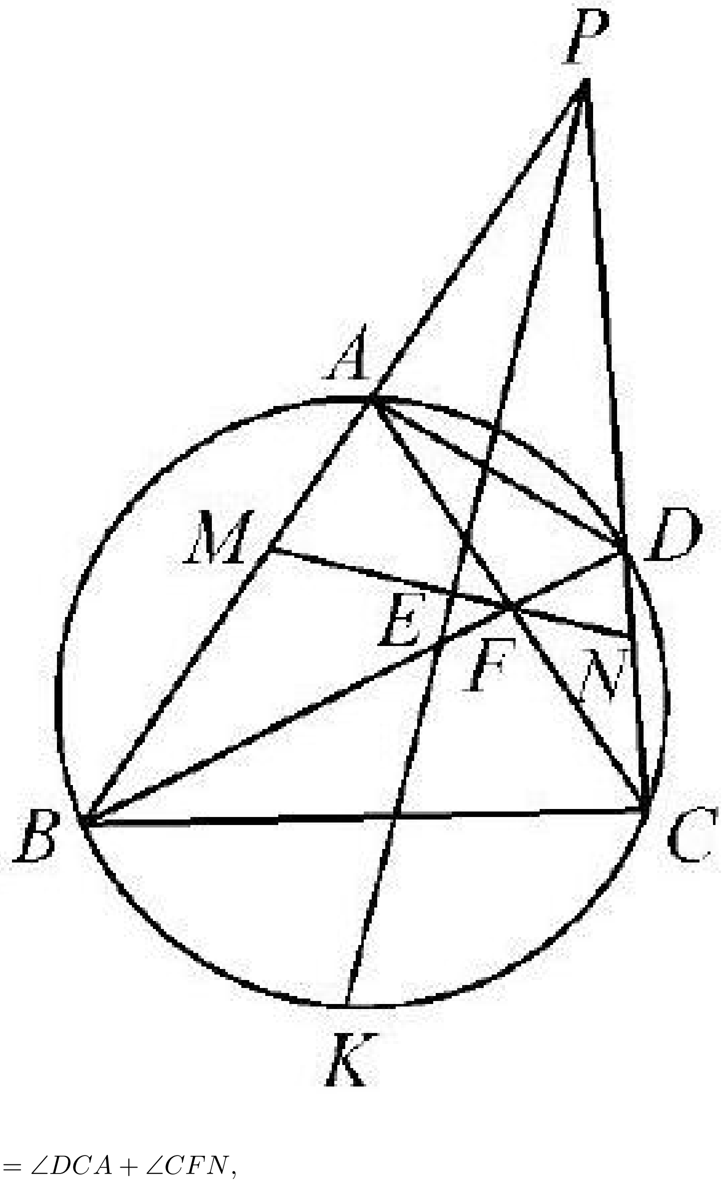
⊠ $\angle G$ $\angle H$ $\angle K$ ⊠⊠⊠ 120° 130° 110° ⊠

⊠5 ⊠⊠3-5, ⊠⊠⊠ $ABCD$ ⊠⊠⊠⊠ O ⊠⊠⊠⊠ AC BD ⊠⊠⊠ F, ⊠⊠ BA CD ⊠⊠ ⊠ P, PK ⊠⊠ $\angle BPC$ ⊠⊠⊠ F ⊠ $EF \perp PK$ ⊠⊠ E ⊠⊠ PB PC ⊠⊠ M ⊠N ⊠⊠ ⊠⊠$\angle AFM = \angle BFM$ ⊠

⊠⊠

⊠⊠ MN ⊠⊠⊠ $\angle BPC$ ⊠⊠⊠⊠⊠⊠⊠⊠ $\triangle PME \cong \triangle PNE$ ⊠⊠⊠ $\angle PMN = \angle PNM$ ⊠

⊠ $\angle PMN = \angle ABD + \angle BFM$,



⊠3-5

$\angle PNM = \angle DCA + \angle CFN$,

⊠⊠⊠
⊠⊠
⊠
⊠⊠

$$\angle ABD = \angle DCA,$$

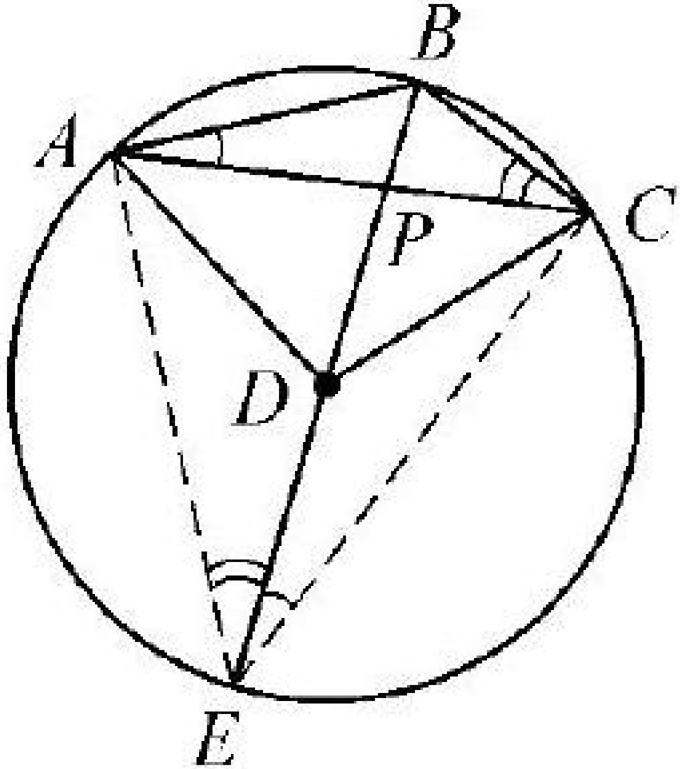
$\angle BFM = \angle CFN.$
$\angle CFN = \angle AFM,$
$\angle BFM = \angle AFM.$

⊠6 ⊠⊠⊠⊠⊠ $ABCD$ ⊠, $\angle BAC = 20^\circ, \angle BCA = 30^\circ, \angle BDC = 40^\circ, \angle BDA = 60^\circ$, ⊠⊠⊠⊠ AC BD ⊠⊠⊠ P, ⊠ $\angle CPB$.

⊠⊠⊠ $\angle BAC = \frac{1}{2}\angle BDC, \angle BCA = \frac{1}{2}\angle BDA$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠

⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠ BD ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠ E, ⊠⊠ AE EC, ⊠ $\angle AEB = \angle ACB = 30^\circ, \angle CEB = \angle BAC = 20^\circ$ ⊠

⊠ $\angle BDA = 60^\circ$ ⊠⊠⊠ $\angle EAD = \angle BDA - \angle AEB = 30^\circ$ ⊠⊠⊠ $DA = DE$ ⊠⊠⊠ $DE = DC$ ⊠⊠ $DA = DE = DC$ ⊠⊠ D ⊠⊠⊠⊠⊠⊠⊠⊠⊠



⊠3-6

$$\angle ADC = 2\angle AEC = 100^\circ,$$

$$\angle DAC = \angle DCA = \frac{180^\circ - \angle ADC}{2} = \frac{180^\circ - 100^\circ}{2} = 40^\circ.$$

⊠⊠

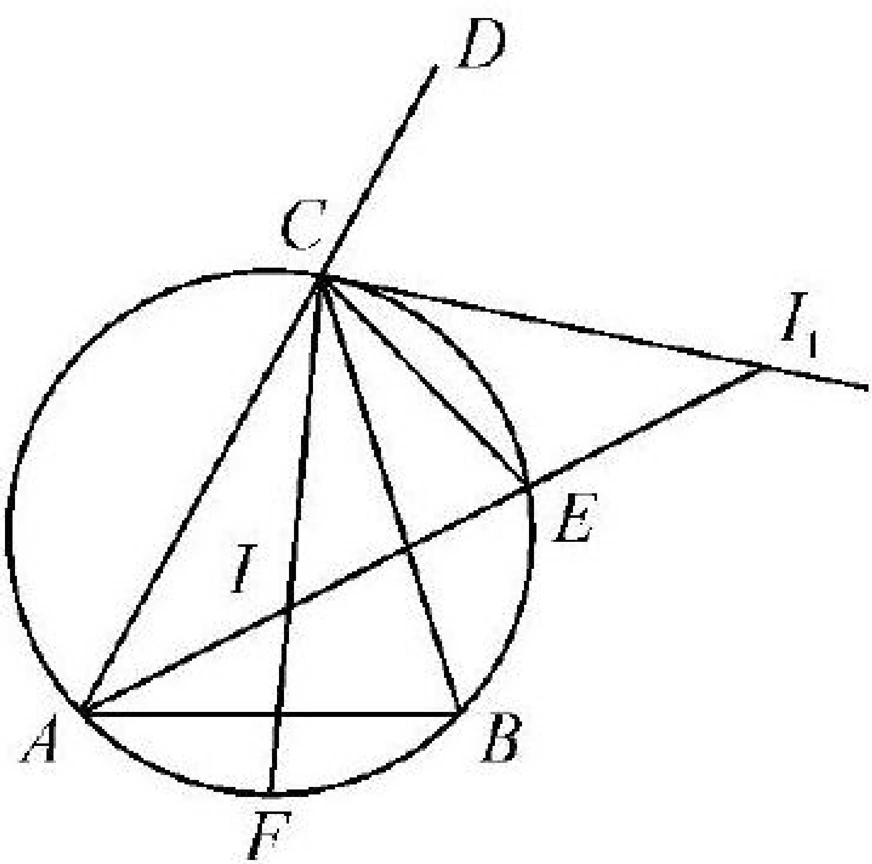
$$\begin{aligned}\angle CPB &= \angle BEC + \angle ECA \\ &= \angle BEC + \angle ECD + \angle DCA \\ &= 20^\circ + 20^\circ + 40^\circ = 80^\circ.\end{aligned}$$

⊠⊠" ⊠⊠⊠⊠⊠⊠⊠⊠⊠" ⊠⊠⊠⊠⊠⊠⊠⊠" ⊠⊠⊠".

⊠7 ⊠⊠⊠I ⊠ $\triangle ABC$ ⊠⊠⊠⊠I_1 ⊠ $\triangle ABC$ ⊠⊠ BC ⊠⊠⊠⊠⊠⊠⊠⊠ II_1 ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠ E ⊠⊠⊠⊠$IE = I_1E$ ⊠

⊠⊠⊠ IC I_1C ⊠⊠⊠⊠ $\angle ACB$ $\angle BCD$ ⊠ $\angle ACB + \angle BCD = 180^\circ$, ⊠ $\angle ICI_1 = 90^\circ$ ⊠⊠⊠ A ⊠I I_1 ⊠⊠⊠

⊠ $\angle CIE = \angle CAE + \angle ACF \overset{m}{=} \frac{1}{2}(AF + CE)$,



☒3-7

$$\angle ICE \overset{m}{=} \frac{1}{2}(BF + BE),$$

☒☒☒ $\angle ACF = \angle FCB$ ☒ $AF = FB, \angle CAE = \angle EAB$ ☒ $CE = BE$ ☒☒☒ $\angle CIE = \angle ICE$ ☒☒☒

$$IE = CE, \tag{1}$$

☒

$$\angle ECI_1 = 90^\circ - \angle ECI = 90^\circ - \angle CIE = \angle II_1C,$$

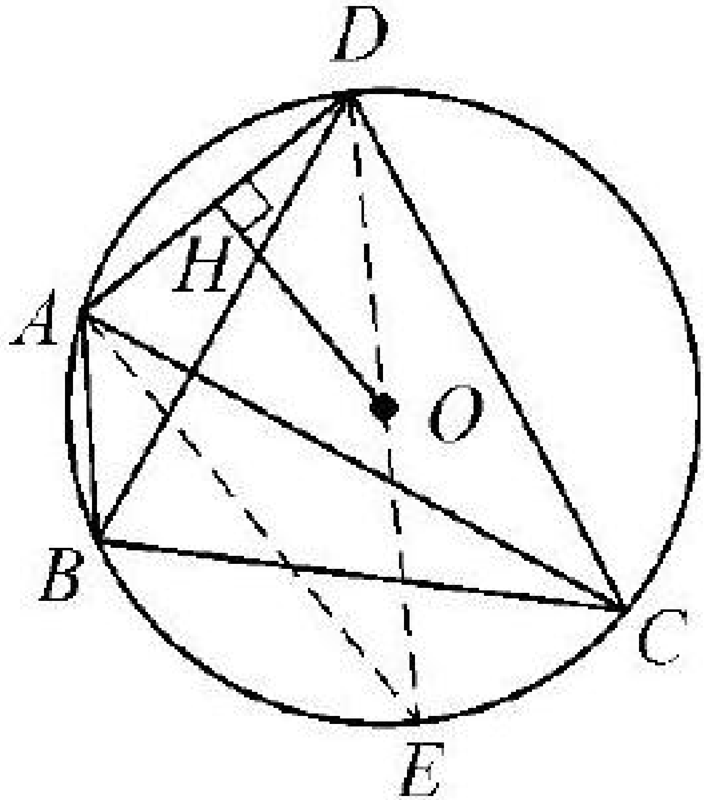
☒ (1)(2) ☒ $IE = I_1E$ ☒

☒8 ☒☒☒ $ABCD$ ☒☒☒ $\odot O$, ☒☒☒☒ AC ☒☒☒☒ BD ☒☒, ☒☒ O ☒ $OH \perp AD$ ☒ H ☒☒☒☒$OH = \frac{1}{2}BC$ ☒

☒☒☒☒ DO ☒☒☒☒ $\odot O$ ☒☒ E ☒☒☒ AE ☒☒☒ DE ☒☒☒☒☒ $AE \perp AD$ ☒☒ $OH \perp AD$ ☒☒ $OH // AE$ ☒☒☒☒ O ☒ DE ☒☒☒☒☒

$$OH = \frac{1}{2}AE \tag{1}$$

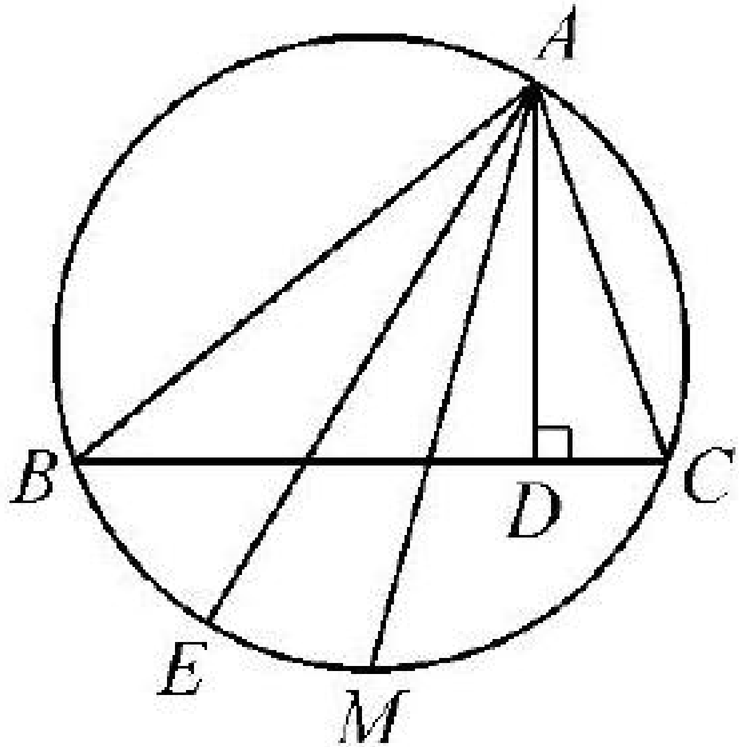
☒ $\frac{1}{2}(AD + AE) \overset{m}{=} 90^\circ$ ☒



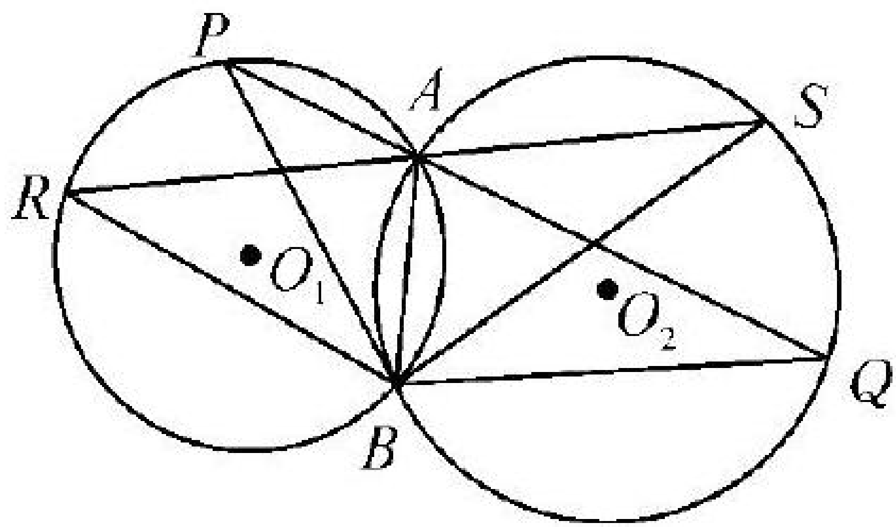
☒3-8

☒☒ $AC \perp BD$, ☒ $\frac{1}{2}(AD+BC) \overset{m}{=} 90^\circ$.
☒☒ $AE = BC$ ☒☒☒ $AE = BC$
☒ (1)☒(2) ☒ $OH = \frac{1}{2}BC$.

1 ☒☒, ☒☒ $\triangle ABC$ ☒, $AD \perp BC$ ☒☒ D, AE ☒ $\triangle ABC$ ☒☒☒☒☒, M ☒ BC ☒☒. ☒☒: $\angle EAM = \angle DAM$.



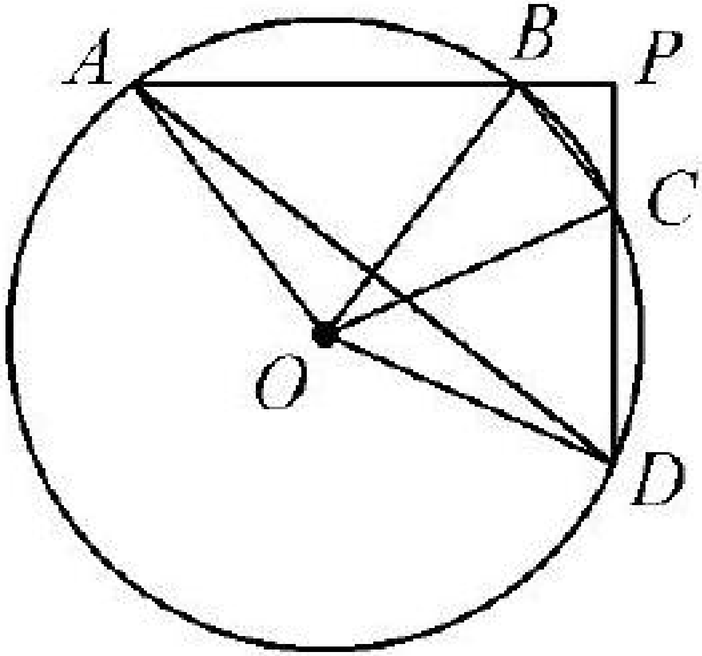
(⊠1 ⊠)



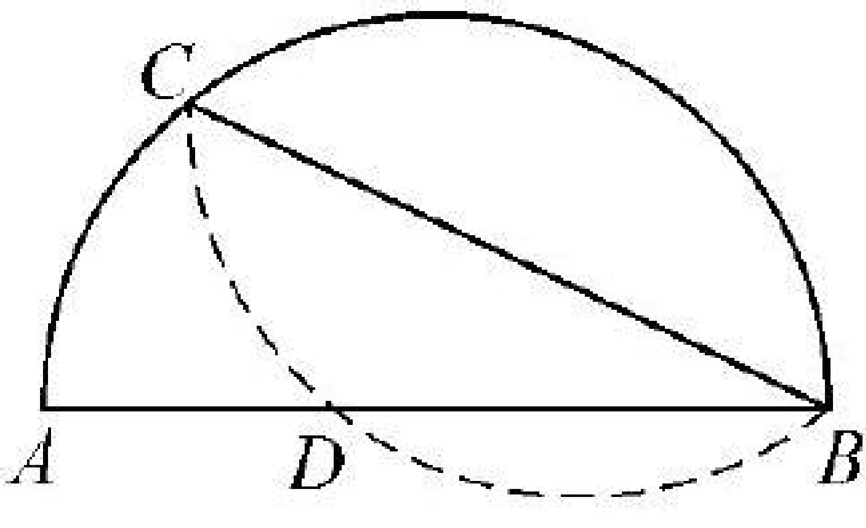
(⊠2 ⊠)

2 ⊠⊠⊠⊠⊠⊠⊠⊠ A B ⊠⊠⊠⊠ A ⊠⊠⊠ PQ ⊠ $\odot O_1$ ⊠ P ⊠⊠ $\odot O_2$ ⊠ Q ⊠⊠ A ⊠⊠⊠ RS ⊠ $\odot O_1$ ⊠ R ⊠⊠ $\odot O_2$ ⊠ S ⊠⊠⊠: $\angle PBR = \angle SBQ$.

3 ⊠⊠⊠$\odot O$ ⊠⊠⊠⊠ AB DC ⊠⊠⊠⊠⊠⊠⊠⊠ P ⊠⊠ $\angle P = 90^\circ$ ⊠⊠⊠ OA, OB, OC, OD, AD, BC, ⊠⊠: $S_{\triangle OAD} = S_{\triangle OBC}$.



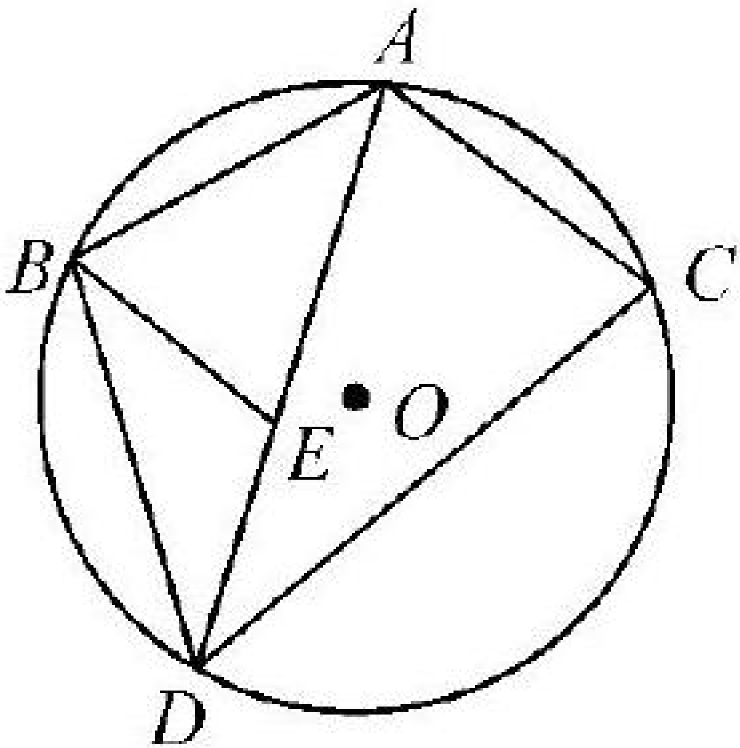
⊠⊠3⊠⊠



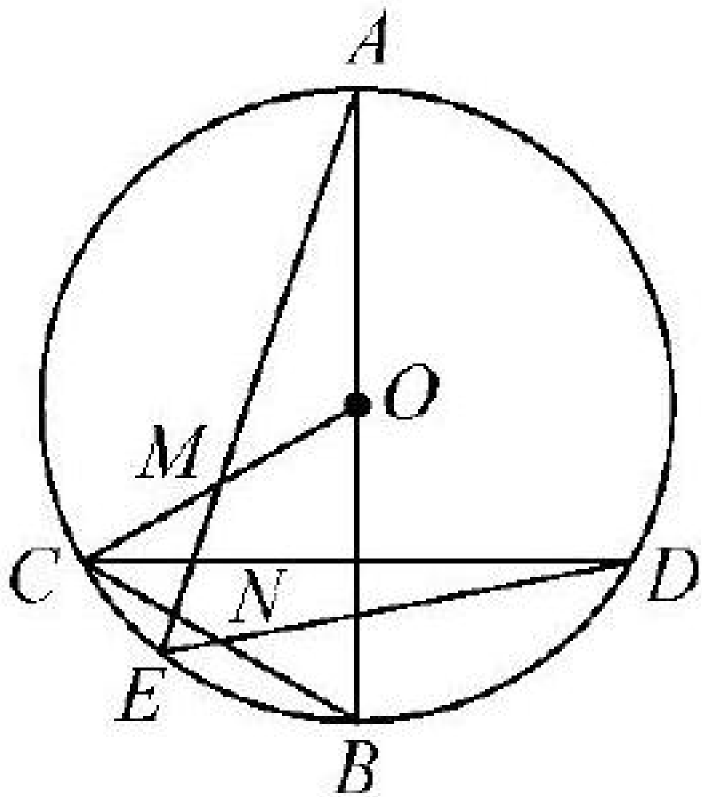
(⊠4 ⊠)

4 ⊠⊠⊠C ⊠⊠⊠⊠⊠⊠⊠AB ⊠⊠⊠⊠⊠⊠⊠⊠ BC ⊠⊠⊠ AB ⊠⊠ D ⊠⊠ $AD = 4, DB = 5$ ⊠⊠ BC ⊠⊠⊠

5 ⊠⊠, ⊠⊠⊠ $ABDC$ ⊠⊠⊠ $\odot O, E$ ⊠ AD ⊠, ⊠ $AB = AE = AC$, ⊠⊠:
(1) $\angle CAD = 2\angle DBE$;
(2) $AD^2 - AB^2 = BD \cdot DC$.



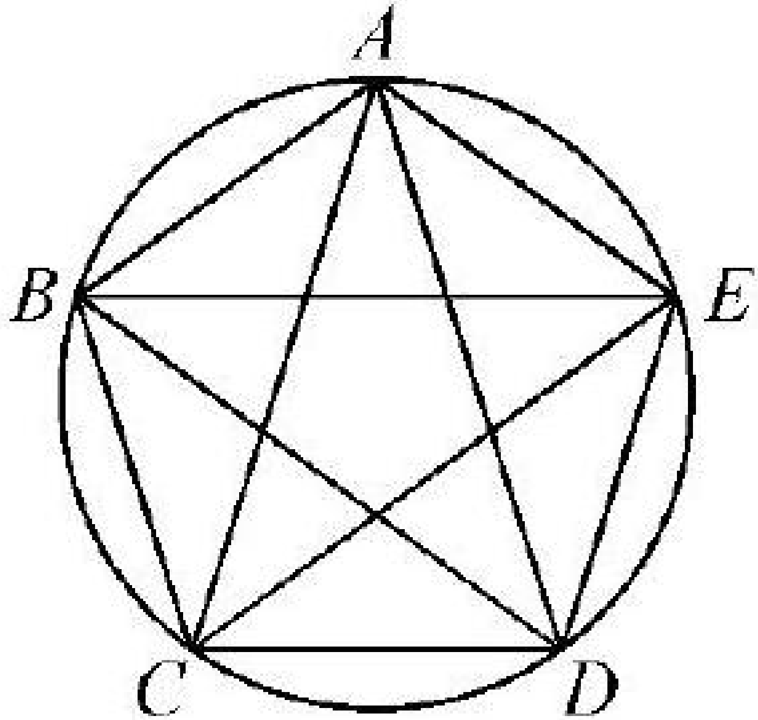
(⊠5 ⊠)



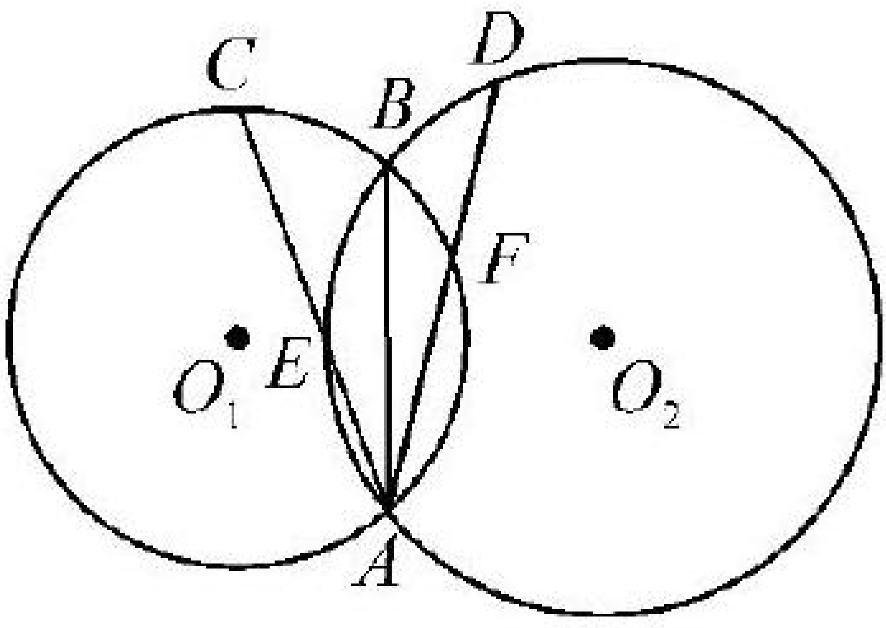
(⊠6 ⊠)

6 ⊠⊠⊠$\odot O$ ⊠⊠ CD ⊠⊠⊠⊠⊠ AB, M ⊠ OC ⊠⊠⊠AM ⊠⊠⊠⊠ $\odot O$ ⊠⊠ E, BC ⊠ ED ⊠⊠ N. ⊠⊠: $BN = CN$.

7 ⊠⊠⊠⊠⊠⊠ $ABCDE$ ⊠⊠⊠⊠⊠⊠ $AC//DE, BD//AE, CE//AB$ ⊠$DA//BC, EB//CD$ ⊠⊠⊠⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠⊠⊠⊠



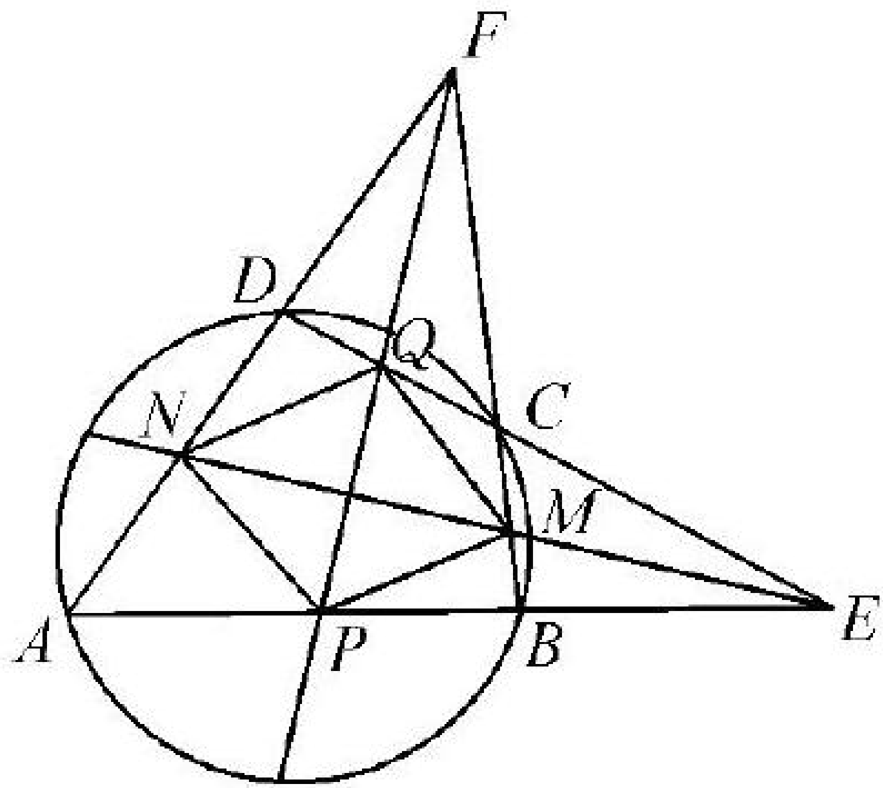
(⊠7⊠)



(☒8☒)

8 ☒☒☒$\odot O_1$ ☒ $\odot O_2$ ☒☒☒ $A\ B, \odot O_1$ ☒☒ AC ☒ $\odot O_2$ ☒☒ $E, \odot O_2$ ☒☒ AD ☒ $\odot O_1$ ☒☒ F, ☒☒: $CE = DF$ ☒☒☒☒☒☒ AB ☒☒ $\angle CAD$.

9 ☒☒☒☒☒☒ $ABCD$ ☒☒☒☒ O ☒☒☒ AD ☒BC ☒☒☒ F, ☒☒ $DC\ AB$ ☒☒☒ E, $\angle AEC$ ☒☒☒☒☒ BC ☒☒ M, ☒ AD ☒☒ $N, \angle BFD$ ☒☒☒☒☒ AB ☒☒ P, ☒ CD ☒☒ Q. ☒☒: ☒☒☒ $MPNQ$ ☒☒☒.



(☒9 ☒)

☒☒☒☒☒☒☒☒☒☒☒☒☒☒: ☒☒☒☒☒☒☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒☒ d ☒☒☒ ☒☒ R ☒☒☒☒☒☒☒☒, ☒

☒☒☒☒ $\Leftrightarrow d > R$ ☒

☒☒☒☒ $\Leftrightarrow d = R$ ☒

☒☒☒☒ $\Leftrightarrow d < R$.

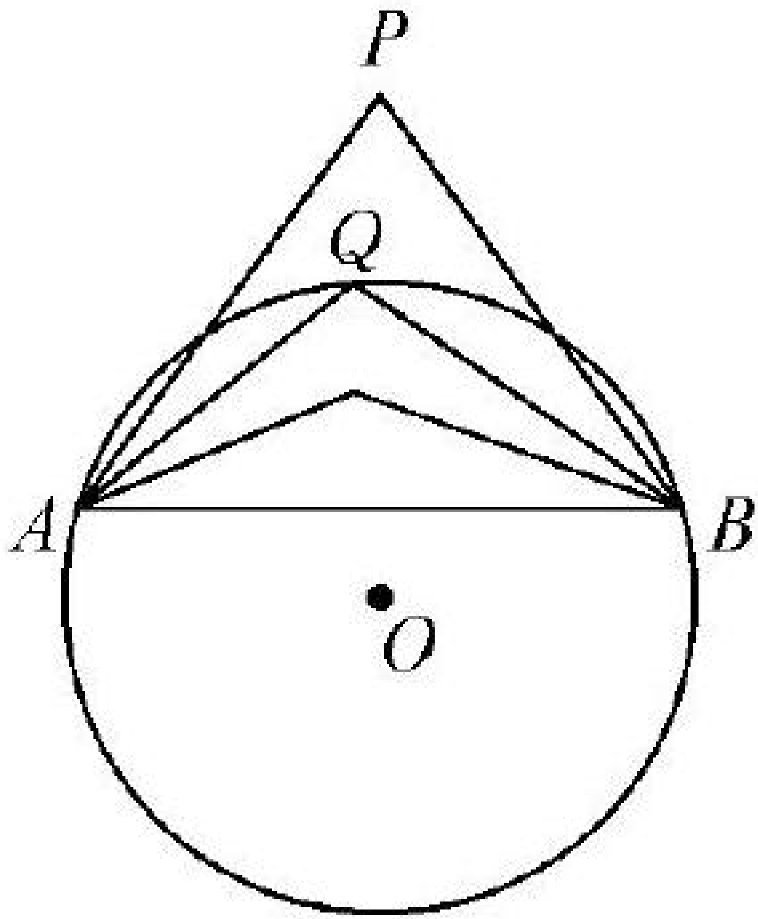
☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒4-1☒AB ☒ $\odot O$ ☒☒☒☒☒☒ Q ☒☒ AB ☒☒☒ $\angle AQB = \alpha$, ☒☒ P ☒☒ Q ☒☒ AB ☒☒☒, ☒

☒ P ☒☒☒ $\Leftrightarrow \angle APB < \alpha$;

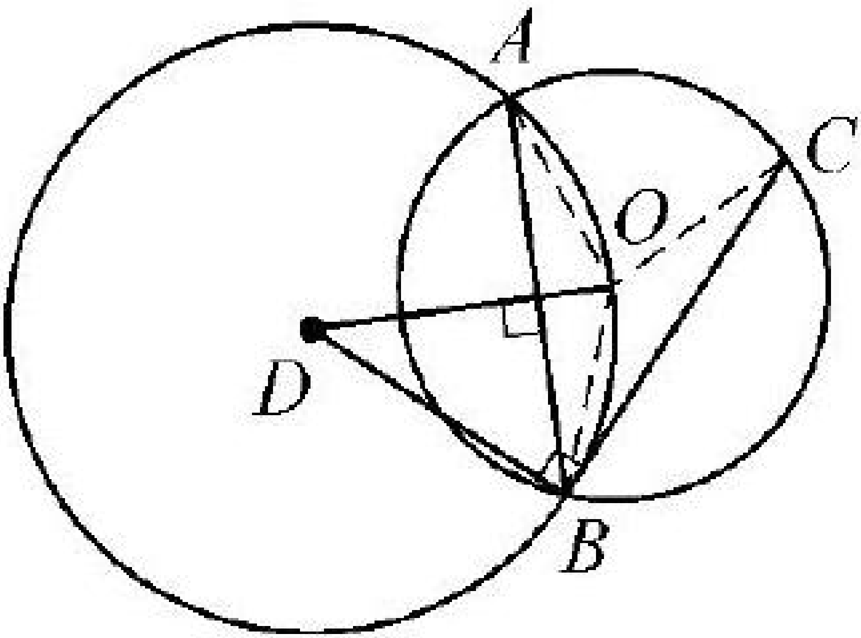
☒ P ☒☒☒ $\Leftrightarrow \angle APB = \alpha$;

☒ P ☒☒☒ $\Leftrightarrow \angle APB > \alpha$.
☒1 ☒☒4-2 ☒☒☒$\odot D$ ☒ $\odot O$ ☒☒☒☒ A ☒B, ☒☒ B ☒ $\odot D$ ☒☒☒☒ $\odot O$ ☒☒ C, ☒ $AB = BC$, ☒☒☒ O ☒ $\odot D$ ☒☒☒☒☒.

☒☒☒☒☒ O ☒ $\odot D$ ☒, ☒☒☒☒: $OA = OB = OC, BC$ ☒☒☒, AB ☒☒☒☒, ☒☒ ☒☒☒☒☒ $OD = DB$ ☒



☒4-1



☒4-2

☒☒☒ OA OB OC, ☒ $\triangle OBC$ ☒ $\triangle OBA$ ☒, $AB = BC, OA = OC$, OB ☒☒☒☒, ☒☒ $\triangle OBC \cong \triangle OBA$, ☒ $\angle OBA = \angle OBC$, ☒ BC ☒ $\odot D$ ☒☒☒, ☒☒ $\angle DBC = 90^\circ, AB$ ☒ $\odot D$ ☒ $\odot O$ ☒☒☒☒, OD ☒☒☒☒☒☒, ☒ $OD \perp AB$, ☒☒

$$\angle DBO = 90^\circ - \angle OBC = 90^\circ - \angle OBA = \angle DOB,$$

☒☒ $DO = DB$, ☒☒☒☒ O ☒☒ D ☒.

☒2 ☒☒☒☒ $2n+3$ ☒☒, ☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒☒☒☒☒, ☒☒ $2n$ ☒☒☒☒ n ☒☒☒☒, ☒ n ☒☒☒☒?

☒☒☒. ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ A B, ☒☒☒☒☒☒☒☒☒☒☒ AB ☒☒☒. ☒☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒ AB ☒☒☒☒☒☒☒☒☒☒☒☒ $\angle AP_1B < \angle AP_2B < \cdots < \angle AP_{n+1}B < \cdots < \angle AP_{2n+1}B$, ☒☒☒☒ A B P_{n+1} ☒☒ O, ☒ $\odot O$ ☒☒☒☒ A B P_{n+1} ☒☒, ☒ $P_1, P_2, \cdots, P_n$ ☒ n ☒☒☒☒☒, $P_{n+2}, \cdots, P_{2n+1}$ ☒ n ☒☒☒☒☒.

☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒. ☒☒☒☒☒☒☒☒☒☒ d ☒☒☒☒☒☒ R ☒☒

$$d > R \Leftrightarrow \text{☒☒} \Leftrightarrow \text{☒☒☒☒☒☒☒☒☒☒},$$
$$d = R \Leftrightarrow \text{☒☒} \Leftrightarrow \text{☒☒☒☒☒☒☒☒☒☒☒},$$
$$d < R \Leftrightarrow \text{☒☒} \Leftrightarrow \text{☒☒☒☒☒☒☒☒☒☒☒}.$$

3 Rt $\triangle ABC$, $\angle C=90^\circ, AC=3, BC=4$ C R
$\odot C$ $\odot C$ AB n, n R
" AB " .

Rt $\triangle ABC$, $\angle C=90^\circ, AC=3, BC=4$ $AB=5$, C
 AB d, $\frac{1}{2}AC\cdot BC=\frac{1}{2}AB\cdot d$, $d=2.\ 4$.

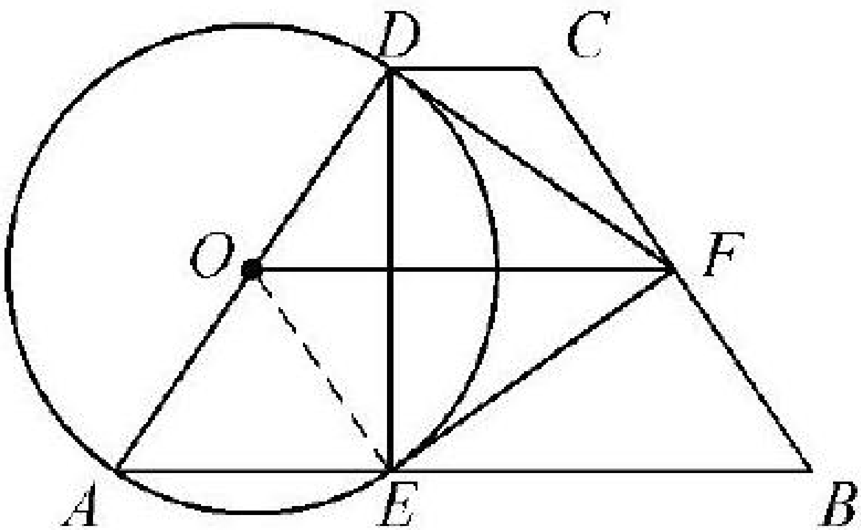
1$n=0$ $0<R<2.4$ $R>4$

2$n=1$ $R=2.4$ $3<R\leqslant 4$

3$n=2$ $2.4<R\leqslant 3$

4 4-3 $ABCD$ $CD//AB, \angle B=60^\circ$ AD O
AB $E, \odot O$ EF BC F, DF $\odot O$ CD AB

 CD AB $\triangle OEF$



4-3

 $CD=x, AB=y$ FD FE $\odot O$ $FO\perp DE$ AD
 $\odot O$ $\angle AED=90^\circ$ $DE\perp AB$ $AB//$ OF // CD.

 O AD , F BC , $OF=\frac{x+y}{2}$.

, $AB//OF, O$ AD , $\angle EOF=\angle OEA=\angle A=60^\circ$. EF $\odot O$
 $E, OE\perp EF$ $2OE=OF$

 $\triangle OAE$, $OE=AE=\frac{y-x}{2}$,

$$y-x=2OE=OF=\frac{x+y}{2}$$

 $y=3x$, $\frac{CD}{AB}=\frac{1}{3}$.

. d
R $r(R\neq r)$

⊠⊠ $\Leftrightarrow d > R + r \Rightarrow$ ⊠⊠⊠⊠,

⊠⊠ $\Leftrightarrow d = R + r \Rightarrow 1$ ⊠⊠⊠⊠,

⊠⊠ $\Leftrightarrow |R - r| < d < R + r \Leftrightarrow 2$ ⊠⊠⊠⊠,

⊠⊠ $\Leftrightarrow d = |R - r| \Rightarrow 1$ ⊠⊠⊠⊠,

⊠⊠ $\Leftrightarrow 0 \leqslant d < |R - r| \Rightarrow$ ⊠⊠⊠⊠.

⊠⊠

⊠5 ⊠⊠ $\odot A$ $\odot B$ ⊠⊠⊠⊠⊠ $1, AB = 10$ ⊠⊠⊠ $\odot B$ ⊠⊠⊠⊠⊠$\odot A$ ⊠⊠⊠ AB ⊠ $\odot B$ ⊠⊠⊠⊠⊠2 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\odot A$ ⊠⊠⊠⊠⊠⊠1 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠

⊠⊠⊠⊠ t ⊠⊠⊠⊠⊠⊠⊠⊠ $d = |10 - 2t|, r_B = 1, r_A = 1 + t$ ⊠

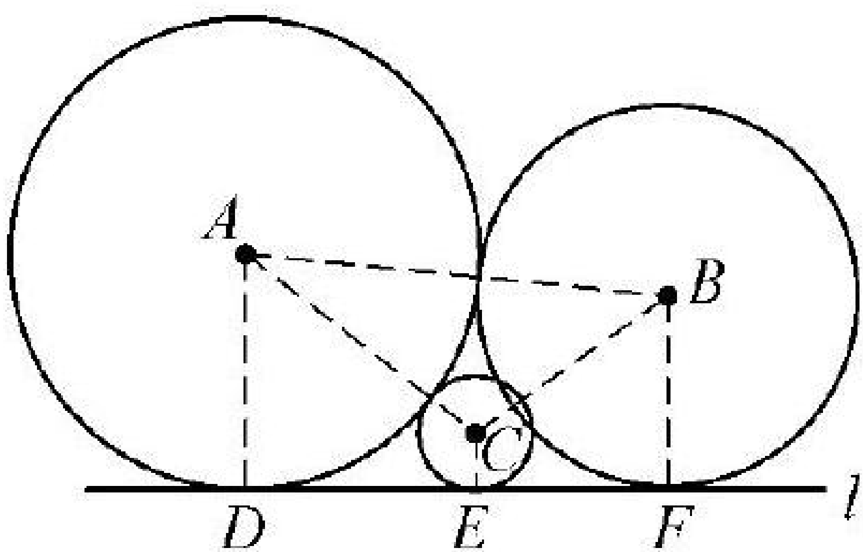
⊠⊠, ⊠⊠⊠⊠⊠⊠, $|10 - 2t| = 2 + t$, ⊠⊠ $t = \frac{8}{3}$ ⊠ $t = 12$.

⊠⊠⊠⊠⊠⊠, $|10 - 2t| = |t|$, ⊠⊠ $t = \frac{10}{3}$ ⊠ $t = 10$.

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠6 ⊠⊠ $4 - 4, \odot A$ $\odot B$ $\odot C$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ l ⊠⊠⊠⊠ $\odot A$ $\odot B$ $\odot C$ ⊠⊠⊠⊠⊠ ⊠ a b c, ⊠⊠: $\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$.

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ l ⊠⊠⊠⊠⊠⊠ AC BC AB ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ DE EF DF ⊠



⊠4-4

⊠ $DF = DE + EF$ ⊠⊠⊠ a b c ⊠⊠⊠⊠⊠⊠⊠

⊠⊠ $\odot A$ $\odot B$ $\odot C$ ⊠⊠⊠⊠⊠ l ⊠⊠ D F E, ⊠⊠ AD BF CE ⊠ AB AC CB ⊠⊠ $AD \perp l, BF \perp l, AD = a, BF = b, AB = a + b$ ⊠⊠⊠

$$DF = \sqrt{(a+b)^2 - (a-b)^2} = 2\sqrt{ab}.$$

⊠⊠ $DE = 2\sqrt{ac}, EF = 2\sqrt{bc}$ ⊠⊠⊠ $DF = DE + EF$ ⊠⊠

$$2\sqrt{ab} = 2\sqrt{ac} + 2\sqrt{bc}$$

⊠⊠

$$\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$$

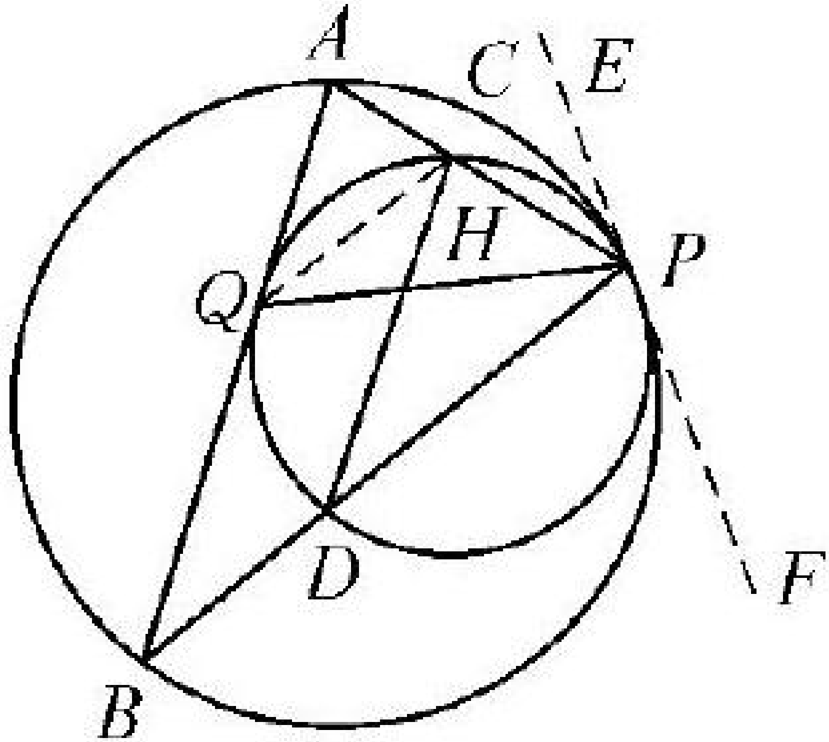
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠

⊠7 ⊠⊠4-5, ⊠⊠⊠⊠⊠⊠ P, ⊠⊠⊠⊠ AB ⊠⊠⊠⊠⊠ Q, ⊠⊠ PA PB ⊠⊠⊠⊠⊠⊠⊠ C D, ⊠⊠ PQ ⊠ CD ⊠⊠⊠ H.

⊠⊠⊠⊠1⊠$\frac{CH}{AQ} = \frac{HD}{QB}$ ⊠

(2) $\angle APQ = \angle QPB$.

⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AB//CD$ ⊠⊠⊠ $\angle ABP =$



⊠4-5

$\angle CDP$, ⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠ P ⊠⊠⊠⊠⊠⊠⊠ EF ⊠⊠⊠ $\angle ABP = \angle CDP = \angle APE$ ⊠

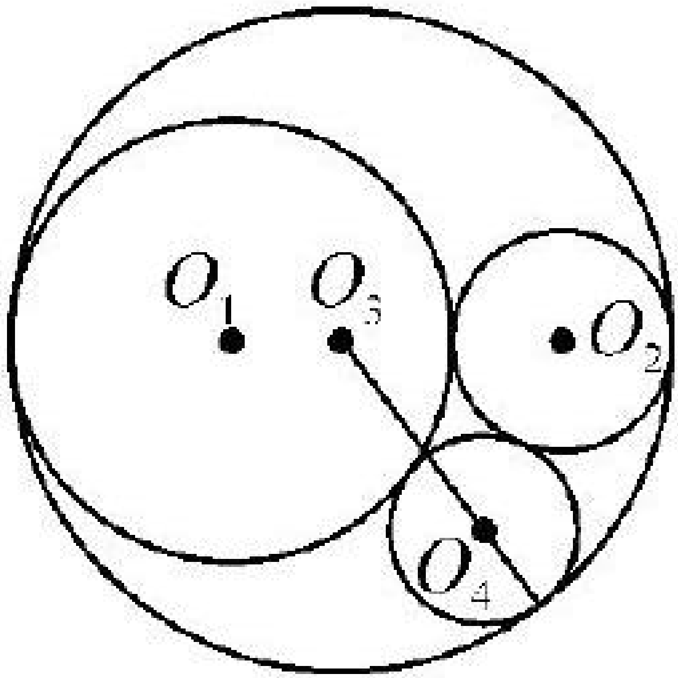
⊠⊠ (1) ⊠⊠ P ⊠⊠⊠⊠⊠⊠ EF ⊠⊠⊠⊠⊠⊠ $\angle CDP = \angle CPE$ ⊠⊠⊠⊠⊠ $\angle ABD = \angle APE$ ⊠⊠⊠ $\angle CDP = \angle ABP$ ⊠⊠⊠ $AB//CD$ ⊠⊠

$$\frac{CH}{AQ} = \frac{PH}{PQ} = \frac{HD}{QB}.$$

⊠2⊠⊠⊠ QC ⊠⊠ $\angle APQ = \angle AQC, \angle QPB = \angle QCH.$
⊠⊠1⊠⊠⊠⊠⊠ $AB//CD$ ⊠⊠ $\angle AQC = \angle QCH$ ⊠⊠⊠ $\angle APQ = \angle QPB$ ⊠
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

4

1 ⊠⊠⊠⊠ $2n+1$ ⊠⊠, ⊠⊠⊠⊠⊠⊠, ⊠⊠⊠ $2n+1$ ⊠⊠⊠ n ⊠⊠⊠⊠⊠⊠n ⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠⊠⊠
2 ⊠⊠⊠2 ⊠⊠⊠⊠⊠⊠⊠8 ⊠⊠⊠⊠⊠⊠⊠⊠1 ⊠⊠⊠⊠?
3 ⊠⊠⊠⊠ $ABCD$ ⊠, $AD//BC, AD = 1, AB = 4, BC = 3,$ ⊠ CD ⊠⊠ O ⊠⊠ ⊠⊠⊠⊠⊠2 ⊠⊠⊠
⊠1⊠⊠ $\odot O$ ⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠⊠ $\angle ABC$ ⊠⊠⊠⊠⊠⊠
⊠2⊠⊠ $\odot O$ ⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠⊠ $\angle ABC$ ⊠⊠⊠⊠⊠⊠
4 ⊠⊠ $\odot A$ $\odot B$ ⊠⊠⊠⊠⊠⊠⊠ l ⊠⊠$AB = 3$ ⊠⊠⊠⊠⊠1 ⊠⊠ $\odot A$ ⊠⊠⊠2 ⊠⊠⊠⊠⊠⊠ ⊠⊠ l ⊠ $\odot B$ ⊠⊠⊠⊠⊠$\odot B$ ⊠⊠⊠⊠⊠⊠⊠ $\odot A$ ⊠⊠⊠⊠⊠⊠2 ⊠⊠⊠⊠⊠⊠⊠⊠⊠
⊠1⊠t ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠
⊠2⊠t ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠
⊠3⊠t ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠
5 ⊠⊠ $\odot O_1$ ⊠ $\odot O_2$ ⊠⊠⊠⊠⊠⊠⊠⊠14 ⊠ $7, \odot O_3$ ⊠ $\odot O_1$ $\odot O_2$ ⊠⊠⊠⊠⊠⊠⊠ O_3 ⊠⊠ ⊠ O_1 ⊠⊠⊠ O_2 ⊠⊠⊠⊠ $\odot O_4$ ⊠⊠⊠ $\odot O_1$ $\odot O_2$ ⊠⊠⊠⊠ $\odot O_3$ ⊠⊠⊠⊠ $\odot O_4$ ⊠⊠⊠⊠

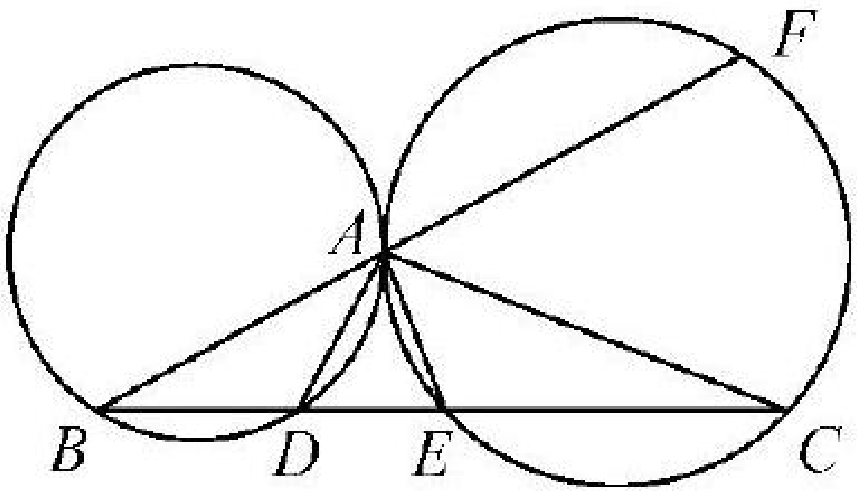


(☒ **5** ☒)

6 ☒☒☒ $ABCD$ ☒, $AB = 1, AD = \sqrt{3}$, ☒☒ B ☒☒☒, BA ☒☒☒☒☒☒☒ BC ☒ ☒ E, ☒ AE ☒☒☒☒ P, ☒ P ☒ $\odot B$ ☒☒☒☒ AD ☒☒ S, ☒ BC ☒☒ T, ☒ ST ☒☒ ☒☒ $ABCD$ ☒☒☒, ☒ ST ☒☒☒

7 ☒☒☒ D E ☒ $\triangle ABC$ ☒☒ BC ☒☒☒☒, ☒ F ☒ BA ☒☒☒☒, ☒ $\angle DAE = \angle CAF$, ☒☒☒$\triangle ABD$ ☒ $\triangle AEC$ ☒☒☒☒☒☒☒☒

8 ☒☒☒☒ a ☒☒☒☒☒☒☒5 ☒☒☒☒1 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒, ☒ a ☒☒☒☒☒



(⊠7⊠)

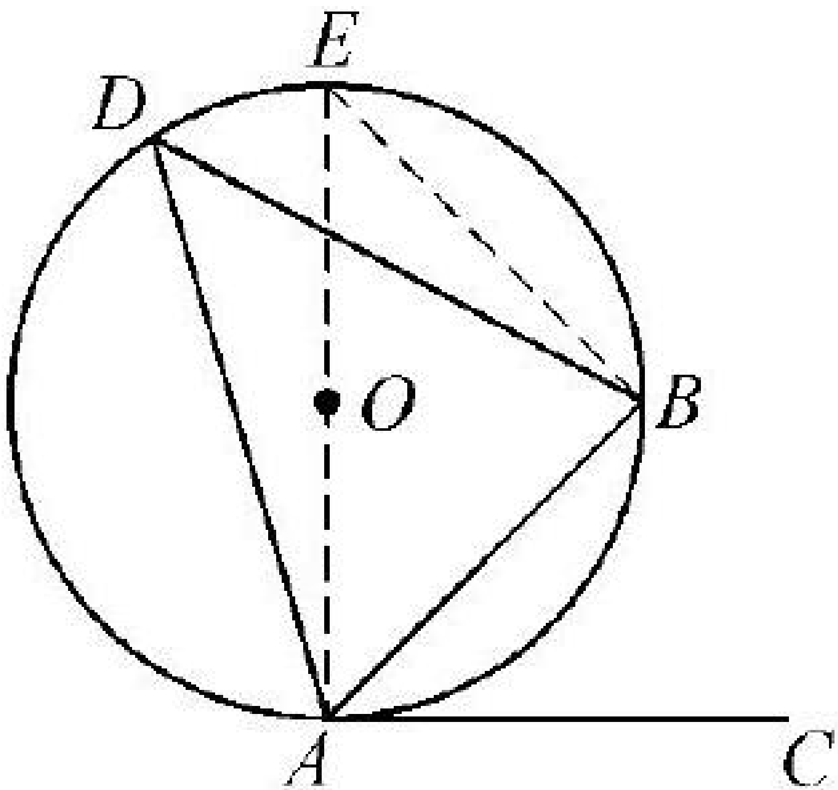
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠. ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠:

(1) ⊠⊠;

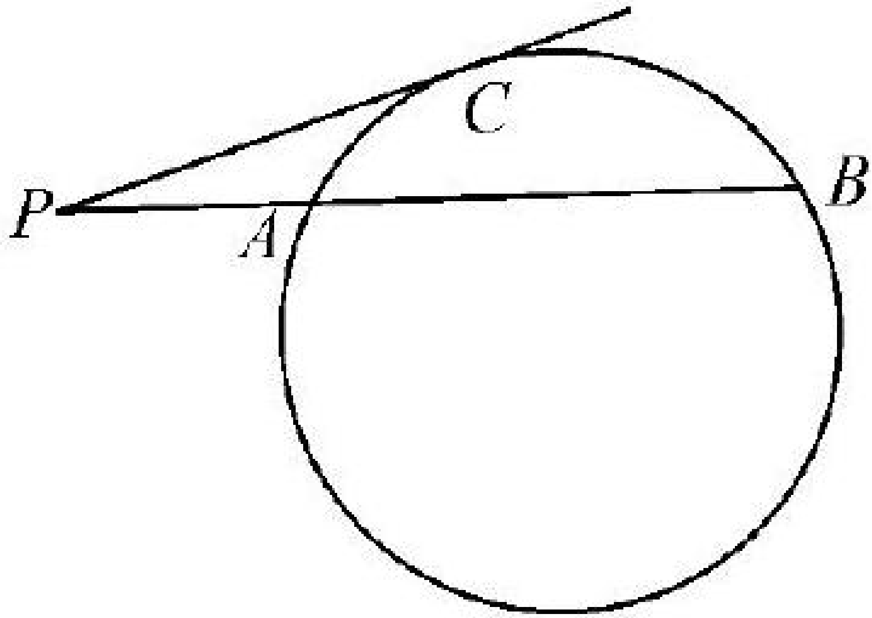
(2) ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠;

(3) ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠;

(4) ⊠⊠5-1, ⊠ A B D ⊠ $\odot O$ ⊠, ⊠ C ⊠ $\odot O$ ⊠, ⊠ $\angle BAC = \angle ADB$, ⊠ AC ⊠ $\odot O$ ⊠⊠⊠ (⊠⊠ AO ⊠⊠⊠ $\odot O$ ⊠⊠⊠⊠ E, ⊠⊠ BE, ⊠⊠⊠⊠);



☒5-1



⊠5-2

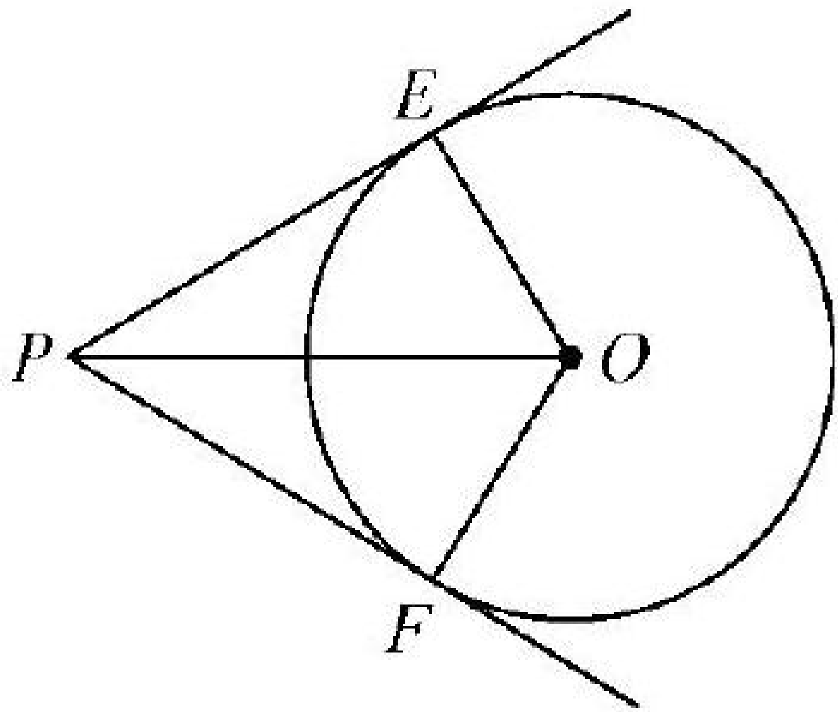
⊠5⊠⊠⊠5-2, ⊠ P ⊠⊠⊠, ⊠ C ⊠⊠⊠, PAB ⊠ $\odot O$ ⊠⊠⊠, ⊠ $PC^2 = PA \cdot PB$ ⊠⊠ PC ⊠ $\odot O$ ⊠⊠⊠⊠⊠⊠ AC BC ⊠⊠ $\triangle PAC \backsim \triangle PCB$ ⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠.

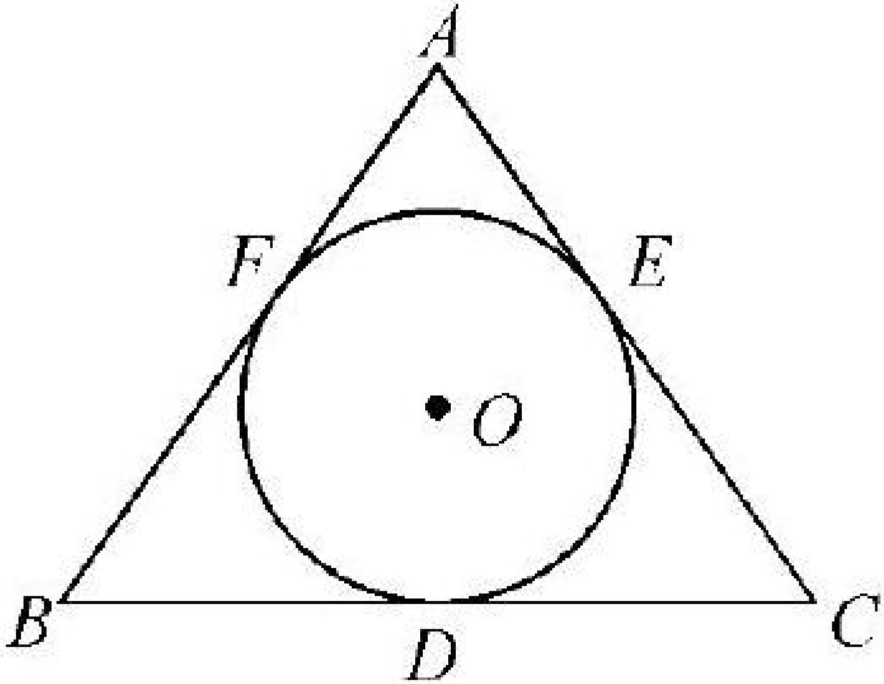
⊠⊠5-3⊠⊠ P ⊠⊠⊠⊠⊠⊠PE PF ⊠⊠⊠ $\odot O$ ⊠⊠ E F, ⊠⊠⊠ P ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠$PE = PF$ ⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

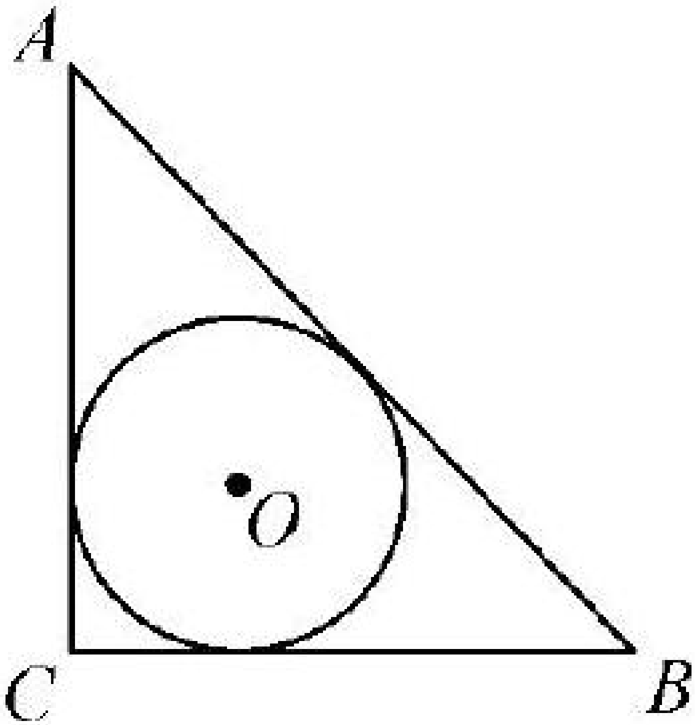
⊠1⊠⊠⊠5-4, $\triangle ABC$ ⊠⊠ $\odot O$ ⊠⊠ D E ⊠F, ⊠ $AB = c, BC = a, CA = b$, ⊠⊠⊠⊠⊠⊠ r,



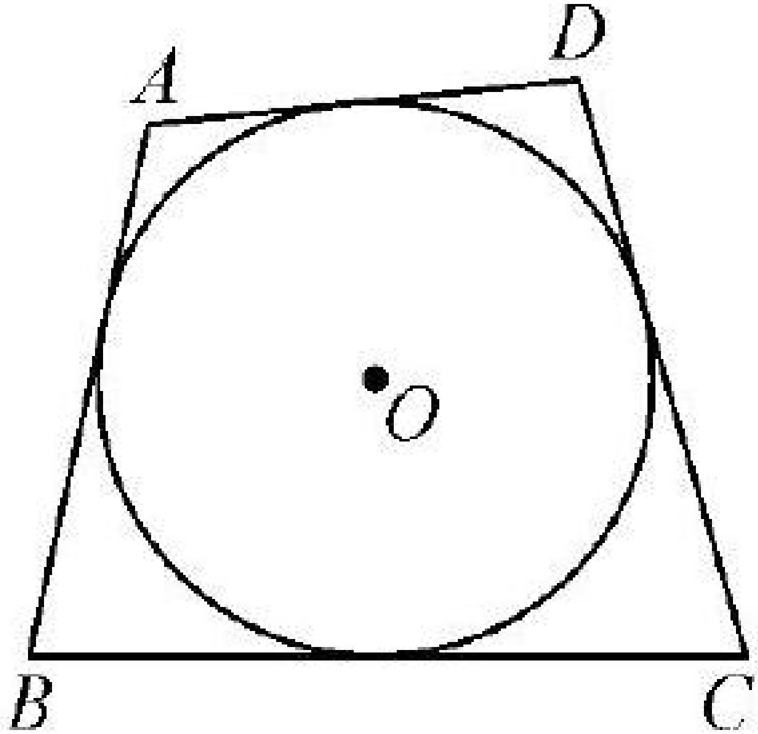
☒5-3



⊠5-4



☒5-5



⊠5-6

⊠ $AE = AF = \frac{b+c-a}{2}, BD = BF = \frac{c+a-b}{2}, CD = CE = \frac{a+b-c}{2}$ ⊠

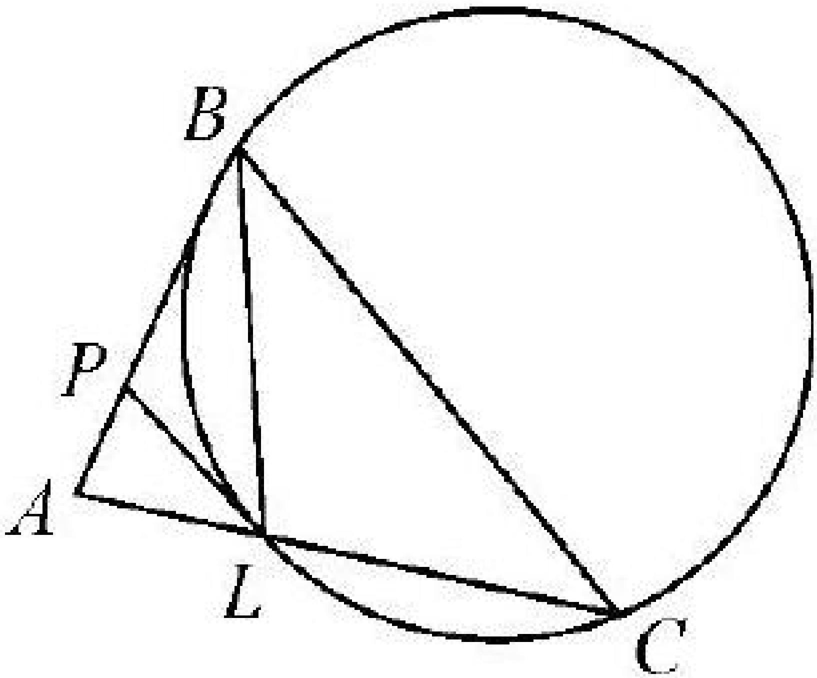
(2) ⊠⊠ $5-5$, Rt$\triangle ABC$ ⊠⊠ $\odot O$, ⊠⊠⊠⊠⊠⊠ $r, \angle C = 90^\circ, AB = c,\ AC = b, BC = a$ ⊠⊠ $r = \frac{a+b-c}{2}$ ⊠

⊠3⊠⊠⊠5-6⊠⊠⊠⊠⊠ $ABCD$ ⊠⊠ $\odot O$ ⊠⊠ $AB + CD = BC + AD$ ⊠⊠⊠⊠⊠⊠⊠⊠ ⊠ $ABCD$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠1 ⊠⊠5-7⊠⊠⊠BL ⊠⊠ $\angle ABC, PL$ ⊠ $\triangle BLC$ ⊠⊠⊠⊠⊠⊠ L, ⊠ AB ⊠⊠ P, ⊠⊠: AC ⊠ $\triangle BLP$ ⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠ $\angle ALP = \angle ABL$ ⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠ PL ⊠ $\triangle BLC$ ⊠⊠⊠⊠⊠ L, ⊠ $\angle PLB = \angle C$, ⊠

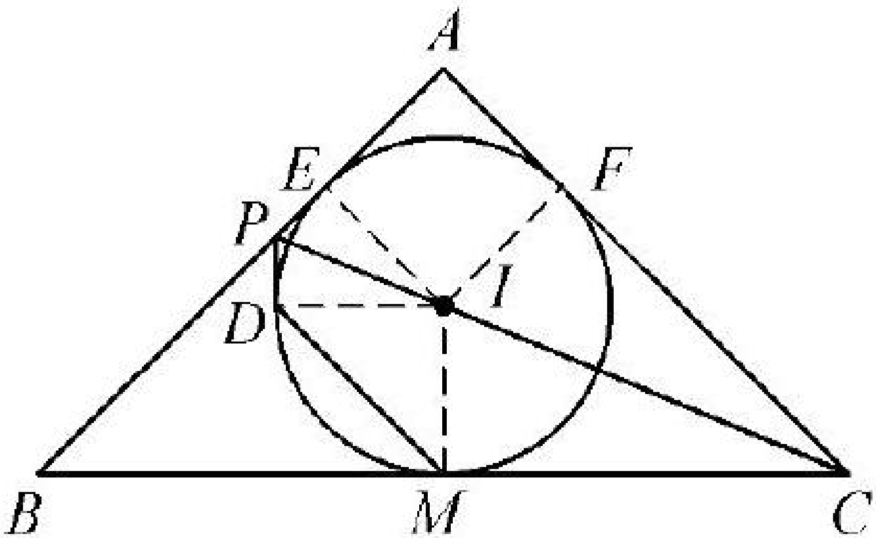


⊠5-7

$$\angle ALP + \angle PLB = \angle ALB = \angle LBC + \angle C,$$

⊠⊠ $\angle ALP = \angle LBC$.
⊠⊠ BL ⊠⊠ $\angle ABC$, ⊠ $\angle ABL = \angle LBC$, ⊠⊠ $\angle ALP = \angle ABL$.
⊠⊠ AC ⊠ $\triangle PBL$ ⊠⊠⊠⊠⊠ L ⊠
⊠2 ⊠⊠5-8⊠⊠⊠$\triangle ABC$ ⊠⊠$AB = AC, \angle C$ ⊠⊠⊠⊠⊠⊠ AB ⊠⊠ P, ⊠ M ⊠ $\triangle ABC$ ⊠⊠⊠⊠ I ⊠ BC ⊠⊠⊠⊠⊠⊠ $MD//\ AC$ ⊠ $\odot I$ ⊠⊠ D ⊠⊠⊠: PD ⊠ $\odot I$ ⊠⊠⊠⊠

⊠⊠⊠ AB ⊠ $\odot I$ ⊠⊠ E, AC ⊠ $\odot I$ ⊠⊠ F ⊠⊠⊠ $IE\ IF\ ID\ IM$ ⊠⊠ $IE \perp AB, IF \perp AC, IM \perp BC$, ⊠ $\angle BCA = 2x$, ⊠ $\angle FIC =$



⊠5-8

$\angle MIC = 90^\circ - x, \angle A = 180^\circ - 4x$ ⊠⊠⊠

$$\angle EIF = 360^\circ - \angle AEI - \angle AFI - \angle A = 4x$$

⊠⊠

$$\angle PIE = 180^\circ - \angle EIF - \angle FIC = 90^\circ - 3x.$$

⊠ $DM//AC$ ⊠⊠

⊠⊠

$$\angle DMC = 180^\circ - \angle ACM = 180^\circ - 2x,$$
$$\angle DMI = 90^\circ - 2x,$$

$$\angle DIM = 180^\circ - 2\angle DMI = 4x$$

⊠⊠

$$\angle DIP = 180^\circ - \angle DIM - \angle MIC = 90^\circ - 3x$$

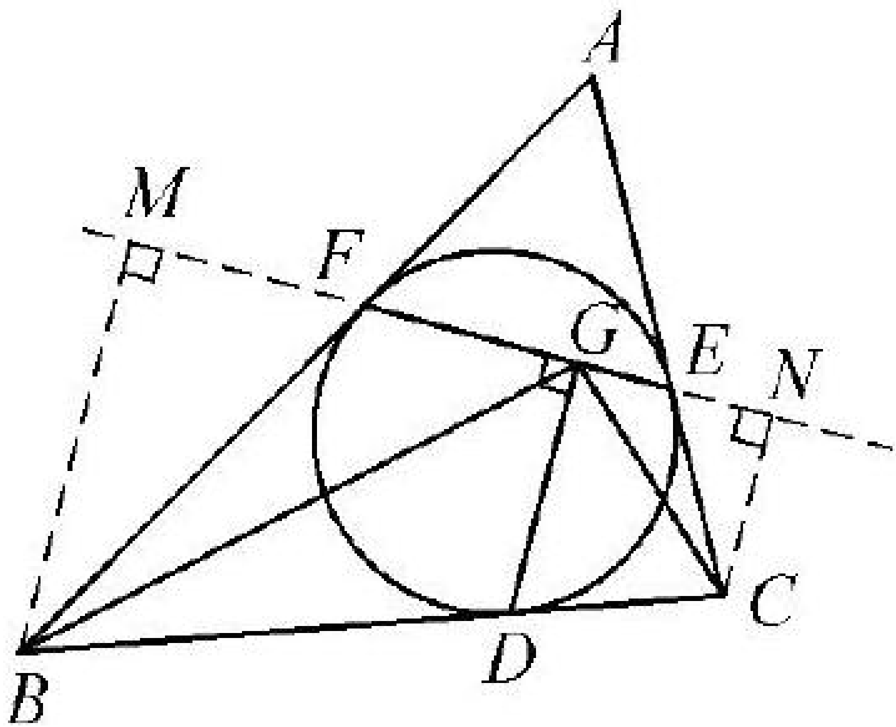
⊠⊠ $\angle PIE = \angle PID.$

⊠⊠ $IE = ID, IP$ ⊠⊠⊠⊠, ⊠ $\triangle PID \cong \triangle PIE$ ⊠⊠⊠ $\angle PDI = \angle PEI = 90^\circ$ ⊠

⊠ ID ⊠ $\odot I$ ⊠⊠⊠, D ⊠ $\odot I$ ⊠, ⊠ PD ⊠ $\odot I$ ⊠⊠⊠.
⊠⊠⊠⊠ PQ ⊠ $\odot I$ ⊠⊠ Q ⊠⊠⊠⊠ BC ⊠⊠ N ⊠⊠⊠ $\triangle NQM \backsim \triangle ABC$ ⊠⊠⊠⊠⊠ $MQ//AC$ ⊠⊠⊠ $MD//AC$ ⊠⊠⊠ Q ⊠ D ⊠⊠⊠⊠⊠ PD ⊠ $\odot I$ ⊠⊠⊠⊠

⊠3 ⊠⊠5-9 ⊠⊠, $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠ BC CA AB ⊠⊠ D E $F, DG \perp EF$ ⊠ G, ⊠⊠: DG ⊠⊠ $\angle BGC$.

⊠⊠⊠⊠⊠ $\angle AFE = \angle AEF$ ⊠⊠ $BM \perp EF$ ⊠⊠ $M, CN \perp EF$ ⊠⊠ N, ⊠ $\triangle BMF \backsim \triangle CNE$ ⊠⊠⊠⊠⊠ $BM//DG//CN$ ⊠⊠⊠ $\frac{MG}{NG}$ ⊠ $\frac{BF}{CE}$ ⊠⊠⊠⊠⊠.



⊠5-9

⊠⊠⊠ $BM \perp EF$ ⊠⊠ $M, CN \perp EF$ ⊠⊠ N, ⊠⊠ AF AE ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠ $AF = AE$ ⊠

⊠⊠ $\angle AFE = \angle AEF$ ⊠⊠⊠ $\angle BFM = \angle CEN$ ⊠⊠⊠ $\angle BMF = \angle CNE$, ⊠ $\triangle BMF \backsim \triangle CNE$ ⊠⊠⊠

$$\frac{BM}{CN} = \frac{BF}{CE} \tag{1}$$

⊠ BM DG CN ⊠⊠⊠⊠ EF, ⊠ $BM//DG//CN$, ⊠⊠ $\frac{MG}{NG} = \frac{BD}{DC}$.
⊠⊠⊠ BF BD CD CE ⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠. ⊠⊠ $BF = BD, CD = CE$ ⊠⊠⊠

$$\frac{MG}{NG} = \frac{BF}{CE} \tag{2}$$

☒ (1)☒(2) ☒ $\frac{BM}{CN}=\frac{MG}{NG}$, ☒☒☒ $\triangle BMG$ $\triangle CNG$ ☒☒☒☒☒☒☒, ☒☒ $\triangle BMG\backsim\triangle CNG$ ☒☒☒ $\angle BGM=\angle CGN$ ☒☒

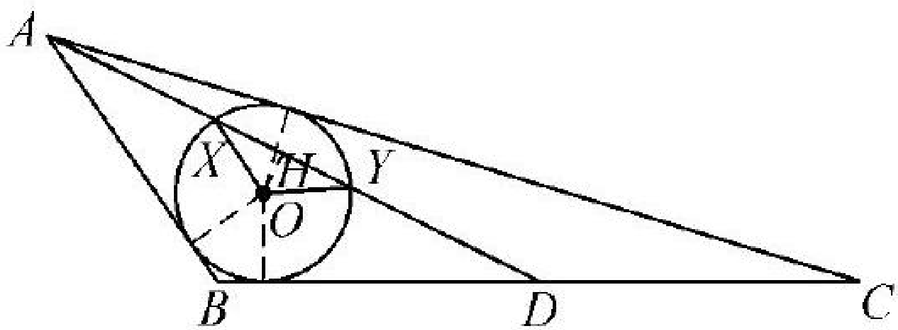
$$\angle BGD=90^\circ-\angle BGM=90^\circ-\angle CGN=\angle CGD,$$

☒ DG ☒☒ $\angle BGC$ ☒

☒☒☒☒☒☒☒, ☒ $\triangle BFG\backsim\triangle CEG$, ☒☒☒☒☒☒ $\frac{BF}{FG}=\frac{CE}{EG}$, ☒☒ DF DE, ☒☒ $\angle BFD=\angle DEG$, ☒☒☒ $BP\perp DF$ ☒☒ P, ☒ $\triangle BFP\backsim\triangle DEG$. ☒☒☒ $CQ\perp DE$ ☒☒ Q, ☒ $\triangle CEQ\backsim\triangle DFG$, ☒☒☒☒☒☒☒☒. ☒☒☒☒☒☒

☒4 ☒☒5-10☒☒☒☒☒☒ ABC ☒☒☒ AD ☒☒☒☒☒☒ O ☒☒☒☒ X ☒Y ☒☒ $AC=AB+AD$ ☒☒ $\angle XOY$ ☒

☒☒☒ $OH\perp XY$ ☒☒ H ☒☒☒ OH ☒☒☒ OX ☒☒☒☒☒☒☒☒☒☒☒☒☒☒



☒5-10

☒ $\odot O$ ☒☒☒ $\triangle ABC$, ☒☒☒☒☒☒☒

☒☒ $AB=c, BC=a, CA=b$, ☒ $AD=b-c$, ☒☒ $\odot O$ ☒☒☒☒ r, ☒ $OH\perp XY$ ☒☒ H, ☒ $S_{\triangle ABC}=\frac{r}{2}(a+b+c)$.

☒☒☒☒☒ $\frac{1}{2}S_{\triangle ABC}=S_{\triangle ABD}$

$$\begin{aligned}&=\frac{rc}{2}+\frac{r}{2}BD+\frac{OH}{2}\cdot AD\\&=\frac{rc}{2}+\frac{ra}{4}+\frac{OH}{2}(b-c)\end{aligned}$$

☒

$$\frac{r}{4}(a+b+c)=\frac{rc}{2}+\frac{ra}{4}+\frac{OH}{2}(b-c),$$

☒☒

$$\frac{r}{2}(b-c)=OH(b-c)$$

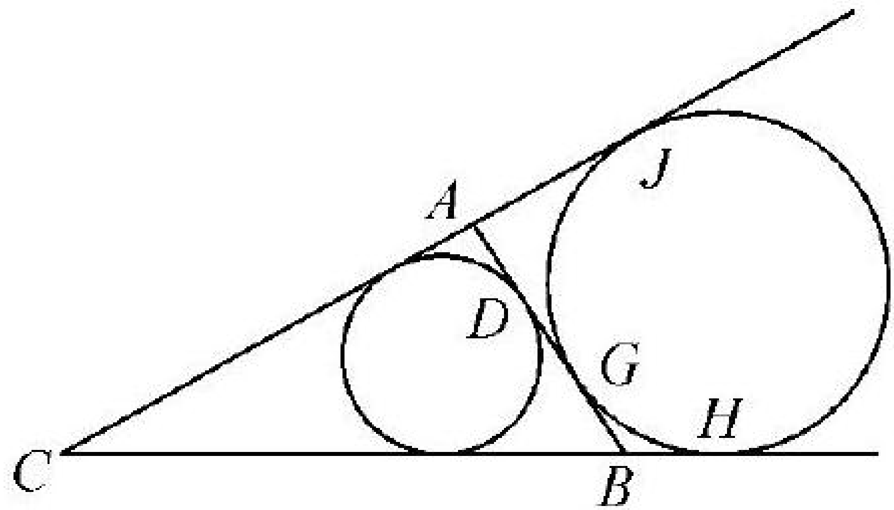
☒☒ $AC=AB+AD>AB$, ☒ $b>c$, ☒ $b-c\neq 0$, ☒☒ $OH=\frac{r}{2}$.
☒ $\angle XOH=60^\circ$, ☒☒ $\angle XOY=120^\circ$.

☒5 ☒☒5-11 ☒☒, $\triangle ABC$ ☒☒☒☒☒ AB ☒☒ D, AB ☒☒☒☒☒☒ AB ☒☒ $G, BC > AC$, ☒ $AD = DG = GB$, ☒☒: $AB = 3(BC - AB)$ ☒ $2AC > BC$.

☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒☒☒ $AB = c, BC = a, CA = b$, ☒☒☒☒☒☒☒ CA ☒☒ J, ☒☒☒ CB ☒☒ H ☒☒ $CJ = CH$ ☒

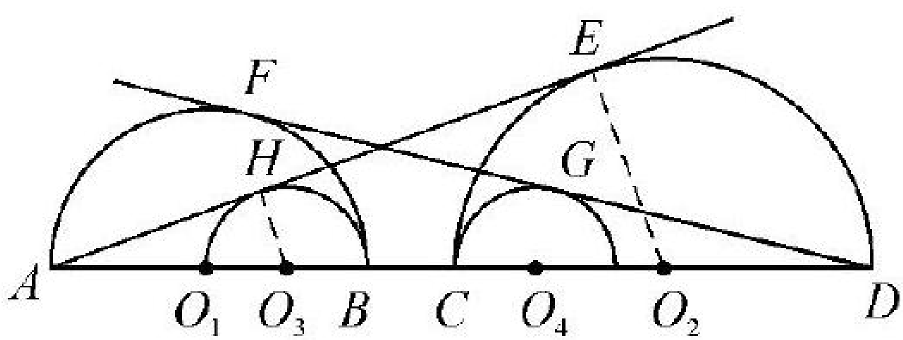
$$☒BH = BG = \frac{1}{3}c, AJ = AG = \frac{2}{3}c,$$



☒5-11

☒ $CJ = CH$, ☒ $b + \frac{2}{3}c = a + \frac{1}{3}c$, ☒ $c = 3(a - b), AB = 3(BC - AC)$.
☒☒ $a + b > c = 3(a - b)$, ☒☒ $2b > a$, ☒ $2AC > BC$.

☒6 ☒☒5-12☒☒☒ O_1 ☒☒☒ O_2 ☒☒☒☒☒☒☒☒ $AB\,CD$ ☒☒☒AE ☒ $\odot O_2$ ☒☒ E, DF ☒ $\odot O_1$ ☒☒ F, $\odot O_3$ ☒☒ $\odot O_1$ ☒☒ B, ☒ AE ☒☒ H, $\odot O_4$ ☒☒ $\odot O_2$ ☒☒ C, ☒ DF ☒☒ G. ☒☒☒$\odot O_3$ ☒ $\odot O_4$ ☒☒☒☒☒.



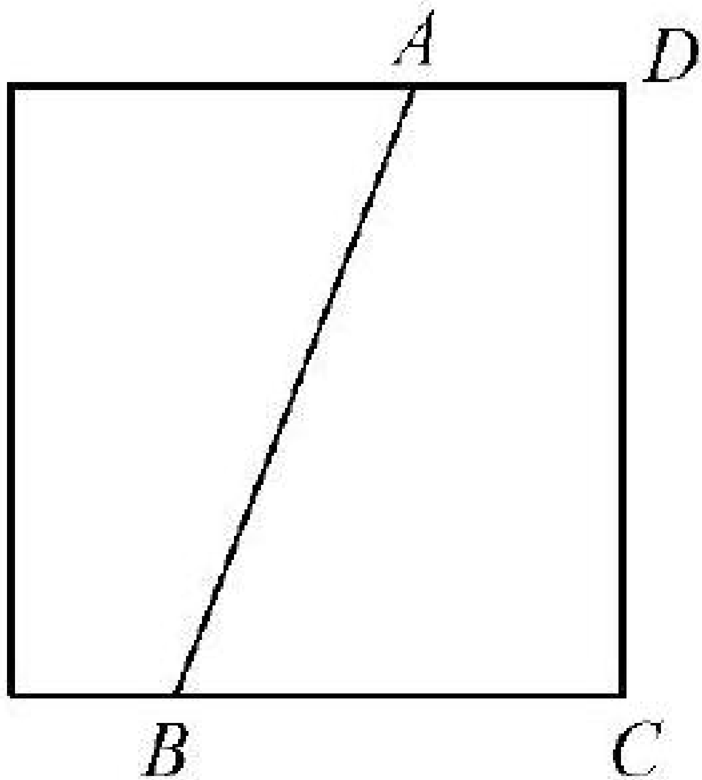
☒5-12

⊠⊠⊠⊠ O_3H O_2E ⊠⊠ $O_3H//O_2E$ ⊠⊠⊠⊠⊠⊠⊠ $\odot O_3$ ⊠⊠⊠⊠
⊠⊠⊠⊠ O_3H O_2E ⊠⊠ $BC = x, \odot O_i$ ⊠⊠⊠⊠ $r_i, i = 1, 2, 3, 4$ ⊠
⊠⊠ $AO_3 = 2r_1 - r_3, AO_2 = 2r_1 + x + r_2$, ⊠⊠ $O_3H \perp AE, O_2E \perp AE$, ⊠ $O_3H//O_2E$ ⊠⊠⊠ $\frac{O_3H}{O_2E} = \frac{AO_3}{AO_2}$ ⊠⊠ $\frac{r_3}{r_2} = \frac{2r_1-r_3}{2r_1+x+r_2}$ ⊠⊠⊠ $r_3 = \frac{2r_1r_2}{2r_1+2r_2+x}$.

⊠⊠⊠⊠⊠⊠ $r_4 = \frac{2r_1r_2}{2r_1+2r_2+x}$ ⊠⊠⊠ $r_3 = r_4$ ⊠
⊠⊠⊠

⊠7 ⊠⊠5-13 ⊠⊠⊠⊠⊠ AB ⊠⊠⊠⊠⊠⊠ n ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ A B ⊠⊠⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠6 cm ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠6 cm ⊠⊠⊠⊠⊠⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠



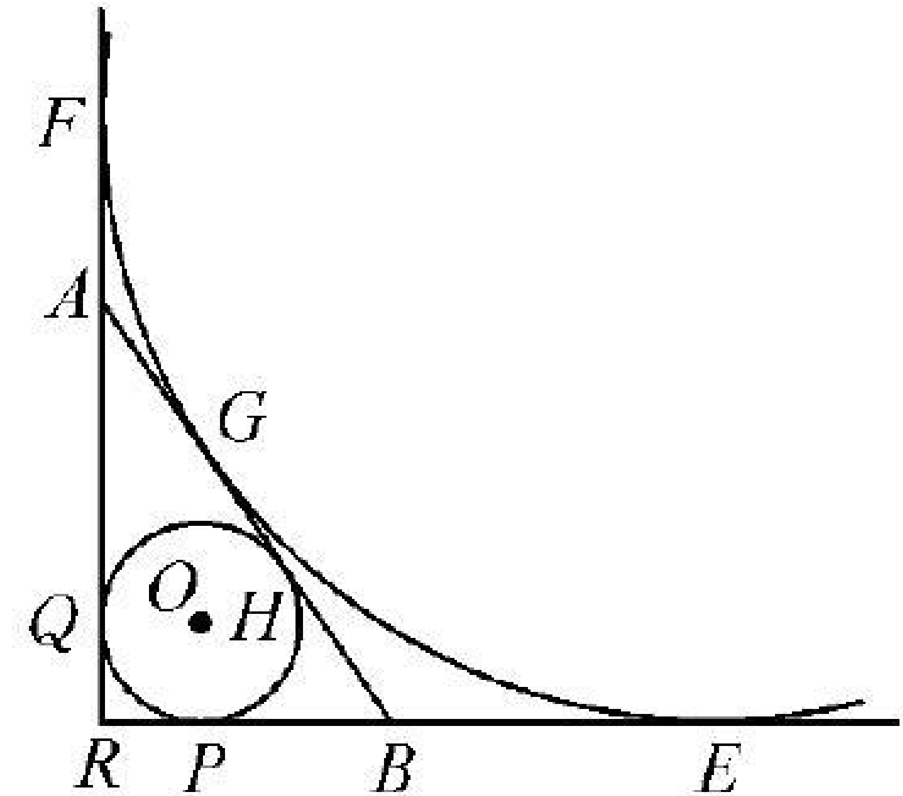
⊠5-13

⊠⊠1⊠⊠⊠⊠ AB ⊠⊠⊠

⊠2⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠5-13⊠⊠ $AB + CD > 2n > AD + BC$ ⊠⊠⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠3⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠ AB ⊠⊠ G, ⊠ A B ⊠⊠⊠⊠⊠⊠⊠⊠⊠ E F (⊠⊠5-14 ⊠⊠), ⊠ $RF = RE = \frac{n}{2}$. ⊠⊠⊠⊠⊠⊠⊠⊠⊠6 cm ⊠⊠ $\triangle ABR$ ⊠⊠⊠⊠⊠⊠ $< AR < RF = 6$ cm ⊠⊠⊠⊠⊠⊠

⊠⊠⊠
⊠ $\triangle ABR$ ⊠⊠⊠⊠⊠⊠6 cm , ⊠⊠⊠⊠⊠⊠



⊠5-14

RE ⊠⊠ P, ⊠ RF ⊠⊠ Q, ⊠ AB ⊠⊠ H, ⊠ $RQ = RP = 6$ cm ⊠⊠⊠

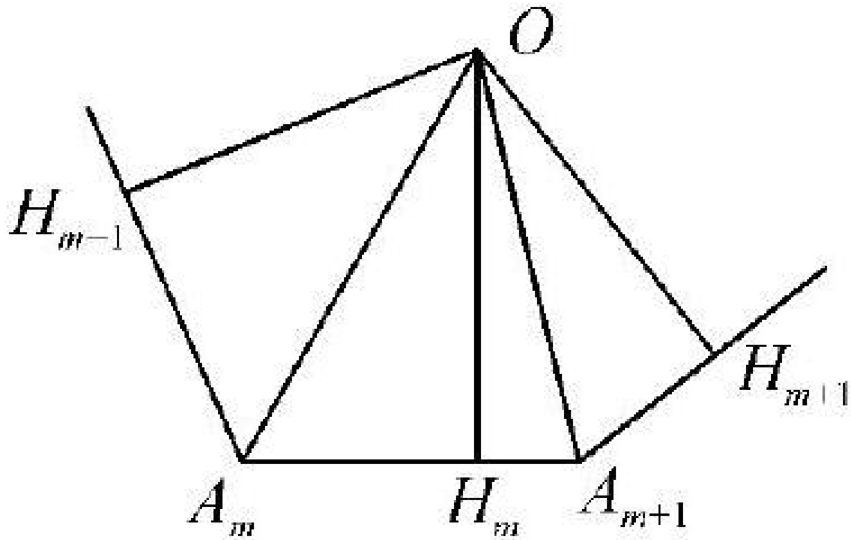
$$\begin{aligned} n - 2AB &= RF + RE - 2AB \\ &= FA + AQ + QR + RP + PB + BE - 2AB \\ &= AG + AH + BH + BG - 2AB + RP + RQ \\ &= AG + GB + AH + HB - 2AB + 12 \\ &= 12(\text{ cm}). \end{aligned}$$

⊠8 ⊠⊠5-15 ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $A_1A_2\cdots A_n$ ⊠⊠ O ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\angle OA_1A_n \leqslant \angle OA_1A_2, \angle OA_2A_1 \leqslant \angle OA_2A_3, \cdots, \angle OA_{n-1}A_{n-2} \leqslant \angle OA_{n-1}A_n, \angle OA_nA_{n-1} \leqslant \angle OA_nA_1$, ⊠⊠⊠⊠⊠⊠⊠⊠. ⊠⊠: O ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠.

⊠⊠⊠⊠ O ⊠ $A_{m-1}A_m, A_mA_{m+1}$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ H_{m-1}, H_m ⊠⊠⊠5-15⊠ ⊠⊠⊠⊠⊠⊠ $\angle OA_mA_{m-1} \leqslant \angle OA_mA_{m+1}$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠

$$\begin{aligned} OH_{m-1} &= OA_m \sin\angle OA_mA_{m-1} \\ &\leqslant OA_m \sin\angle OA_mA_{m+1} = OH_m \end{aligned}$$

⊠⊠⊠ $OH_1 \leqslant OH_2 \leqslant \cdots \leqslant OH_n \leqslant OH_1$,



⊠5-15
⊠⊠

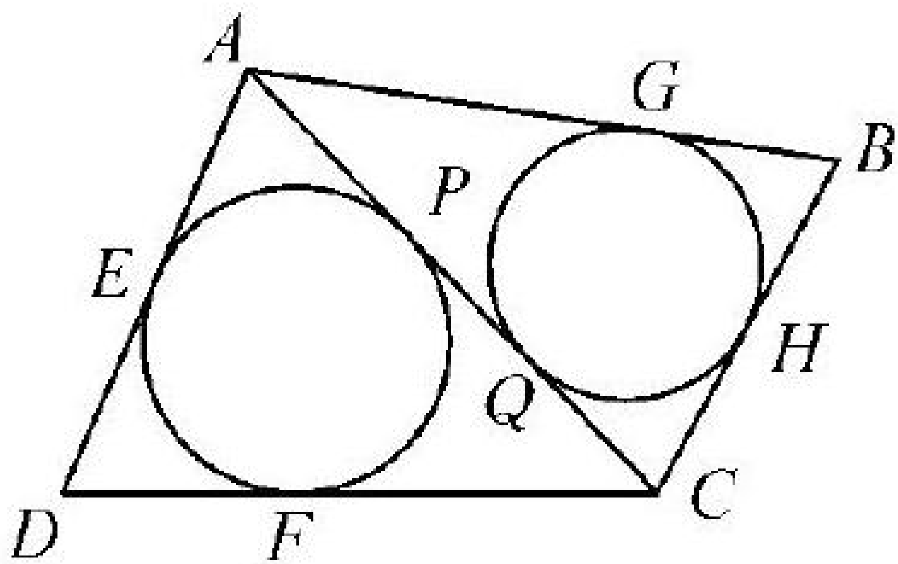
$$OH_1 = OH_2 = \cdots = OH_n$$

⊠⊠⊠⊠⊠⊠ $A_1A_2\cdots A_n$ ⊠⊠⊠⊠⊠⊠⊠⊠ O ⊠⊠⊠⊠ OH_1 ⊠

⊠9 ⊠⊠⊠⊠ $ABCD$ ⊠ $AB+CD=BC+AD$ ⊠⊠⊠⊠$\triangle ABC$ ⊠⊠⊠⊠⊠ $\triangle ACD$ ⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ AC ⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠5-16, ⊠ $\triangle ACD$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ E F P,$\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ G H ⊠ Q, ⊠⊠ P Q ⊠⊠ AC ⊠, ⊠ $AE=AP, AG=AQ, CQ=CH, CF=CP$ ⊠



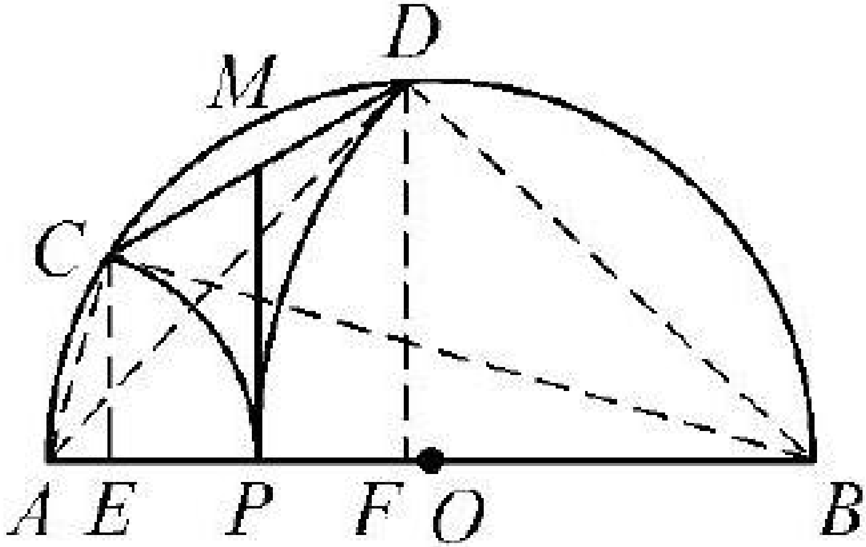
☒5-16
☒☒☒☒☒☒

$$
\begin{aligned}
PQ &= AQ - AP = AG - AE \\
&= (AB - GB) - (AD - DE) \\
&= AB - AD - GB + DE \\
&= AB - AD - BH + DF \\
&= AB - AD - (BC - CH) + (CD - CF) \\
&= AB - AD - BC + CD + CH - CF \\
&= AB + CD - BC - AD + CQ - CP \\
&= AB + CD - (BC + AD) - PQ
\end{aligned}
$$

☒

$$2PQ = AB + CD - (BC + AD).$$

☒ $AB + CD = BC + AD$ ☒☒☒ $PQ = 0$ ☒☒ P Q ☒☒☒
☒☒☒☒☒, AC ☒☒☒☒☒☒☒☒
☒10 ☒☒5-17, AB ☒☒☒ O ☒☒☒☒P ☒☒☒ AB ☒☒☒☒☒ A ☒☒☒ AP ☒☒☒☒☒☒ ☒☒ O ☒☒ C ☒☒ B ☒☒☒☒BP ☒☒☒☒☒☒☒☒☒ O ☒☒ D, M ☒ CD ☒☒☒ ☒☒☒MP ☒ $\odot A$ ☒ $\odot B$ ☒☒☒☒.
☒☒☒☒ AC BD AD BC, ☒☒ $CE \perp AB$



⊠5-17

⊠⊠ $E, DF \perp AB$ ⊠⊠ F, ⊠ $CE//DF$.

⊠⊠ AB ⊠⊠⊠, ⊠ $\angle ADB = \angle ACB = 90°$, ⊠⊠⊠Rt $\triangle ABC$ ⊠Rt $\triangle ADB$ ⊠⊠

$$PA^2 = AC^2 = AE \cdot AB, PB^2 = BD^2 = BF \cdot AB$$

⊠⊠

$$PA^2 - PB^2 = AB(AE - BF),$$

⊠

$$PA^2 - PB^2 = (PA + PB)(PA - PB) = AB(PA - PB),$$

⊠⊠

$$AE - BF = PA - PB,$$

⊠

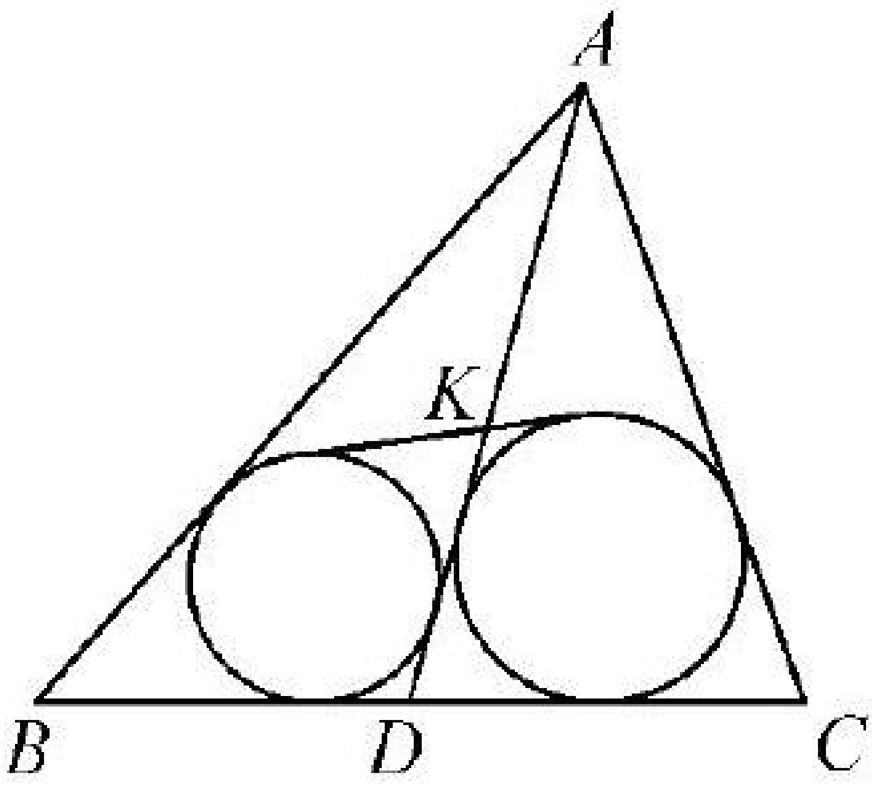
$$PA - AE = PB - BF, \text{⊠⊠} PE = PF.$$

⊠ MP ⊠⊠⊠ $CEFD$ ⊠⊠⊠⊠⊠⊠⊠ $CE//DF$ ⊠⊠ $MP \perp EF$ ⊠⊠⊠ MP ⊠ $\odot A$ ⊠ $\odot B$ ⊠⊠⊠⊠⊠

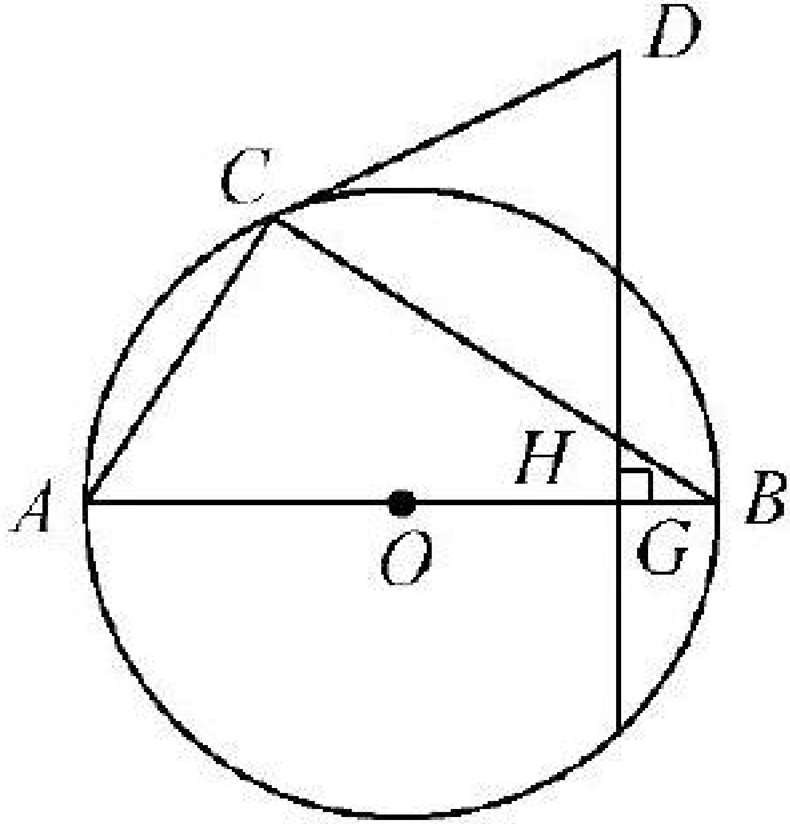
习 题 5

1.$\triangle ABC$ ⊠, $AB=c, BC=a, CA=b, D$ ⊠ BC ⊠⊠⊠⊠, $\triangle ABD$ ⊠$\triangle ADC$ ⊠⊠⊠⊠⊠⊠⊠ BC ⊠⊠⊠⊠⊠⊠ AD ⊠⊠ K, ⊠ AK ⊠

2 ⊠ $\triangle ABC$ ⊠⊠⊠⊠ AM ⊠⊠⊠⊠⊠⊠⊠" $\triangle ABM$ ⊠⊠⊠⊠⊠⊠⊠ $\triangle ACM$ ⊠⊠⊠⊠⊠⊠⊠⊠" ⊠⊠⊠⊠



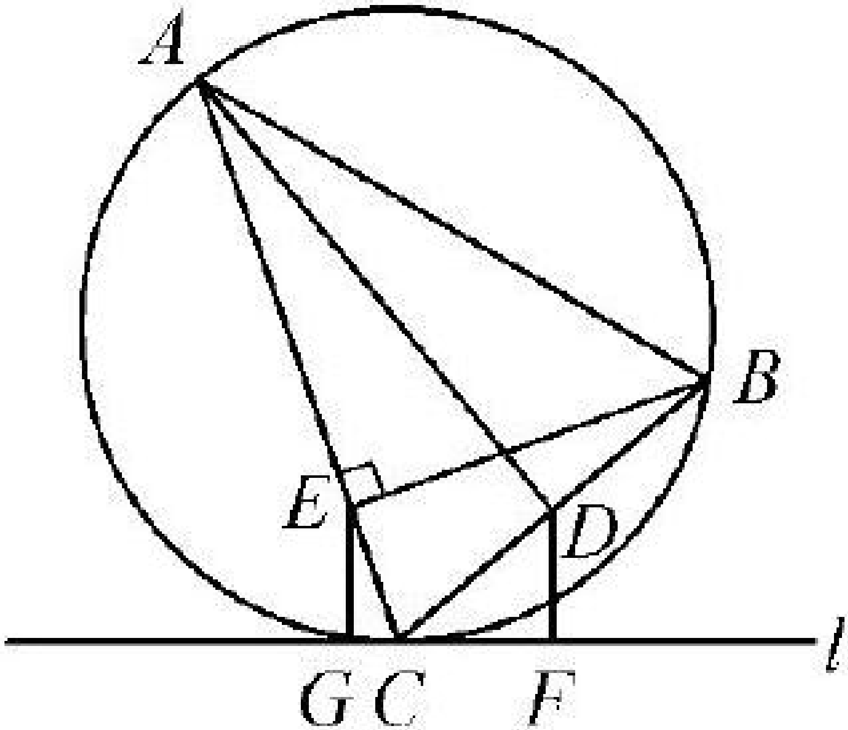
(⊠1 ⊠)



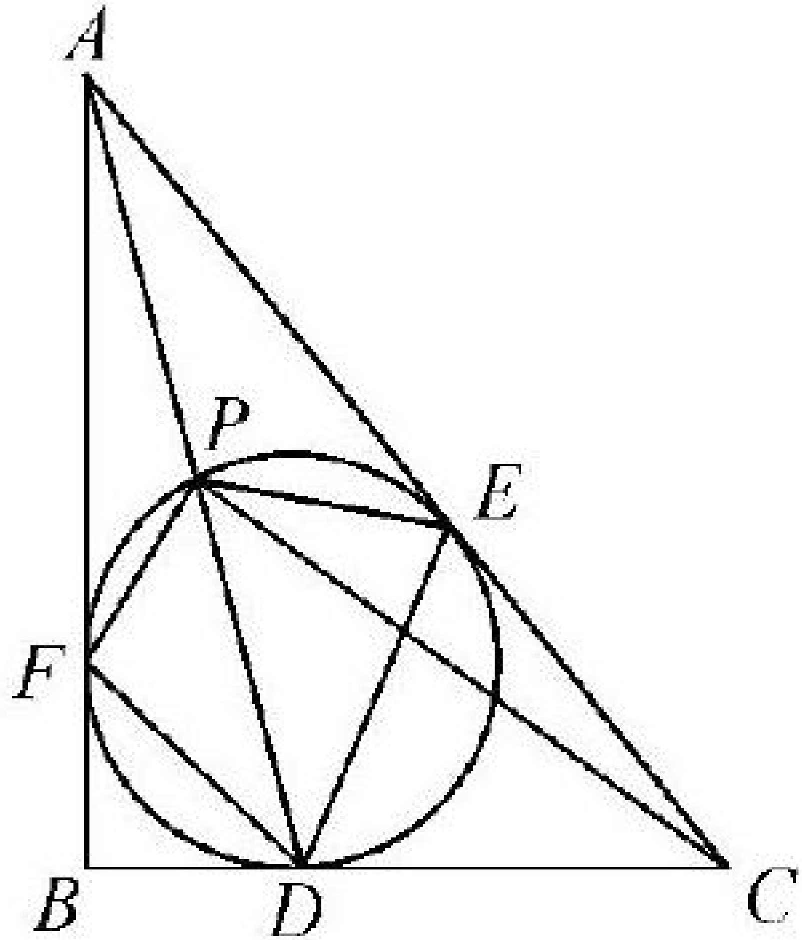
(⊠3 ⊠)

3 ⊠⊠ AB ⊠ $\odot O$ ⊠⊠⊠⊠BC ⊠ $\odot O$ ⊠⊠⊠⊠ BC ⊠⊠⊠ H ⊠ $HG \perp AB$ ⊠⊠ G, ⊠ D ⊠⊠⊠⊠, ⊠ $DH = DC$ (⊠⊠), ⊠⊠: DC ⊠ $\odot O$ ⊠⊠⊠.

4 ⊠⊠⊠⊠ l ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠ $C, AD \perp BC$ ⊠⊠ $D, BE \perp AC$ ⊠⊠ $E, EG \perp l$ ⊠⊠ $G, DF \perp l$ ⊠⊠ F. ⊠⊠: $EG = DF$.



(⊠4⊠)

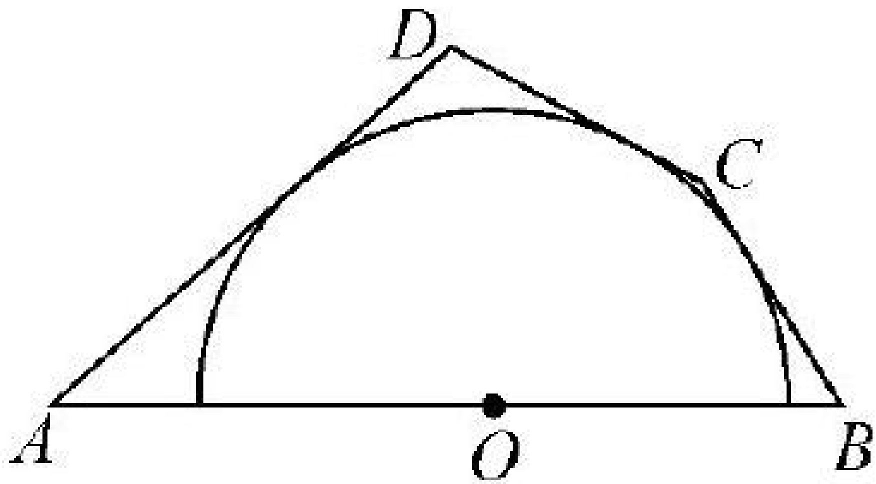


(⊠ **5** ⊠)

5Rt$\triangle ABC$ ⊠, $\angle B = 90°$. ⊠⊠⊠⊠⊠⊠⊠ BC CA AB ⊠⊠ D E F, ⊠ AD ⊠⊠⊠⊠ ⊠⊠⊠⊠ P, ⊠⊠ PC PE PF, ⊠ $PC \perp PF$. ⊠⊠⊠⊠1⊠$\triangle PFD \backsim \triangle PDC$ ⊠
(2) $PE \cdot DC = PD \cdot DE$.

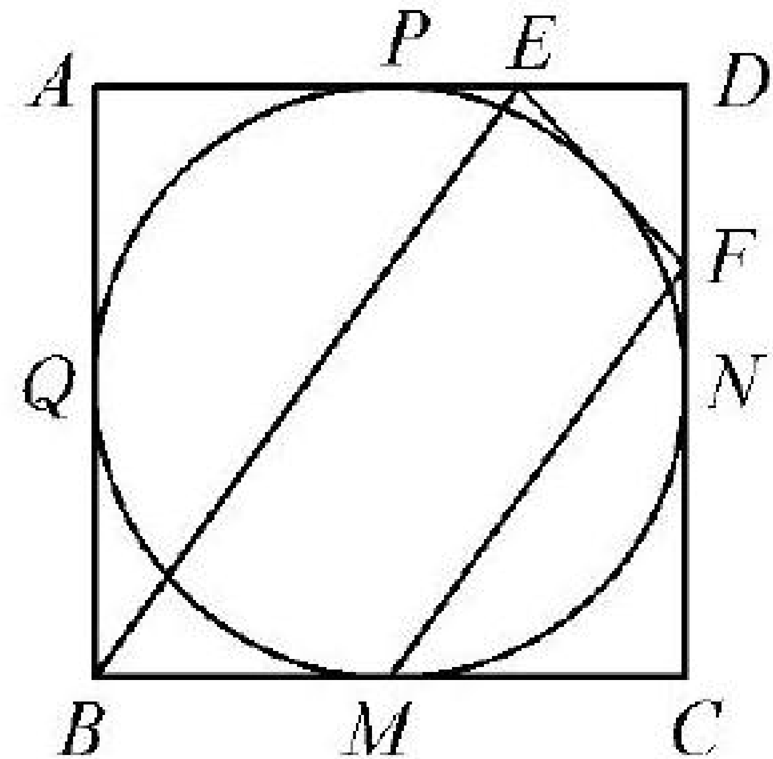
6 ⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ O ⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AD + BC = AB$.

7Rt$\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠ AB ⊠ D. ⊠ $AD = 10, BD = 3$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠

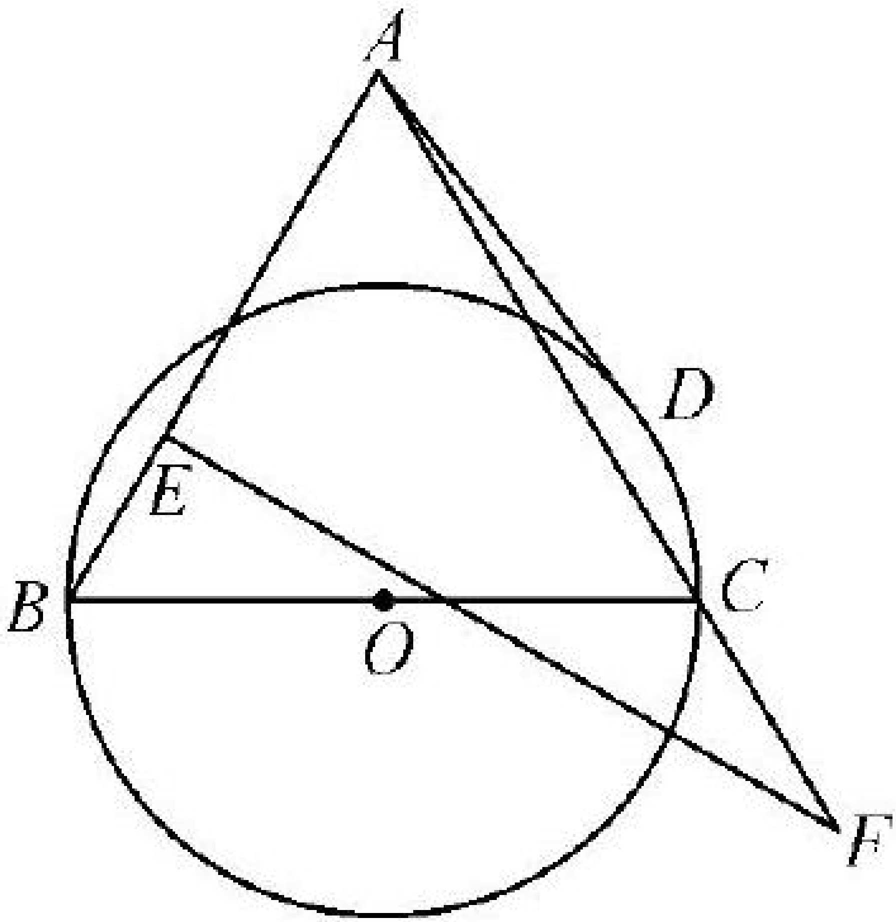


(⊠6 ⊠)

8 ⊠⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠⊠⊠⊠⊠ BC CD DA AB ⊠ M N P Q, EF ⊠⊠⊠⊠ ⊠⊠⊠⊠⊠ AD ⊠ E, DC ⊠ F. ⊠⊠: $BE//MF$.



(⊠8 ⊠)



(⊠9 ⊠)

9$\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠ BC ⊠⊠⊠⊠ $\odot O, AD$ ⊠ $\odot O$ ⊠ $D.E$ ⊠ AB ⊠⊠⊠⊠⊠ $EF \perp AB$ ⊠ AC ⊠⊠⊠⊠ F ⊠⊠ $\frac{AB}{AF} = \frac{AE}{AC}$. ⊠⊠: $AD = AE$.

10 ⊠⊠ B_1 C_1 ⊠⊠⊠ $\triangle ABC$ ⊠⊠ AC AB ⊠, BB_1 ⊠ CC_1 ⊠⊠⊠ D, ⊠⊠: ⊠⊠⊠⊠ AB_1DC_1 ⊠⊠⊠⊠, ⊠ $\triangle ABD$ ⊠⊠⊠⊠⊠ $\triangle ACD$ ⊠⊠⊠⊠⊠⊠⊠⊠

" ⊠⊠⊠⊠" ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠

" ⊠⊠⊠⊠" ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ (⊠⊠⊠⊠) ⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠. ⊠⊠" ⊠⊠⊠⊠" ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

(1) ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠;

⊠2⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

(3) ⊠⊠ C D ⊠⊠⊠ AB ⊠⊠⊠, ⊠ $\angle ACB = \angle ADB$, ⊠ A B C D ⊠⊠⊠⊠;

(4) ⊠⊠⊠⊠ AB, CD ⊠⊠⊠⊠ E, ⊠ $AE \cdot EB = CE \cdot ED$, ⊠ A B C ⊠D ⊠⊠⊠⊠⊠

(5) ⊠⊠⊠⊠ PA PB ⊠⊠⊠⊠⊠⊠ P A B ⊠⊠ C D, ⊠ $PA \cdot PC = PB \cdot PD$, ⊠ A B C D ⊠⊠⊠⊠.

☒1 ☒ $\triangle ADE$ ☒☒☒☒ O, ☒ BC ☒☒☒ AD AE ☒☒☒ F G, ☒ $AB = AC$ ☒☒☒☒F, D, E, G ☒☒☒☒☒

☒☒☒☒☒☒☒☒☒☒☒☒, ☒☒, ☒☒☒☒☒☒☒☒☒, ☒☒☒☒ $AF \cdot AD = AG \cdot AE$ ☒☒☒☒ F, D, E, G ☒☒☒☒

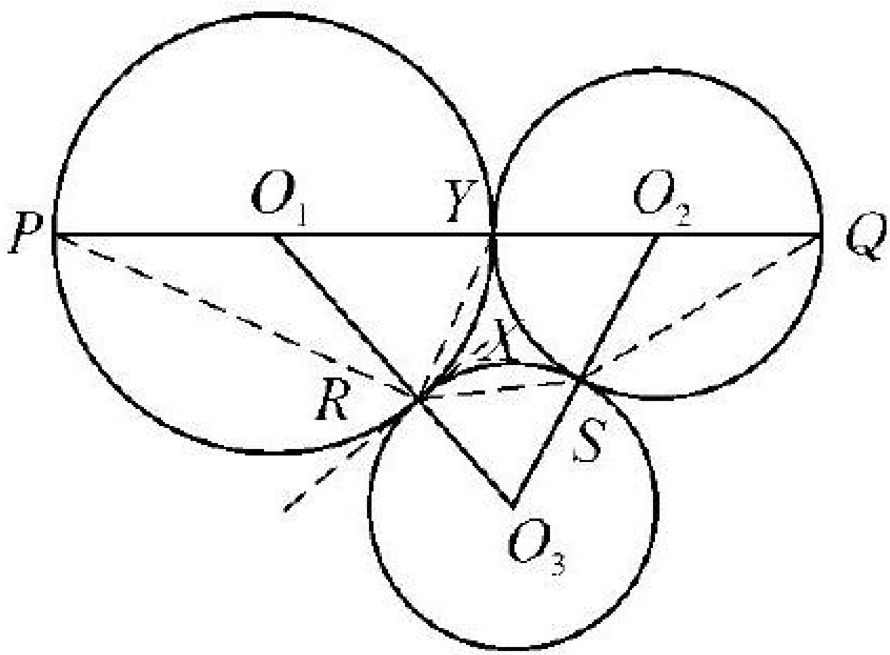
☒☒☒ $AM \perp BC$ ☒☒ M, ☒☒ O ☒ N, ☒☒ $AB = AC$, ☒ AN ☒☒☒ O ☒☒☒, ☒☒ $\angle FDN = \angle FMN = 90^\circ$, ☒☒ F, D, N, M ☒☒☒☒, ☒☒☒ $AD \cdot AF = AN \cdot AM$ ☒☒☒☒$AG \cdot AE = AN \cdot AM$ ☒☒☒ $AD \cdot AF = AG \cdot AE$, ☒ F, D, E, G ☒☒☒☒.

☒2 ☒ $\odot O_1, \odot O_2, \odot O_3$ ☒☒☒☒, Y ☒ $\odot O_1$ ☒ $\odot O_2$ ☒☒☒, R S ☒☒☒ $\odot O_1$ $\odot O_2$ ☒ $\odot O_3$ ☒☒☒, ☒☒☒ O_1O_2 ☒ $\odot O_1$ ☒☒ P, ☒ $\odot O_2$ ☒☒ Q. ☒☒: P, Q, R, S ☒☒☒☒.

☒☒☒☒6-1, ☒☒☒☒ $\angle Q$ ☒ $\angle PRS$ ☒☒, ☒ P, R, S, Q ☒☒.

☒☒☒ RY, PR, RS, SQ, ☒☒ $\odot O_1$ ☒ $\odot O_3$ ☒☒☒☒ RX ☒☒☒☒☒☒ $PRSQ$ ☒☒

$$\begin{aligned}
\angle Q &= \frac{1}{2}\angle O_1O_2O_3, \\
\angle PRS &= \angle PRY + \angle YRX + \angle XRS \\
&= 90^\circ + \angle P + \frac{1}{2}\angle O_3 \\
&= 90^\circ + \frac{1}{2}\angle O_2O_1O_3 + \frac{1}{2}\angle O_3,
\end{aligned}$$



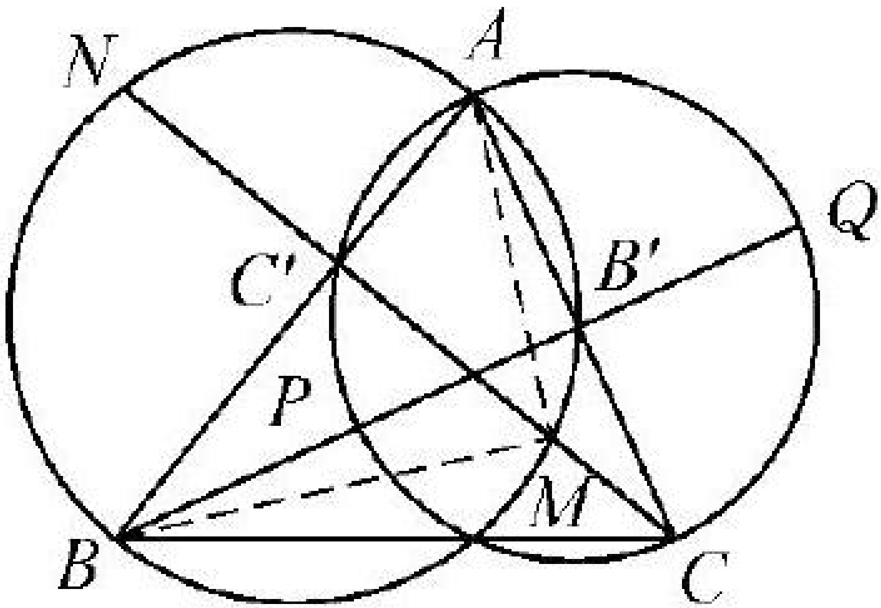
☒6-1
☒☒

$$\begin{aligned}\angle Q+\angle PRS &= 90^\circ+\frac{1}{2}\left(\angle O_1O_2O_3+\angle O_2O_1O_3+\angle O_3\right)\\ &= 90^\circ+90^\circ=180^\circ\end{aligned}$$

⊠⊠ P Q R S ⊠⊠⊠⊠⊠

⊠3 ⊠⊠6-2⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ABC ⊠⊠ AB ⊠⊠⊠⊠⊠⊠ AB ⊠⊠⊠⊠⊠ CC' ⊠⊠⊠ ⊠⊠⊠⊠⊠ M N ⊠⊠ AC ⊠⊠⊠⊠⊠⊠⊠ AC ⊠⊠⊠⊠⊠ BB' ⊠⊠⊠⊠⊠⊠⊠⊠⊠ P Q. ⊠⊠⊠ M N ⊠P Q ⊠⊠⊠⊠⊠

⊠⊠⊠⊠ AB ⊠ AC ⊠⊠⊠⊠⊠⊠⊠⊠⊠



⊠6-2

MN ⊠ PQ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ AB ⊠ AC. ⊠ $AM=AN, AP=AQ$.

⊠⊠ AM BM, ⊠Rt $\triangle ABM$ ⊠, MC' ⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠ ⊠⊠

$$\begin{aligned}AM^2&=AC'\cdot AB,\\ AP^2&=AB'\cdot AC\end{aligned}$$

⊠⊠ B,C,B',C' ⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠

$$AC'\cdot AB=AB'\cdot AC,$$

⊠ $AM^2=AP^2$ ⊠⊠ $AM=AP$ ⊠

⊠⊠ M N P Q ⊠⊠⊠⊠⊠ A ⊠⊠⊠⊠⊠⊠⊠⊠⊠.

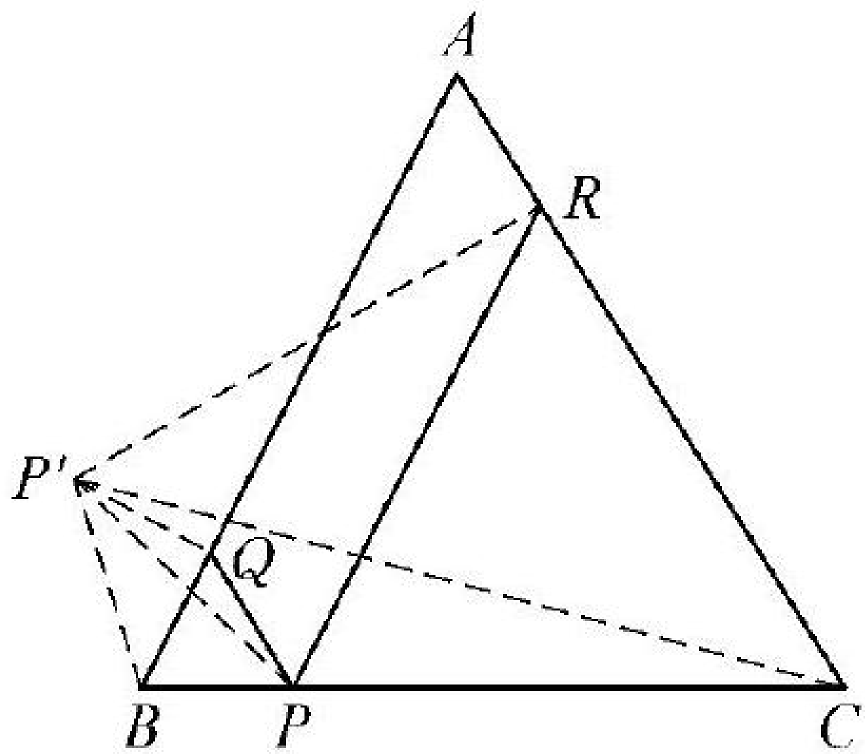
⊠4 ⊠⊠6-3⊠⊠⊠⊠⊠⊠⊠⊠ ABC ⊠⊠P ⊠⊠⊠ BC ⊠⊠⊠⊠⊠⊠⊠⊠⊠ P ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠ AB AC ⊠⊠⊠ Q R ⊠⊠, ⊠⊠ P' ⊠⊠ P ⊠⊠⊠⊠ QR ⊠⊠⊠⊠, ⊠⊠: ⊠ P' ⊠⊠⊠⊠ ABC ⊠⊠⊠⊠⊠.

⊠⊠⊠⊠⊠ A, P', B, C ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\angle BP'C = \angle BAC$ ⊠

⊠⊠⊠⊠ $P'B, P'C, P'Q, P'R$ ⊠⊠⊠ $PQ//AC, PR//AB$ ⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠ ⊠⊠⊠⊠

$$QP = QB = QP', RP = RC = RP'$$

⊠⊠, ⊠ Q ⊠ $\triangle PBP'$ ⊠⊠⊠, ⊠ R ⊠



⊠6-3

$\triangle PP'C$ ⊠⊠⊠. ⊠⊠

$$\angle BP'P = \frac{1}{2}\angle BQP = \frac{1}{2}\angle A,$$
$$\angle CP'P = \frac{1}{2}\angle CRP = \frac{1}{2}\angle A$$

⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠

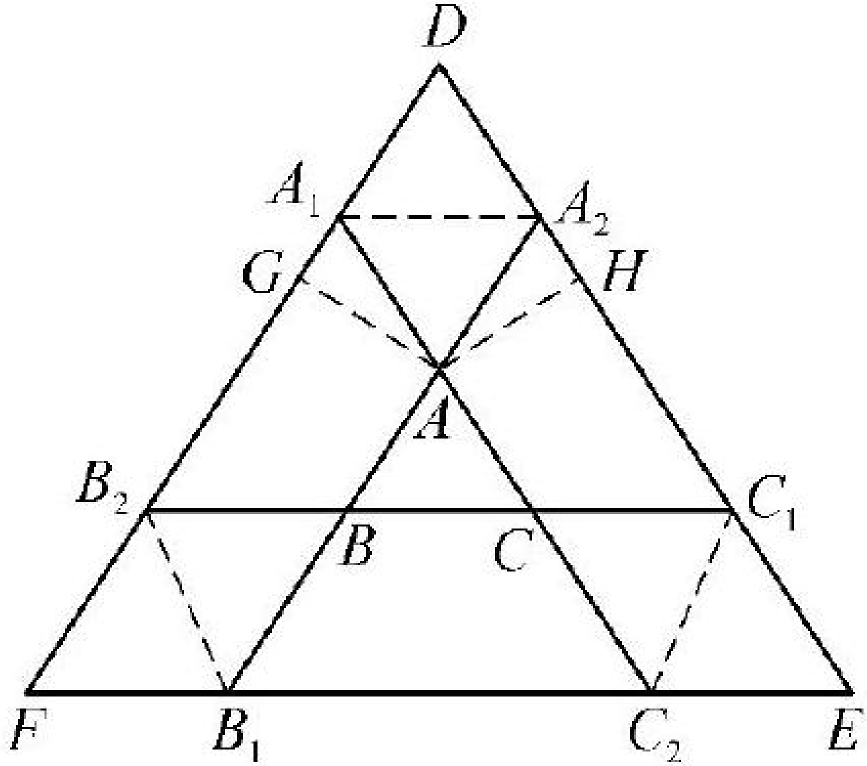
$$\angle BP'P + \angle CP'P = \angle A,$$
$$\angle BP'C = \angle A$$

⊠⊠, A, P', B, C ⊠⊠⊠⊠.

⊠5 ⊠⊠6-4, $AB//DF$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AB; AC//DE$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AC; BC//EF$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ BC ⊠⊠⊠⊠$A_1, B_1, C_1, A_2, B_2, C_2$ ⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠⊠⊠ A_1, A_2, C_1, B_2 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠ $AB//DF$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AB; AC//DE$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ AC ⊠ ⊠⊠ $AG \perp DF$ ⊠⊠ $G, AH \perp DE$ ⊠⊠ H ⊠⊠



⊠6-4

⊠⊠ $\angle GA_1A = \angle HA_2A = \angle D$ ⊠⊠ $AG = A_1A\sin\angle D, AH = A_2A\sin\angle D$, ⊠⊠ $\frac{AA_1}{AB} = \frac{AA_2}{AC}$, ⊠⊠ $\triangle AA_1A_2 \backsim \triangle ABC$, ⊠ $\angle AA_2A_1 = \angle ACB = \angle E$ ⊠⊠⊠

$$\begin{aligned}\angle A_1A_2C_1 + \angle A_1B_2C_1 &= \angle A_1A_2A + \angle AA_2C_1 + A_1B_2C_1\\ &= \angle E + \angle D + \angle F = 180^\circ\end{aligned}$$

⊠ A_1, A_2, C_1, B_2 ⊠⊠⊠⊠, ⊠⊠ A_1, A_2, B_1, B_2 ⊠⊠⊠⊠, A_1, A_2, C_1, C_2 ⊠⊠⊠⊠⊠⊠⊠ $A_1, B_1, C_1, A_2, B_2, C_2$ ⊠⊠⊠⊠.

☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ A, B ☒C, D ☒☒☒☒, ☒☒ B, C, D, E ☒☒☒☒, ☒☒ A, B, C, D, E ☒☒☒☒.

☒6 ☒☒ $PQRS$ ☒☒☒☒☒☒☒, $\angle PSR = 90°$, ☒☒ Q ☒ PR PS ☒☒☒☒☒☒☒☒ ☒☒ H K ☒☒☒☒HK ☒☒ QS ☒

☒☒1 ☒ HK ☒ QS ☒☒☒☒ T ☒☒☒6-5☒☒

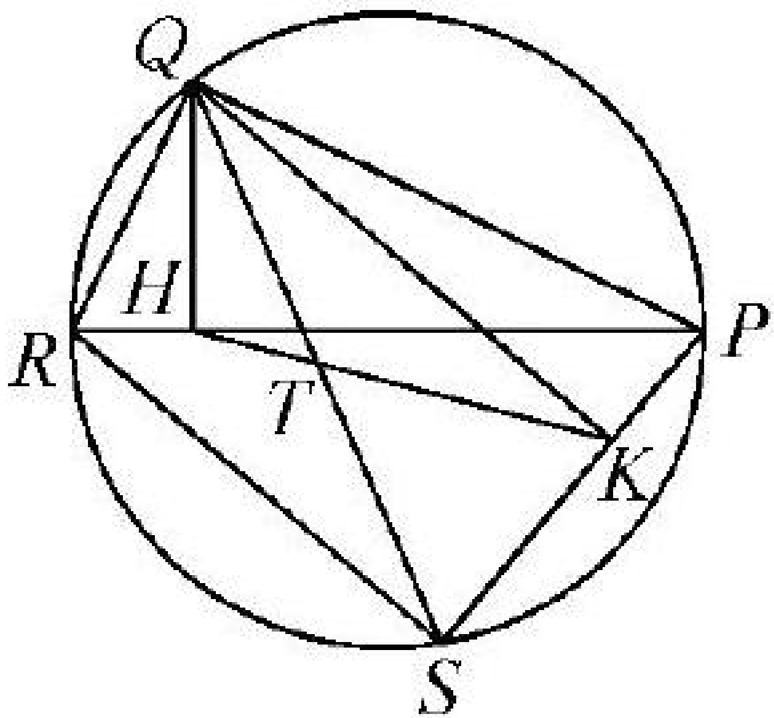
$$\angle TSK = 90° - \angle RSQ = 90° - \angle RPQ = \angle TKS,$$

☒☒ $TS = TK$ ☒

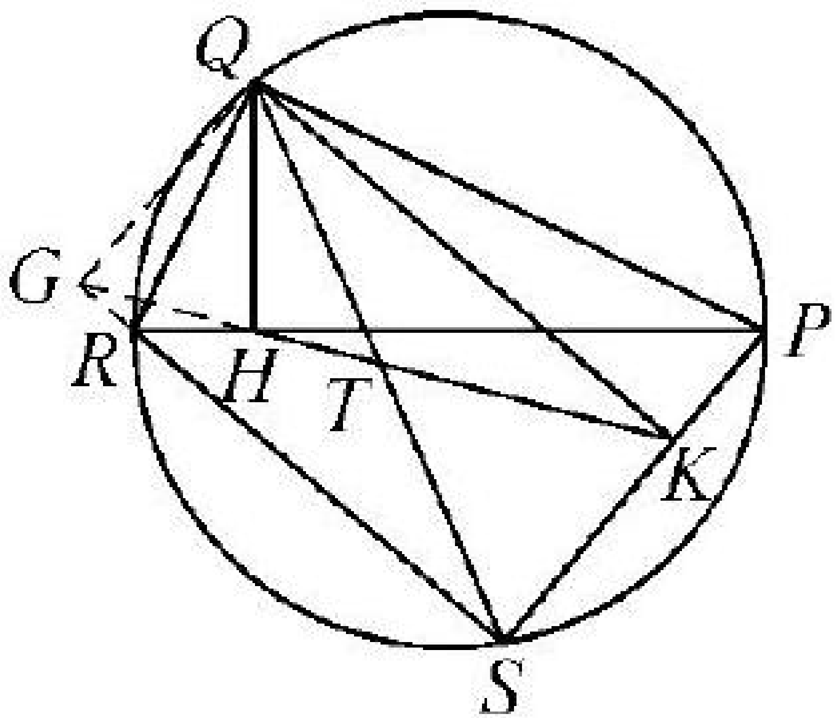
☒ $\angle TQK = 90° - \angle TSK = 90° - \angle TKS = \angle TKQ$ ☒☒☒ $TQ = TK$ ☒

☒ $TS = TQ$ ☒☒ HK ☒☒ QS ☒

☒☒☒☒1☒☒☒☒ T ☒☒☒☒☒☒ QSK ☒☒☒☒☒☒☒☒☒☒☒☒☒ $TS = TK$ ☒ $TS = TQ$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒2 ☒



☒6-5



☒6-6

☒☒2 ☒☒☒☒ KH ☒ SR ☒☒☒☒☒☒☒ G ☒☒☒6-6☒☒☒ QG ☒
☒☒ $\angle QHP = \angle QKP = 90^\circ$ ☒☒☒ Q, H, K, P ☒☒☒☒☒☒☒

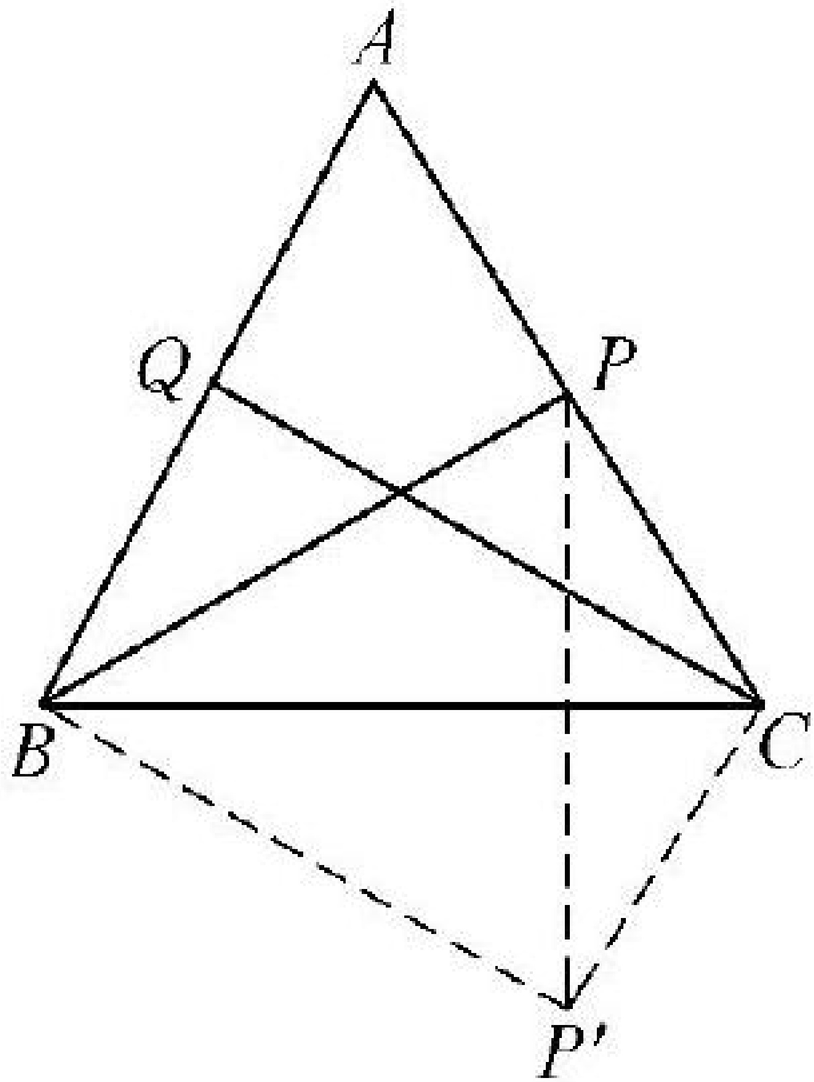
$$\angle QKH = \angle QPH = \angle QSR,$$

☒☒, Q, K, S, G ☒☒☒☒, ☒☒ $QKSG$ ☒☒☒☒☒, ☒☒ HK ☒☒ QS.

☒7 ☒☒6-7 ☒☒, ☒ $\triangle ABC$ ☒☒☒ AB, AC ☒☒☒☒☒ Q, P, ☒☒ $\angle PBC = \angle QCB = \frac{1}{2}\angle A$. ☒☒: $BQ = CP$.

☒☒☒☒☒☒ $\angle PBC = \angle QCB = \frac{1}{2}\angle A$, ☒☒

$$\angle BQC + \angle CPB = \left(\angle A + \angle C - \frac{1}{2}\angle A\right) + \left(\angle A + \angle B - \frac{1}{2}\angle A\right)$$



☒6-7

$$= \angle A + \angle B + \angle C = 180°.$$

☒ P ☒☒ BC ☒☒☒☒ P', ☒☒ BP', CP'. ☒☒ $\angle BQC + \angle BP'C = 180°$ ☒☒ ☒ B, Q, C, P' ☒☒☒☒.

☒ $\angle P'BC = \angle PBC = \angle QCB$, ☒☒ $BP'//QC$, ☒ $BQ = P'C$ ☒☒☒ $BQ = CP$ ☒

⊠⊠ (1) $\angle BQC$ ⊠ $\angle CPB$ ⊠⊠⊠⊠ BC ⊠⊠⊠⊠⊠, ⊠ $\angle BQC + \angle CPB = 180°$ ⊠⊠⊠ P Q ⊠⊠⊠ BC ⊠⊠⊠⊠⊠B Q P C ⊠⊠⊠⊠⊠⊠ P Q ⊠ BC ⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

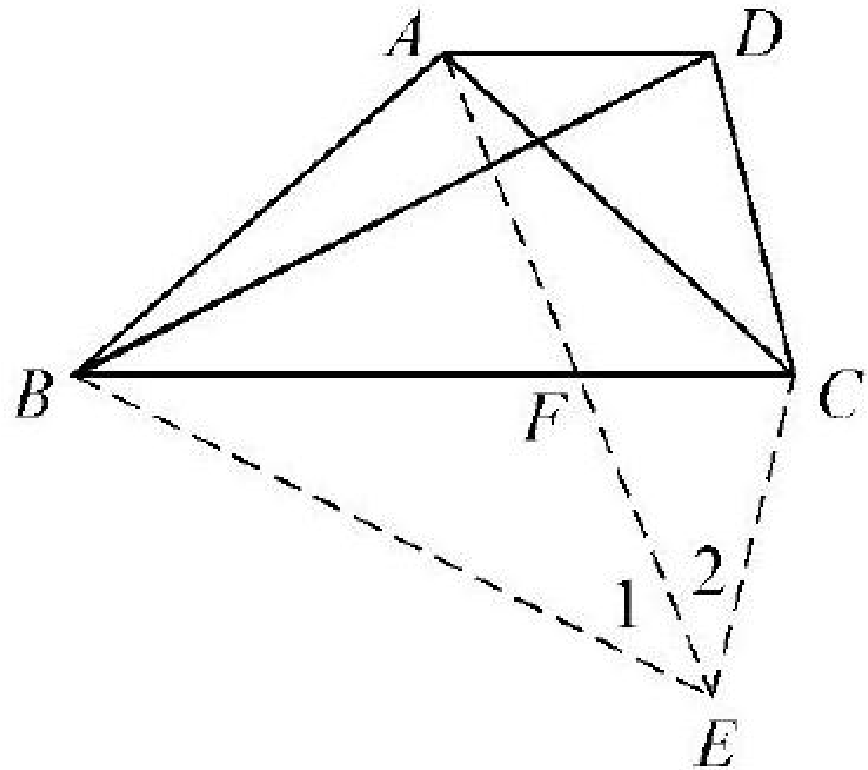
⊠2⊠⊠ $BP = CQ$ ⊠⊠ $\triangle BPC \cong \triangle CQB$ ⊠⊠ $BP \neq CQ$ ⊠⊠⊠⊠ $CQ > BP$ ⊠⊠ CQ ⊠⊠⊠ $CD = BP$ ⊠⊠ $\triangle DCB \cong \triangle PBC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $BQ = CP$ ⊠⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠8 ⊠⊠6-8 ⊠⊠⊠⊠⊠⊠ $ABCD$ ⊠⊠$AD// BC, BC = BD = 1, AB = AC, CD < 1$ ⊠⊠ $\angle BAC + \angle BDC = 180°$, ⊠ CD ⊠⊠.

⊠⊠ D ⊠⊠ BC ⊠⊠⊠⊠ E, ⊠⊠ AE, BE ⊠CE, ⊠ AE ⊠ BC ⊠⊠⊠ F.

⊠ $AD//BC$ ⊠: A E ⊠ BC ⊠⊠⊠⊠⊠, ⊠⊠⊠⊠ $AF = FE$ ⊠

⊠ $CD = CE = x, AF = FE = m$. ⊠ $\angle BAC +$



⊠6-8

$\angle BDC = 180°$ ⊠⊠ $\angle BAC + \angle BEC = 180°$ ⊠⊠⊠ A B E C ⊠⊠⊠⊠. ⊠ $AB = AC$ ⊠ $\angle 1 = \angle 2$.

⊠ $\angle EBF = \angle EAC$, ⊠⊠ $\triangle BFE \backsim \triangle ACE$, ⊠⊠ $\frac{BE}{FE} = \frac{AE}{CE}$,

$$2m^2 = AE \cdot FE = BE \cdot CE = x \tag{1}$$

⊠⊠⊠⊠⊠⊠⊠⊠⊠: $\frac{BF}{CF} = \frac{BE}{CE} = \frac{1}{x}$, ⊠ $BF + CF = 1$, ⊠

$$BF = \frac{1}{x+1}, CF = \frac{x}{x+1} \tag{2}$$

☒ (2) ☒☒☒☒☒☒☒

$$m^2 = AF \cdot FE = BF \cdot FC = \frac{x}{(x+1)^2} \tag{3}$$

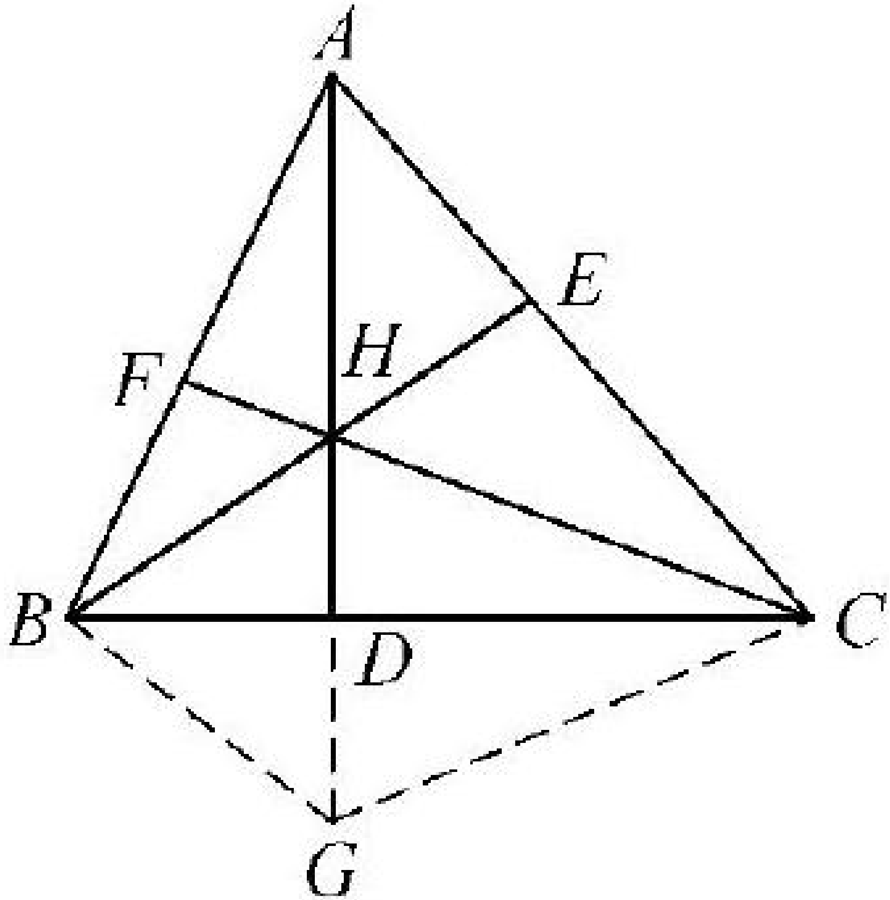
☒ (3) ☒☒ (1), ☒ $\frac{2x}{(x+1)^2} = x$, ☒☒ $x = \sqrt{2} - 1$. ☒☒, $CD = \sqrt{2} - 1$.

☒9 ☒☒☒☒☒☒ ABC ☒☒$AB \neq AC, AD$ ☒☒☒H ☒ AD ☒☒☒☒☒☒ BH ☒☒☒☒ AC ☒☒ E, ☒☒ CH ☒☒☒☒ AB ☒☒ F, ☒☒ B, C, E, F ☒☒☒☒☒☒☒☒H ☒☒☒☒☒☒☒☒ ABC ☒☒☒☒☒☒☒☒☒☒☒

☒☒☒☒☒☒☒☒

☒ AD ☒☒☒☒☒☒☒☒☒ G ☒☒☒

$$AH \cdot AG = AF \cdot AB = AE \cdot AC$$



⊠6-9

⊠1⊠⊠ G D ⊠⊠⊠, ⊠

$$\angle AFH=\angle AGB, \angle AEH=\angle AGC.$$

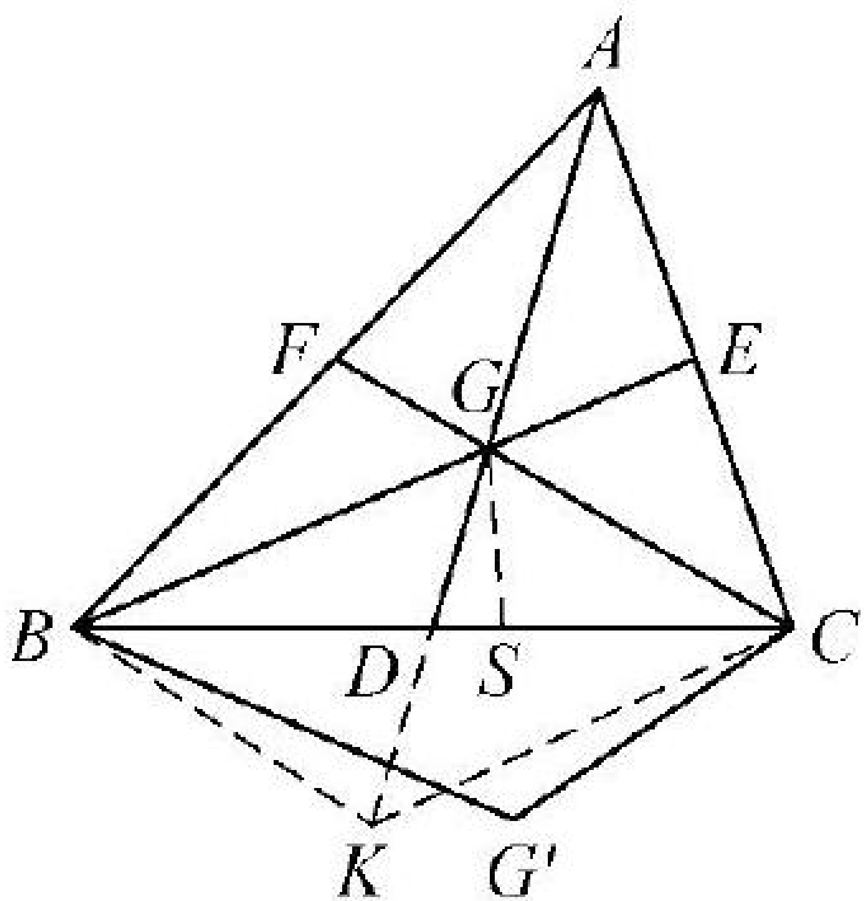
⊠⊠ B,C,E,F ⊠⊠⊠⊠, ⊠⊠ $\angle BFC=\angle CEB$, ⊠⊠ $\angle AFH=\angle AEH$ ⊠⊠ $\angle AGB=\angle AGC$ ⊠⊠⊠⊠ $GD\perp BC$ ⊠⊠⊠ $\triangle GBD\cong\triangle GCD$ ⊠⊠⊠ $\triangle GBA\cong\triangle GCA$ ⊠⊠⊠⊠$AB=AC$ ⊠⊠⊠⊠

⊠2⊠⊠ G D ⊠⊠⊠⊠⊠ $AH\cdot AD=AF\cdot AB$ ⊠ F B D H ⊠⊠⊠⊠⊠⊠⊠ $\angle AFH=\angle ADB=90^\circ$ ⊠⊠⊠⊠$\angle AEH=\angle ADC=90^\circ$ ⊠⊠⊠⊠H ⊠⊠⊠⊠⊠⊠ ABC ⊠⊠⊠⊠

⊠10 $\triangle ABC$ ⊠⊠⊠ G ⊠⊠⊠ BC ⊠⊠⊠⊠⊠ G', ⊠⊠: A,B,G',C ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AB^2+AC^2=2BC^2$ ⊠

⊠⊠⊠ AD,BE,CF ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠, ⊠⊠ G' ⊠ G ⊠⊠ BC ⊠⊠⊠⊠⊠ $\angle BGC=\angle BG'C$ ⊠

(1) ☒ A, B, G', C ☒☒☒☒, ☒ $\angle BG'C + \angle BAC = 180°$, ☒☒☒ $\angle EGF = \angle BGC = \angle BG'C$, ☒☒ $\angle EGF + \angle EAF = 180°$, ☒ A, F, G, E ☒☒☒



☒6-10

☒☒☒☒ $\angle BGF = \angle BAC$ ☒☒☒

$$\angle BGC = 180° - \angle BGF = 180° - \angle BAC = \angle ABC + \angle ACB.$$

☒ G ☒☒☒ GS ☒☒ BC ☒☒ S, ☒☒ $\angle CGS = \angle ABC$, ☒ $\angle BGS = \angle ACB$.

☒☒ $\angle CGS = \angle ABC = \angle FBS$, ☒☒, B, F, G, S ☒☒☒☒, ☒ $\angle BGS = \angle ACB = \angle ECS$ ☒☒ C, E, G, S ☒☒☒☒☒

☒☒☒☒☒☒☒☒

$$BF \cdot BA = BG \cdot BE = BS \cdot BC$$

$$CE \cdot CA = CG \cdot CF = CS \cdot CB,$$

⊠⊠

$$BF \cdot BA + CE \cdot CA = BC(BS + CS),$$

⊠

$$AB^2 + AC^2 = 2BC^2$$

⊠2⊠⊠ $AB^2 + AC^2 = 2BC^2$ ⊠⊠⊠ AD ⊠⊠ K ⊠⊠⊠ $DK = DG$ ⊠⊠⊠ BK ⊠ CK ⊠⊠ $BGCK$ ⊠⊠⊠⊠⊠⊠⊠$\angle BKC = \angle BGC$ ⊠

⊠⊠⊠⊠⊠⊠⊠, $DK = DG = \frac{1}{3}AD$ ⊠

⊠⊠ AD ⊠ $\triangle ABC$ ⊠⊠⊠, ⊠⊠

$$AB^2 + AC^2 = 2AD^2 + 2BD^2 = 2AD^2 + \frac{1}{2}BC^2,$$

⊠⊠ $AB^2 + AC^2 = 2BC^2$, ⊠ $AD^2 = \frac{3}{4}BC^2$, ⊠⊠,

$$AD \cdot DK = AD \cdot \frac{1}{3}AD = \frac{1}{3} \cdot \frac{3}{4}BC^2 = \frac{1}{4}BC^2 = BD \cdot DC,$$

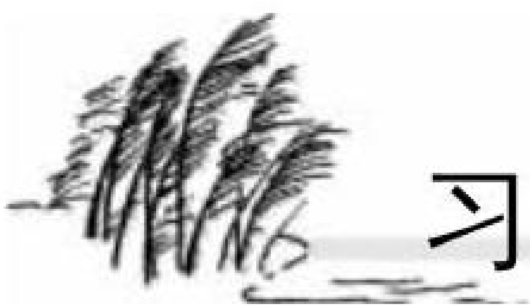
⊠⊠ A, B, K, C ⊠⊠⊠⊠, ⊠

$$\angle BKC + \angle BAC = 180°,$$

⊠⊠⊠ $\angle BKC = \angle BGC = \angle BG'C$, ⊠⊠

$$\angle BG'C + \angle BAC = 180°,$$

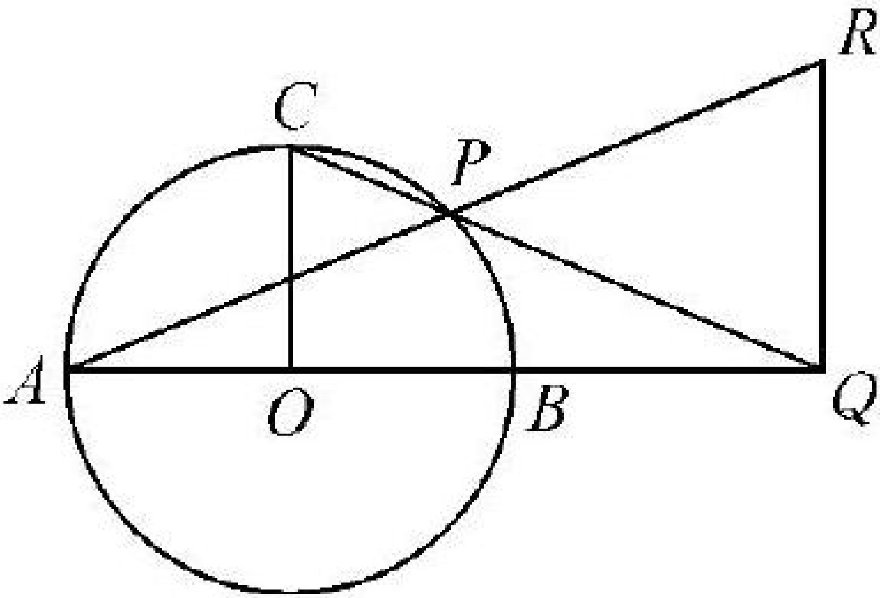
⊠⊠ A, B, G', C ⊠⊠⊠⊠.



习 题 6

1 ☒ D ☒☒☒☒☒☒☒☒ ABC ☒☒ BC ☒☒☒☒☒ C, D ☒☒☒☒☒☒☒ A ☒☒☒☒☒ ☒☒☒ AC ☒☒ E, ☒☒ BE ☒☒☒☒☒ F, ☒☒☒$AF \perp BE$ ☒

2AB ☒☒ O ☒☒☒☒C ☒☒ O ☒☒☒ $OC \perp AB$, P ☒☒ O ☒☒☒☒☒☒ B, C ☒☒☒☒ ☒

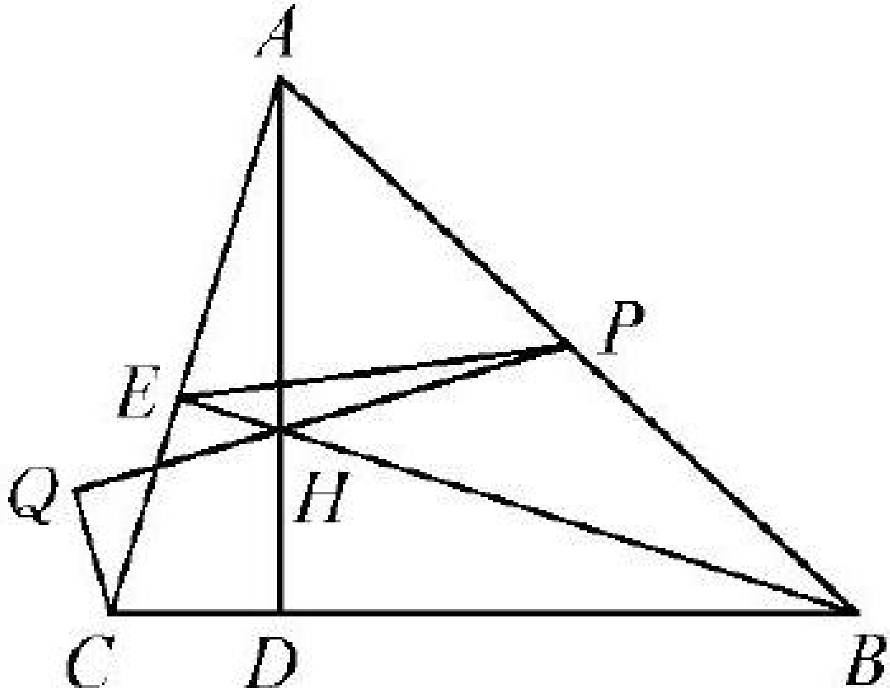


(☒2☒)

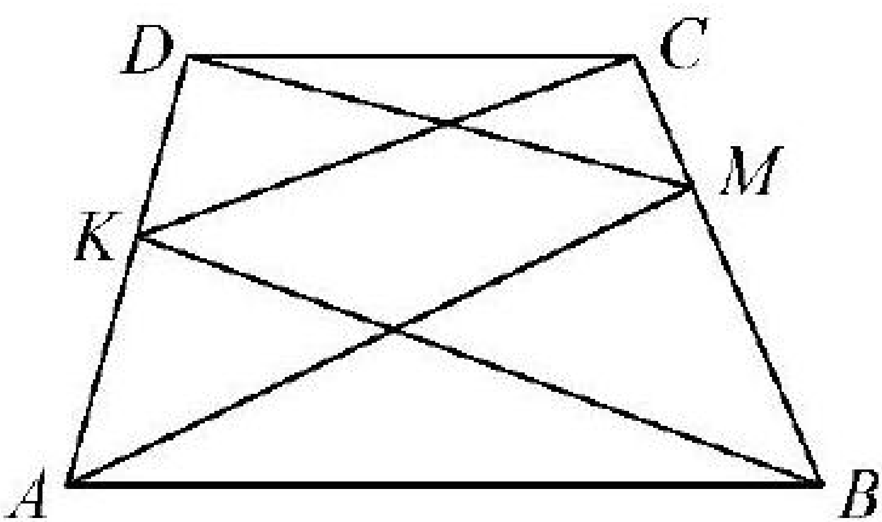
CP ☒ AB ☒☒☒☒ Q, ☒ Q ☒☒☒☒ AB ☒☒, ☒☒☒ AP ☒☒ R. ☒☒: $BQ = QR$.

3 ☒☒, ☒ $\triangle ABC$ ☒, ☒☒ $AD \perp BC, BE \perp CA, AD$ ☒ BE ☒☒☒☒ H, P ☒☒ AB ☒☒☒, ☒☒ C ☒ $CQ \perp PH$, ☒☒☒☒ Q. ☒☒: $PE^2 = PH \cdot PQ$ ☒

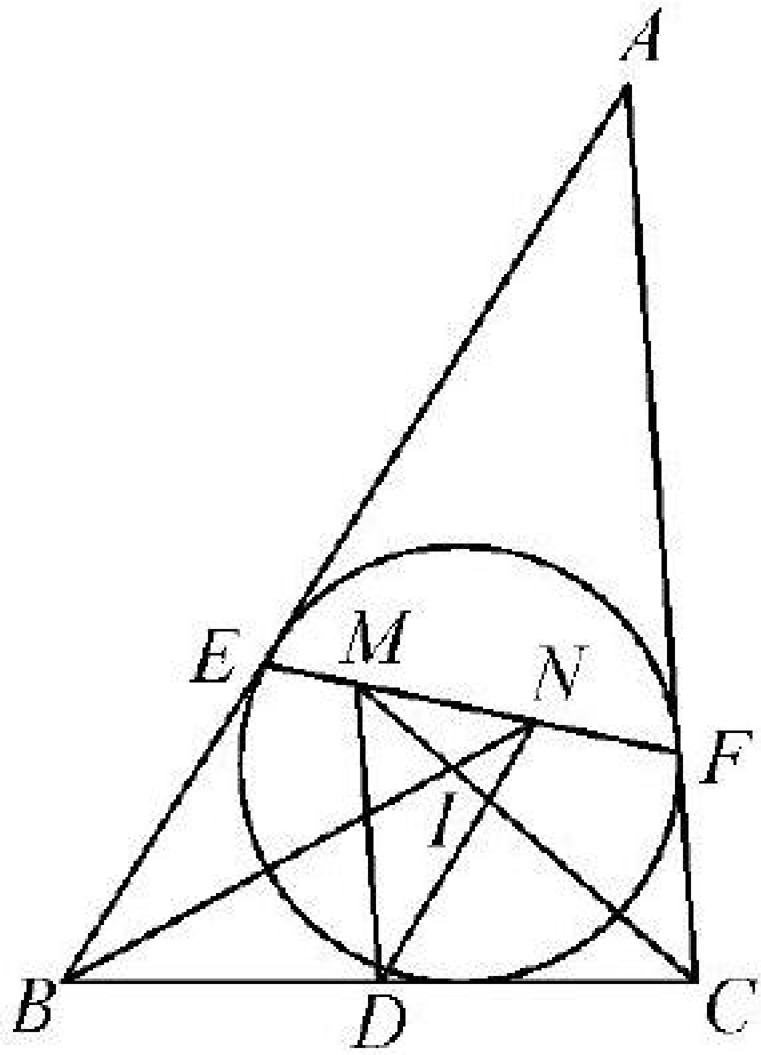
4 ☒☒☒ $ABCD$ ☒, $AB//CD, AB > CD, K\ M$ ☒☒☒☒ $AD\ CB$ ☒☒☒☒$\angle DAM = \angle CBK$ ☒☒☒: $\angle DMA = \angle CKB$.



(⊠3⊠)



(⊠4⊠)

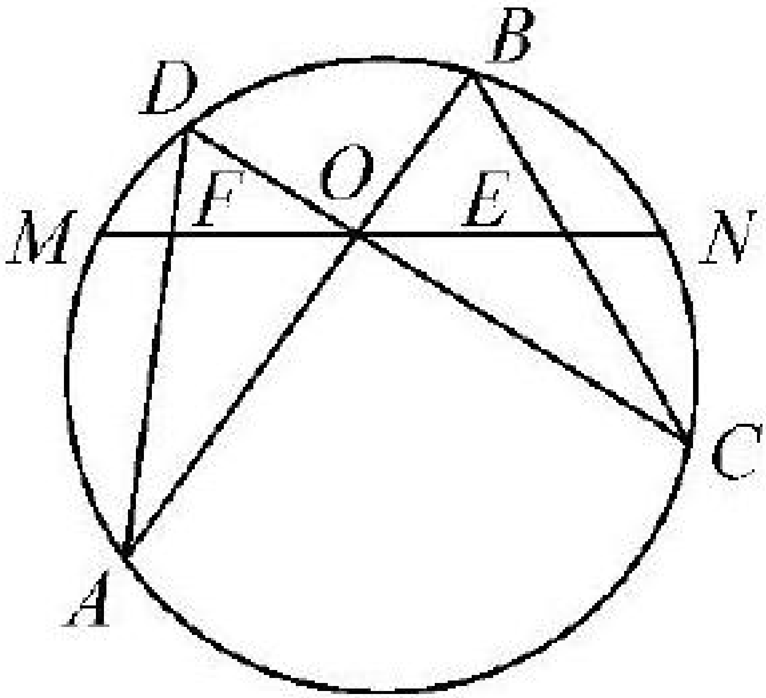


(⊠5 ⊠)

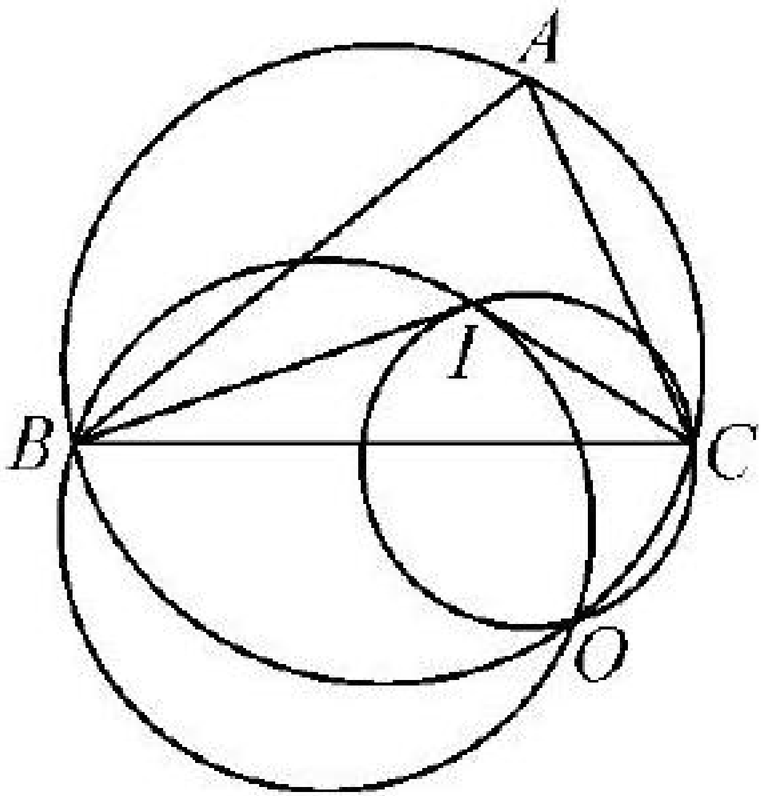
5 ⊠⊠⊠$\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠ AB AC ⊠⊠ E F, D ⊠ BC ⊠⊠⊠⊠$\angle B$ $\angle C$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ EF ⊠⊠⊠ N M. ⊠⊠: $DM = DN$.

6 ⊠ O ⊠⊠⊠⊠ MN ⊠⊠⊠, ⊠ O ⊠⊠ AB CD, ⊠⊠ AD BC ⊠ MN ⊠⊠ F E ⊠⊠⊠⊠$EO = OF$ ⊠

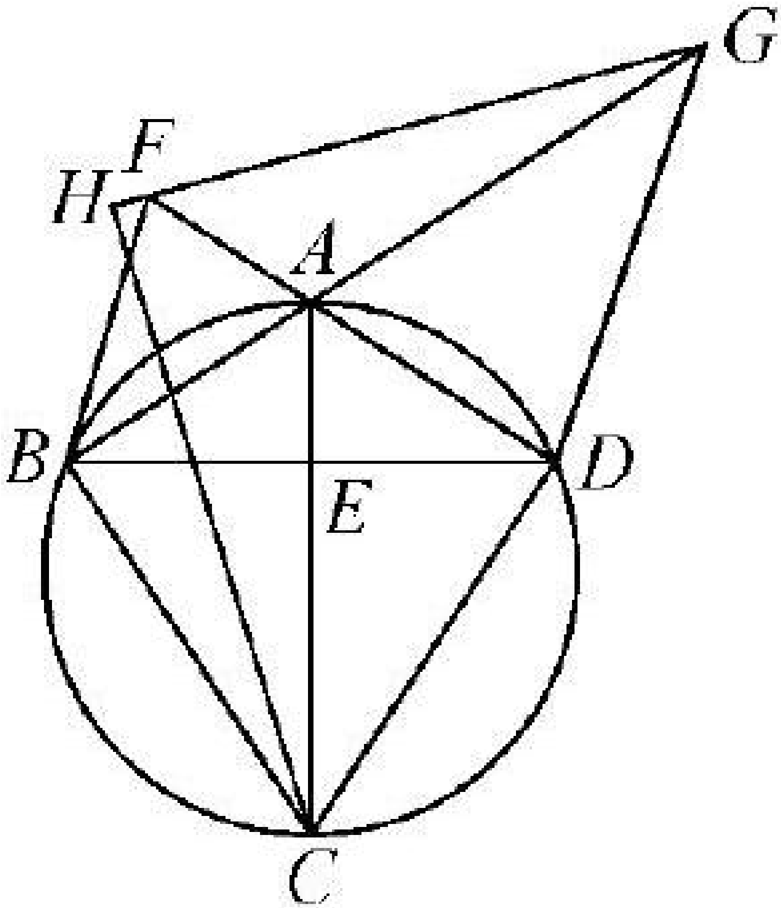
7 ⊠⊠ I ⊠ $\triangle ABC$ ⊠⊠⊠, ⊠⊠ B ⊠⊠⊠ CI ⊠⊠ I, ⊠⊠ C ⊠⊠⊠ BI ⊠⊠ I ⊠⊠⊠⊠⊠⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠



(⊠6 ⊠)



(⊠7⊠)

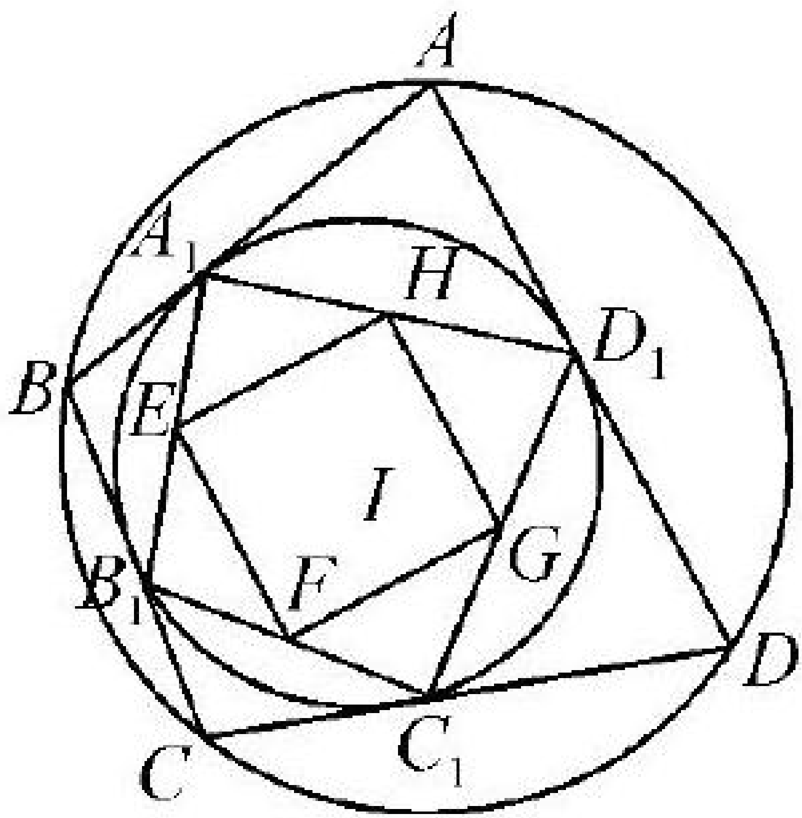


(⊠8 ⊠)

8 ⊠⊠, $ABCD$ ⊠⊠⊠⊠⊠⊠⊠, AC ⊠⊠⊠⊠⊠, $BD \perp AC, AC$ ⊠ BD ⊠⊠⊠⊠ E, F ⊠ DA ⊠⊠⊠⊠⊠, ⊠⊠ BF, G ⊠ BA ⊠⊠⊠⊠⊠, ⊠⊠ $DG//BF, H$ ⊠ GF ⊠⊠⊠⊠⊠, $CH \perp GF$. ⊠⊠: B, E, F, H ⊠⊠⊠⊠⊠

9 ⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠ AB, BC, CD, DA ⊠⊠⊠⊠⊠⊠ A_1, B_1, C_1, D_1, ⊠⊠ $A_1B_1, B_1C_1, C_1D_1, D_1A_1$, ⊠ E, F, G, H ⊠⊠⊠ $A_1B_1, B_1C_1, C_1D_1, D_1A_1$ ⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $EFGH$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ A, B, C, D ⊠⊠⊠⊠⊠

10 ⊠⊠⊠⊠⊠⊠ ABC ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ D E F ⊠



(☒9 ☒)

☒☒☒☒☒☒ DEF ☒☒☒☒ ABC ☒☒☒☒☒☒

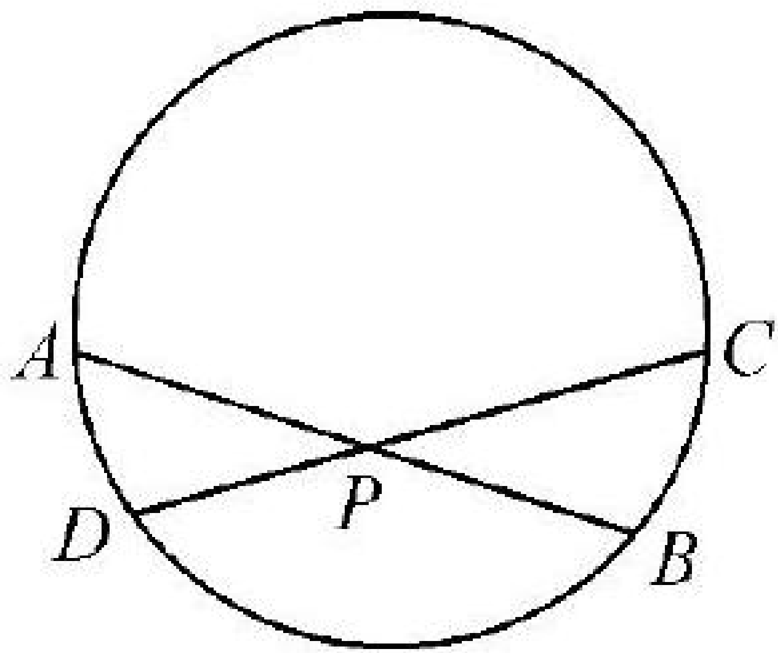
☒1☒☒☒1☒

(2) ☒☒☒2.

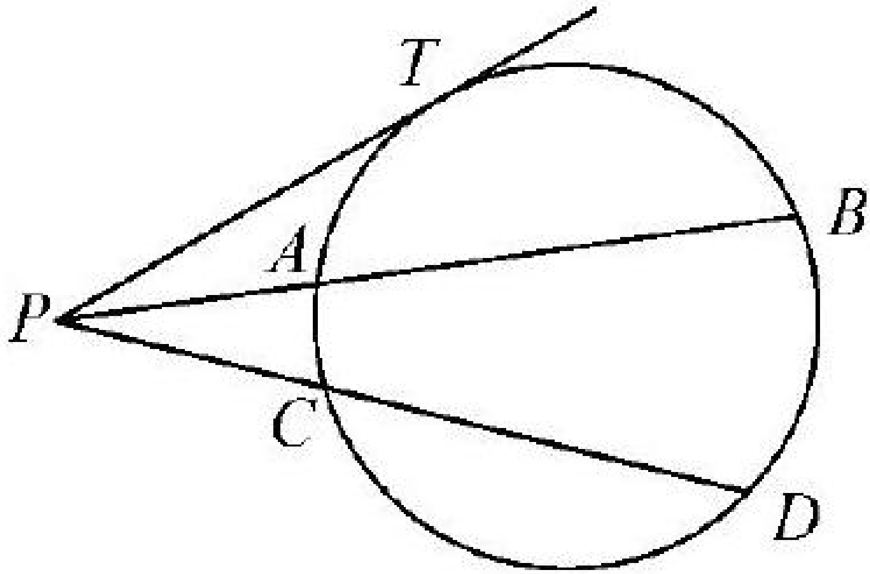
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠: ⊠⊠ AB CD ⊠⊠⊠ P, ⊠⊠7-1, ⊠ $PA \cdot PB = PC \cdot PD$.

⊠⊠⊠⊠⊠: ⊠ PT ⊠⊠⊠⊠⊠, T ⊠⊠⊠, ⊠⊠ PAB ⊠⊠⊠⊠ A B, ⊠⊠ PCD ⊠⊠⊠⊠ C D, ⊠⊠7-2, ⊠ $PC^2 = PA \cdot PB = PC \cdot PD$.



⊠7-1



☒7-2

☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒☒☒☒☒

☒☒☒☒☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

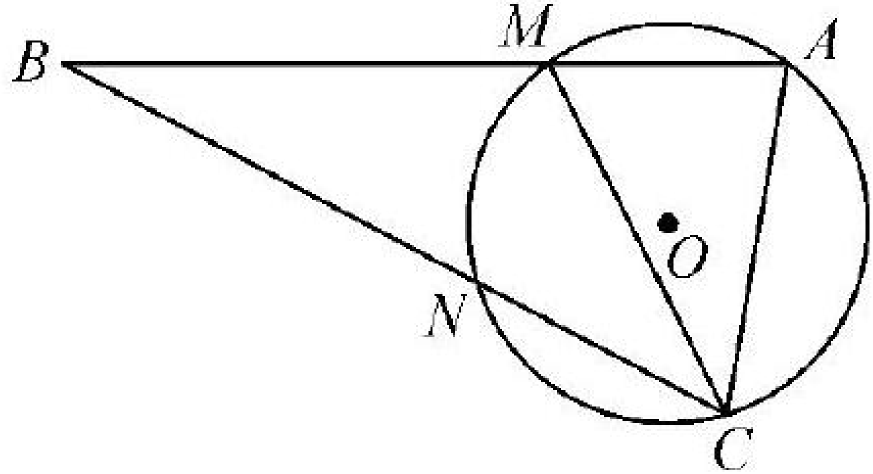
☒1 ☒ $\triangle ABC$ ☒, ☒☒ CM ☒ $\angle C$ ☒☒☒☒, $\triangle AMC$ ☒☒☒☒☒ BC ☒☒ N, ☒ $AC = \frac{1}{2}AB$, ☒☒: $BN = 2AM$ ☒

☒☒☒ CM ☒ $\angle C$ ☒☒☒☒, ☒

$$\frac{AC}{BC} = \frac{AM}{BM} \tag{1}$$

☒☒☒☒☒☒☒ $BM \cdot BA = BN \cdot BC$ ☒☒☒

$$\frac{AB}{BN} = \frac{BC}{BM} \tag{2}$$

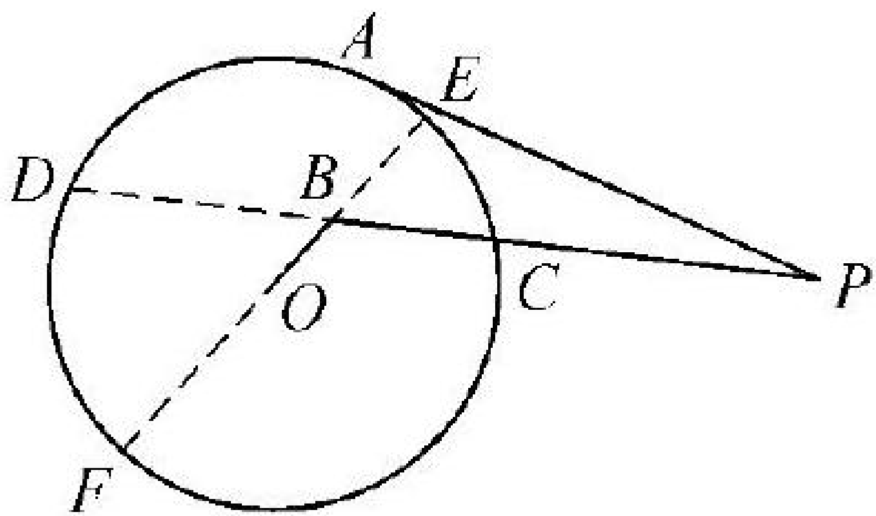


⊠7-3

⊠ (1)⊠(2) ⊠: $\frac{AC}{AM}=\frac{AB}{BN}$, ⊠ $AC=\frac{1}{2}AB$, ⊠⊠ $BN=2AM$.

⊠2 ⊠⊠7-4, PA ⊠ $\odot O$ ⊠⊠ A, ⊠ B ⊠ $\odot O$ ⊠, PB ⊠ $\odot O$ ⊠⊠ C, ⊠ $PA=6, PC=4, CB=2, OB=1$ ⊠⊠ $\odot O$ ⊠⊠⊠⊠

⊠⊠

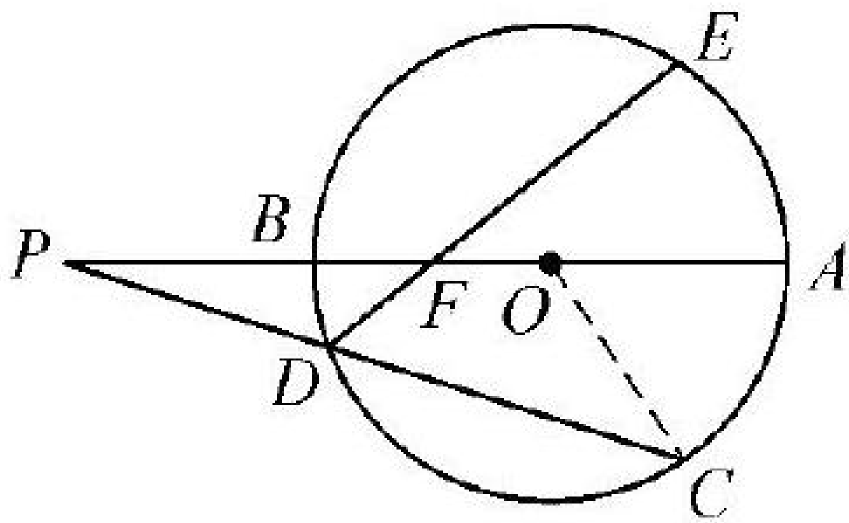


⊠7-4

⊠⊠⊠ PC ⊠ $\odot O$ ⊠⊠ D ⊠⊠⊠⊠⊠ OB ⊠ $\odot O$ ⊠⊠ E F ⊠⊠ $PA^2=PC\cdot PD, BC\cdot BD=BE\cdot BF$ ⊠⊠⊠ $\odot O$ ⊠⊠⊠⊠ r ⊠⊠ $6^2=4(6+BD)$ ⊠$2BD=(r-1)(r+1)$ ⊠⊠⊠ $BD=3$ ⊠⊠⊠ $r=\sqrt{7}$ ⊠

⊠3 ⊠⊠7-5, $\odot O$ ⊠⊠⊠ AB ⊠⊠⊠⊠⊠⊠ CD ⊠⊠⊠⊠⊠⊠⊠⊠ P, E ⊠ $\odot O$ ⊠⊠⊠⊠, $AE=AC, DE$ ⊠ AB ⊠⊠ F, ⊠⊠: $PF\cdot PO=PA\cdot PB$.

⊠⊠⊠⊠ OC, ⊠ $AE = AC$, ⊠ $\angle EDC =$



⊠7-5

$\angle AOC$ ⊠⊠⊠

$$\angle PDF = 180^\circ - \angle EDC = 180^\circ - \angle AOC = \angle POC.$$

⊠⊠ D F O C ⊠⊠⊠⊠⊠⊠⊠

$$PD \cdot PC = PF \cdot PO \qquad (1)$$

⊠ B A C D ⊠⊠⊠⊠, ⊠⊠

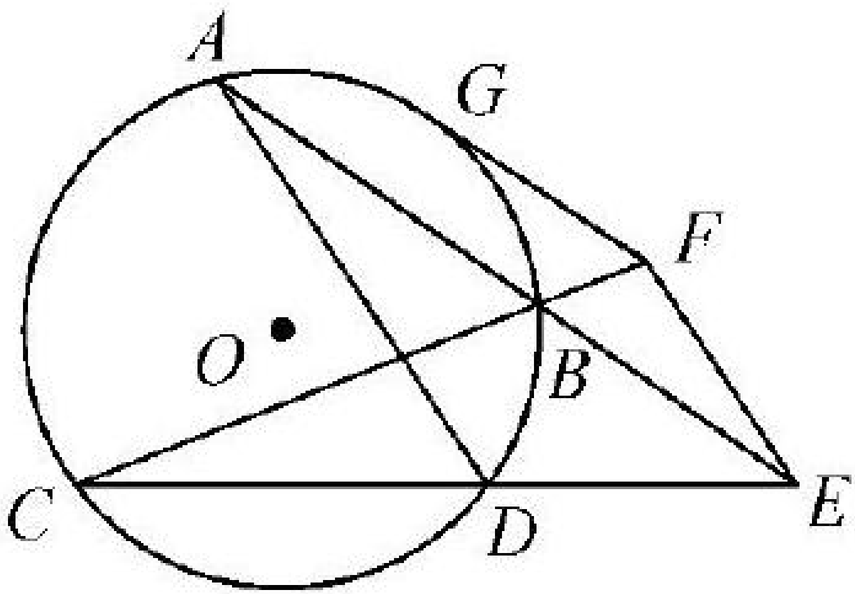
$$PD \cdot PC = PB \cdot PA \qquad (2)$$

⊠ (1)⊠(2) ⊠ $PF \cdot PO = PA \cdot PB$ ⊠

⊠4 ⊠⊠ $7-6, \odot O$ ⊠⊠⊠ AB CD ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ E, ⊠ E ⊠ AD ⊠⊠⊠⊠⊠⊠ ⊠ BC ⊠⊠⊠ F, ⊠⊠⊠ FG, G ⊠⊠⊠, ⊠⊠: $EF = FG$ ⊠

⊠⊠⊠⊠ FG ⊠⊠ O ⊠⊠ G ⊠⊠⊠ $FG^2 = FB \cdot FC$ ⊠⊠⊠⊠⊠⊠⊠⊠ $FE^2 = FB \cdot FC$ ⊠⊠⊠⊠⊠

⊠⊠⊠⊠ $EF//AD$ ⊠⊠⊠⊠ $\triangle BFE$ ⊠ $\triangle EFC$ ⊠⊠ $\angle BEF = \angle A = \angle C$, ⊠ $\angle BFE = \angle EFC$ ⊠⊠⊠ $\triangle BFE \backsim \triangle EFC$ ⊠⊠⊠ $\frac{FE}{FB} = \frac{FC}{FE}$ ⊠⊠ $FE^2 = FB \cdot FC$ ⊠



☒7-6

☒ $FG^2 = FB \cdot FC$ ☒☒☒ $FE^2 = FG^2$ ☒☒☒ $FE = FG$ ☒

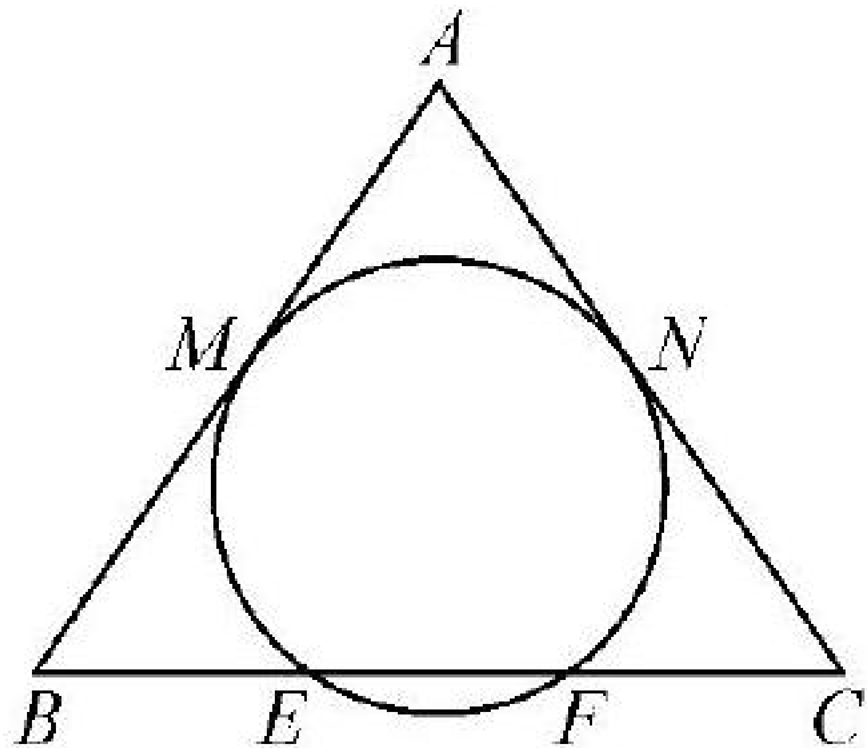
☒5 ☒☒7-7 ☒☒, ☒☒ $\odot O$ ☒☒☒ $\triangle ABC$ ☒ AB AC ☒☒☒☒ M ☒N, ☒ BC ☒☒ E F, ☒ $BE = EF = FC$, ☒☒: $\angle B = \angle C$.

☒☒☒☒☒☒☒☒☒

$$BM^2 = BE \cdot BF = BE \cdot (BE + EF),$$
$$CN^2 = CF \cdot CE = CF \cdot (CF + EF).$$

☒ $BE = EF = CF$ ☒☒☒ $BM = CN$ ☒

☒☒ AM, AN ☒☒☒☒☒☒☒☒ $AM = AN$ ☒☒☒ $AB = AM + BM = AN + CN = AC$ ☒



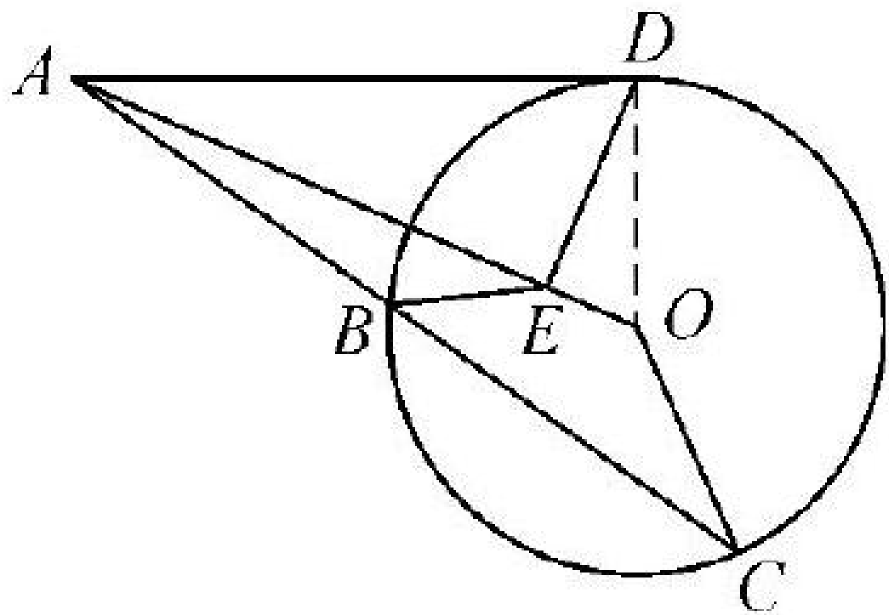
☒7-7

☒☒ $\angle B = \angle C$.

☒6 ☒☒ $7-8, AD$ ☒ $\odot O$ ☒☒☒, D ☒☒☒, ABC ☒☒☒, $DE \perp AO$ ☒☒ E ☒☒☒: $\angle AEB = \angle C$.

☒☒☒☒ $\angle AEB = \angle C$ ☒☒☒☒ O, E, B ☒C ☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $AD^2 = AE \cdot AO$ ☒☒☒☒ $\triangle ODA$ ☒☒☒☒☒☒☒☒ $DE \perp AO$ ☒☒☒☒☒☒☒☒☒☒☒☒

☒☒☒☒ OE, ☒ $\triangle ODA$ ☒☒☒☒☒☒, ☒ $DE \perp AO$ ☒☒☒ $AD^2 = AE \cdot AO$ ☒☒ $AD^2 = AB \cdot AC$ ☒☒☒ $AE \cdot AO = AB \cdot AC$ ☒☒☒ O E B C ☒

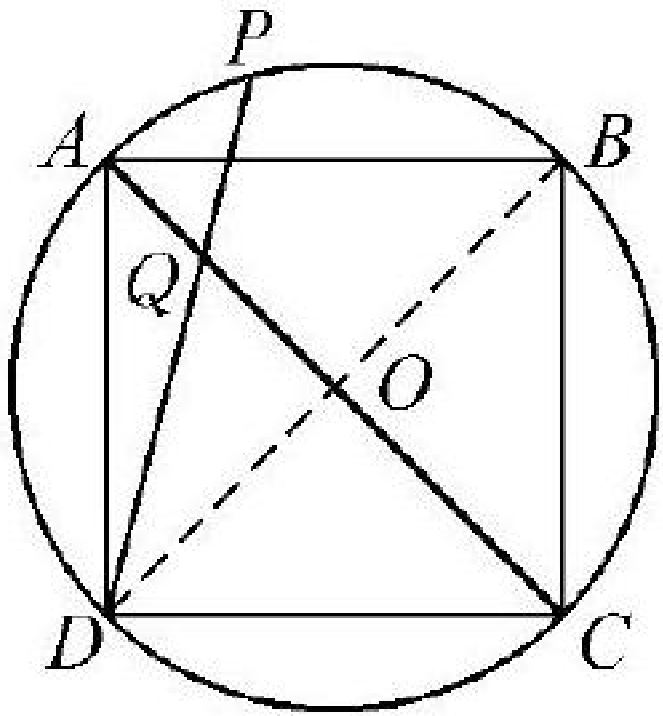


☒7-8

☒☒☒☒☒ $\angle AEB = \angle C$ ☒

☒7 ☒☒7-9 ☒☒, ☒☒☒ $ABCD$ ☒☒☒ $\odot O$, ☒ P ☒☒☒ AB ☒☒☒, ☒☒ DP ☒ AC ☒☒ Q, ☒ $QP = QO$, ☒ $\frac{QC}{QA}$ ☒☒.

☒☒☒ BD, ☒ BD ☒☒ O, ☒ $AO = r, QP = QO = m$ ☒☒ $DQ \cdot QP = AQ \cdot QC = (AO - OQ) \cdot (OC + OQ) = (r - m)(r + m)$, ☒☒ $DQ = \frac{r^2 - m^2}{m}$, ☒☒☒☒ $\triangle QOD$ ☒Rt $\triangle$, ☒ $DQ^2 = OD^2 + OQ^2$, ☒ $\left(\frac{r^2 - m^2}{m}\right)^2 = r^2 + m^2$ ☒☒☒ $r = \sqrt{3}m$ ☒☒☒ $\frac{QC}{QA} = \frac{r+m}{r-m} = 2 + \sqrt{3}$.



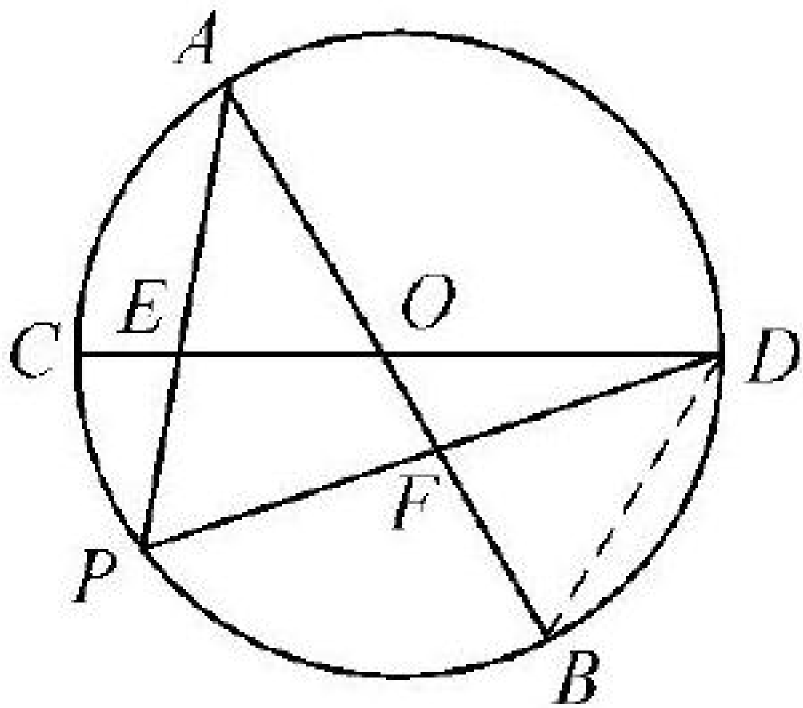
⊠7 - 9

⊠8 ⊠⊠ $7-10$, ⊠⊠ $AB\ CD$ ⊠⊠⊠⊠ R ⊠ $\odot O$ ⊠⊠⊠⊠⊠, ⊠ $\angle AOC = 60^\circ$, ⊠ P ⊠⊠⊠ BC ⊠, ⊠⊠ $PA\ PD$ ⊠⊠⊠ $CD\ AB$ ⊠⊠ $E\ F$, ⊠⊠⊠$AE \cdot AP + DF \cdot DP$ ⊠⊠⊠⊠

⊠⊠⊠⊠⊠ $AE \cdot AP, DF \cdot DP$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $O\ E\ P\ F$ ⊠⊠⊠⊠, ⊠⊠⊠⊠⊠ $OE + OF = R$.

⊠⊠⊠⊠ BD ⊠⊠ $\angle DBF \overset{m}{=} \frac{1}{2} AD = 60^\circ = \angle AOE$ ⊠⊠⊠⊠ $\angle EAO = \angle BDF\ OA = BD$ ⊠⊠⊠ $\triangle EAO \cong \triangle FDB$ ⊠⊠⊠ $OE = BF, OE + OF = BF + OF = OB = R$.

⊠⊠ $\angle P \overset{m}{=} \frac{1}{2} AD = 60^\circ = \angle AOE$ ⊠⊠⊠ $O, E, P,$



☒7-10

F ☒☒☒☒☒☒ $AE \cdot AP + DF \cdot DP = AO \cdot AF + DO \cdot DE = R(AO + OF + DO + OE) = 3R^2$.

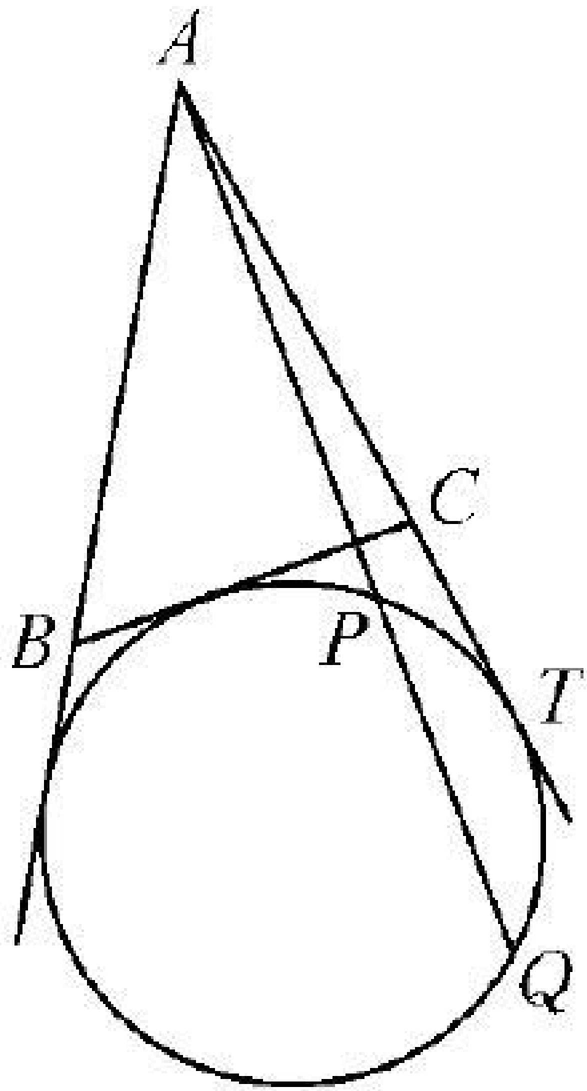
☒9 ☒☒☒ $\triangle ABC$ ☒☒☒ A ☒☒☒☒☒☒☒ $\angle A$ ☒☒ $\triangle ABC$ ☒☒☒☒☒☒☒ P Q, ☒☒: $AP + AQ > AB + BC + CA$.

☒☒☒ AC ☒☒☒☒ T, ☒

$$AP \cdot AQ = AT^2 = \left(\frac{AB+BC+CA}{2}\right)^2,$$

☒
$$\left(\frac{AP+AQ}{2}\right)^2 \geqslant AP \cdot AQ,$$

☒☒ $\frac{AP+AQ}{2} \geqslant \frac{AB+BC+CA}{2}$, ☒☒☒☒☒☒ $AP = AQ$



⊠7-11

⊠⊠⊠⊠⊠ P Q ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AP+AQ > AB+BC+CA$ ⊠

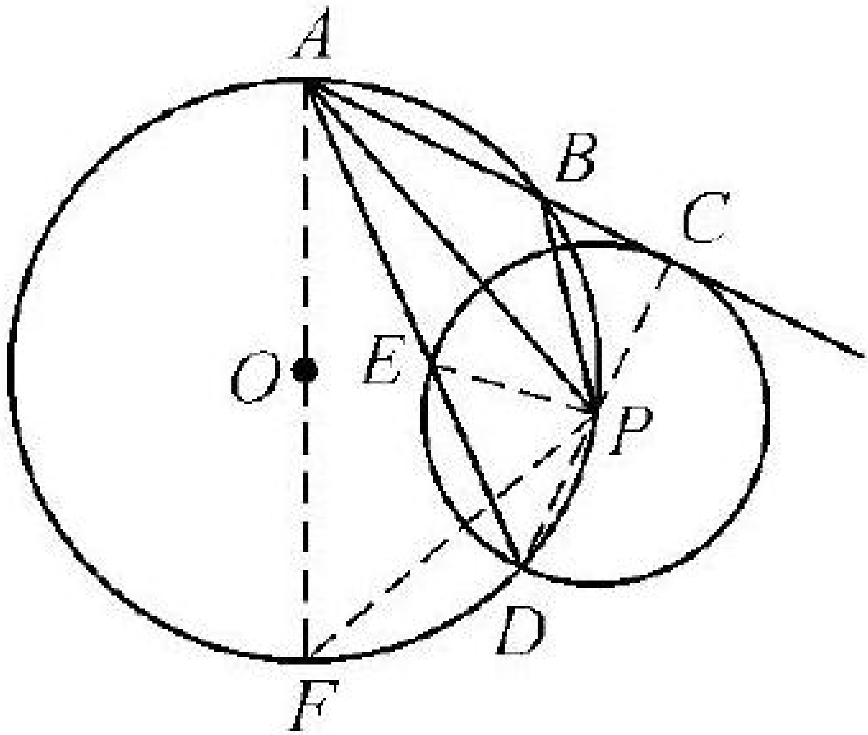
⊠10 ⊠⊠7-12 ⊠⊠. $\odot P$ ⊠⊠⊠ P ⊠ $\odot O$ ⊠, $\odot O$ ⊠⊠ AB ⊠⊠⊠⊠⊠ $\odot P$ ⊠⊠⊠ C ⊠⊠ $\odot P$ ⊠⊠⊠⊠ $r, \odot O$ ⊠⊠⊠⊠ R ⊠

(1) ⊠⊠⊠$PA \cdot PB = 2Rr$ ⊠

⊠2⊠$\odot O$ ⊠ $\odot P$ ⊠⊠⊠ D, AD ⊠ $\odot P$ ⊠⊠ E, ⊠ $\odot O$ ⊠ $\odot P$ ⊠⊠⊠⊠⊠ $9:4$, ⊠ $PA = 10, PB = 4.8$ ⊠⊠ DE ⊠ AE ⊠⊠⊠

⊠⊠ (1) ⊠⊠⊠ $PA \cdot PB = 2Rr$ ⊠⊠⊠ $\frac{PA}{2R} = \frac{r}{PB}$ ⊠⊠⊠⊠⊠⊠ $\triangle PCB \backsim$

$\triangle APF$.



⊠7 - 12

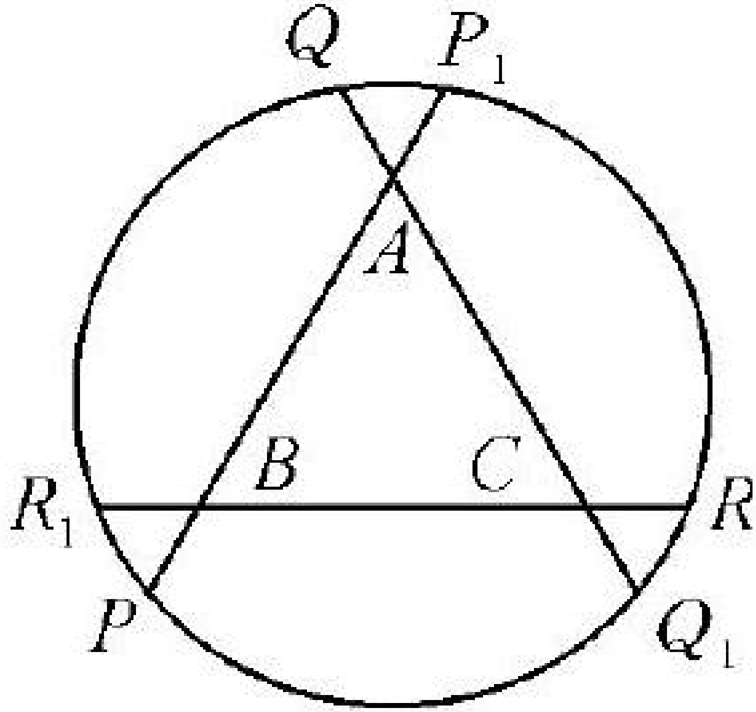
⊠2⊠⊠⊠⊠⊠⊠⊠ $9:4$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $3:2$ ⊠⊠⊠⊠ $2Rr = PA\cdot PB$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠Rt $\triangle PAC$ ⊠Rt $\triangle PAF$ ⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ AC ⊠ PF ⊠⊠⊠⊠⊠ PE ⊠⊠⊠⊠ $\triangle PED$ ⊠⊠⊠⊠⊠ DE ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AC^2 = AE\cdot AD$ ⊠⊠⊠ AE ⊠

⊠⊠⊠1⊠⊠ A ⊠ $\odot O$ ⊠⊠⊠ AF ⊠⊠⊠ PF PC ⊠⊠⊠ AF ⊠ $\odot O$ ⊠⊠

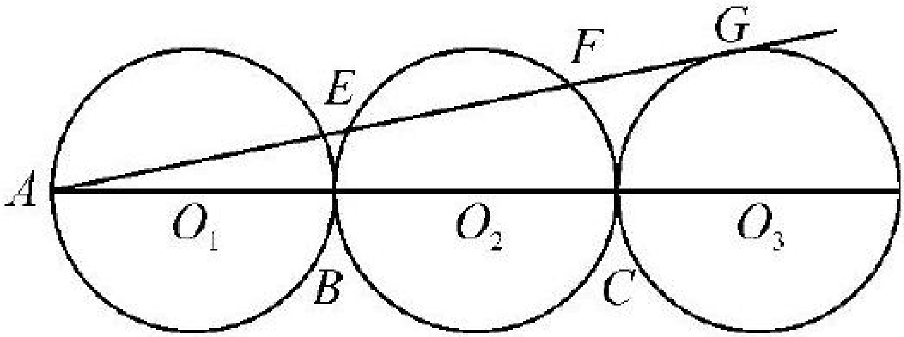
⊠, ⊠⊠ $\angle FPA = 90^\circ$. ⊠⊠ AC ⊠ $\odot P$ ⊠⊠ C, ⊠ $\angle PCB = 90^\circ$. ⊠⊠ $\angle PBC = \angle F$, ⊠⊠ $\triangle PCB \backsim \triangle APF$ ⊠⊠⊠ $\frac{PB}{PC} = \frac{AF}{PA}$ ⊠⊠ $PA\cdot PB = 2Rr$ ⊠ ⊠2⊠⊠⊠ $\odot O$ ⊠ $\odot P$ ⊠⊠⊠⊠⊠⊠ $9:4$ ⊠⊠ $2R = 3r$ ⊠⊠ $R = 3k$ $k > 0$ ⊠⊠ $r = 2k$. ⊠⊠ $PA\cdot PB = 2Rr$ ⊠ $PA = 10, PB = 4.8$, ⊠⊠ $k = 2$ ⊠⊠⊠ $R = 6, r = 4$. ⊠⊠ $PA = 10, PC = 4$, ⊠⊠ $AC = \sqrt{PA^2 - PC^2} = 2\sqrt{21}$, ⊠⊠ $AF = 2R = 12$, ⊠⊠ $PF = \sqrt{AF^2 - PA^2} = 2\sqrt{11}$, ⊠⊠ PD, PE, ⊠ $PD = PE = r = 4$, ⊠⊠ $\cos\angle ADP = \cos\angle F = \frac{\sqrt{11}}{6}$, ⊠⊠⊠⊠⊠⊠ $PE^2 = PD^2 + DE^2 - 2PD\cdot DE\cos\angle ADP$, ⊠⊠ $DE^2 - \frac{4}{3}\sqrt{11}DE = 0$, ⊠⊠ $DE = \frac{4}{3}\sqrt{11}\,(DE = 0,$ ⊠$)$, ⊠⊠ $AC^2 = AE\cdot AD = AE(AE + DE), (2\sqrt{21})^2 = AE\left(AE + \frac{4}{3}\sqrt{11}\right)$, ⊠⊠ $AE = \frac{2}{3}(10\sqrt{2} - \sqrt{11})$.

习 题 7

1 ⊠⊠⊠$\odot O$ ⊠⊠⊠⊠ PP_1, QQ_1, RR_1 ⊠⊠⊠⊠⊠⊠⊠⊠ A, B, C ⊠⊠ $AP_1 = BR_1 = CQ_1, AQ = BP = CR$, ⊠⊠: $\triangle ABC$ ⊠⊠⊠⊠⊠.



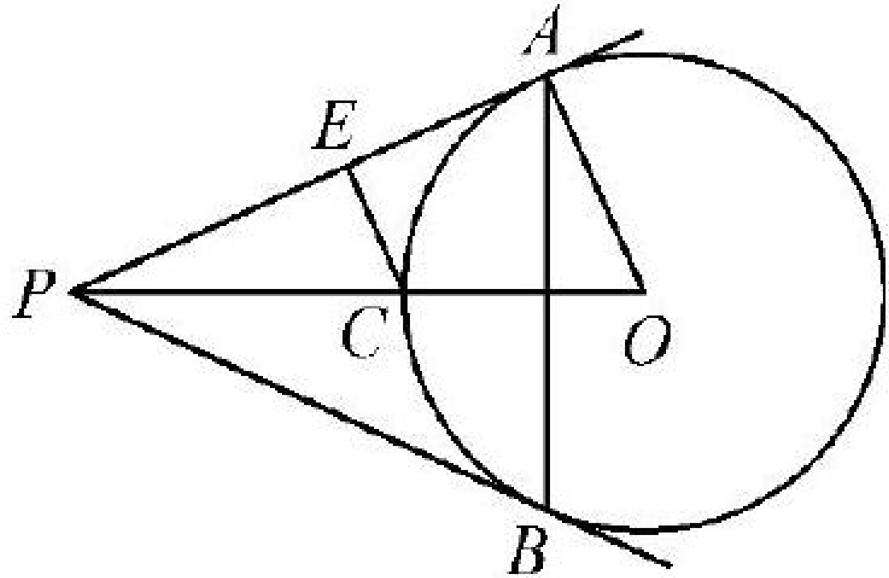
(⊠1 ⊠)



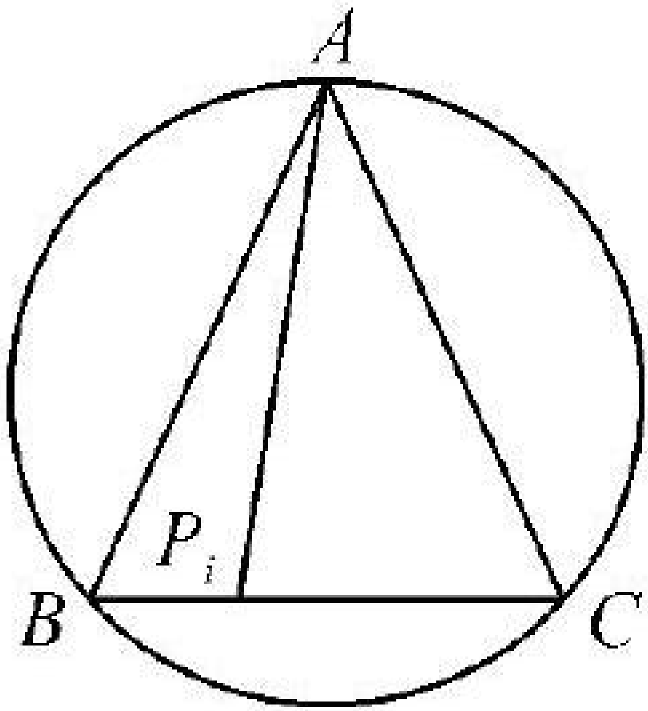
(⊠2 ⊠)

2 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠1 ⊠⊠⊠⊠⊠ O_1, O_2, O_3 ⊠⊠⊠⊠ $\odot O_1$ ⊠ $\odot O_2$ ⊠⊠⊠⊠ $B, \odot O_2$ ⊠ $\odot O_3$ ⊠⊠⊠⊠ C, ⊠ A ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\odot O_1$ ⊠⊠⊠, ⊠⊠ AG ⊠ $\odot O_3$ ⊠⊠ G, ⊠ $\odot O_2$ ⊠⊠ E F, ⊠ EF ⊠⊠.

3 ⊠ $\odot O$ ⊠⊠⊠ P ⊠ $\odot O$ ⊠⊠⊠⊠⊠ PA PB, ⊠⊠ OP, ⊠ $\odot O$ ⊠⊠⊠ C, ⊠⊠ C ⊠ AP ⊠⊠, ⊠⊠⊠⊠ E, ⊠ $PA = 10, PC = 5$, ⊠ CE ⊠⊠.



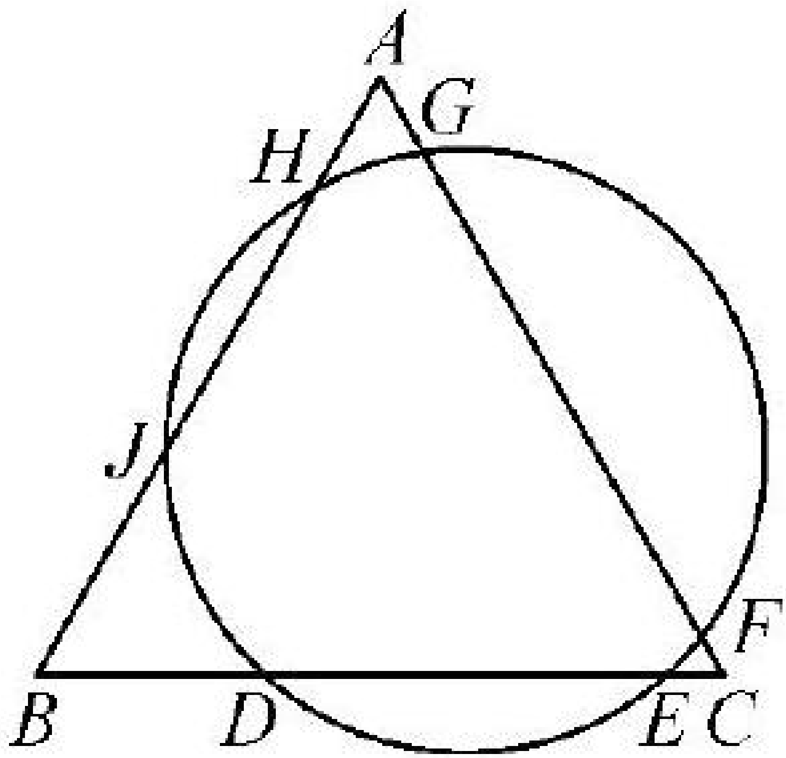
(⊠3⊠)



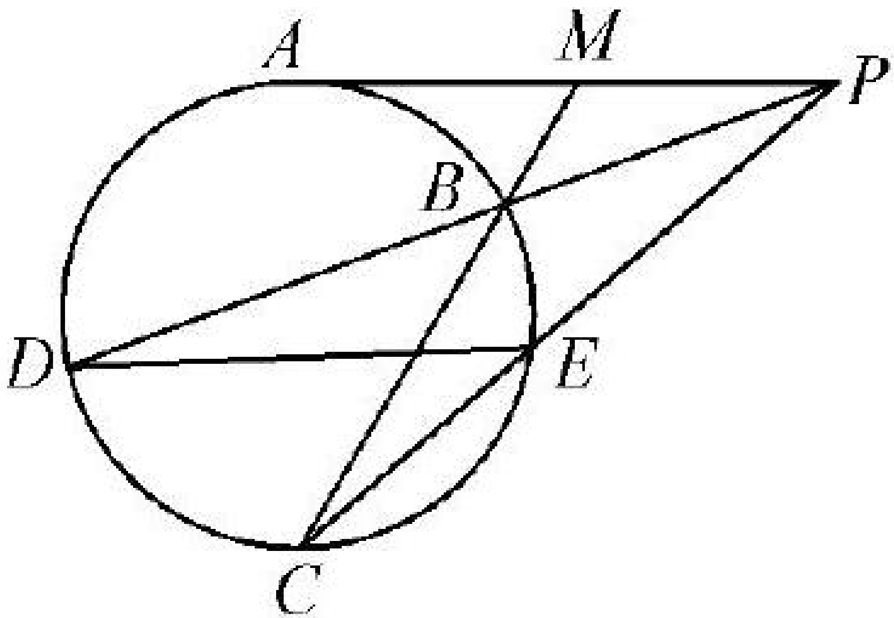
(⊠4⊠)

4$\triangle ABC$ ⊠, $AB = AC = 2, BC$ ⊠⊠⊠100 ⊠⊠⊠⊠⊠ $P_1, P_2, \cdots, P_{100}$, ⊠ $m_i = AP_i^2 + BP_i \cdot CP_i (i = 1, 2, \cdots, 100)$ ⊠⊠ $m_1 + m_2 + \cdots + m_{100}$ ⊠

5 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠6 ⊠⊠, ⊠⊠⊠⊠. ⊠ $AG = 2, GF = 13, FC = 1$, $HJ = 7$ ⊠⊠ DE ⊠⊠⊠



(⊠5⊠)



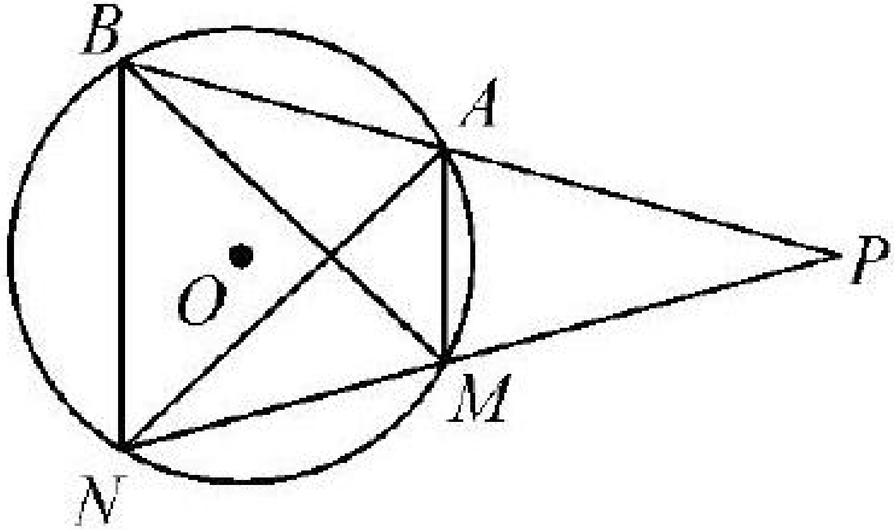
(⊠6⊠)

6 ⊠⊠⊠PA ⊠⊠⊠⊠⊠⊠A ⊠⊠⊠⊠M ⊠ PA ⊠⊠⊠⊠⊠⊠ M ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $B\ C$, ⊠ PB ⊠⊠⊠⊠⊠⊠⊠⊠⊠ D, ⊠ PC ⊠⊠⊠⊠ E, ⊠⊠ DE ⊠⊠⊠⊠$DE//PA$ ⊠

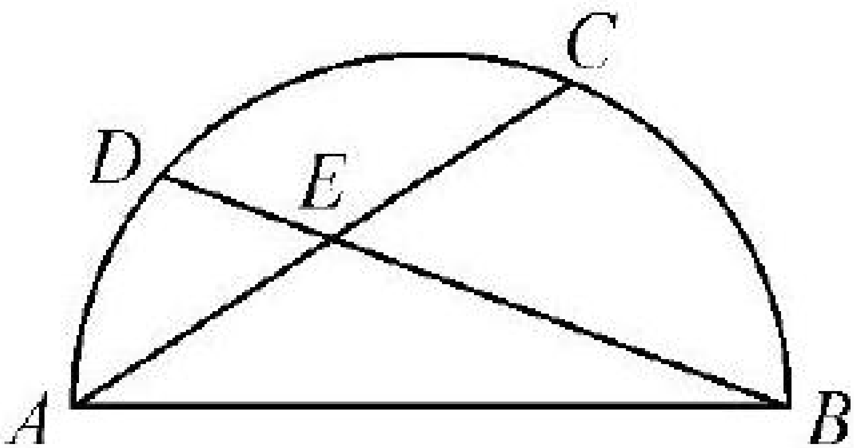
7 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ D ⊠⊠⊠⊠⊠⊠ ABC ⊠⊠⊠ BC ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AD^2 - AB^2 = BD \cdot CD$ ⊠

8 ⊠⊠⊠ I ⊠ $\triangle ABC$ ⊠⊠⊠, ⊠⊠ AI ⊠⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠ D, ⊠ BC ⊠⊠ E, ⊠ $AB = 3, AC = 4$, ⊠ $IE = ED$, ⊠ BC ⊠⊠.

9 ⊠⊠⊠⊠⊠⊠ $ABNM$ ⊠⊠⊠⊠ O, BA ⊠ NM ⊠⊠⊠⊠⊠⊠⊠⊠ P ⊠⊠⊠⊠$AM \cdot BM \cdot PN = AN \cdot BN \cdot PM$ ⊠



(⊠9 ⊠)



(⊠10 ⊠)

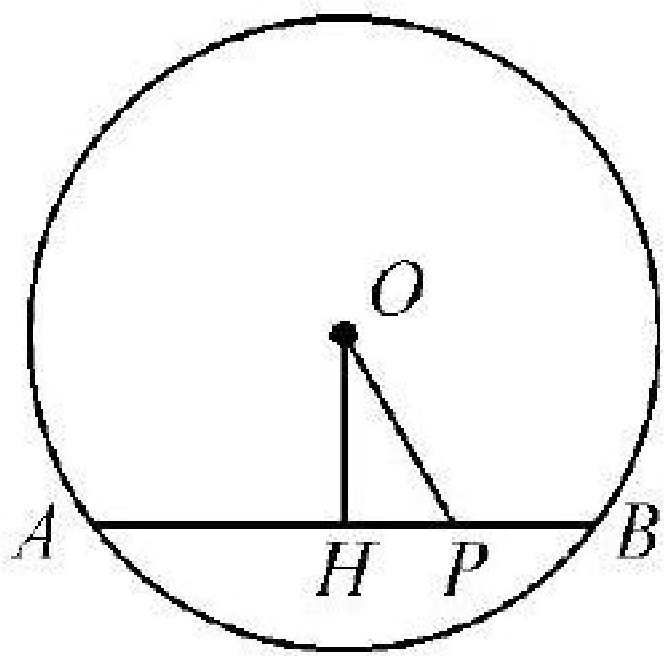
10 ⊠⊠, AB ⊠⊠⊠⊠⊠, ⊠ AC ⊠ BD ⊠⊠⊠ E, ⊠⊠: $AB^2 = AE\cdot AC+BE\cdot BD$.

⊠⊠⊠⊠⊠ r ⊠ $\odot O$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ P, ⊠⊠ $PO^2 - r^2$ ⊠⊠ P ⊠ $\odot O$ ⊠⊠⊠⊠⊠ ⊠ P ⊠⊠⊠

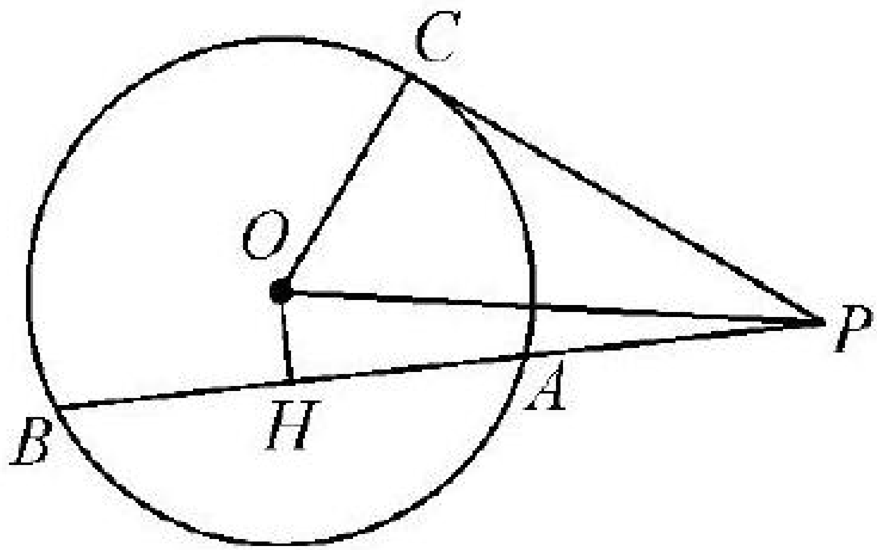
$$P\ ⊠⊠ \begin{cases} > 0, & ⊠P\ ⊠⊠⊠, \\ = 0, & ⊠P\ ⊠⊠⊠, \\ < 0, & ⊠P\ ⊠⊠⊠. \end{cases}$$

⊠⊠ $8-1, AB$ ⊠ $\odot O$ ⊠⊠, ⊠ P ⊠ AB ⊠. ⊠ $OH \perp AB$ ⊠⊠ H, ⊠

$$
\begin{aligned}
PA \cdot PB &= (PH + HA)(HB - HP) \\
&= (AH + PH)(AH - PH) \\
&= AH^2 - PH^2 \\
&= OA^2 - OH^2 - PH^2 \\
&= r^2 - PO^2.
\end{aligned}
$$



☒8-1



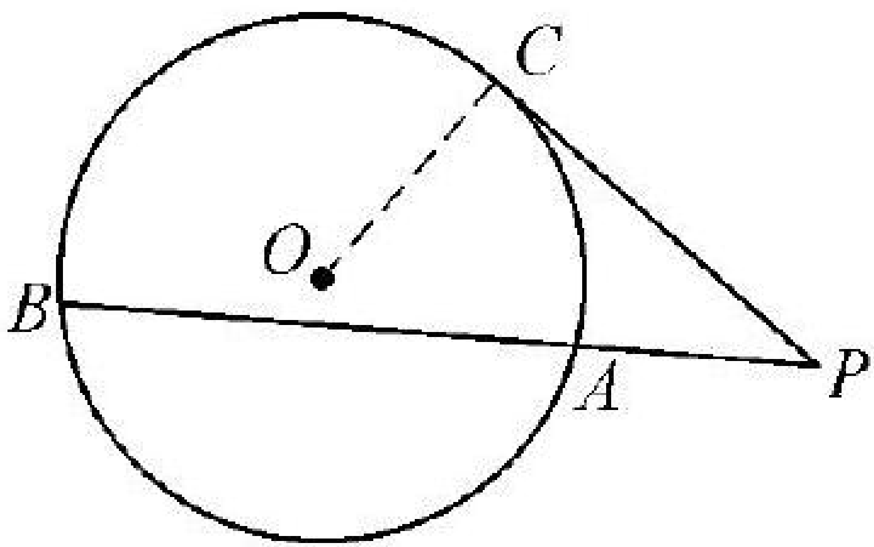
⊠8-2

⊠⊠⊠⊠⊠ P ⊠⊠⊠⊠⊠⊠8-2⊠$PA \cdot PB = PO^2 - r^2$ ⊠⊠ PC ⊠ $\odot O$ ⊠⊠ C ⊠⊠ $PC^2 = PO^2 - r^2$ ⊠

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠. ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠1 ⊠⊠8-3, PAB ⊠ $\odot O$ ⊠⊠⊠⊠⊠, ⊠ C ⊠ $\odot O$ ⊠⊠⊠ $PC^2 = PA \cdot PB$ ⊠

⊠⊠⊠PC ⊠ $\odot O$ ⊠⊠⊠⊠



⊠8-3

⊠⊠⊠ $\odot O$ ⊠⊠⊠⊠ r ⊠⊠ $PC^2 = PA\cdot$

$PB = PO^2 - r^2$, ☒ $PO^2 = OC^2 + PC^2$, ☒ $OC \perp PC$ ☒ C, ☒ $OC = r$, ☒☒ PC ☒ $\odot O$ ☒☒☒☒

☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

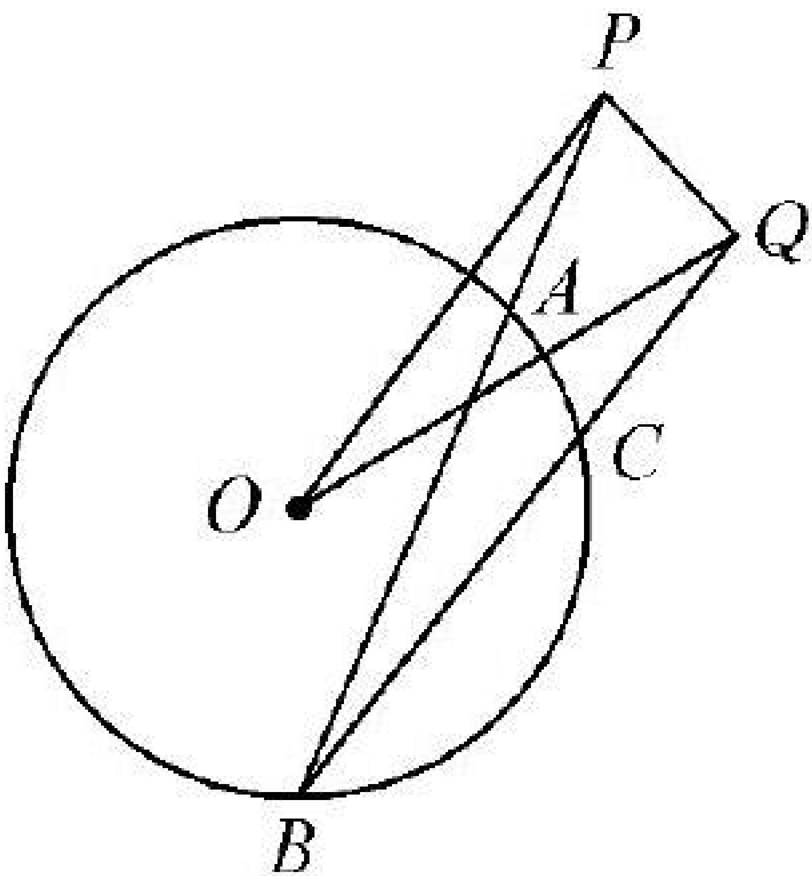
☒2 ☒☒8-4, PAB QCB ☒ $\odot O$ ☒☒☒☒☒☒☒ $PA \cdot PB = QC \cdot QB$ ☒☒☒☒ $\triangle POQ$ ☒☒☒☒☒☒☒

☒☒☒ $\odot O$ ☒☒☒☒ r, ☒ $PA \cdot PB = PO^2 - r^2, QC \cdot QB = QO^2 - r^2$ ☒

☒☒ $PA \cdot PB = QC \cdot QB$ ☒☒ $PO^2 = QO^2$ ☒

☒☒ $PO = QO$ ☒☒ $\triangle POQ$ ☒☒☒☒☒☒☒

☒☒☒☒☒☒☒☒☒☒☒ $PA \cdot PB$ ☒☒☒☒☒



☒8-4

☒☒☒☒

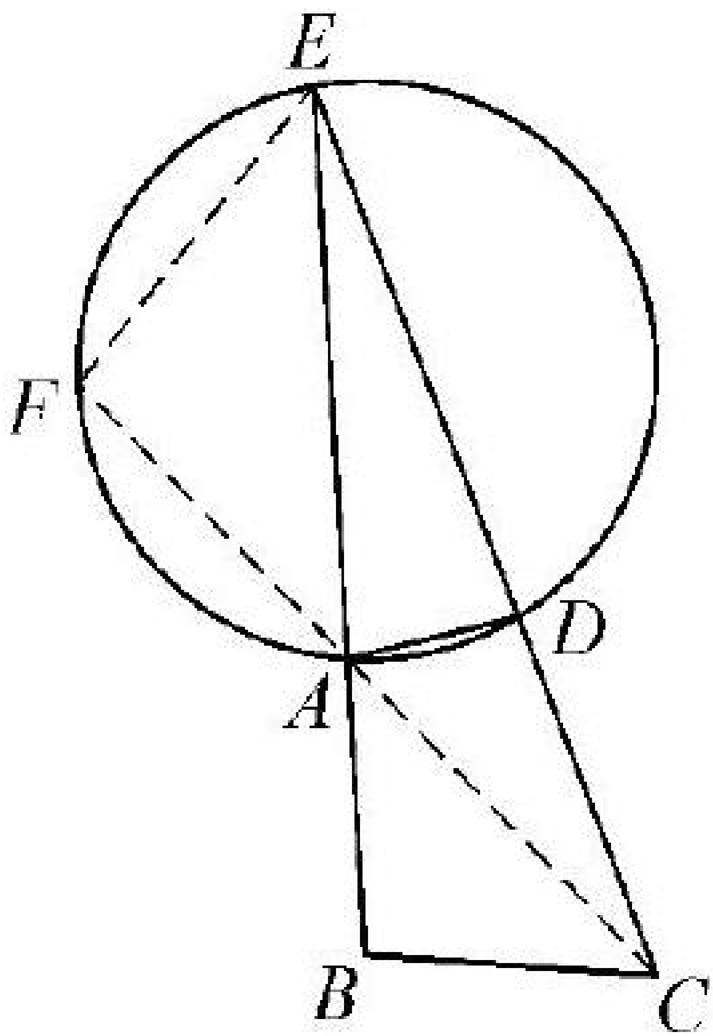
☒3 ☒☒8-5 ☒☒☒☒☒☒☒ $ABCD$ ☒☒$\angle ABC + \angle BCD < 180°$ ☒☒☒ AB ☒ CD ☒☒☒ E ☒☒☒: $\angle ABC = \angle ADC$ ☒☒☒☒☒☒ $AC^2 = CD \cdot CE - AB \cdot AE$ ☒

☒☒☒☒☒☒ $CD \cdot CE$ ☒☒ C ☒ $\triangle ADE$ ☒☒☒☒☒☒☒☒☒☒☒☒☒ C ☒☒☒☒☒☒☒☒☒☒☒ CAF ☒ $\triangle ADE$ ☒☒☒☒☒ F, ☒☒ A D E F ☒☒☒☒, ☒☒ $\angle ADC = \angle AFE$ ☒ $AB \cdot AE$ ☒☒☒☒☒☒☒☒☒

☒☒☒ CA ☒☒☒☒☒☒☒☒ F ☒☒ $CA \cdot CF = CD \cdot CE$, ☒☒ A D E F ☒☒☒☒,

⊠⊠ $\angle ADC = \angle AFE$.

⊠⊠ $\angle ABC = \angle ADC \Leftrightarrow \angle AFE = \angle ABC \Leftrightarrow F$ ⊠E C B ⊠⊠⊠⊠ $\Leftrightarrow AB \cdot AE = AC \cdot AF \Leftrightarrow AB \cdot AE =$



⊠8-5

$CA \cdot CF - AC^2 = CD \cdot CE - AC^2 \Leftrightarrow AC^2 = CD \cdot CE -$ $AB \cdot AE$.

⊠⊠⊠⊠⊠⊠⊠ A D E F ⊠⊠⊠⊠⊠⊠ C ⊠⊠⊠$CD \cdot CE$)-⊠ C ⊠⊠ $(CA \cdot CF) = 0$ ⊠⊠⊠⊠⊠⊠ E F B C ⊠⊠⊠⊠⊠⊠ A ⊠⊠ $(AC \cdot AF)-$ ⊠ A ⊠⊠ $(AB \cdot AE) = 0$ ⊠

⊠4 ⊠⊠8-6, ⊠⊠⊠⊠⊠⊠ $ABCD$ ⊠, AC ⊠ BD ⊠⊠⊠ Q, BA ⊠ CD ⊠⊠⊠ P, ⊠⊠: $PQ^2 = PA \cdot PB - QB \cdot QD$ ⊠

⊠⊠⊠⊠⊠⊠⊠⊠ $PA \cdot PB$ $QB \cdot QD$ ⊠⊠⊠⊠⊠ PQ ⊠.

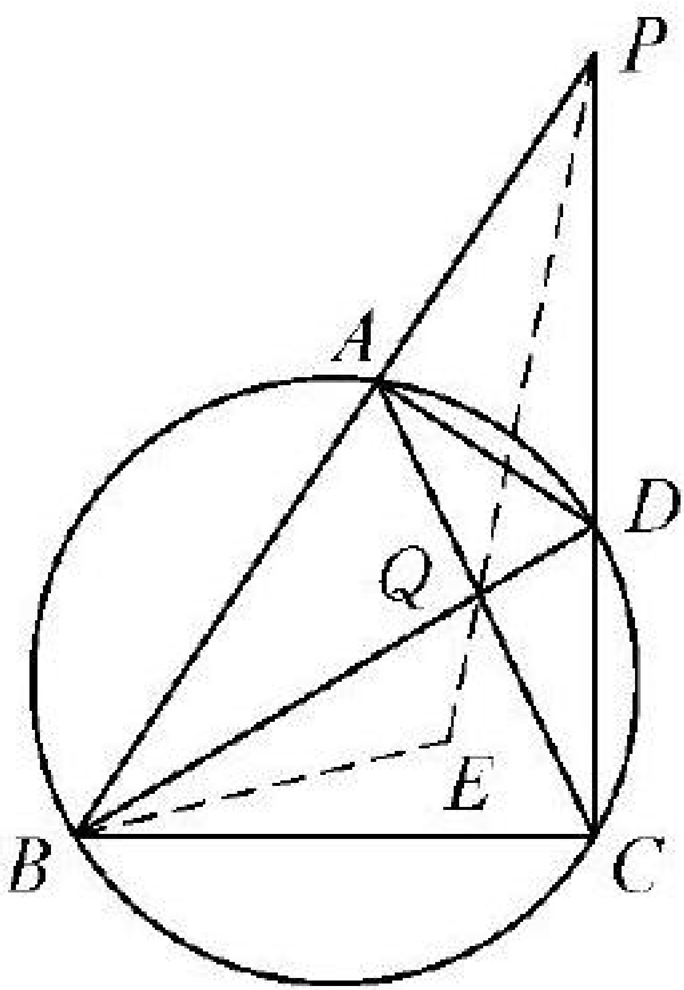
⊠⊠⊠⊠ PQ ⊠⊠ E, ⊠ $PQ \cdot PE = PA \cdot PB$,

⊠ A Q E B ⊠⊠⊠⊠⊠⊠ $\angle QEB = \angle PAQ$ ⊠
⊠⊠
$\angle QDP = 180° - \angle BDC = 180° - \angle BAC = \angle PAQ$, ⊠⊠ $\angle QEB = \angle QDP$, ⊠
⊠ B E D P ⊠⊠⊠⊠, ⊠⊠

$$PQ \cdot QE = QB \cdot QD \tag{2}$$

⊠ (1)-(2) ⊠ $PQ^2 = PQ(PE - QE)$

$$= PQ \cdot PE - PQ \cdot QE$$
$$= PA \cdot PB - QB \cdot QD$$



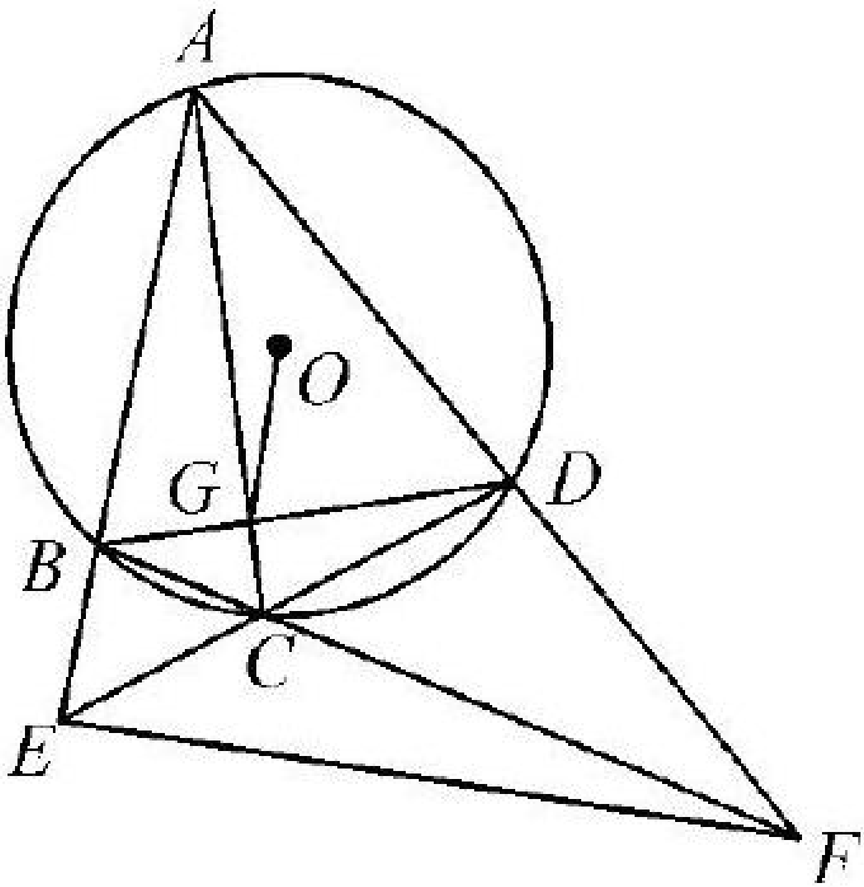
⊠8-6

⊠⊠⊠⊠⊠⊠⊠ $PQ^2 = P$ ⊠⊠ $+Q$ ⊠⊠. ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠5 ⊠⊠⊠ $ABCD$ ⊠⊠⊠ $\odot O$ ⊠⊠⊠8-7 ⊠⊠⊠AC ⊠ BD ⊠⊠⊠ G, AB ⊠ DC ⊠ ⊠⊠ E, BC ⊠ AD ⊠⊠⊠ F, ⊠⊠ EF, OG ⊠

⊠⊠⊠$OG \perp EF$ ⊠

⊠⊠⊠ $\odot O$ ⊠⊠⊠⊠ r, ⊠ $EG^2 = E$ ⊠⊠ $+G$ ⊠⊠⊠
$FG^2 = F$ ⊠⊠ $+G$ ⊠⊠⊠



⊠8-7

⊠⊠

(1)

$= EO^2 - r^2 - FO^2 + r^2$ $= EO$

⊠⊠8-8, ⊠⊠ E ⊠ $EH_1 \perp OG$ ⊠⊠ H_1, ⊠

$$EG^2 = EH_1^2 + GH_1^2,$$
$$EO^2 = EH_1^2 + (OG + GH_1)^2,$$

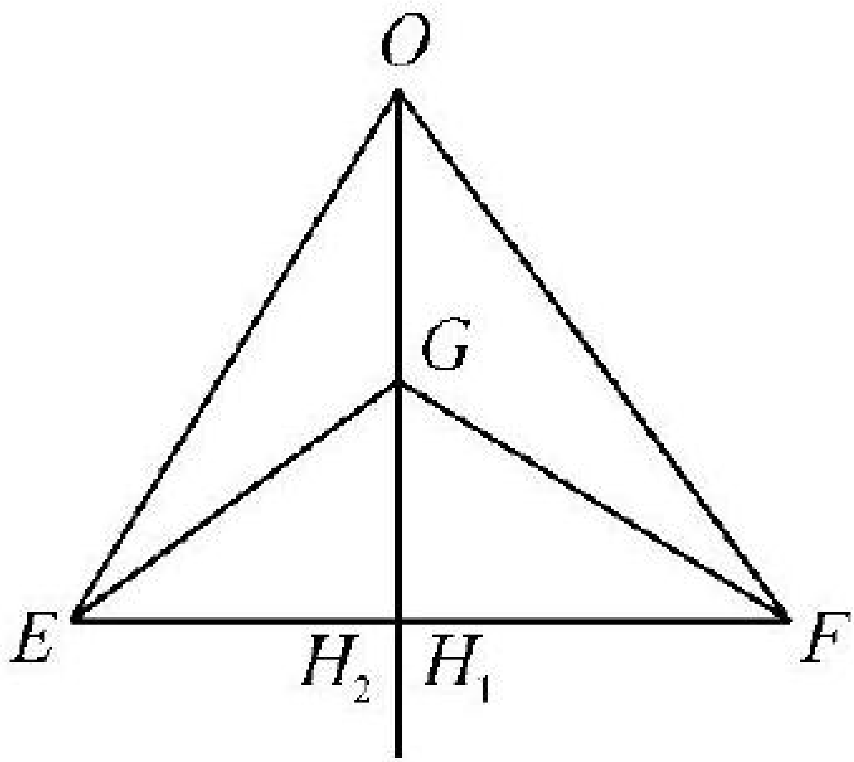
⊠

$$EO^2 - EG^2 = OG^2 + 2OG \cdot GH_1 \tag{2}$$

⊠⊠ F ⊠ $FH_2 \perp OG$ ⊠⊠ H_2, ⊠

(3)

⊠⊠ $FO^2 - FG^2 = OG^2 + 2OG \cdot GH_2$ ⊠
⊠ (2)⊠(3) ⊠⊠ (1), ⊠⊠⊠⊠ $OG \cdot GH_1 = OG\cdot$



⊠8-8

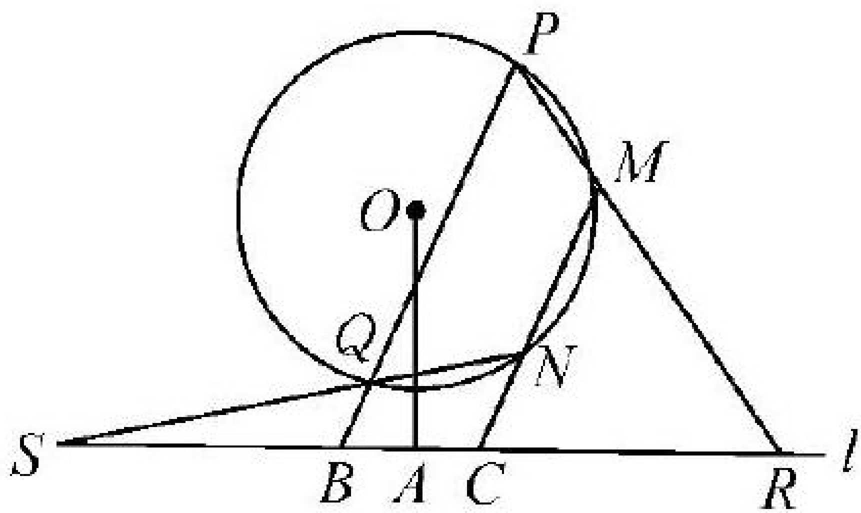
GH_2,

⊠⊠ $GH_1 = GH_2$, ⊠ H_1 ⊠ H_2 ⊠⊠.

⊠ $OG \perp EF$ ⊠

⊠6 ⊠⊠8-9⊠⊠⊠⊠ $\odot O$ ⊠⊠⊠⊠ $OA \perp l$ ⊠⊠ A ⊠⊠ l ⊠⊠ $AB = AC$ ⊠⊠⊠ B ⊠⊠⊠ BQP ⊠ $\odot O$ ⊠⊠ Q P ⊠⊠⊠ C ⊠⊠⊠ CNM ⊠ $\odot O$ ⊠⊠ N M, ⊠⊠ PM ⊠ l ⊠⊠ R, ⊠⊠ NQ ⊠ l ⊠⊠ S, ⊠ $MN//PQ$, ⊠⊠: $AS = AR$ ⊠

⊠⊠ $AS = AR \Leftrightarrow SO^2 - OA^2 = RO^2 -$



⊠8-9

$OA^2 \Leftrightarrow SQ \cdot SN = RM \cdot RP$.

⊠⊠⊠⊠ $MN//PQ$, ⊠ $PM = QN$.

⊠⊠ $BQ \cdot BP = BO^2 - r^2 = OC^2 - r^2 = CN \cdot CM$ ⊠⊠⊠ r ⊠⊠⊠⊠⊠⊠⊠ ⊠ $\frac{BQ}{CN} = \frac{MC}{PB}$ ⊠⊠⊠⊠ $\frac{BQ}{CN} = \frac{SQ}{SN}, \frac{MC}{PB} = \frac{RM}{RP}$ ⊠⊠ $\frac{RM}{RP} = \frac{SQ}{SN}$ ⊠⊠⊠

$$\frac{RP - RM}{RP} = \frac{SN - SQ}{SN}$$

⊠⊠

$$RP - RM = PM = QN = SN - SQ,$$

⊠

$$SN = RP, SQ = RM$$

⊠⊠

$$SN \cdot SQ = RP \cdot RM$$

⊠⊠

$$\begin{aligned} AS^2 &= SO^2 - OA^2 = SO^2 - r^2 + r^2 - OA^2 \\ &= SQ \cdot SN + r^2 - OA^2 \\ &= RP \cdot RM + r^2 - OA^2 \\ &= RO^2 - r^2 + r^2 - OA^2 \\ &= RO^2 - OA^2 = RA^2, \end{aligned}$$

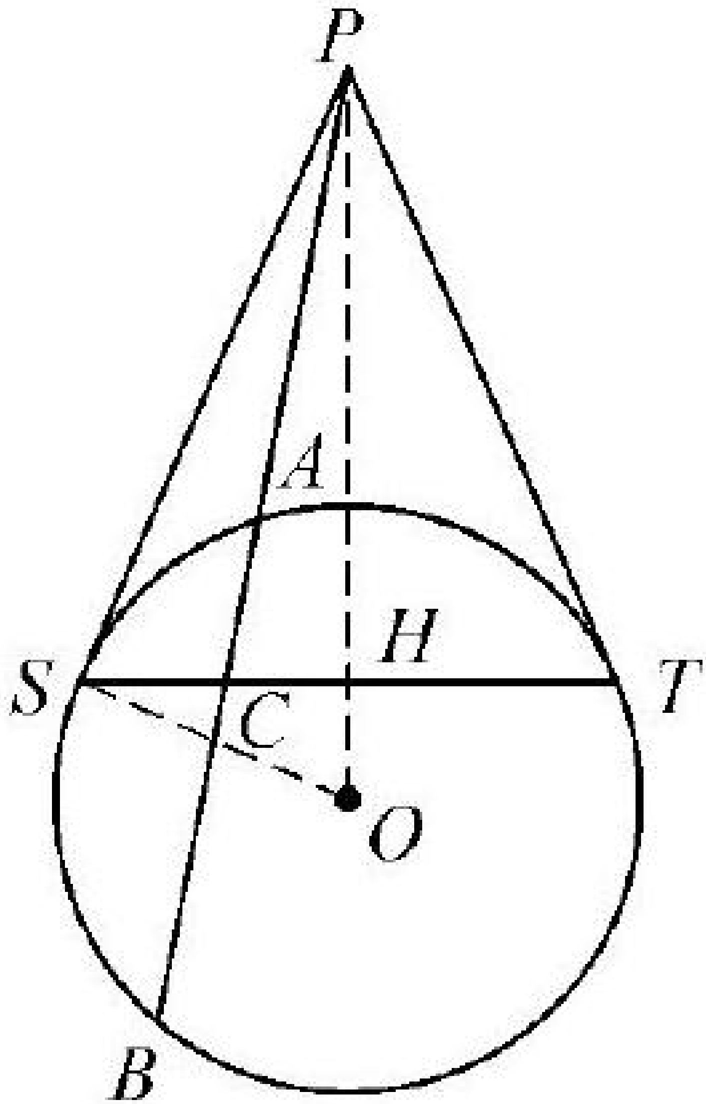
⊠⊠ $AS = AR$ ⊠

⊠7 ⊠⊠8-10 ⊠⊠⊠⊠⊠⊠ P ⊠ $\odot O$ ⊠⊠⊠⊠PS PT ⊠ $\odot O$ ⊠⊠ S T, ⊠ P ⊠ $\odot O$ ⊠⊠⊠ PAB ⊠ $\odot O$ ⊠⊠ A ⊠B, ⊠ ST ⊠⊠ C, ⊠⊠: $\frac{2}{PC} = \frac{1}{PA} + \frac{1}{PB}$.

⊠⊠⊠ PC^2 ⊠⊠⊠⊠ P ⊠⊠⊠⊠ C ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ PC, PA, PB ⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠ PO ⊠ ST ⊠⊠ H ⊠⊠ $PO \perp ST$ ⊠⊠ $SH = HT$ ⊠⊠⊠ SO ⊠⊠

$$PC^2 = PH^2 + CH^2 = PS^2 - SH^2 + CH^2$$



⊠8-10

$$
\begin{aligned}
&= PS^2 + (CH - SH)(CH + SH) \\
&= PS^2 - SC \cdot CT = PA \cdot PB - AC \cdot CB \\
&= PA \cdot PB - (PC - PA)(PB - PC) \\
&= PA \cdot PB + PC^2 - PC(PA + PB) + PA \cdot PB,
\end{aligned}
$$

⊠

$$2PA \cdot PB = PC(PA + PB)$$

⊠

$$\frac{2}{PC} = \frac{1}{PA} + \frac{1}{PB}$$

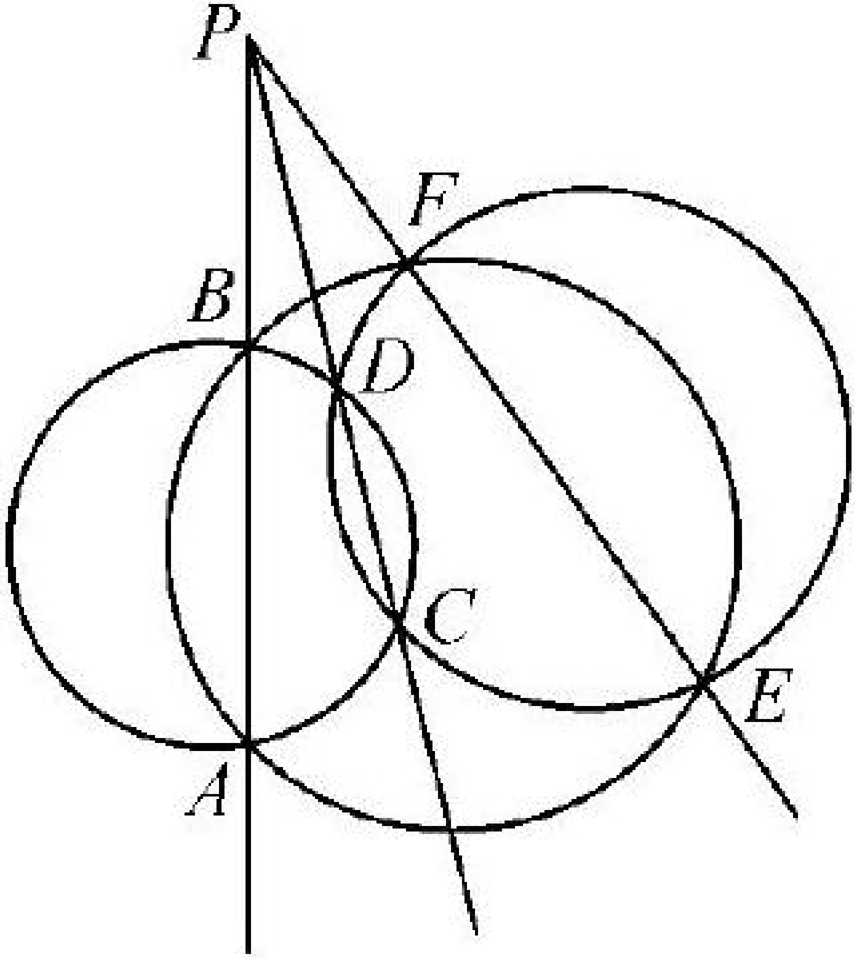
⊠⊠⊠⊠ $PC^2 = PA \cdot PB - AC \cdot CB$ ⊠⊠ $PC^2 = P$ ⊠⊠ $+C$ ⊠⊠⊠⊠⊠⊠⊠⊠

⊠8 ⊠⊠8-11⊠⊠⊠⊠⊠⊠⊠⊠⊠ A B C ⊠D E F ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ AB CD EF ⊠⊠⊠⊠.

⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠ AB ⊠ CD ⊠⊠⊠ P, ⊠⊠ PE ⊠ $\odot CDE$ ⊠⊠ E ⊠⊠⊠⊠ F' ⊠⊠

$$PB \cdot PA = PF' \cdot PE = PD \cdot PC$$



⊠8-11

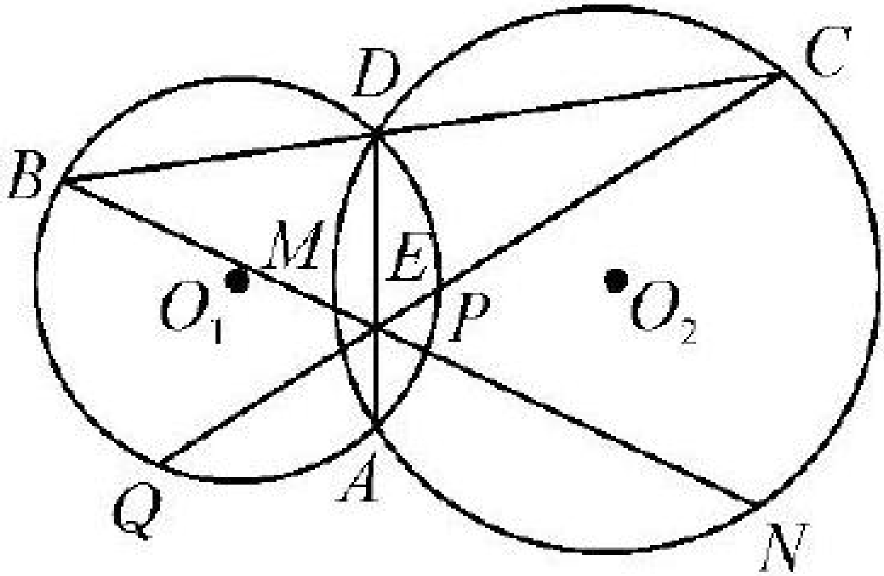
⊠⊠ F' ⊠ $\odot CDE$ ⊠, ⊠⊠ $\odot ABE$ ⊠, ⊠ F' ⊠ $\odot CDE$ ⊠ $\odot ABE$ ⊠⊠⊠, ⊠⊠ F' ⊠ F ⊠⊠⊠⊠⊠ EF ⊠ EF' ⊠⊠⊠⊠ AB CD EF ⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠. ⊠⊠

⊠9 ⊠⊠8-12⊠$\odot O_1$ ⊠ $\odot O_2$ ⊠ A, D ⊠⊠, ⊠⊠ D ⊠⊠⊠⊠ $\odot O_1$ ⊠⊠ B, ⊠ $\odot O_2$ ⊠⊠ C, E ⊠ AD ⊠⊠⊠ A D ⊠⊠, ⊠⊠ BE ⊠ $\odot O_2$ ⊠⊠ M N, ⊠⊠ CE ⊠ $\odot O_1$ ⊠⊠ P Q.

(1) ⊠⊠⊠P M Q N ⊠⊠⊠⊠;

(2) ⊠⊠⊠ P M Q N ⊠⊠⊠⊠⊠ O_3, ⊠



☒8-12

☒: $DO_3 \perp BC$ ☒

☒☒☒1☒☒ $EP\cdot EQ = ED\cdot EA, EM\cdot EN = ED\cdot EA$ ☒☒ $EP\cdot EQ = EM\cdot EN$, ☒☒ P M Q N ☒☒☒☒☒

(2) ☒☒

$$BM \cdot BN = BD \cdot BC,$$

☒

$$BD(BD+DC) = BD^2 + BD \cdot DC,$$

$$BM \cdot BN = BO_3^2 - r_3^2 \,(r_3 \text{ ☒} \odot O_3 \text{ ☒☒☒}),$$

☒

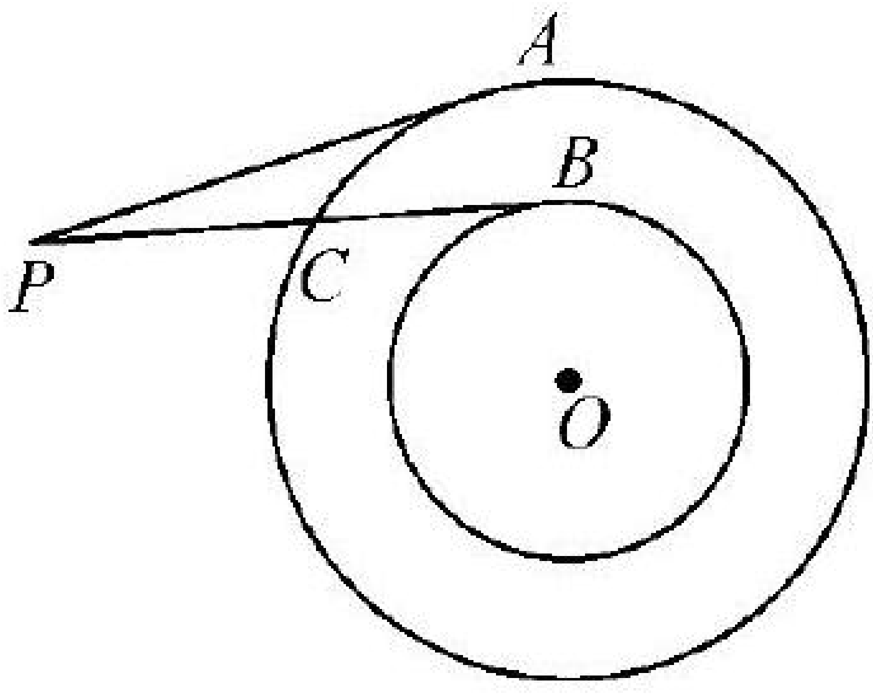
$$BD^2 + BD \cdot DC = BO_3^2 - r_3^2,$$

☒☒

$$CD^2 + BD \cdot DC = CO_3^2 - r_3^2,$$

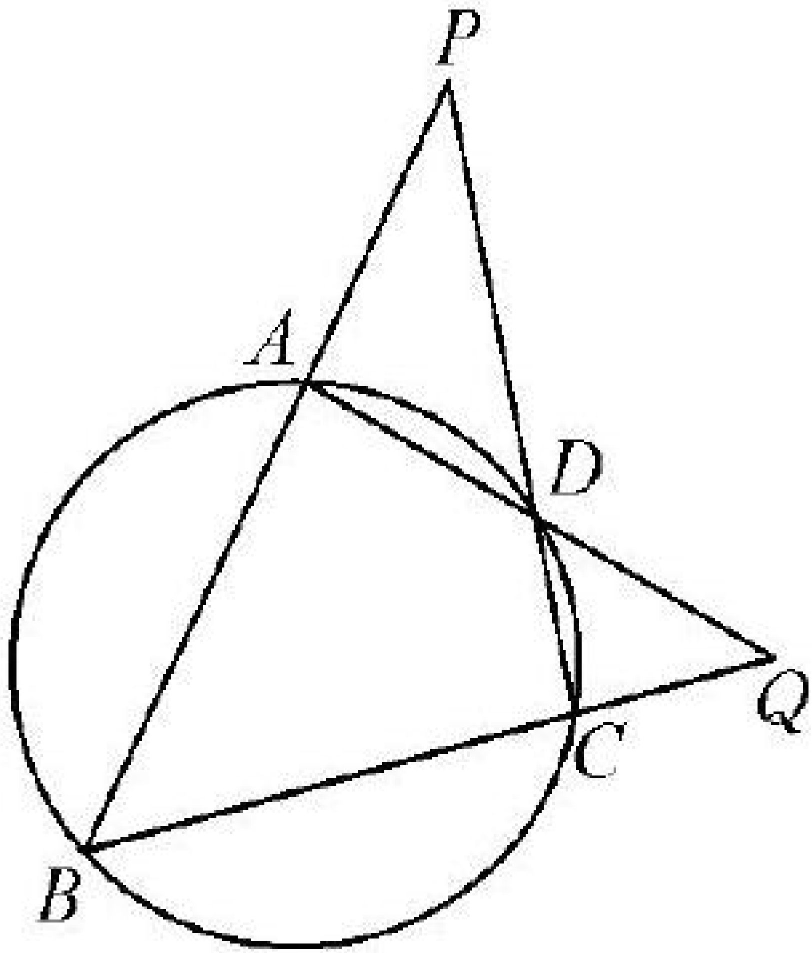
⊠ $BD^2 - CD^2 = BO_3^2 - CO_3^2$ ⊠⊠⊠ $DO_3 \perp BC$ ⊠
⊠⊠⊠⊠ MN ⊠ $\odot O_2$ ⊠ $\odot O_3$ ⊠⊠⊠⊠PQ ⊠ $\odot O_1$ ⊠ $\odot O_3$ ⊠⊠⊠⊠E ⊠ $\odot O_1, \odot O_2, \odot O_3$
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

1 ⊠ PA PB ⊠ $\odot O$ ⊠⊠⊠⊠⊠⊠ PEC ⊠ $\odot O$ ⊠⊠ E C, ⊠ P E ⊠ AB ⊠⊠, ⊠ AB ⊠⊠ D ⊠⊠⊠ $PE = 2, CD = 1$ ⊠⊠ DE ⊠⊠⊠
2 ⊠⊠⊠PA PB ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ PB ⊠⊠⊠⊠⊠ B ⊠
⊠⊠⊠$PB^2 - PA^2 = CB^2$ ⊠

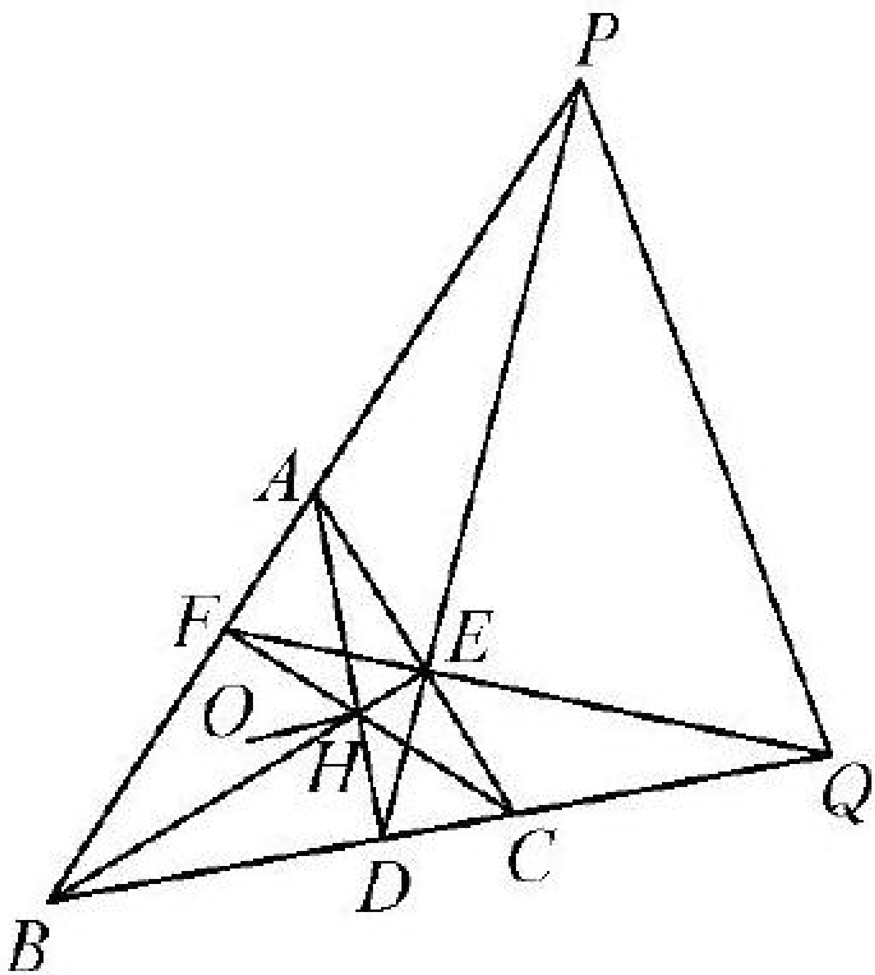


(⊠2 ⊠)

3 ⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠ $\odot O$ ⊠⊠ BA ⊠ CD ⊠⊠ P, AD ⊠ BC ⊠⊠ Q ⊠⊠⊠), ⊠⊠: $PQ^2 = P$ ⊠⊠ $+Q$ ⊠⊠.



(⊠3 ⊠)



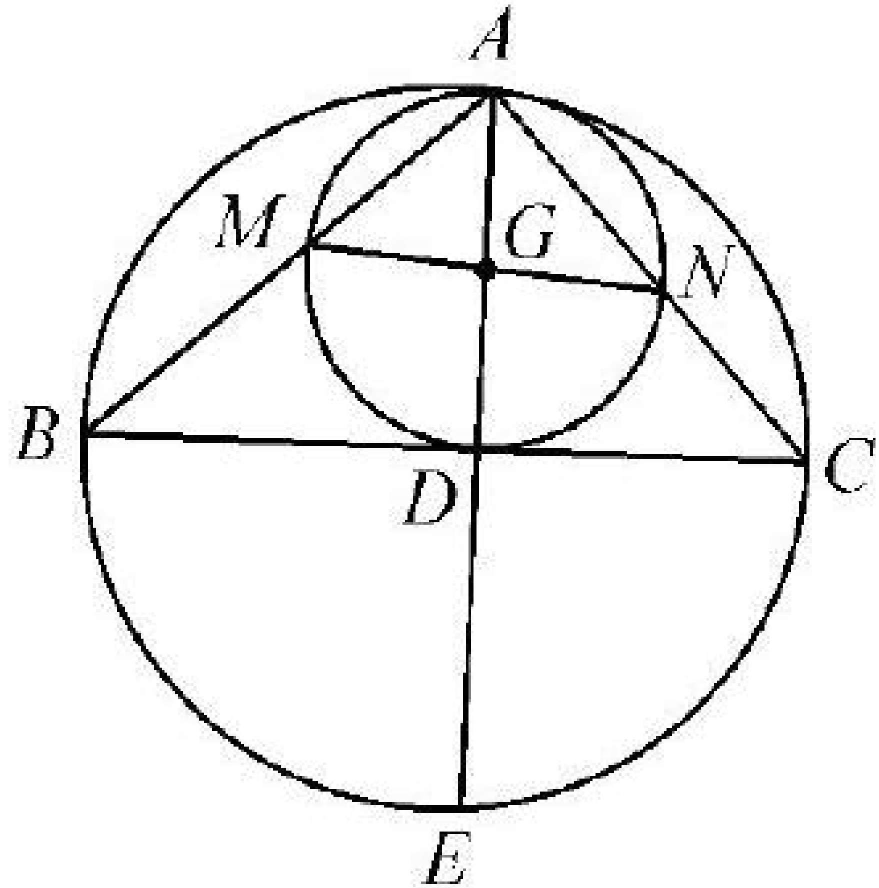
(⊠4 ⊠)

4 ⊠⊠⊠⊠⊠ ABC ⊠⊠$AD \perp BC$ ⊠⊠ $D, BE \perp AC$ ⊠⊠ $E, CF \perp AB$ ⊠⊠ F, ⊠ H ⊠⊠⊠, ⊠ O ⊠⊠⊠, DE ⊠ BA ⊠⊠ P, FE ⊠ BC ⊠⊠ Q, ⊠⊠ $PQ\ OH$, ⊠⊠: $PQ \perp OH$ ⊠

5 ⊠⊠⊠$\triangle ABC$ ⊠⊠$AD \perp BC$ ⊠⊠ D ⊠⊠ AD ⊠⊠⊠⊠⊠⊠ AB ⊠⊠ M, ⊠ AC ⊠⊠ N, ⊠⊠ MN ⊠ AD ⊠⊠ G, ⊠⊠ AD ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠ E. ⊠⊠⊠(1) $M\ N\ C\ B$ ⊠⊠⊠⊠⊠

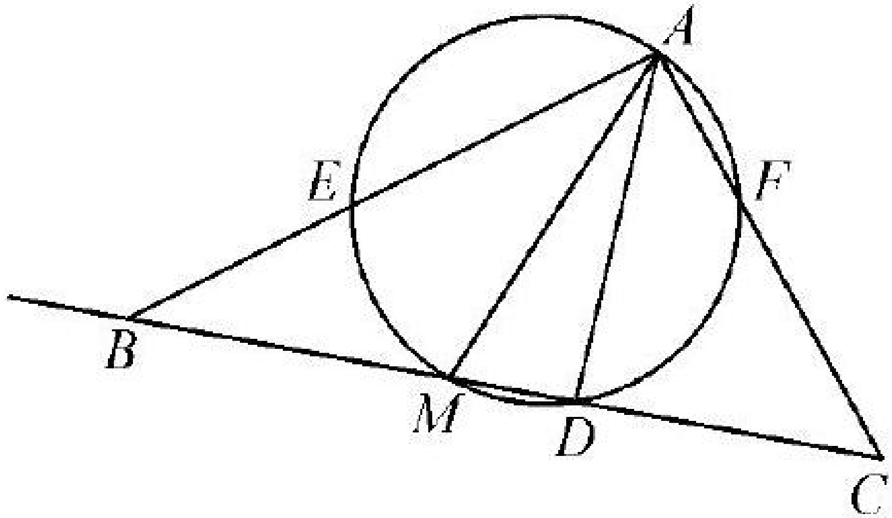
(2) $AD^2 = AG \cdot AE$.

6 ⊠ $ABCD$ ⊠⊠⊠⊠ O, AD ⊠ BC ⊠⊠ E, AB ⊠ CD ⊠⊠ F, ⊠ $\odot O$ ⊠⊠⊠⊠ $10, OE = 23, OF = 20$, ⊠ EF ⊠

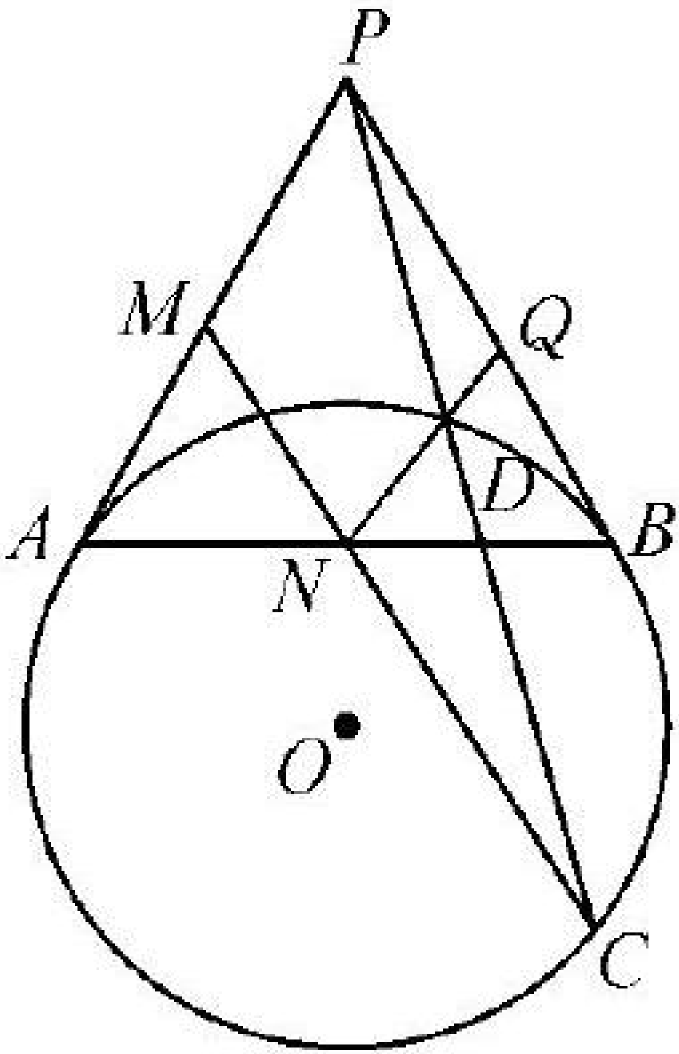


(☒5 ☒)

7 ☒☒☒$\triangle ABC$ ☒☒AD ☒☒ $\angle BAC, AM$ ☒ BC ☒☒☒☒☒☒$\triangle AMD$ ☒☒☒☒☒ AB ☒☒ E, ☒ AC ☒☒ F, ☒☒: $BE = CF$.



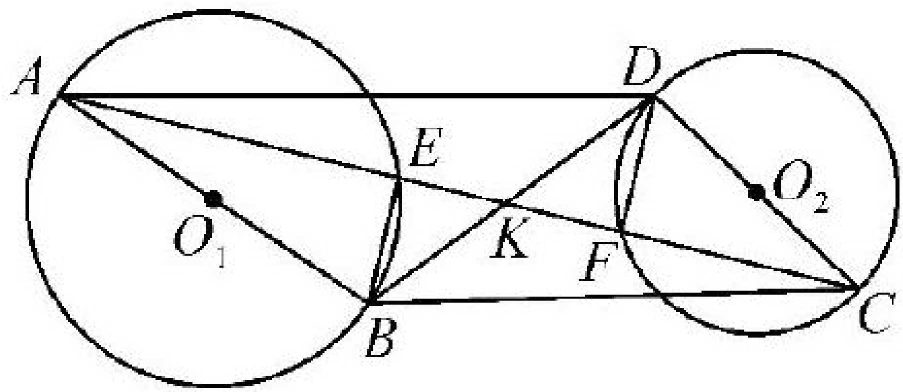
(⊠7⊠)



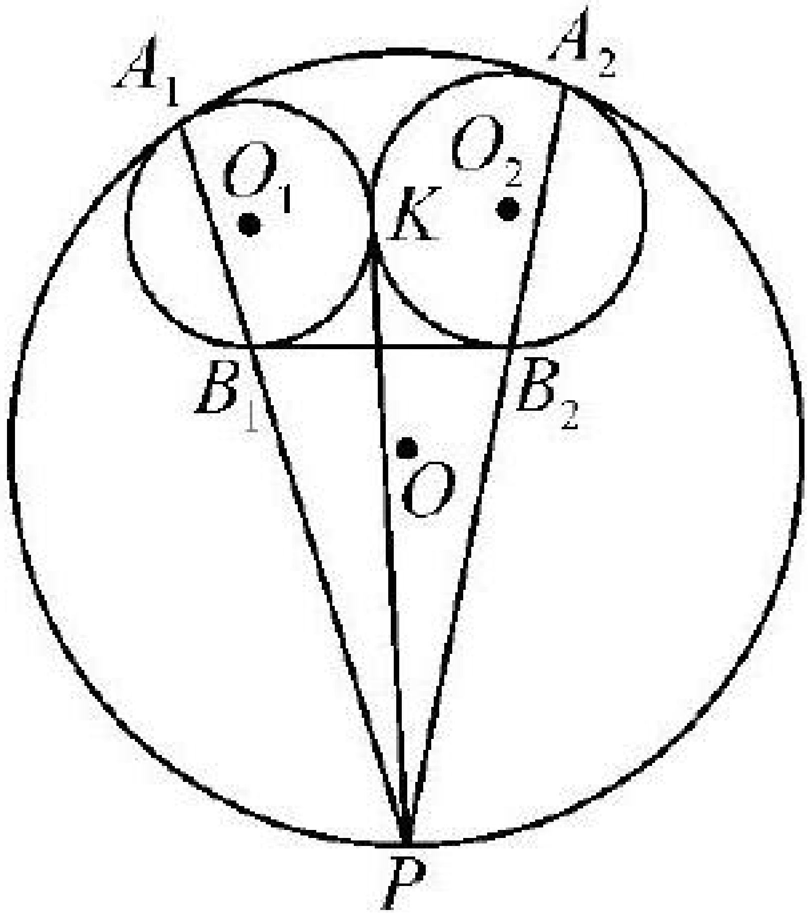
(⊠8 ⊠)

8 ⊠⊠⊠⊠⊠ $\odot O$ ⊠⊠⊠ $P, PA\ PB$ ⊠ $\odot O$ ⊠⊠ $A\ B, M$ ⊠ AP ⊠⊠⊠N ⊠ AB ⊠ ⊠, MN ⊠ $\odot O$ ⊠⊠ C, ⊠⊠ PC ⊠ $\odot O$ ⊠⊠ D, ⊠⊠ ND ⊠ PB ⊠⊠ Q. ⊠⊠⊠⊠⊠⊠ $PMNQ$ ⊠⊠⊠⊠

9 ⊠⊠ $\odot O_1$ ⊠ $\odot O_2$ ⊠⊠ (⊠⊠), $AB\ CD$ ⊠⊠⊠ $\odot O_1, \odot O_2$ ⊠⊠⊠, ⊠ $AD//BC$, ⊠ AC ⊠ BD ⊠⊠⊠ K. ⊠⊠: ⊠ K ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠.



(⊠9 ⊠)



(⊠10 ⊠)

10 ⊠⊠ $\odot O_1$ ⊠ $\odot O_2$ ⊠⊠⊠⊠ K, ⊠⊠⊠ $\odot O$ ⊠⊠, ⊠⊠⊠⊠⊠ A_1 A_2, ⊠⊠⊠⊠⊠⊠⊠ K ⊠ $\odot O_1$ $\odot O_2$ ⊠⊠⊠⊠⊠ $\odot O$ ⊠⊠ P, PA_1 ⊠ $\odot O_1$ ⊠⊠ B_1, PA_2 ⊠ $\odot O_2$ ⊠⊠ B_2, ⊠⊠: B_1B_2 ⊠ $\odot O_1$ $\odot O_2$ ⊠⊠⊠⊠.

⊠⊠⊠ (Ptolemy) ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠9-1, $ABCD$ ⊠⊠⊠⊠⊠⊠⊠, AC BD ⊠⊠⊠⊠⊠. ⊠⊠: $AC\cdot BD = AB\cdot CD + AD\cdot BC$ ⊠

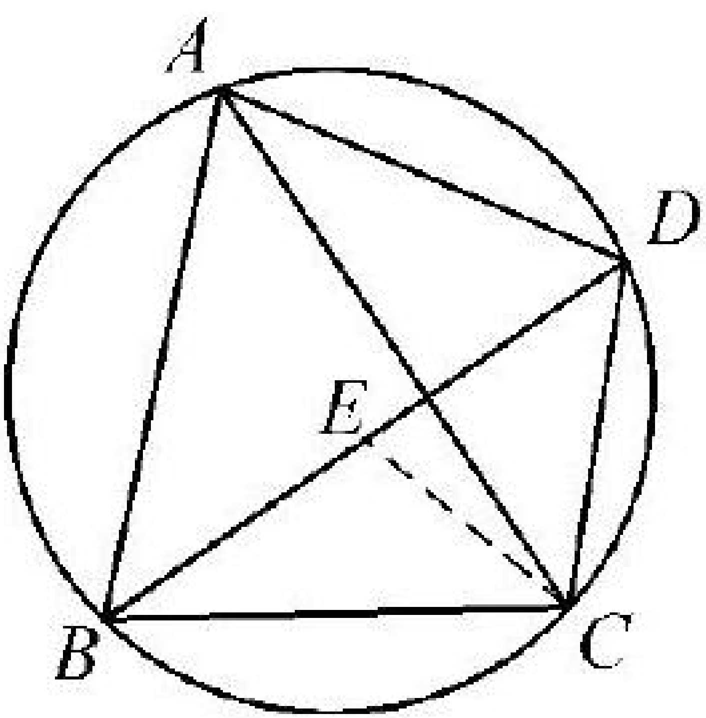
⊠⊠⊠⊠9-1 ⊠⊠⊠⊠ CE ⊠⊠ $\angle BCE = \angle ACD$ ⊠⊠⊠ $\angle CAD = \angle CBD$ ⊠⊠⊠ $\triangle ACD \backsim \triangle BCE$ ⊠⊠ $\frac{AC}{BC} = \frac{AD}{BE}$ ⊠⊠

$$AD \cdot BC = AC \cdot BE \tag{1}$$

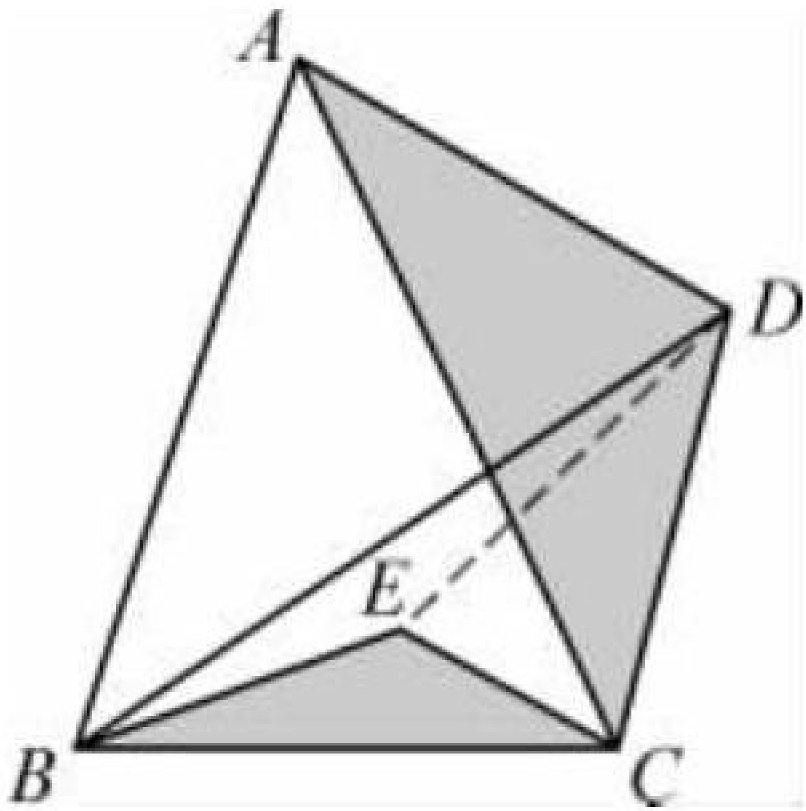
⊠ $\angle BAC = \angle BDC, \angle ACB = \angle DCE$, ⊠⊠ $\triangle ACB \backsim \triangle DCE$. ⊠ $\frac{AC}{CD} = \frac{AB}{DE}$, ⊠

$$AB \cdot CD = AC \cdot DE \tag{2}$$

⊠ (1) +(2) ⊠ $AB \cdot CD + AD \cdot BC = AC \cdot DE + AC \cdot BE = AC \cdot BD$.



⊠9-1



☒9-2

☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $ABCD$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $ABCD$ ☒☒☒☒☒

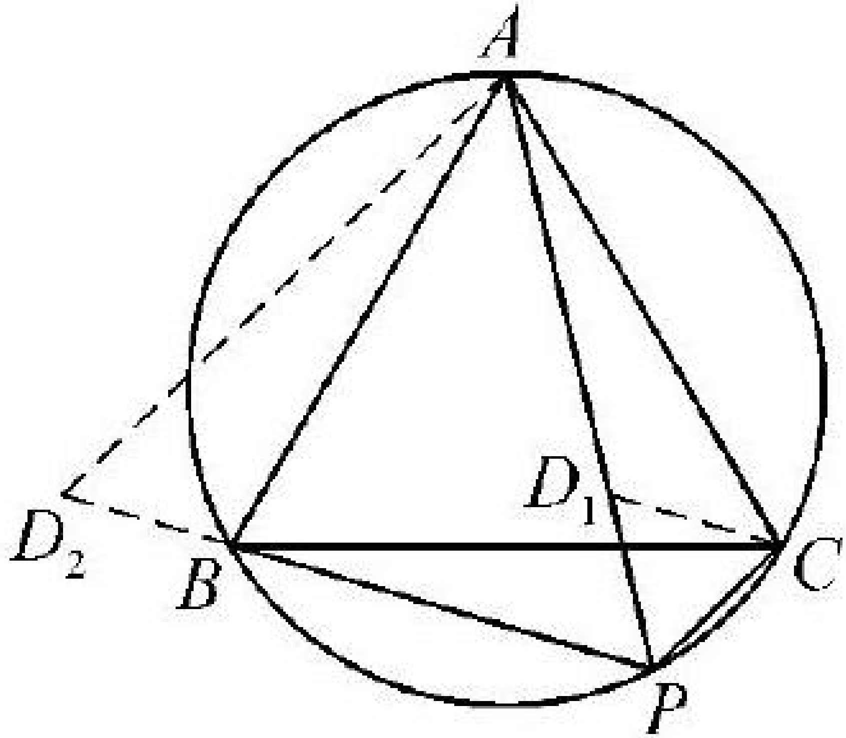
☒☒☒☒☒☒☒☒☒☒☒☒☒ $ABCD$ ☒☒$AB \cdot CD + AD \cdot BC \geqslant AC \cdot BD$ ☒☒☒☒☒☒☒☒☒☒☒☒☒9-2☒☒☒☒
☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒☒☒☒☒

☒1 ☒☒9-3, $\triangle ABC$ ☒☒☒☒☒, P ☒ BC ☒☒☒☒☒. ☒☒ $PA = PB + PC$ ☒

☒☒☒☒☒☒☒☒ $ABPC$ ☒☒☒☒☒☒☒☒ $AB\ CP + AC \cdot BP = AP \cdot BC$ ☒☒ $AB = AC = BC$ ☒☒☒ $CP + BP = AP$.

☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒9-3☒☒☒☒1☒☒☒ PB ☒ D_2 ☒



⊠9-3

$BD_2 = PC$ ⊠⊠ $PA = PD_2$ ⊠⊠2⊠⊠ PA ⊠⊠⊠ $PD_1 = CP$ ⊠⊠ $PB = D_1A$ ⊠
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠2 ⊠ $ABCDEF$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AB = BC = CD, DE = EF = FA, \angle BCD = \angle EFA = 60°$. ⊠ G H ⊠⊠⊠⊠⊠⊠⊠, ⊠⊠ $\angle AGB = \angle DHE = 120°$ ⊠⊠⊠: $AG + GB + GH + DH + HE \geqslant CF$ ⊠

⊠⊠⊠ X Y ⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠ $\triangle ABX$ $\triangle DEY$ ⊠⊠⊠⊠⊠. ⊠⊠⊠⊠ $ABCDEF$ ⊠ $DBXAEY$ ⊠⊠, $CF = XY$ ⊠

⊠⊠ $\angle AXB + \angle AGB = \angle DYE + \angle DHE = 180°$ ⊠⊠⊠⊠⊠⊠ $AXBG$ ⊠ $DHEY$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠1⊠⊠⊠⊠ $XG = AG + GB, HY = DH + HE$ ⊠

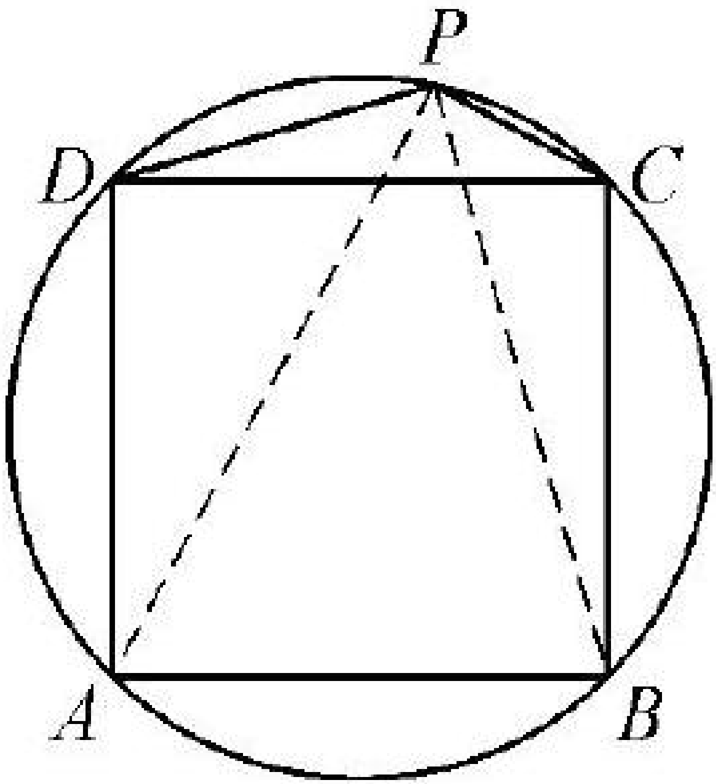
⊠⊠ $AG + GB + GH + DH + HE = XG + GH + HY \geqslant XY = CF$.

⊠3 ⊠⊠9-4, ⊠ P ⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠⊠⊠⊠ DC ⊠⊠⊠⊠⊠

$$PA(PA + PC) = PB(PB + PD).$$

⊠⊠⊠⊠ PA PB. ⊠⊠⊠⊠⊠⊠⊠ a, ⊠⊠⊠⊠⊠⊠⊠⊠ $ABCP$ ⊠,

$$PA \times a + PC \times a = PB \times \sqrt{2}a,$$



⊠9-4
⊠

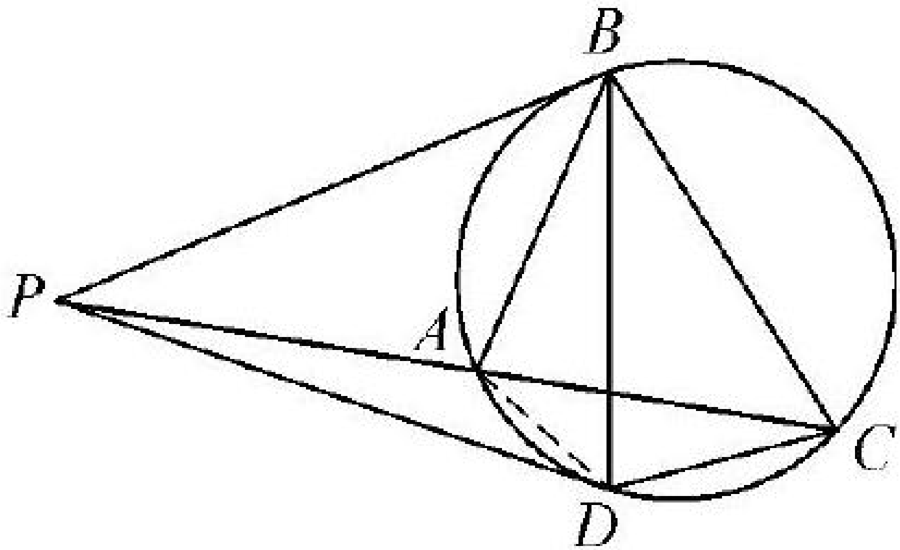
$$PA + PC = \sqrt{2}PB$$

⊠⊠⊠⊠⊠⊠⊠ $ABPD$ ⊠⊠

$$PB + PD = \sqrt{2}PA$$

⊠⊠ $\frac{PA+PC}{PB} = \frac{PB+PD}{PA}$, ⊠ $PA(PA + PC) = PB(PB + PD)$.
⊠4 ⊠⊠9-5, ⊠⊠ $\triangle ABC$ ⊠, $AB = AC$, ⊠⊠ B ⊠ $\triangle ABC$ ⊠⊠⊠⊠
⊠⊠⊠ CA ⊠⊠⊠⊠⊠⊠ P, ⊠⊠ P ⊠⊠⊠⊠⊠⊠⊠⊠ PD, ⊠⊠⊠ D, ⊠ $\frac{DB}{DC}$ ⊠⊠.
⊠⊠⊠ $\angle PBA = \angle PCB, \angle BPA = \angle CPB$, ⊠⊠ $\triangle PBA \backsim \triangle PCB$, ⊠⊠⊠⊠ AD, ⊠

$$\triangle PDA \backsim \triangle PCD,$$



☒9-5

☒☒ $PB = PD$ ☒☒☒☒ $\frac{DA}{CD} = \frac{PA}{PD} = \frac{PA}{PB} = \frac{BA}{CB}$,

☒

$$\frac{DA \cdot CB}{BA \cdot CD} = 1$$

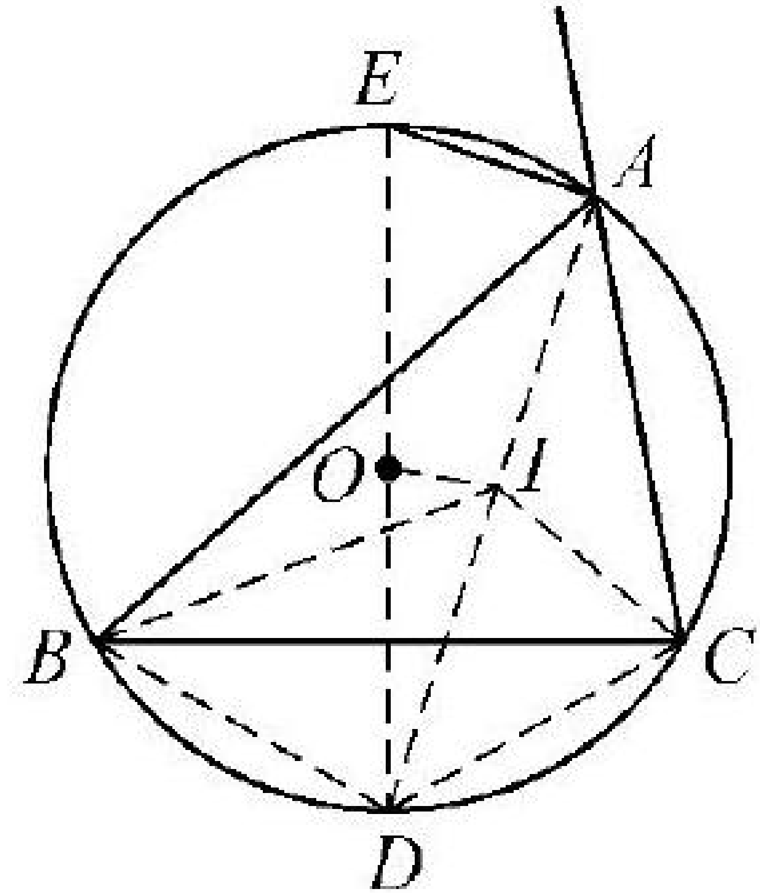
☒☒ $AC \cdot BD = AB \cdot CD + AD \cdot BC$, ☒ $AB = AC$, ☒☒

$$\frac{DB}{DC} = \frac{AB \cdot CD + AD \cdot BC}{AC \cdot DC} = \frac{AC \cdot DC}{AC \cdot DC} + \frac{AD \cdot BC}{AB \cdot DC} = 2.$$

☒5 ☒☒9-6☒☒☒☒☒☒☒ ABC ☒☒$BC = \frac{1}{2}(AB + AC)$, ☒ O I ☒☒☒ $\triangle ABC$ ☒☒☒☒☒☒. $\angle BAC$ ☒☒☒☒☒☒☒ $\triangle ABC$ ☒☒☒☒☒☒ E. ☒☒: $OI = \frac{1}{2}AE$ ☒

☒☒☒☒ AI ☒☒☒☒ $\triangle ABC$ ☒☒☒☒☒☒ D.

☒☒ AI ☒ AE ☒☒☒ $\angle BAC$ ☒☒☒☒☒☒☒☒☒ $\angle EAD = 90^\circ$ ☒☒☒ DE ☒☒☒☒☒ ☒☒, ☒ O ☒ ED



⊠9-6

⊠. ⊠⊠⊠ AD ⊠⊠⊠⊠⊠, I ⊠⊠⊠⊠⊠⊠ $BD = DI = DC$ ⊠⊠⊠⊠3⊠⊠2⊠
⊠⊠⊠⊠⊠⊠⊠⊠⊠ $ABDC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

$$CD \cdot AB + BD \cdot AC = BC \cdot AD,$$

⊠

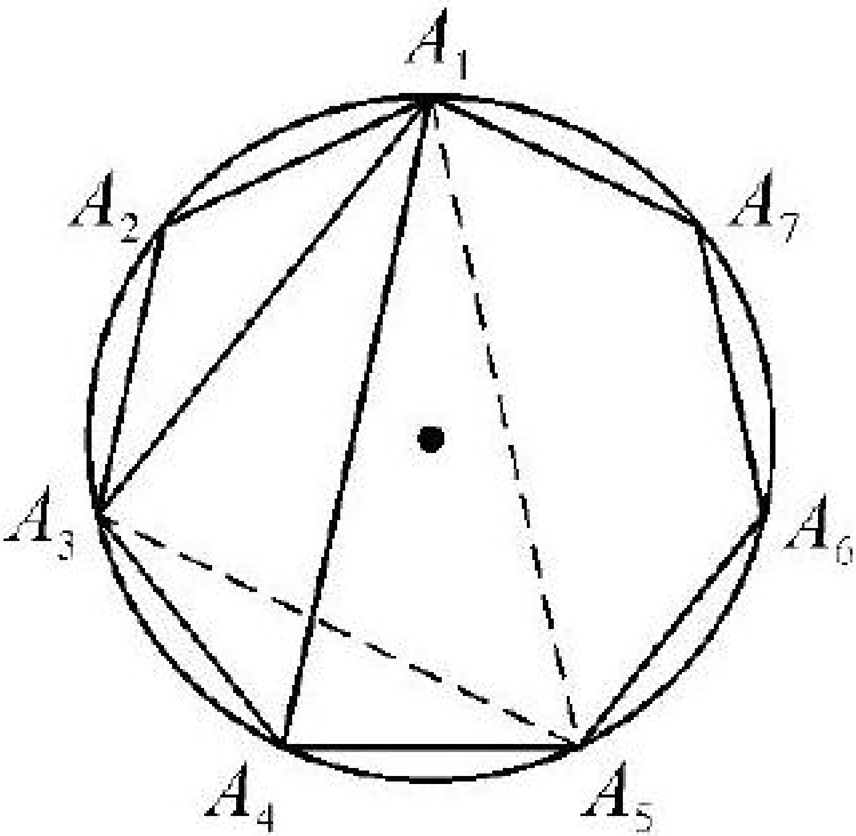
$$BD \cdot (AB + AC) = BC \cdot AD,$$

⊠⊠⊠⊠⊠⊠, ⊠ $AD = 2BD$, ⊠⊠ I ⊠ AD ⊠⊠.
⊠⊠ O I ⊠⊠⊠ DE ⊠ AD ⊠⊠⊠, ⊠ $OI = \frac{1}{2}AE$.
⊠6 ⊠⊠ $A_1A_2A_3A_4A_5A_6A_7$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠: $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$.
⊠⊠⊠⊠9-7, ⊠⊠ $A_1A_2A_3A_4A_5A_6A_7$ ⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠.
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $A_1A_3A_4A_5$ ⊠⊠
$A_1A_4 \cdot A_3A_5 = A_1A_3 \cdot A_4A_5 + A_3A_4 \cdot A_1A_5$,
⊠ $A_3A_5 = A_1A_3, A_4A_5 = A_3A_4 = A_1A_2$,

$$A_1A_4 = A_1A_5$$

⊠⊠ $A_1A_4 \cdot A_1A_3 = A_1A_3 \cdot A_1A_2 + A_1A_2 \cdot A_1A_4$, ⊠ $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$.

⊠7 ⊠⊠ $\triangle ABC$ ⊠ $\triangle A'B'C'$ ⊠⊠⊠⊠⊠ a ⊠



⊠9-7

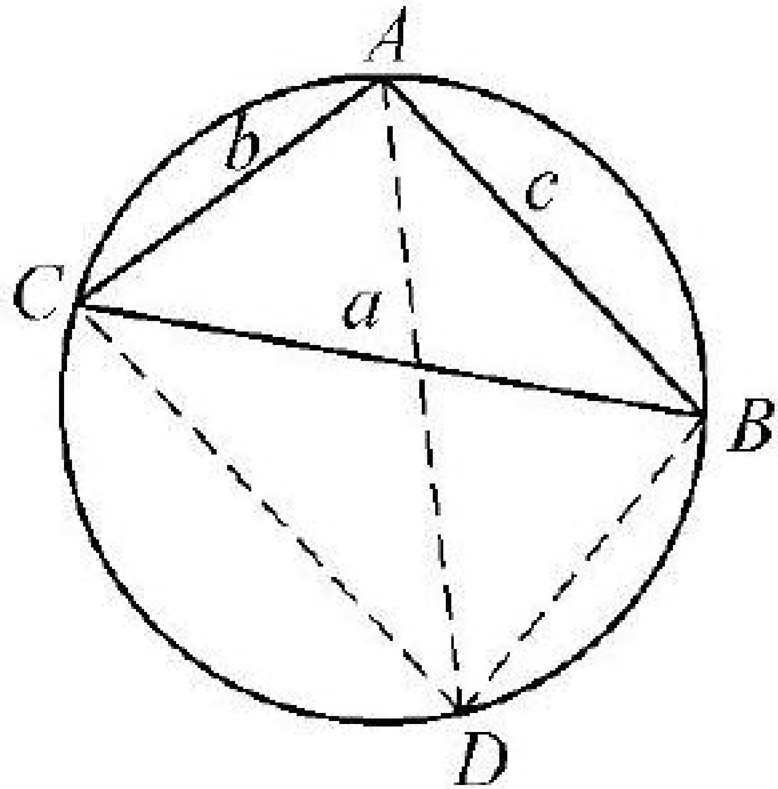
b c ⊠ a' b' c' ⊠⊠ $\angle B = \angle B', \angle A + \angle A' = 180°$ ⊠

⊠⊠⊠$aa' = bb' + cc'$ ⊠

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\angle B = \angle B', \angle A + \angle A' = 180°$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠9-8, ⊠ $\triangle ABC$ ⊠⊠⊠⊠, ⊠⊠ C ⊠ $CD//AB$ ⊠⊠⊠⊠ D, ⊠⊠ AD BD ⊠⊠⊠

$$\angle A + \angle A' = 180° = \angle A + \angle CDB,$$
$$\angle B' = \angle B = \angle BCD$$



☒9-8
☒☒

$$\angle A' = \angle CDB, \angle B' = \angle BCD,$$

☒☒

$$\triangle A'B'C' \backsim \triangle DCB,$$

☒☒

$$\frac{A'B'}{DC} = \frac{B'C'}{CB} = \frac{A'C'}{DB}$$

☒

$$\frac{c'}{DC} = \frac{a'}{a} = \frac{b'}{DB}$$

⊠⊠

$$DC = \frac{c'a}{a'}, DB = \frac{b'a}{a'},$$

⊠⊠⊠⊠ $ACDB$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠

$$AC \cdot BD + AB \cdot CD = AD \cdot BC,$$
$$b \cdot \frac{b'a}{a'} + c \cdot \frac{c'a}{a'} = AD \cdot a.$$

⊠

⊠⊠ $AB//CD$, ⊠⊠ $AD = BC = a$. ⊠⊠

$$b \cdot \frac{b'a}{a'} + c \cdot \frac{c'a}{a'} = a \cdot a$$

⊠

$$aa' = bb' + cc'.$$

⊠8 ⊠⊠ $\triangle ABC$ ⊠ $\angle A$ $\angle B$ $\angle C$ ⊠⊠⊠⊠⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠ A_1 B_1 C_1, ⊠ $m = AA_1 + BB_1 + CC_1, n = AB + BC + CA$ ⊠⊠⊠⊠ m ⊠n ⊠⊠⊠⊠

⊠⊠⊠ A_1B, A_1C ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

$$AA_1 \cdot BC = AB \cdot A_1C + AC \cdot A_1B$$

⊠⊠⊠ $A_1B = A_1C, A_1C + A_1B > BC$, ⊠

$$2AA_1 = \frac{(AB + AC) \cdot 2A_1B}{BC} > AC + AB$$

⊠⊠

$$2BB_1 > BC + AB,$$
$$2CC_1 > CA + CB,$$

⊠⊠⊠⊠⊠ $m > n$.

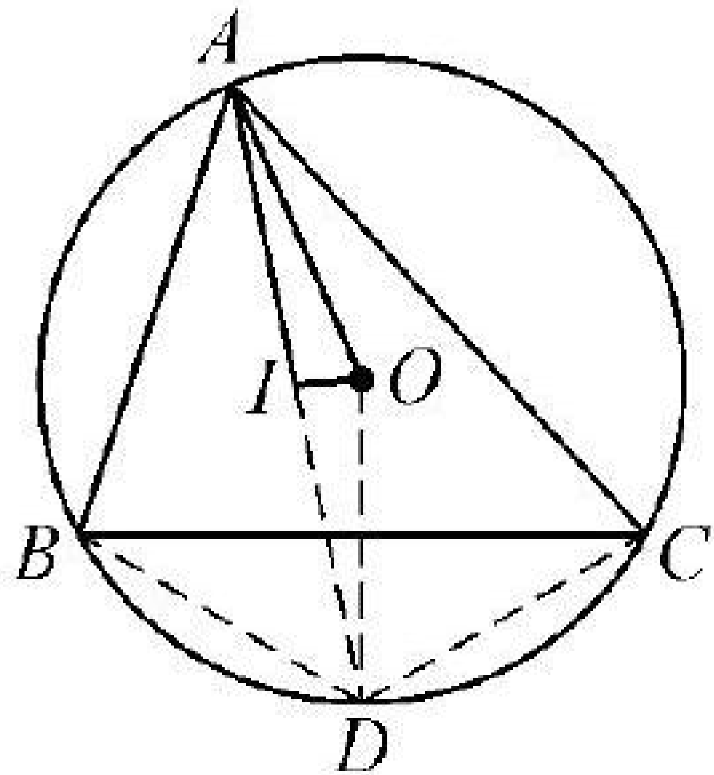
⊠9 ⊠⊠9-9, ⊠ I ⊠ O ⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠$\angle AIO \leqslant 90^\circ$ ⊠⊠⊠⊠⊠⊠ $2BC \leqslant AB + AC$ ⊠

⊠⊠⊠⊠ AI ⊠⊠⊠⊠⊠⊠⊠ D, ⊠⊠ BD, CD, OD,

⊠

$$
\begin{aligned}
\angle AIO \leqslant 90^\circ \Leftrightarrow AI \geqslant ID \\
\Leftrightarrow 2 \leqslant \frac{AD}{DI}.
\end{aligned}
$$

⊠⊠⊠⊠⊠⊠⊠⊠⊠3⊠⊠2⊠⊠⊠DI=DB=DC.



⊠9-9
⊠⊠⊠⊠⊠⊠⊠⊠

$$
\begin{aligned}
AD \cdot BC &= AB \cdot CD + AC \cdot BD \\
&= AB \cdot DI + AC \cdot DI,
\end{aligned}
$$

⊠⊠

$$
\frac{AD}{DI} = \frac{AB + AC}{BC}
$$

⊠⊠

$$\angle AIO \leqslant 90^\circ \Leftrightarrow 2 \leqslant \frac{AB+AC}{BC},$$

⊠ $\angle AIO \leqslant 90^\circ$ ⊠⊠⊠⊠⊠⊠ $2BC \leqslant AB+AC$.
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\angle AIO \leqslant 90^\circ$ ⊠⊠⊠ $AI \geqslant ID$ ⊠
⊠10 ⊠⊠ $\triangle DEF$ ⊠⊠⊠ D E F ⊠⊠⊠ $\triangle ABC$ ⊠⊠ BC CA AB ⊠, ⊠⊠⊠$AD+BE+CF<AB+BC+CA$ ⊠

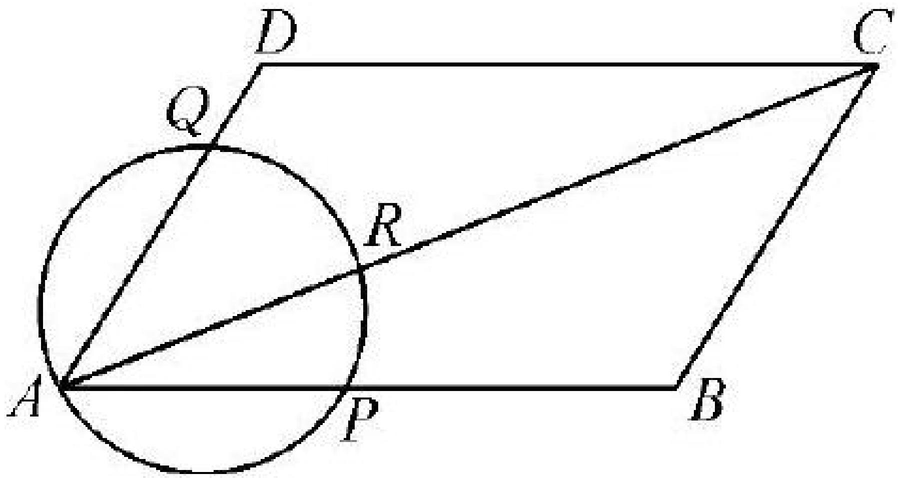
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠
⊠⊠⊠⊠⊠ $AEDF$ ⊠⊠$AD\cdot EF \leqslant AE\cdot DF+AF\cdot DE$ ⊠⊠⊠⊠ $DE=DF=EF$ ⊠⊠⊠ $AD \leqslant AE+AF$ ⊠⊠⊠ $BE \leqslant BF+BD, CF \leqslant CE+CD$ ⊠⊠

⊠ $AD+BE+CF \leqslant AE+AF+BF+BD+CE+CD=AB+BC+CA$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $AEDF$ ⊠⊠⊠⊠ $BDEF$ ⊠⊠⊠⊠ $CDFE$ ⊠⊠⊠⊠⊠, ⊠⊠ $\angle A=\angle B=\angle C=120^\circ$ ⊠⊠⊠ $\angle A+\angle B+\angle C=180^\circ$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠ $AD+BE+CF<AB+BC+CA$.

9

1 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠
2 ⊠1⊠⊠⊠⊠ P ⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠O ⊠⊠⊠⊠⊠⊠⊠⊠ $AP \perp PB$ ⊠⊠ $AP=4, PO=6\sqrt{2}$, ⊠⊠⊠ AB;
(2) ⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠ $\odot O$, P ⊠⊠ DC ⊠⊠⊠, $PA=\sqrt{2}, PC=\frac{\sqrt{2}}{2}$, ⊠ $PB\cdot PD$ ⊠
3 ⊠⊠ O ⊠⊠ AB ⊠⊠ C ⊠⊠ CD ⊠ CE, ⊠⊠⊠ AB ⊠⊠ F G. ⊠⊠: $FD\cdot GE+DE\cdot FG=DG\cdot EF$.
4$\triangle ABC$ ⊠, $AB<AC<BC$, ⊠ D ⊠ BC ⊠, ⊠ E ⊠ BA ⊠⊠⊠⊠, ⊠ $BD=BE=AC$, ⊠ F ⊠ $\triangle BDE$ ⊠⊠⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠$BF=AF+CF$ ⊠

5 ⊠⊠⊠⊠ $ABCDEFG$ ⊠, $AD=a, BG=b$, ⊠⊠: $(a+b)^2(a-b)=ab^2$.
6 ⊠⊠⊠⊠⊠⊠ $ABCD$ ⊠⊠ A ⊠⊠⊠⊠⊠ AB AC AD ⊠⊠ P Q R, ⊠⊠: $AR\cdot AC=AB\cdot AP+AD\cdot AQ$.
7 ⊠⊠ P ⊠⊠⊠⊠⊠ $ABCDE$ ⊠⊠⊠⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠$PC+PE=PA+PB+PD$ ⊠
8 ⊠⊠⊠⊠⊠⊠⊠ $ABCDEF$ ⊠⊠



(☒6 ☒)

☒1☒☒ $BA = AF = FE = a, ED = DC = CB = b$ ☒☒ CF, BF ☒

(2) ☒☒☒$AD \cdot BE \cdot CF = AB \cdot CD \cdot EF + BC \cdot DE \cdot AF + AB \cdot FC \cdot ED + BC \cdot AD \cdot EF + CD \cdot BE \cdot AF$.

9 ☒ $\triangle ABC$ ☒☒☒☒☒ BC ☒☒☒ P ☒☒☒☒ BC AC ☒ AB ☒☒☒ PK PL ☒ PM ☒☒☒☒$\frac{BC}{PK} = \frac{AC}{PL} + \frac{AB}{PM}$.

10$\triangle ABC$ ☒ $AB = 1, AC = 2$ ☒☒ BC ☒☒☒☒☒☒☒☒☒☒☒ $BCDE$ ☒☒☒ AD AE, ☒ $AD + AE$ ☒☒☒☒.

☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒; ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒.

☒☒. ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒☒☒☒☒☒

☒☒1 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒.

☒☒2 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒☒3 ☒☒

☒☒4 ☒☒ O ☒☒☒☒ ABC ☒☒☒, ☒ $\angle BOC = 2\angle A$ ☒ $\angle BOC = 360^\circ - 2\angle A$ ☒

☒☒☒☒☒:

☒☒1 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒.

☒☒2 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

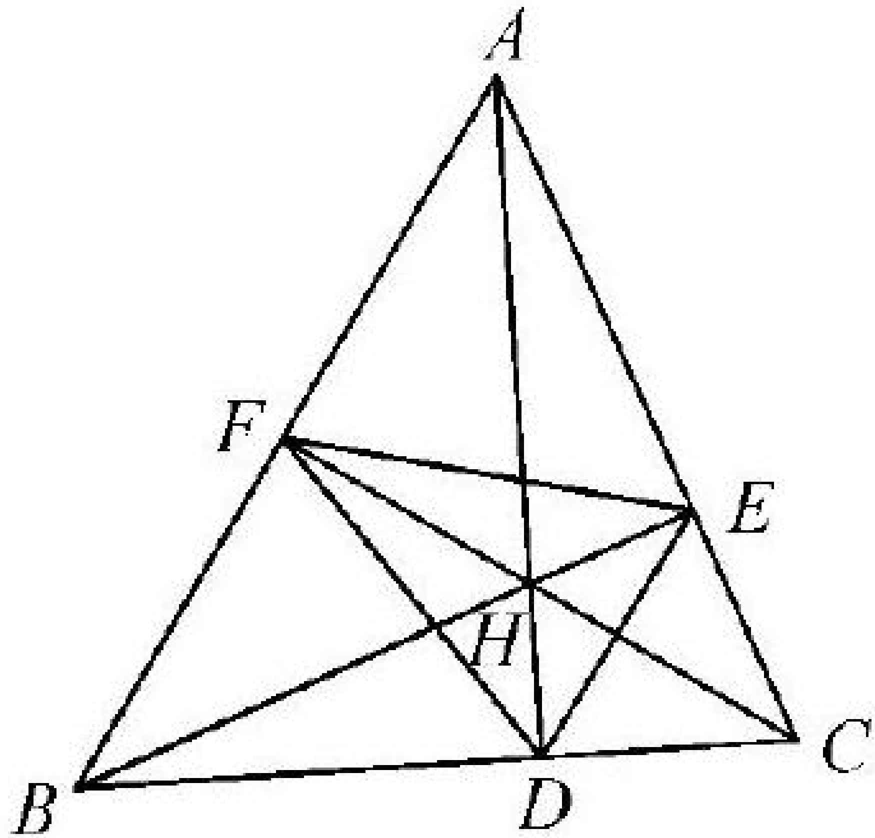
☒☒3 ☒ I ☒ $\triangle ABC$ ☒☒☒, AI ☒☒☒☒☒ $\triangle ABC$ ☒☒☒☒☒☒ D. I ☒ $\triangle ABC$ ☒☒ ☒☒☒☒☒☒ $ID = DB = DC$.

☒☒4 ☒ I ☒ $\triangle ABC$ ☒☒☒, ☒

$$\angle BIC = 90^\circ + \frac{1}{2}\angle A, \angle CIA = 90^\circ + \frac{1}{2}\angle B, \angle AIB = 90^\circ + \frac{1}{2}\angle C.$$

⊠1 ⊠⊠10-1, $\triangle ABC$ ⊠, ⊠⊠⊠⊠⊠ AD ⊠BE CF ⊠⊠⊠ H, ⊠⊠ DE EF FD, ⊠⊠: H ⊠ $\triangle EFD$ ⊠⊠⊠⊠

⊠⊠⊠⊠ $CF \perp AB$ ⊠⊠ $F, BE \perp AC$ ⊠⊠ E, ⊠ B F E C ⊠⊠⊠⊠, ⊠ $\angle AFE = \angle ACB$, ⊠⊠ $\angle BFD = \angle ACB$, ⊠ $\angle AFE = \angle BFD$, ⊠ $\angle CFE = \angle CFD$ ⊠⊠⊠ CF ⊠⊠ $\angle DFE$ ⊠⊠

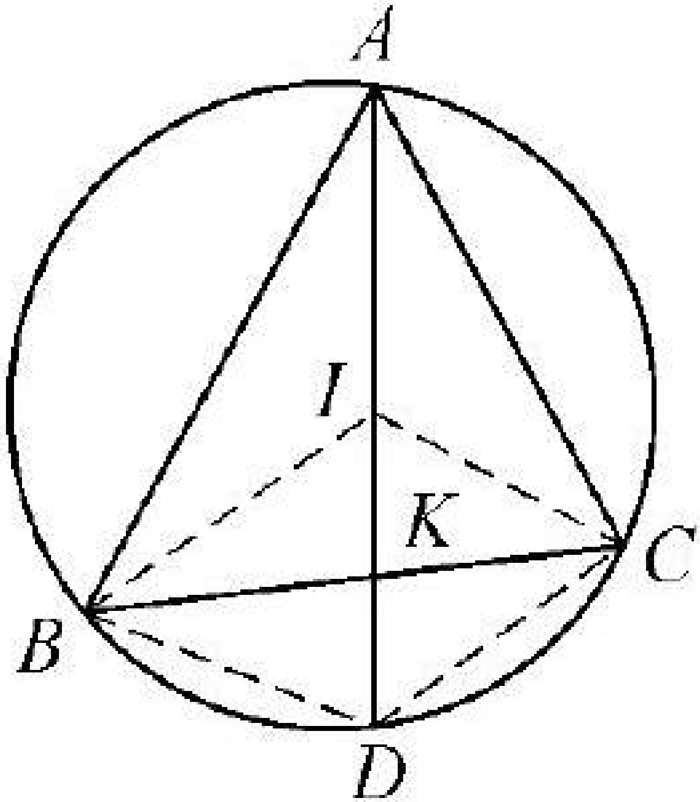


⊠10-1

⊠ BE ⊠⊠ $\angle DEF, AD$ ⊠⊠ $\angle FDE$ ⊠⊠⊠ H ⊠ $\triangle EFD$ ⊠⊠⊠⊠
⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠" ⊠" ⊠⊠⊠⊠⊠⊠⊠⊠

⊠2 ⊠⊠ $10-2, I$ ⊠ $\triangle ABC$ ⊠⊠⊠, $BC = a,\ AC = b, AB = c, \angle A$ ⊠⊠⊠⊠⊠ BC ⊠⊠ K, ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠ D, ⊠⊠: $\frac{AI}{KI} = \frac{AD}{DI} = \frac{DI}{DK} = \frac{b+c}{a}$.

⊠⊠⊠⊠⊠⊠⊠ DB DC BI CI ⊠⊠⊠⊠⊠⊠⊠3 ⊠ $DB = DI = DC$ ⊠



☒10 - 2

☒☒☒$\angle ADB = \angle ACK, \angle BAD = \angle KAC$ ☒☒☒ $\triangle DBA \backsim \triangle CKA$, ☒☒☒ $\frac{DB}{CK} = \frac{DA}{CA}$ ☒ $KC = \frac{CA\cdot DB}{AD} = \frac{b\cdot DI}{AD}$.

☒☒☒ $\triangle BAK \backsim \triangle DAC$ ☒ $KB = \frac{BA\cdot DC}{AD} = \frac{c\cdot DI}{AD}$.

☒☒ $a = BC = KC + KB = \frac{b\cdot DI}{AD} + \frac{c\cdot DI}{AD} = \frac{(c+b)\cdot DI}{AD}$, ☒☒ $\frac{AD}{DI} = \frac{b+c}{a}$.

☒☒, ☒☒☒ $\angle KCD = \angle DBC = \angle DAC$, ☒ $\triangle CDK \backsim \triangle ADC$, ☒☒ $\frac{AD}{DI} = \frac{DC}{DK} = \frac{DI}{DK}$.

☒☒☒☒☒☒☒☒☒☒☒☒☒ $\frac{AI}{IK} = \frac{BA}{BK} = \frac{AC}{CK} = \frac{AB+AC}{BC} = \frac{b+c}{a}$.

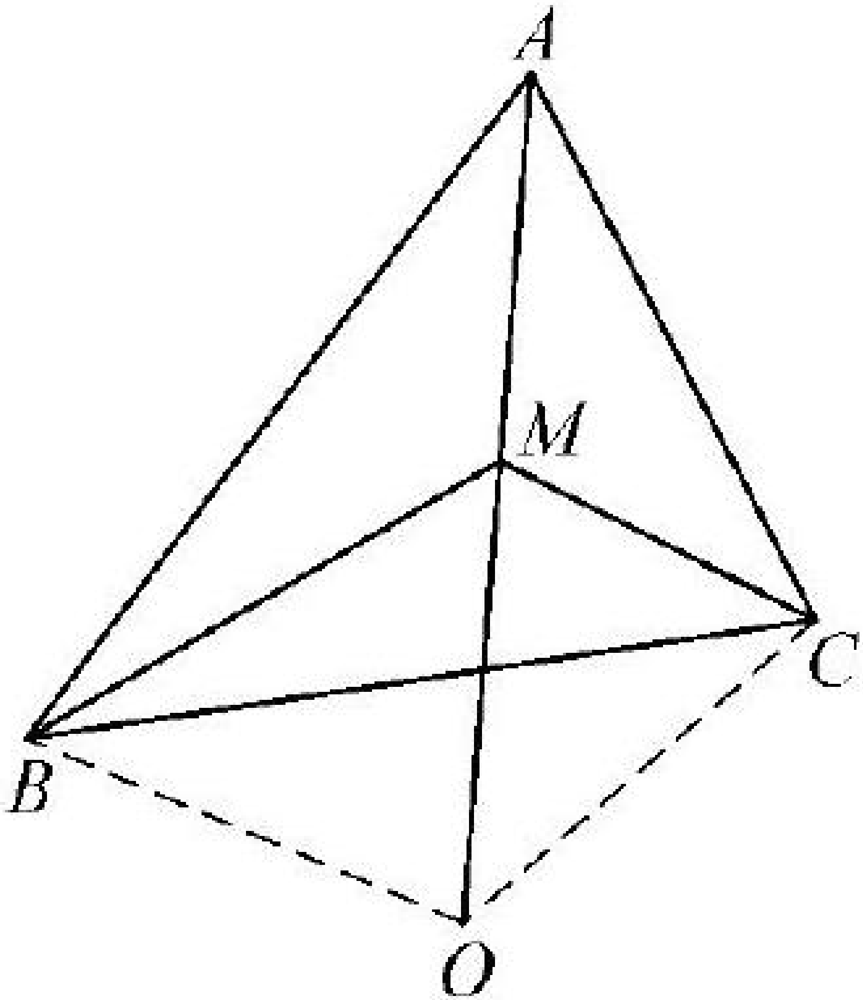
☒☒☒$\frac{AI}{KI} = \frac{AD}{DI} = \frac{DI}{DK} = \frac{b+c}{a}$.

☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒3 ☒☒ $10-3$ ☒☒☒☒ M ☒ $\triangle ABC$ ☒☒☒☒ $\angle BMC = 90^\circ + \frac{1}{2}\angle BAC$ ☒☒☒☒ AM ☒☒ $\triangle BMC$ ☒☒☒☒☒☒ O, ☒☒: ☒ M ☒ $\triangle ABC$ ☒☒.

☒☒☒☒ BO CO ☒☒ $BO = OM = CO$ ☒☒☒ $\angle OBM = \angle OMB, \angle OCM = \angle OMC$ ☒☒☒

$$
\begin{aligned}
\angle BOC &= \angle BOM + \angle COM \\
&= 180^\circ - 2\angle OMB + 180^\circ - 2\angle OMC \\
&= 360^\circ - 2\angle BMC
\end{aligned}
$$



⊠10 - 3

$$= 180^\circ - \angle BAC$$

⊠⊠ A B O C ⊠⊠⊠⊠.

⊠⊠ $OB = OC$ ⊠⊠ OA ⊠⊠ $\angle BAC \cdots$ (1),

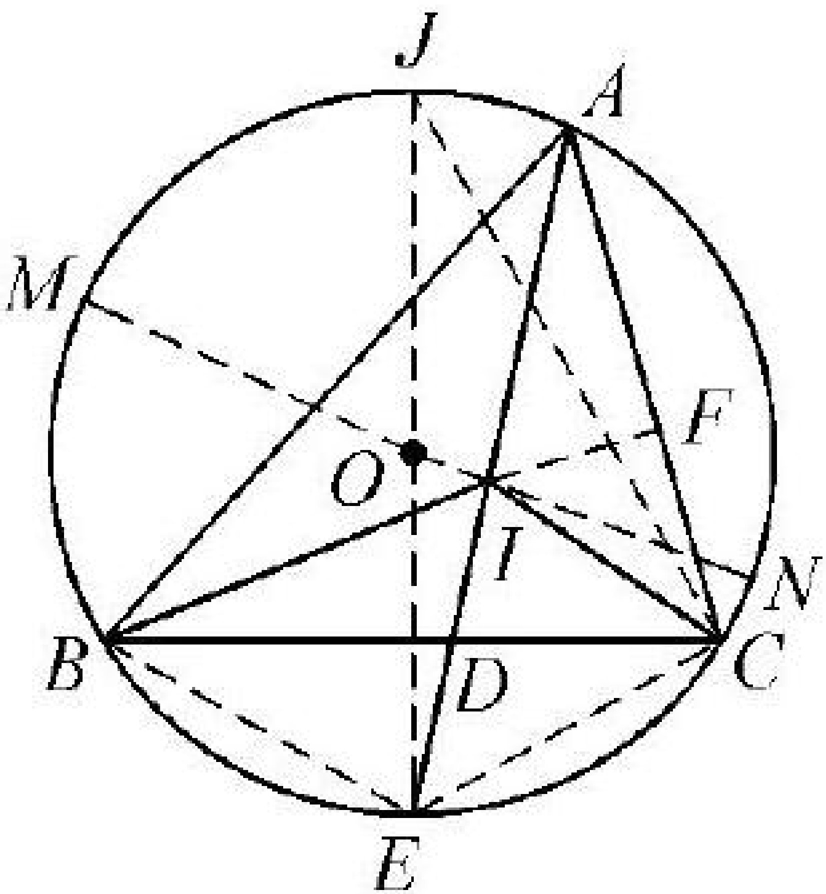
⊠⊠⊠ $OB = OC$ ⊠⊠ $\angle OBC = \angle OCB = \angle OAB$ ⊠
⊠⊠ $\angle ABM+\angle MAB = \angle OMB = \angle MBO = \angle MBC+\angle OBC$ ⊠⊠⊠ $\angle ABM = \angle MBC$.

⊠ BM ⊠⊠ $\angle ABC \cdots$ (2).

⊠ (1)⊠(2) ⊠⊠ M ⊠ $\triangle ABC$ ⊠⊠⊠
⊠4 ⊠⊠10-4, ⊠ $\triangle ABC$ ⊠⊠⊠⊠ I, ⊠⊠ AI ⊠ $\triangle ABC$ ⊠⊠⊠⊠ O ⊠⊠⊠⊠⊠ E, AE ⊠ BC ⊠⊠⊠ D ⊠⊠ R r ⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠: (1) E ⊠ $\triangle BCI$ ⊠⊠⊠⊠⊠2⊠$AD\cdot AE = AB\cdot AC$ 3 $AI\cdot IE = 2Rr$ 4 $OI^2 = R^2 - 2Rr$.

⊠⊠⊠1⊠⊠⊠ BE CE ⊠⊠⊠ $EB = EC = EI$ ⊠⊠⊠ E ⊠ $\triangle BCI$ ⊠⊠⊠⊠



⊠10-4

⊠2⊠⊠⊠ $\triangle ABD \backsim \triangle AEC$ ⊠⊠⊠ $\frac{AD}{AC} = \frac{AB}{AE}$ ⊠⊠ $AD \cdot AE = AB \cdot AC$ ⊠
⊠3⊠⊠⊠ I ⊠ $IF \perp AC$ ⊠⊠⊠⊠ F ⊠⊠ $IF = r$ ⊠⊠⊠ EO ⊠⊠⊠⊠⊠⊠ O ⊠⊠ J, ⊠⊠ JC, ⊠ $\mathrm{Rt}\triangle AIF \backsim \mathrm{Rt}\triangle JEC$ ⊠⊠⊠

$$\frac{AI}{JE} = \frac{IF}{EC}$$

☒☒

$$AI \cdot EC = JE \cdot IF,$$

☒☒1☒☒☒$EC = IE$ ☒☒ $AI \cdot IE = 2Rr$ ☒
☒4☒☒☒☒ OI ☒☒ O ☒☒☒ M N, ☒☒3☒☒☒☒☒☒☒☒☒

$$\begin{aligned} 2Rr &= AI \cdot IE = IM \cdot IN \\ &= (R + OI)(R - OI) = R^2 - OI^2 \end{aligned}$$

☒☒

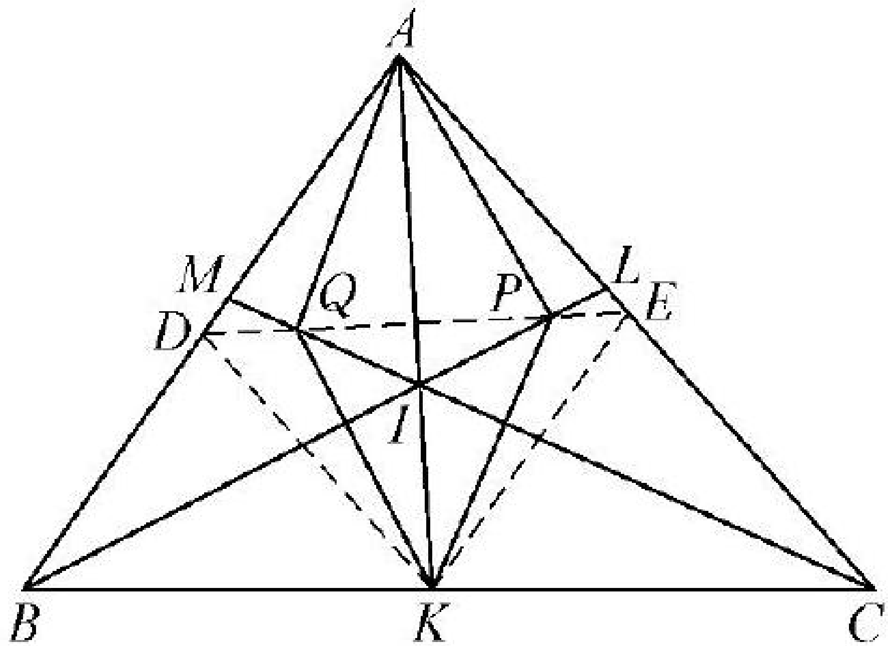
$$OI^2 = R^2 - 2Rr$$

☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒4☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒☒☒☒☒☒☒☒☒☒☒☒☒

☒5 ☒☒10-5☒☒ AK BL CM ☒☒☒☒ ABC ☒☒☒☒☒☒K L ☒M ☒☒☒ BC CA AB ☒, ☒ P Q ☒☒☒ BL CM ☒, ☒☒ $AP = PK$, $AQ = QK$ ☒☒☒☒$\angle PAQ = 180^\circ - \angle BAC$ ☒

☒☒☒☒ PQ ☒☒☒☒ AB ☒☒ D ☒☒ AC ☒☒ E, ☒☒ KD KE ☒☒☒ $AQ = QK, AP = PK$ ☒☒ PQ ☒☒☒☒ AK ☒☒ DE ☒☒☒☒ AK, ☒☒ $\triangle ADE \cong \triangle KDE$ ☒

☒ $\angle ADE = \angle KDE, \angle AED = \angle KED$, ☒ DE ☒☒ $\angle ADK$ $\angle AEK$, ☒☒ BP ☒☒ $\angle ABC, CQ$ ☒☒ $\angle ACB$ ☒☒ P ☒ $\triangle BDK$ ☒☒☒, Q ☒ $\triangle CEK$ ☒☒☒, ☒



⊠10-5

⊠ PK ⊠⊠ $\angle DKC, QK$ ⊠⊠ $\angle EKB$.

⊠⊠⊠⊠⊠ $\angle DKP + \angle EKQ = \angle DKE + \angle PKQ = \angle BAC + \angle PAQ$,
⊠⊠⊠⊠ $\angle DKP + \angle EKQ = \frac{1}{2}(\angle DKC + \angle EKB)$

$$= \frac{1}{2}\left(180^\circ + \angle DKE\right)$$
$$= \frac{1}{2}\left(180^\circ + \angle BAC\right),$$

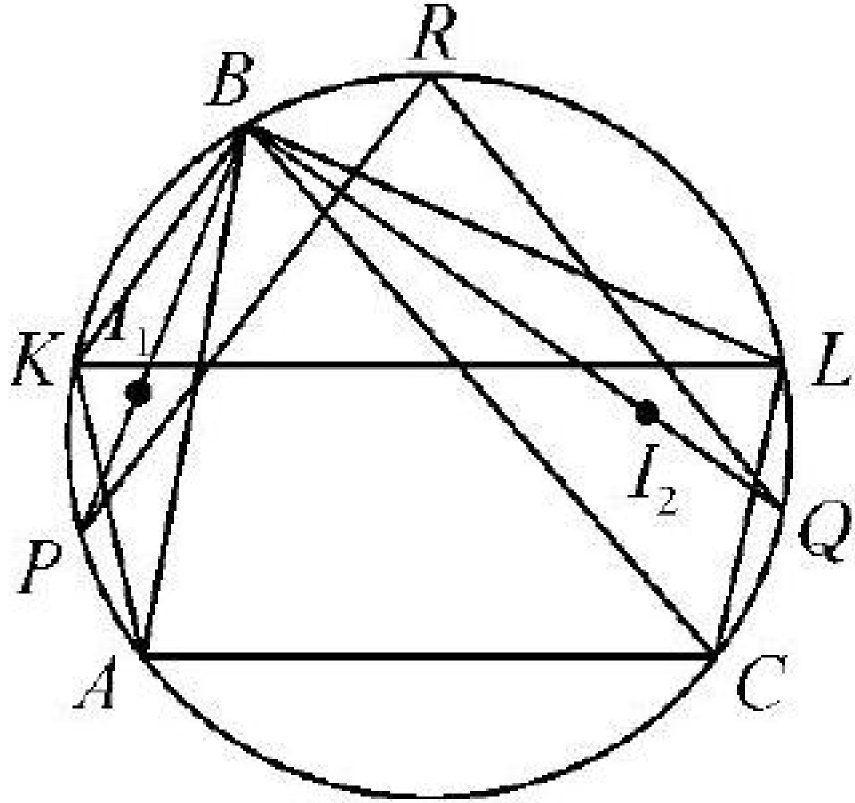
⊠⊠ $2\angle PAQ = 180^\circ - \angle BAC$ ⊠

⊠⊠⊠⊠" ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠" ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠ ⊠⊠⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠6 ⊠⊠ $10-6$ ⊠⊠⊠⊠⊠ ABC ⊠⊠⊠⊠⊠⊠⊠ AB ⊠⊠⊠⊠ C ⊠⊠⊠⊠ BC ⊠⊠⊠ ⊠ A ⊠⊠⊠⊠⊠⊠ K ⊠ L ⊠⊠⊠⊠⊠ KL ⊠⊠⊠ AC ⊠⊠⊠⊠⊠: $\triangle ABK$ ⊠ $\triangle CBL$ ⊠ ⊠⊠⊠⊠⊠ AC ⊠⊠⊠⊠ B ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠ $AB = BC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠ $AB < BC$ ⊠⊠ I_1 I_2 ⊠⊠⊠⊠ $\triangle AKB$ ⊠ $\triangle CLB$ ⊠⊠⊠⊠⊠⊠⊠⊠ BI_1 BI_2 ⊠⊠



⊠10-6

⊠⊠⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ P Q, ⊠⊠ ABC ⊠⊠⊠⊠ R ⊠

⊠ $KL//AC$, ⊠ $AK = CL$, ⊠ P Q ⊠⊠⊠ AK CL ⊠⊠⊠, ⊠⊠, $RP = RQ, PA = QC$.

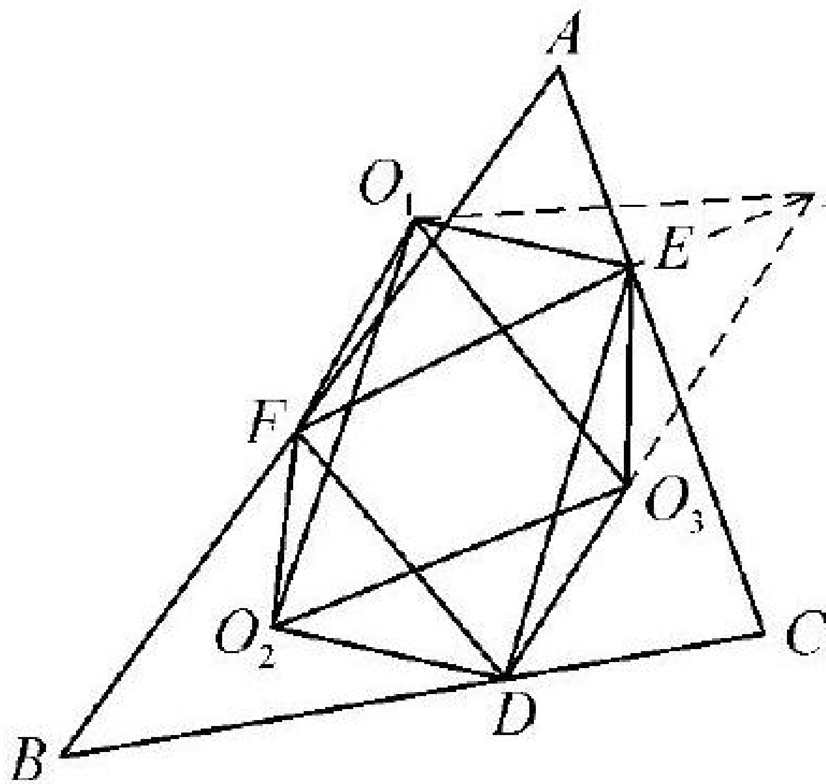
⊠⊠⊠⊠⊠⊠⊠⊠3 ⊠

$$PI_1 = PA = QC = QI_2$$

⊠ $\angle I_1PR = \angle I_2QR$ ⊠⊠⊠⊠$\triangle I_1PR \cong \triangle I_2QR$ ⊠⊠⊠ $RI_1 = RI_2$ ⊠

⊠7 ⊠⊠10-7, ⊠ $\triangle ABC$ ⊠⊠ AB BC ⊠CA ⊠⊠⊠⊠⊠ F D E ⊠⊠⊠⊠⊠ $\triangle AEF$ ⊠ $\triangle BFD$ $\triangle CDE$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\triangle ABC$ ⊠⊠.

⊠⊠⊠⊠⊠ $\angle EO_1F = 2\angle A, \angle DO_2F = 2\angle B, \angle DO_3E = 2\angle C$ ⊠⊠⊠ $\angle EO_1F + \angle FO_2D + \angle DO_3E = 360^\circ$ ⊠⊠⊠ $\angle O_1FO_2 + \angle O_2DO_3 + \angle O_3EO_1 = 360^\circ$ ⊠ $\triangle DEF$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠



⊠10-7

⊠⊠⊠ O_1 O_2 O_3 ⊠ $\triangle AEF$ $\triangle BFD$ $\triangle CDE$ ⊠⊠⊠⊠⊠⊠ O_1F ⊠O_1E O_2F O_2D O_3D O_3E, ⊠⊠⊠⊠⊠⊠⊠ $\angle EO_1F = 2\angle A, \angle DO_2F = 2\angle B, \angle DO_3E = 2\angle C$.

⊠⊠ $\angle EO_1F + \angle FO_2D + \angle DO_3E = 360°$ ⊠⊠⊠ $\angle O_1FO_2 + \angle O_2DO_3 + \angle O_3EO_1 = 360°$ ⊠

⊠ $\triangle O_2DO_3$ ⊠⊠⊠ O_3 ⊠⊠⊠ $\triangle PEO_3$, ⊠⊠ PO_1, ⊠ $\angle O_1EP = 360° - \angle PEO_3 - \angle O_1EO_3 = 360° - \angle O_2DO_3 - \angle O_1EO_3 = \angle O_1FO_2$ ⊠

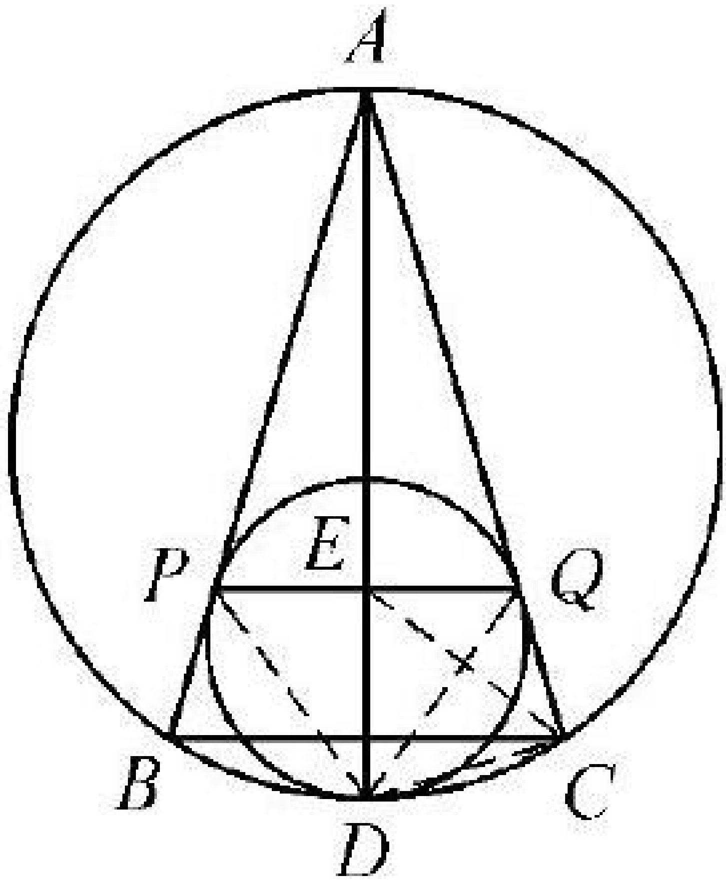
⊠ $O_1E = O_1F, PE = O_2D = O_2F$, ⊠⊠ $\triangle PEO_1 \cong \triangle O_2FO_1$, ⊠⊠⊠⊠ $\triangle O_1O_2O_3 \cong \triangle O_1PO_3$.

⊠ $\angle O_2O_1O_3 = \angle PO_1O_3 = \frac{1}{2}\angle O_2O_1P = \frac{1}{2}(\angle O_2O_1E + \angle EO_1P) = \frac{1}{2}(\angle O_2O_1E + \angle FO_1O_2) = \frac{1}{2}\angle EO_1F = \angle A$ ⊠⊠⊠⊠ $\angle O_2O_3O_1 = \angle C$ ⊠⊠ $\triangle O_1O_2O_3 \backsim \triangle ABC$ ⊠

⊠8 ⊠⊠ $10-8$ ⊠⊠⊠⊠⊠⊠⊠ ABC ⊠⊠$AB = AC$ ⊠⊠⊠⊠⊠⊠⊠⊠ $\triangle ABC$ ⊠⊠ ⊠⊠⊠⊠⊠⊠ AB AC ⊠⊠⊠⊠⊠ P Q, ⊠⊠: P Q ⊠⊠⊠⊠⊠⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠.

⊠⊠⊠ D ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠AD ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠ E ⊠ AD ⊠ PQ ⊠⊠⊠⊠⊠ $AP = AQ$ ⊠⊠E ⊠ PQ ⊠⊠⊠⊠⊠ $AE \perp PQ$ ⊠

⊠⊠⊠ $PD =$ ⊠ QD ⊠⊠⊠



⊠10 - 8

$$\angle DQC = \angle DPQ = \angle DQE$$

⊠ DQ ⊠⊠⊠⊠, ⊠⊠Rt $\triangle DQE \cong \text{Rt}\triangle DQC$ ⊠⊠⊠ $EQ = QC$ ⊠⊠⊠ $\angle QEC = \angle QCE$ ⊠

⊠ $PQ//BC$ ⊠⊠⊠ $\angle BCE = \angle QEC = \angle QCE$ ⊠ CE ⊠ $\angle BCA$ ⊠⊠⊠⊠⊠⊠ E ⊠ $\triangle ABC$ ⊠⊠⊠⊠

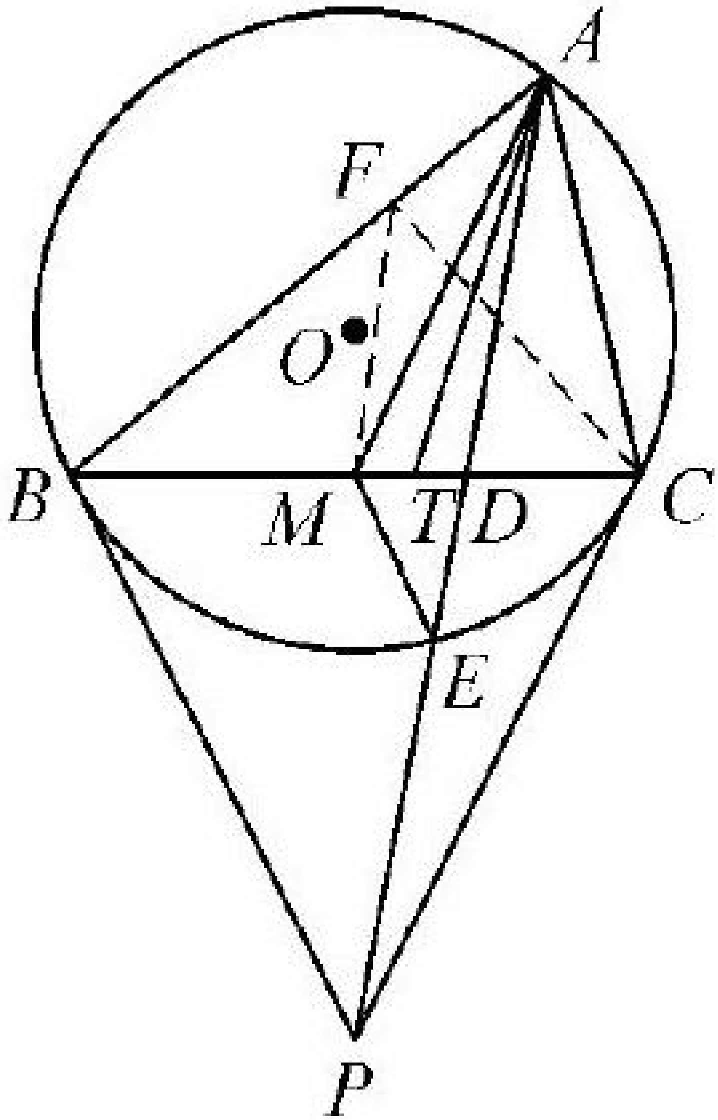
⊠⊠⊠⊠⊠, ⊠ $AB \neq AC$ ⊠, ⊠⊠⊠⊠⊠, ⊠⊠⊠Mannheim ⊠⊠, ⊠⊠⊠⊠⊠⊠⊠

⊠9 ⊠⊠10-9 ⊠⊠, ⊠ O ⊠ $\triangle ABC$ ⊠⊠⊠⊠, AM AT ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠ B ⊠⊠ C ⊠⊠ O ⊠⊠⊠⊠⊠⊠⊠ P, ⊠⊠ AP, ⊠ BC ⊠⊠ O ⊠⊠⊠⊠⊠⊠ D E ⊠⊠⊠: ⊠ T ⊠⊠⊠⊠ AME ⊠⊠⊠.

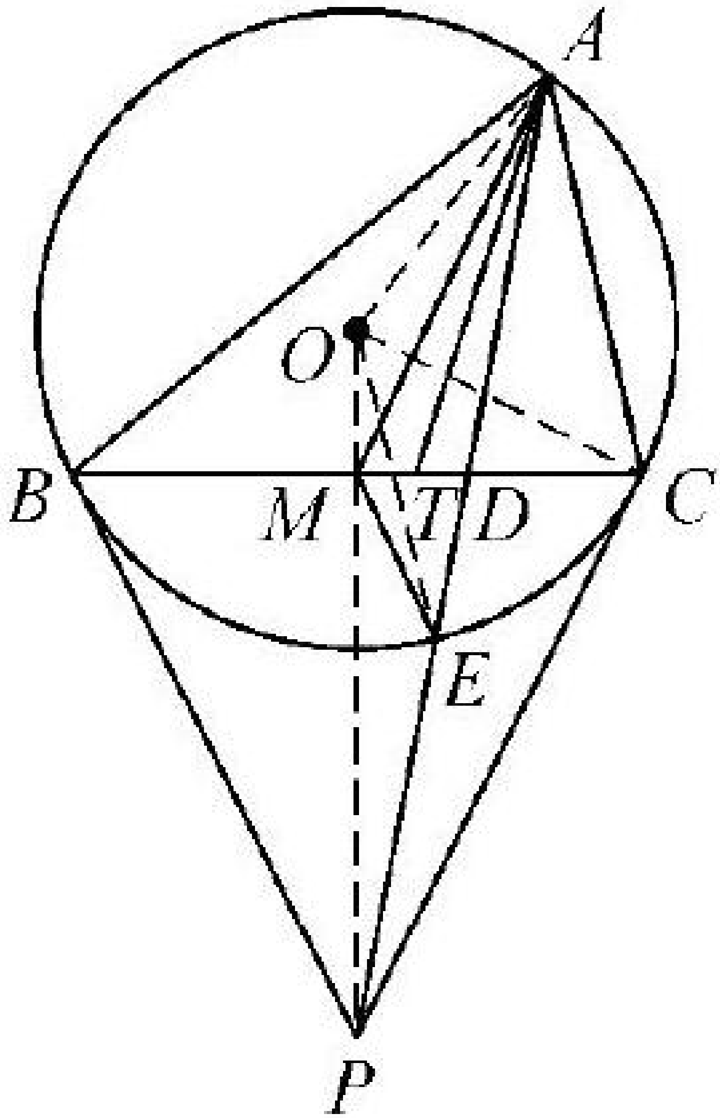
⊠⊠⊠⊠⊠ AT ⊠ $\angle MAE$ ⊠⊠⊠⊠⊠. ⊠⊠ $\angle BAM = \angle CAP$ ⊠

⊠ $CF \perp AB$ ⊠⊠⊠⊠ F ⊠⊠⊠ MF ⊠⊠⊠ $10-9$ ⊠⊠ $FM = \frac{1}{2}BC = MC$ ⊠ ⊠ $\angle BAC = \angle BCP$ ⊠⊠⊠ $\frac{FA}{AC} = \cos\angle BAC = \cos\angle BCP = \frac{CM}{PC} = \frac{FM}{PC}$, ⊠⊠

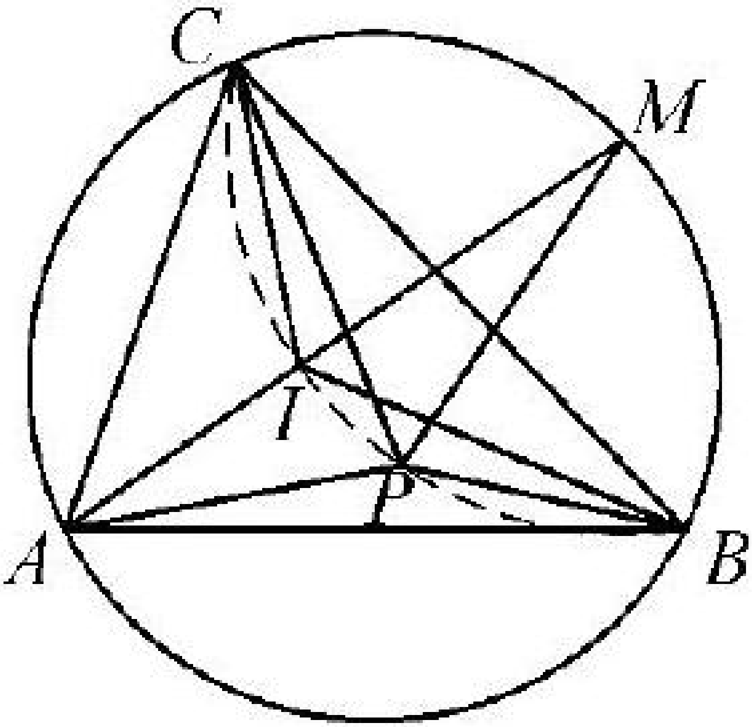
$\frac{FA}{FM} = \frac{CA}{CP}$.



⊠10-9



⊠10 - 10



☒10 - 11

$$\angle PBA+\angle PCA+\angle PBC+\angle PCB=\alpha+\beta,$$

☒☒☒☒☒

$$\angle PBC+\angle PCB=\frac{\alpha+\beta}{2}.$$

☒☒, $\angle BPC=\pi-\frac{\alpha+\beta}{2}=\angle BIC$ ☒☒☒ P I ☒☒ BC ☒☒☒☒, ☒ B ☒C I P ☒☒☒☒, ☒☒ P ☒ $\triangle BCI$ ☒☒☒☒☒. ☒☒ $MC=MB=MI$, ☒☒ M ☒ $\triangle BCI$ ☒ ☒☒☒☒☒, ☒☒ $MP=MI$ ☒

☒ $\triangle APM$ ☒, ☒

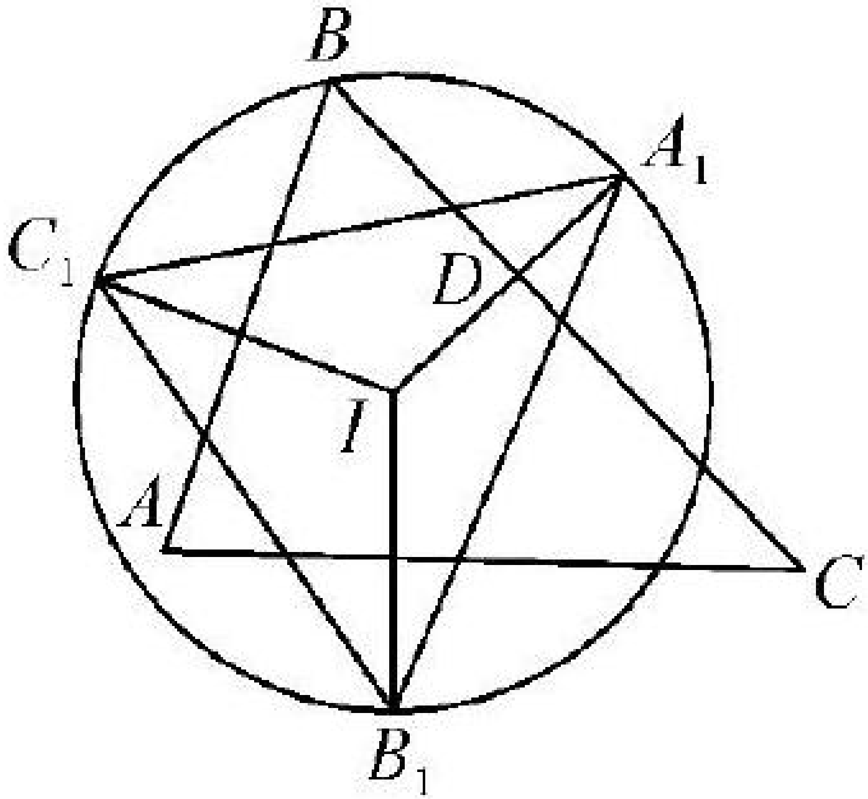
$$AP+PM\geqslant AM=AI+IM=AI+PM$$

☒ $AP\geqslant AI$ ☒

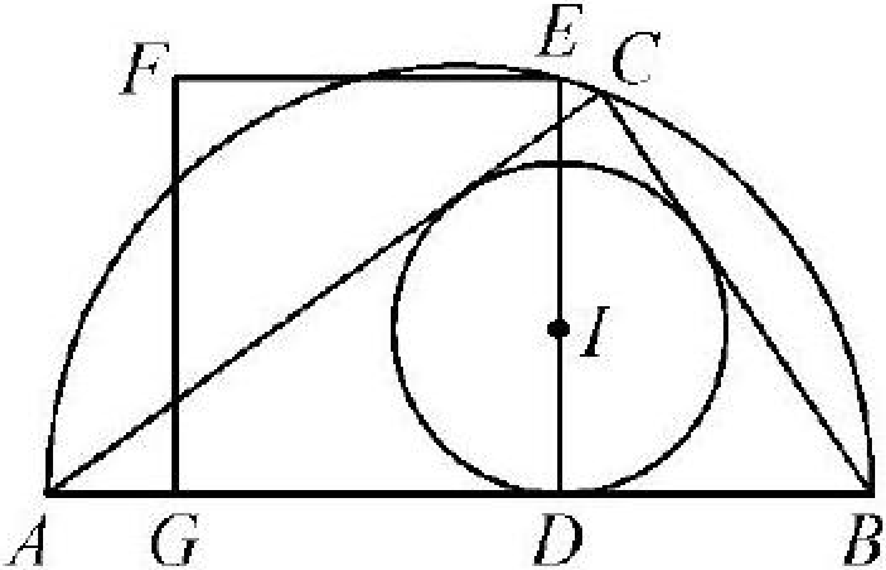
☒☒☒☒☒☒☒☒☒☒☒☒☒ P ☒☒☒☒ AI ☒, ☒ $P=I$.

10

1 ⊠⊠ AD BE ⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠, ⊠ DE ⊠⊠ $\angle ADC$, ⊠ $\angle BAC$.
2 ⊠⊠⊠ I ⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠A_1 B_1 C_1 ⊠⊠⊠ I ⊠⊠⊠ BC CA AB ⊠⊠⊠⊠, ⊠ ⊠ B ⊠ $\triangle A_1B_1C_1$ ⊠⊠⊠⊠⊠, ⊠ $\angle ABC$.
3 ⊠⊠



(⊠2 ⊠)

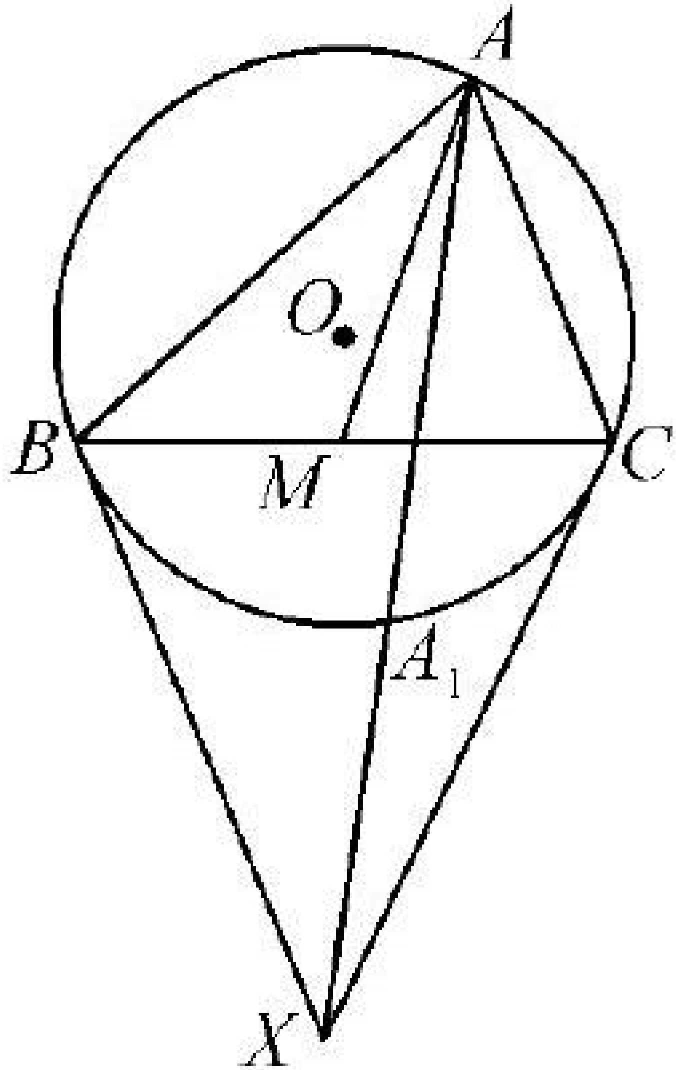


(⊠4⊠)

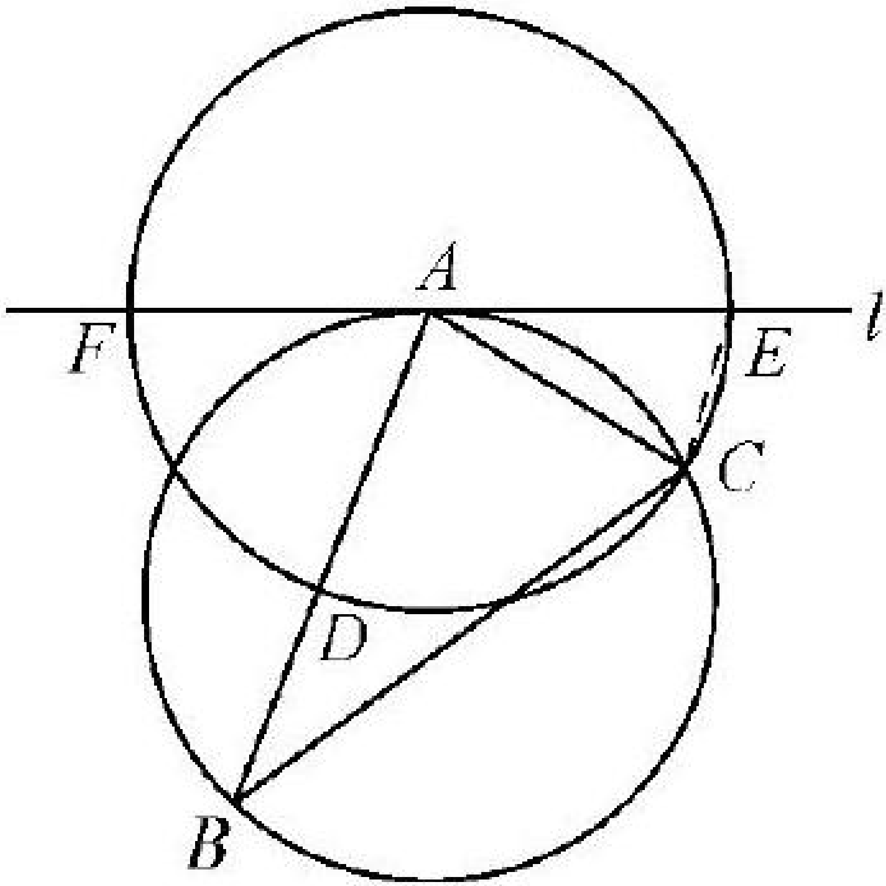
4 ⊠⊠, AB ⊠⊠⊠⊠⊠⊠, C ⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠ $DEFG$ ⊠⊠⊠ DG ⊠⊠⊠ AB ⊠⊠⊠⊠⊠ DE ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠ I ⊠⊠⊠ E ⊠⊠⊠⊠⊠. ⊠⊠⊠⊠⊠ $DEFG$ ⊠⊠ ⊠⊠100 , ⊠ $\triangle ABC$ ⊠⊠⊠⊠

5 ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\triangle ABC$ ⊠⊠AM ⊠ BC ⊠⊠⊠⊠⊠⊠X ⊠⊠⊠⊠⊠ B ⊠C ⊠⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠ O ⊠⊠⊠⊠⊠⊠⊠⊠. ⊠⊠⊠

$$\frac{AM}{AX} = \cos \angle BAC$$



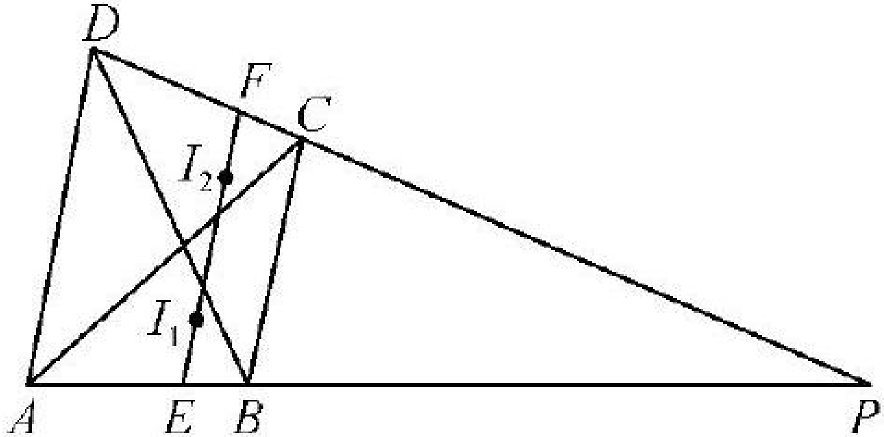
(⊠5⊠)



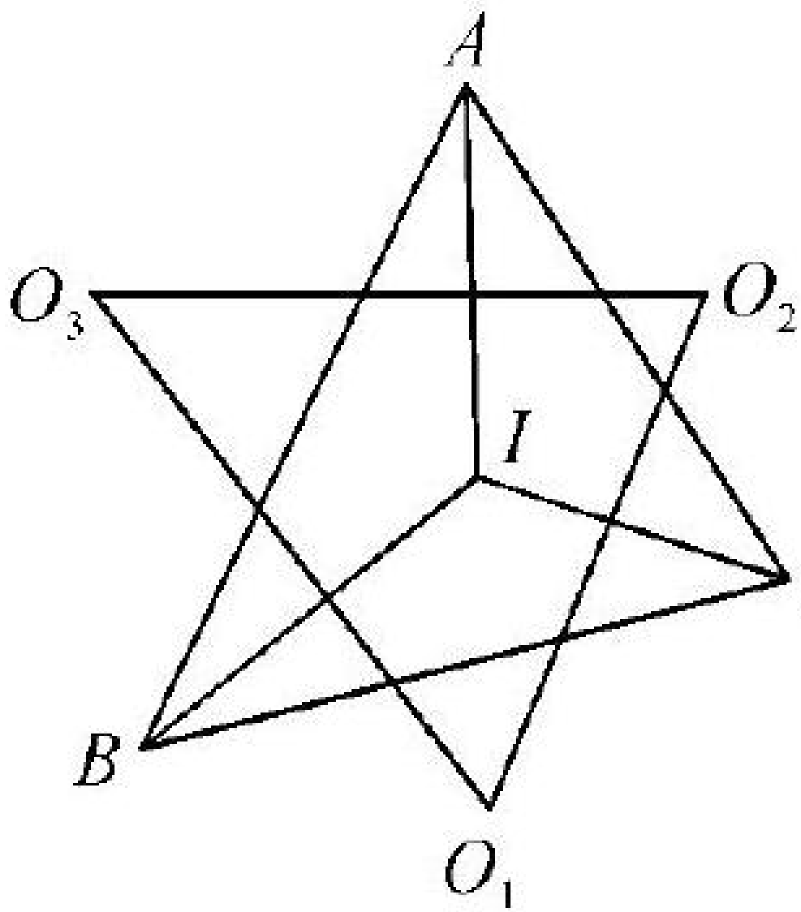
(☒6☒)

6 ☒☒☒☒ $\triangle ABC$ ☒☒☒ $AB > AC$ ☒☒ A ☒ $\triangle ABC$ ☒☒☒☒☒☒☒ l. ☒☒ A ☒ ☒☒☒AC ☒☒☒☒☒☒☒☒☒☒ AB ☒☒ D ☒☒☒☒ l ☒☒ E F ☒☒☒☒☒☒ DE DF ☒ ☒☒☒ $\triangle ABC$ ☒☒☒☒☒ BC ☒☒☒☒☒☒

7$ABCD$ ☒☒☒☒☒☒☒I_1 I_2 ☒☒☒ $\triangle ABC$ $\triangle DBC$ ☒☒☒☒☒☒ I_1 I_2 ☒☒☒☒☒☒ AB DC ☒☒ E F ☒☒☒☒☒ AB ☒ DC ☒☒☒☒☒☒☒ P, ☒ $PE = PF$, ☒☒: A B C D ☒☒☒☒.



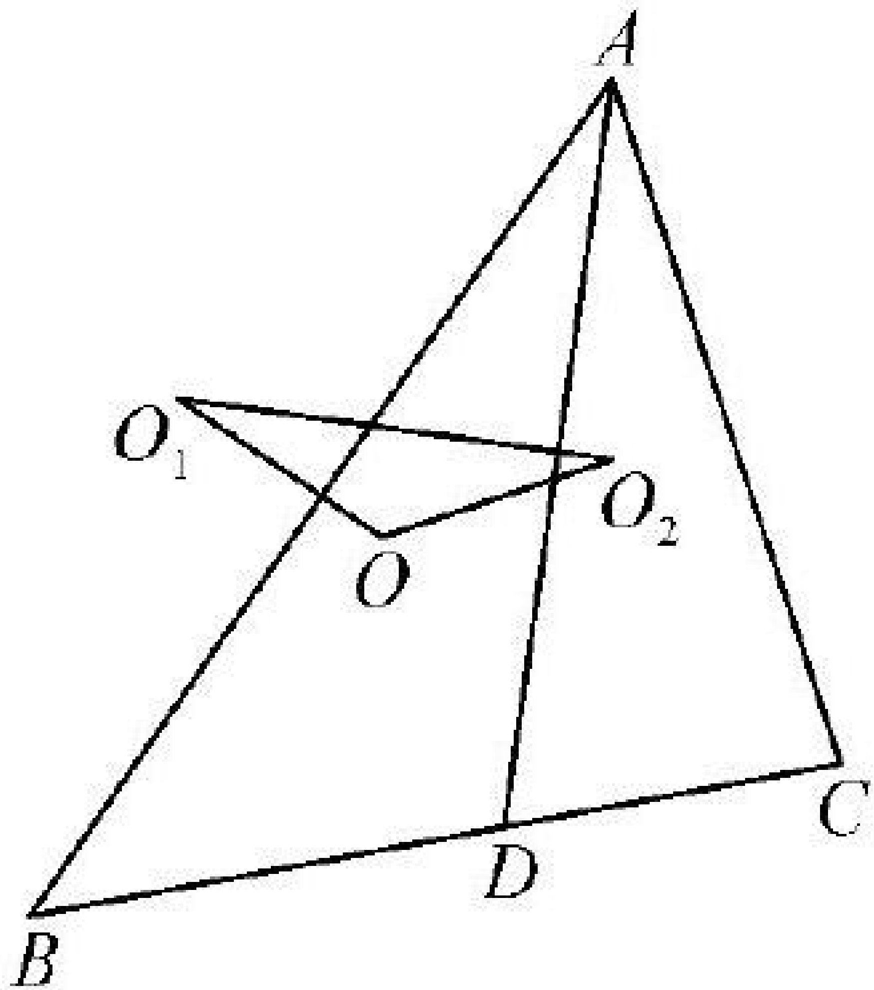
(⊠7⊠)



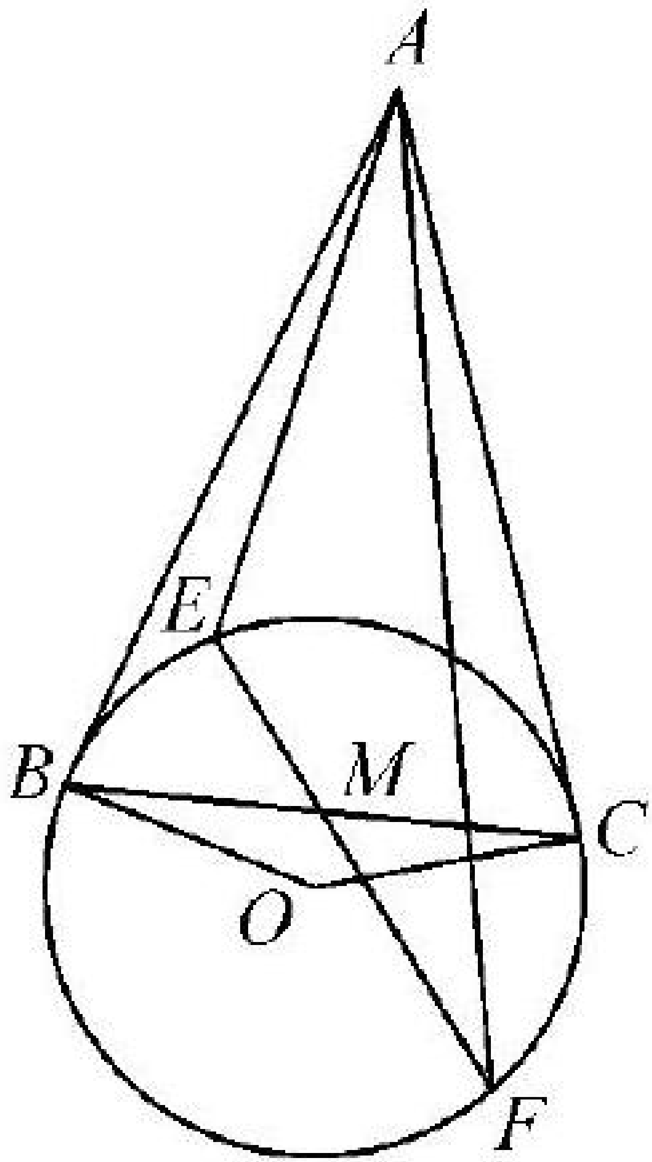
(⊠8 ⊠)

8I ⊠ $\triangle ABC$ ⊠⊠⊠. ⊠ $\triangle IBC$ $\triangle ICA$ $\triangle IAB$ ⊠⊠⊠ O_1 O_2 O_3. ⊠⊠: $\triangle O_1O_2O_3$ ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠

9 ⊠⊠ AD ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠. ⊠ $\triangle ABC$ $\triangle ABD$ $\triangle ADC$ ⊠⊠⊠ O O_1 O_2 ⊠⊠ ⊠: $\triangle OO_1O_2$ ⊠⊠⊠⊠⊠⊠.

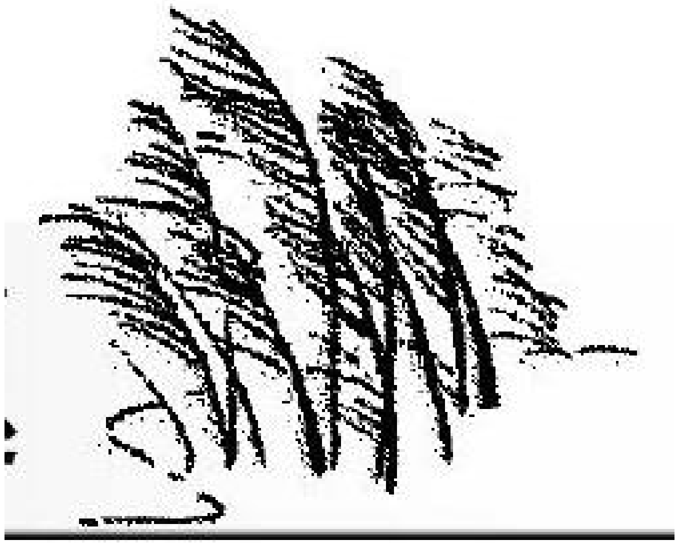


(⊠9 ⊠)



(⊠10 ⊠)

10 ⊠⊠ AB AC ⊠ $\odot O$ ⊠ B C ⊠⊠ OA ⊠ BC ⊠⊠⊠ M ⊠⊠ $\odot O$ ⊠⊠ EF ⊠⊠ ⊠⊠⊠1⊠$\triangle AEF$ ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠(2) $\triangle AEF$ ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠



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☒1 ☒☒☒☒☒1007 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒1☒☒☒☒☒☒☒☒2011 ☒☒☒☒☒2☒☒☒☒☒☒☒☒2011 ☒☒☒☒

☒☒☒1☒☒☒☒1007 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ A B ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ A B ☒☒☒☒☒☒ $\frac{1}{2}AB$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ C, ☒☒ $CA \leqslant AB, CB \leqslant AB$, ☒☒☒ CA CB ☒☒☒☒☒☒☒☒ $\odot A$ $\odot$ B ☒☒☒☒☒☒☒. ☒☒☒☒☒☒☒☒☒ D E ☒ DA EA ☒☒☒☒☒☒☒ D E ☒☒☒☒☒☒☒☒☒☒ $\odot A$ ☒ $\odot B$ ☒☒☒☒☒☒☒☒☒☒☒☒ $\odot A$ ☒☒☒☒☒☒ $\odot B$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $\odot A$ ☒$\odot B$ ☒☒☒☒☒☒☒☒☒☒☒☒☒ $2\times 1005+1=2011$ ☒☒☒☒

☒2☒☒1007 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $1,2,\cdots,1007$ ☒☒☒☒☒☒☒☒☒ $2,3,\cdots,1006$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $1.5, 2.5, \cdots$ ☒1006.5 ☒☒☒☒☒☒☒☒☒☒☒☒☒ $1005+1006=2011$ ☒☒☒☒

☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒2 ☒☒ $2n-1$ $n>2$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒1 ☒☒☒☒☒☒☒☒☒☒☒ n ☒☒☒☒☒☒☒1 ☒☒☒☒

☒☒☒☒ $2n-1$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ AB ☒☒☒☒☒☒☒ A B ☒☒☒☒☒☒1 ☒☒☒☒ $AB<1$ ☒☒☒☒☒☒☒☒☒☒ C ☒☒☒ $CA \leqslant AB < 1, CB \leqslant AB < 1$ ☒☒ C ☒ $\odot A$ ☒☒☒☒ $\odot B$ ☒☒☒ $AB \geqslant 1$ ☒☒☒☒☒☒☒☒☒ C, ☒☒ $CA<1$ ☒ $CB<1$, ☒☒ C ☒☒ $\odot A$ ☒, ☒☒ $\odot B$ ☒. ☒☒☒☒☒☒☒☒☒☒☒ n ☒☒☒ $\odot A$ ☒☒ $\odot B$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ n ☒☒☒☒☒☒☒☒1 ☒☒☒☒

☒3 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒1. ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $\frac{1}{9}$ ☒

☒☒☒☒☒☒☒ $\frac{1}{9}$ ☒☒☒☒☒☒☒☒☒

☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $\odot A_1$ ☒☒☒☒☒☒ r_1 ☒☒ A_1 ☒☒☒☒☒☒☒☒☒ $3r_1$ ☒☒☒☒☒☒☒☒☒☒ $\odot A_1$ ☒☒☒☒☒☒☒☒☒☒☒ A_1 ☒☒☒, ☒☒☒ $3r_1$ ☒☒☒☒☒. ☒☒ $\odot A_1$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ S_1, ☒ $\odot A_1$ ☒☒☒☒☒ S_1', ☒ $9S_1' = \pi\left(3r_1\right)^2 \geqslant S_1$, ☒

$$S_1' \geqslant \frac{1}{9} S_1$$

⊠⊠⊠⊠ $\odot A_1$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ n ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠

$$S_1' + S_2' + \cdots + S_n' \geqslant \frac{1}{9}\left(S_1 + \cdots + S_n\right)$$
$$S_1 + S_2 + \cdots + S_n = 1$$

⊠ $S_1' + S_2' + \cdots + S_n' \geqslant \frac{1}{9}$ ⊠⊠⊠⊠ $\odot A_1$ ⊠⊠ $\odot A_2, \cdots,$ ⊠ $\odot A_n$ ⊠⊠⊠⊠⊠⊠.

⊠4 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠1 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠, ⊠ $\angle O_1OO_2 + \angle O_2OO_3 + \cdots + \angle O_7OO_1 = 360°$, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $60°$ ⊠⊠⊠⊠ $\angle O_1OO_2 < 60°$. ⊠ $\angle O_1OO_2 = 0°$, ⊠⊠⊠ $OO_1 < 1, OO_2 < 1$ ⊠⊠⊠ $O_1O_2 = |OO_1 - OO_2| < 1$ ⊠⊠⊠ $O_1O_2 \geqslant 1$ ⊠⊠.

⊠ $\angle O_1OO_2 \neq 0°$ ⊠⊠ $\triangle O_1OO_2$ ⊠⊠$\angle O_1OO_2$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\angle O_1O_2O$ ⊠⊠⊠⊠ $\angle O_1O_2O > 60° > \angle O_1OO_2$ ⊠⊠ $O_1O_2 < OO_1 \leqslant 1$ ⊠⊠ $O_1O_2 < 1$, ⊠⊠⊠ $O_1O_2 \geqslant 1$ ⊠⊠.

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠5 ⊠⊠⊠⊠2012 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠1 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\frac{1}{2}$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠⊠ A_1A_2 ⊠⊠⊠⊠ A_1A_2 ⊠⊠ O ⊠⊠⊠⊠⊠ $\frac{1}{2}A_1A_2$ ⊠⊠⊠⊠⊠. ⊠⊠⊠⊠⊠⊠⊠⊠ P, ⊠⊠ $PA_1 \leqslant A_1A_2, PA_2 \leqslant A_1A_2$ ⊠ $\triangle A_1PA_2$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\angle A_1PA_2 > 90°$ ⊠⊠⊠ P ⊠ $\odot O$ ⊠⊠⊠⊠⊠ $\frac{A_1A_2}{2} \leqslant \frac{1}{2}$ ⊠⊠⊠⊠

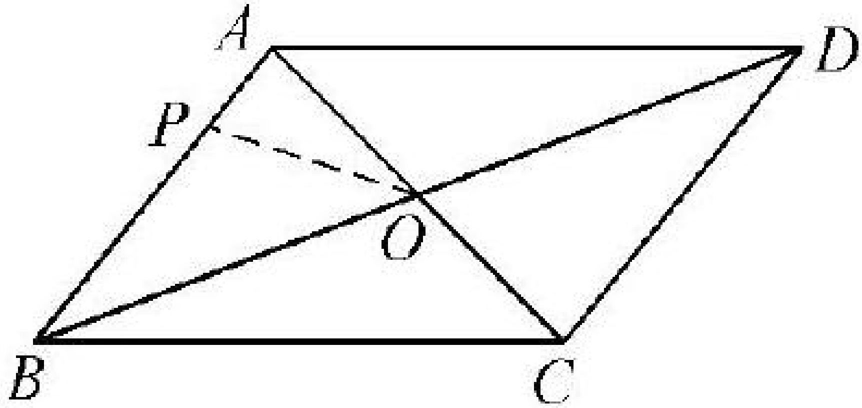
⊠⊠⊠⊠⊠⊠ O ⊠⊠⊠⊠$\frac{1}{2}$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠.

⊠⊠⊠⊠⊠⊠⊠⊠ F ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ O, ⊠⊠⊠ F ⊠⊠⊠⊠⊠ O ⊠⊠⊠⊠⊠⊠⊠⊠⊠ r ⊠⊠ F ⊠⊠⊠⊠⊠ r ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ A B ⊠⊠⊠ α ⊠⊠⊠⊠ F ⊠⊠⊠⊠⊠ AB ⊠⊠⊠⊠⊠ AB ⊠⊠⊠⊠⊠⊠ α ⊠⊠⊠⊠ F ⊠⊠⊠ AB ⊠⊠⊠⊠ AB ⊠⊠⊠ α ⊠⊠⊠⊠⊠⊠⊠

⊠6 ⊠⊠ $11-1$ ⊠⊠⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠⊠⊠ $BD > AC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠⊠⊠ BD ⊠⊠⊠⊠⊠

⊠⊠⊠ AB ⊠⊠⊠⊠⊠ P, ⊠ AC ⊠ BD ⊠⊠ O, ⊠⊠ OP ⊠

⊠⊠ $BD > AC$ ⊠⊠ $OB > OA$ ⊠⊠⊠ $\angle OAB >$



☒11 - 1

$\angle OBA$ ☒☒ $\angle BPO > \angle OAB$ ☒☒ $\angle BPO > \angle OBA$ ☒☒☒ $OB > OP$ ☒

☒☒☒☒ P ☒ AD CD BC ☒☒☒ $OB > OP$ ☒

☒☒☒☒☒☒☒☒☒☒☒ O ☒☒☒☒☒☒☒ $\frac{BD}{2}$ ☒☒☒☒☒

☒7 9☒☒☒☒☒1☒☒☒☒☒☒☒☒☒☒5☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒ $\frac{1}{9}$.

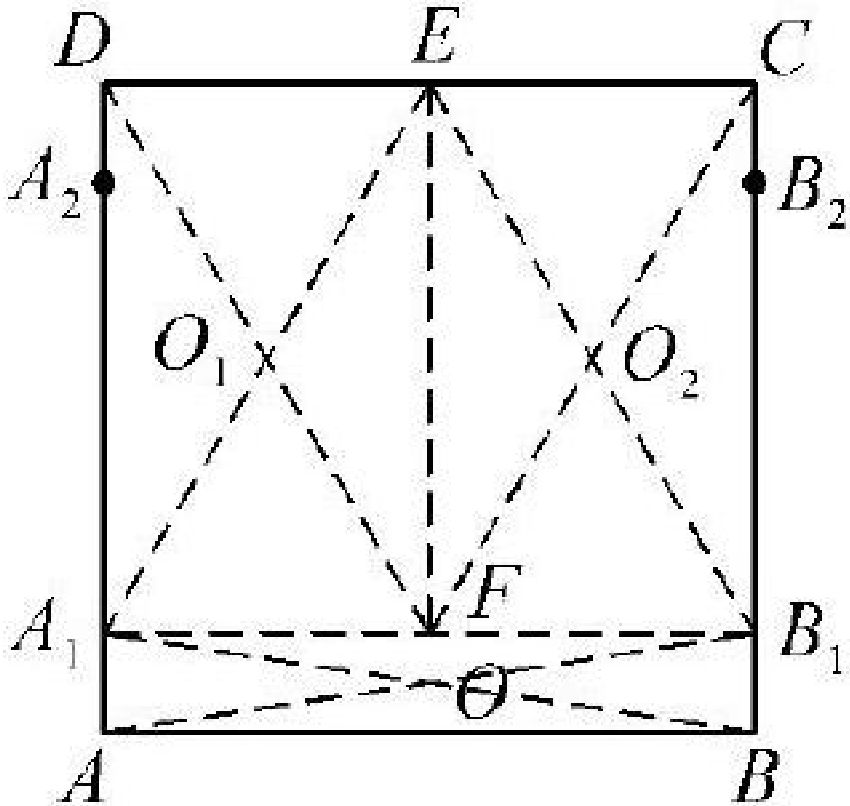
☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $\frac{1}{9}, 9$ ☒☒☒☒☒☒☒ $\mathrm{C}_9^2 = 36$ ☒☒☒☒☒☒☒☒☒☒ ☒☒☒☒☒☒☒☒☒☒☒☒ $\frac{1}{9} \times 36 = 4$ ☒☒☒☒☒☒☒☒☒☒☒☒☒ $9 - 4 = 5$ ☒☒☒☒☒☒☒☒ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $\frac{1}{9}$ ☒

☒8 ☒☒☒☒☒ r ☒☒☒☒☒☒☒☒16 ☒☒☒☒☒☒ r ☒☒☒☒☒

☒☒☒☒☒☒☒☒☒☒☒☒ r ☒☒☒ r ☒☒☒☒☒

☒☒☒11-2☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ E F ☒☒☒ CD A_1B_1 ☒☒☒☒ $A_1B_1 \perp AB$ ☒☒☒ A_1DEF ☒ $\odot O_1$ ☒☒☒☒☒ $FECB_1$ ☒ $\odot O_2$ ☒☒☒☒☒ A_1B_1BA ☒ $\odot O$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $AA_1 = x$ ☒☒

$$\sqrt{8^2 + (16 - x)^2} = 2\sqrt{8^2 + \left(\frac{x}{2}\right)^2}$$



⊠11-2

⊠⊠ $x = 2$. ⊠⊠⊠⊠⊠⊠ $r = \sqrt{8^2 + 1^2} = \sqrt{65}$.

⊠⊠: $r = \sqrt{65}$ ⊠⊠⊠⊠.

⊠ $r' < r = \sqrt{65}$, ⊠⊠⊠⊠⊠⊠⊠ A B C D ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ (⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠⊠⊠), ⊠⊠⊠ A B ⊠ $\odot O$ ⊠, ⊠⊠ $r' < r$, ⊠ A_1D CD B_1C ⊠⊠⊠ $\odot O$ ⊠, ⊠ ⊠⊠⊠⊠⊠⊠⊠⊠:

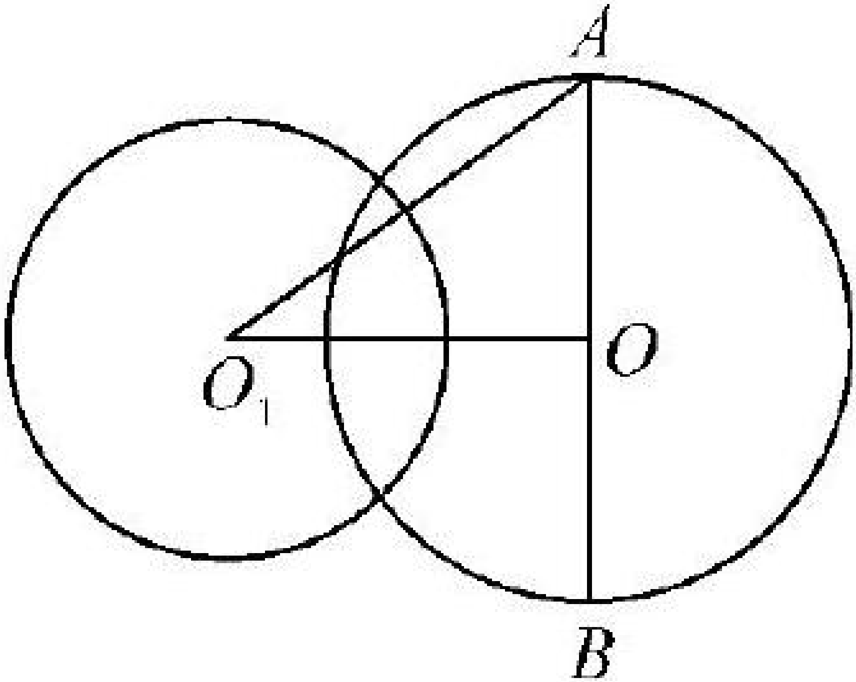
⊠ C D ⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠ $\odot O_3$ ⊠, ⊠⊠⊠ AD BC ⊠⊠ A_2, B_2, ⊠⊠ $11-2$, ⊠ $DA_2 = AA_1, CB_2 = BB_1$, ⊠ A_1A_2 B_1B_2 ⊠⊠⊠ $\odot O_3$ ⊠, ⊠⊠⊠⊠⊠ O_4 ⊠⊠⊠⊠ ⊠⊠⊠ A_1A_2 ⊠ B_1B_2 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $ABCD$ ⊠

⊠ C D ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\odot O_3$ ⊠⊠ D ⊠⊠⊠⊠⊠ $C, \odot O_4$ ⊠⊠ C ⊠⊠⊠⊠⊠ ⊠ D ⊠⊠ $\odot O_3, \odot O_4$ ⊠⊠⊠⊠⊠⊠⊠ E A_1 B_1 ⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠

⊠⊠⊠⊠⊠$r' < r = \sqrt{65}$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠16 ⊠⊠⊠⊠⊠⊠ r ⊠⊠⊠⊠ ⊠ $\sqrt{65}$ ⊠

⊠9 ⊠⊠⊠⊠⊠⊠1 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠1 ⊠⊠⊠⊠⊠

⊠⊠⊠. ⊠⊠ $11-3$ ⊠⊠ $\odot O$ ⊠⊠⊠⊠ $1, \odot O_1$ ⊠⊠⊠⊠⊠1 , ⊠ $\odot O_1$ ⊠⊠⊠⊠ $\odot O$ ⊠ ⊠⊠⊠, ⊠⊠ O_1O ⊠⊠ O ⊠ $AB \perp OO_1$ ⊠ $\odot O$ ⊠ A B ⊠⊠ $O_1A > OA = \frac{1}{2}$ ⊠⊠⊠ A B ⊠⊠ $\odot O_1$ ⊠⊠⊠⊠ $AB = 1$ ⊠⊠



☒11-3

☒☒ $\odot O_1$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒1 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒ A B ☒☒☒☒

☒☒

☒10 ☒☒☒☒2014 ☒☒, ☒☒☒☒12 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒1 , ☒☒: ☒☒☒☒☒1 ☒☒☒☒☒☒☒☒☒☒184 ☒☒☒

☒☒☒2014 ☒☒☒, ☒☒☒☒☒ O_1, ☒ O_1 ☒☒☒, 1 ☒☒☒☒☒. ☒ $\odot O_1$ ☒☒☒☒☒☒☒☒, ☒☒☒☒☒. ☒☒☒☒ O_2, ☒ $O_1O_2 > 1$, ☒ O_2 ☒☒☒, 1 ☒☒☒☒ $\odot O_2$, ☒☒☒☒☒ O_3, ☒☒☒ $\odot O_1$ $\odot O_2$ ☒☒, ☒☒☒☒☒ $\odot O_3$ ☒☒☒☒☒ $\odot O_4, \cdots, \odot O_{11}$ ☒☒☒ $O_iO_j > 1 (1 \leqslant i < j \leqslant 11)$ ☒☒☒☒☒2014 ☒☒☒☒☒11 ☒☒☒☒, ☒☒, ☒☒ O_{12} ☒☒☒☒11 ☒☒☒, ☒ $O_1, O_2, \cdots, O_{12}$ ☒☒☒☒☒☒☒1 , ☒☒☒☒☒.

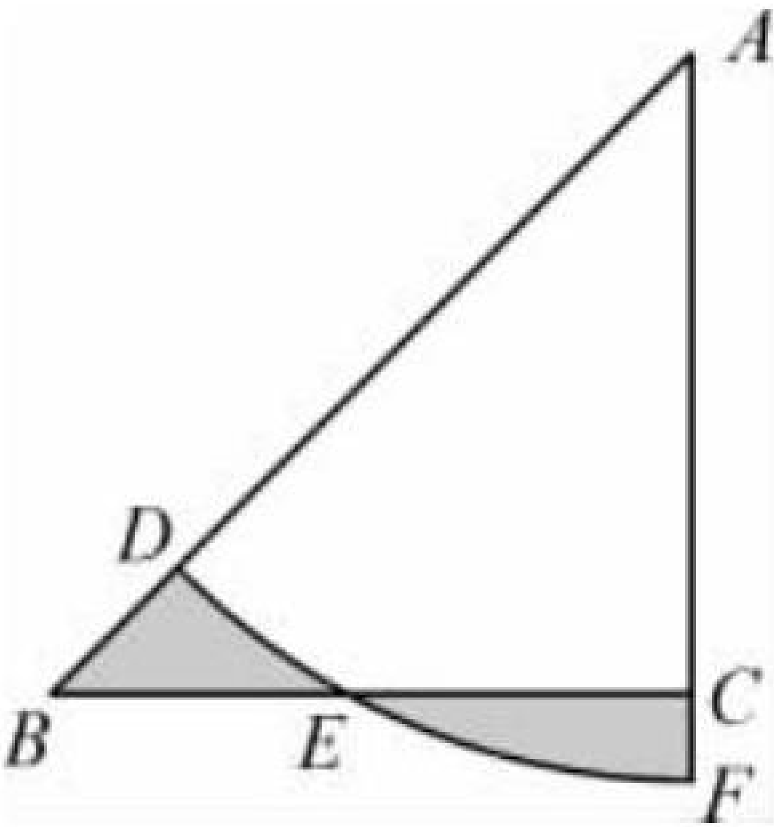
☒☒☒☒ $2014 = 11 \times 183 + 1$, ☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒184 ☒☒☒

1 ☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒, ☒☒: ☒☒☒☒☒☒☒☒☒☒☒☒☒☒

☒☒☒☒☒☒☒☒
2 ☒☒☒☒☒☒ $4l$ ☒☒☒☒☒☒☒☒☒☒☒☒ l ☒☒☒☒☒
3 ☒☒☒☒☒☒1 , ☒☒☒☒☒2 , ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒?
4 ☒7 ☒☒☒☒1 ☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒4 . ☒☒: ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒☒☒ $\frac{1}{7}$ ☒
5 ☒☒☒☒☒☒6 ☒☒☒, ☒☒☒☒6 ☒☒☒☒1 ☒☒☒, ☒☒: ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒☒☒1 ☒☒☒
6 ☒☒☒25 ☒☒, ☒☒☒☒☒☒☒☒☒☒☒☒☒☒1 , ☒☒: ☒☒☒☒1 ☒☒☒☒☒☒☒☒☒☒13 ☒☒.
7 ☒☒☒☒☒2012 ☒☒, ☒☒☒☒☒☒☒☒☒2012 , ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒, ☒☒: ☒☒ ☒☒☒☒☒2012 ☒☒☒☒☒2012 ☒☒.
8 ☒☒☒☒1 ☒☒☒☒4 ☒☒☒☒1 ☒☒☒, ☒☒: ☒☒☒☒☒☒☒ l, ☒☒☒☒☒☒☒☒☒ l ☒☒☒ ☒☒, ☒☒☒☒☒☒2 ☒☒☒☒☒.
9 ☒☒☒☒☒, ☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒, ☒☒: ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒
10 ☒☒☒☒5 ☒☒, ☒☒☒☒☒☒☒, ☒☒: ☒☒☒☒☒☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒, ☒☒☒☒ ☒.
11 ☒☒☒☒1 ☒☒☒☒☒, ☒☒☒☒☒☒☒0.001 ☒☒, ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒0.001 . ☒☒: ☒☒☒☒☒☒☒☒☒☒☒0.34 .
12 ☒☒☒☒50 ☒☒☒☒☒☒☒☒, ☒☒: ☒☒☒☒, ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒☒☒☒☒☒☒☒☒☒

1

1. ☒☒ $ADEC$ ☒☒☒ DAF ☒ $\triangle ACB$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $S_{\triangle ABC} = S_{\text{☒☒AF}}$ ☒☒ $\frac{1}{2}AC^2 = \frac{45}{360}\pi \cdot AD^2$ ☒☒ $2AC^2 = AB^2$ ☒☒ $\frac{1}{4}AB^2 = \frac{\pi}{8} \cdot AD^2$, ☒☒ $\frac{AD}{AB} = \sqrt{\frac{2}{\pi}}$, ☒☒ $\frac{AD}{DB} = \frac{\sqrt{2\pi}+2}{\pi-2}$.

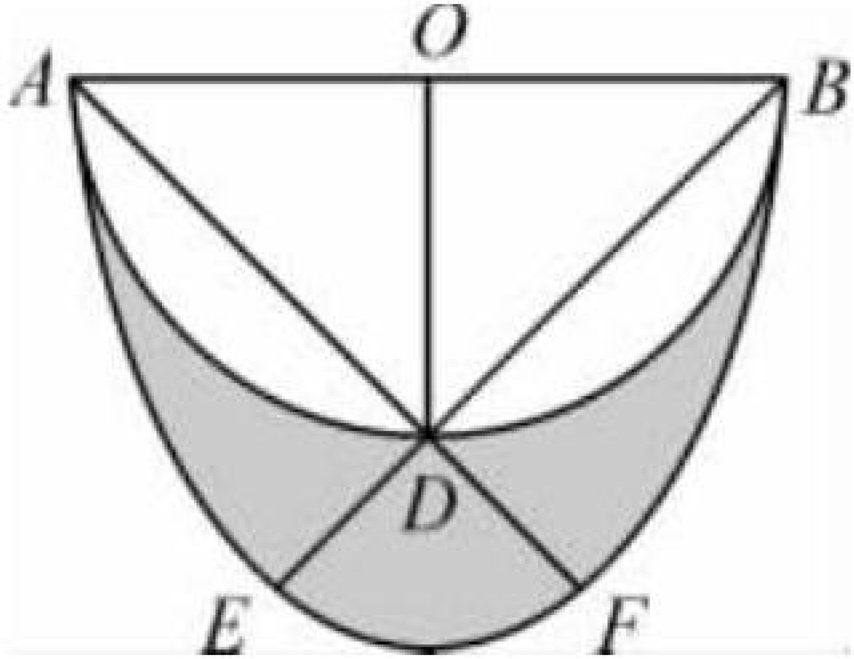


(⊠1 ⊠)

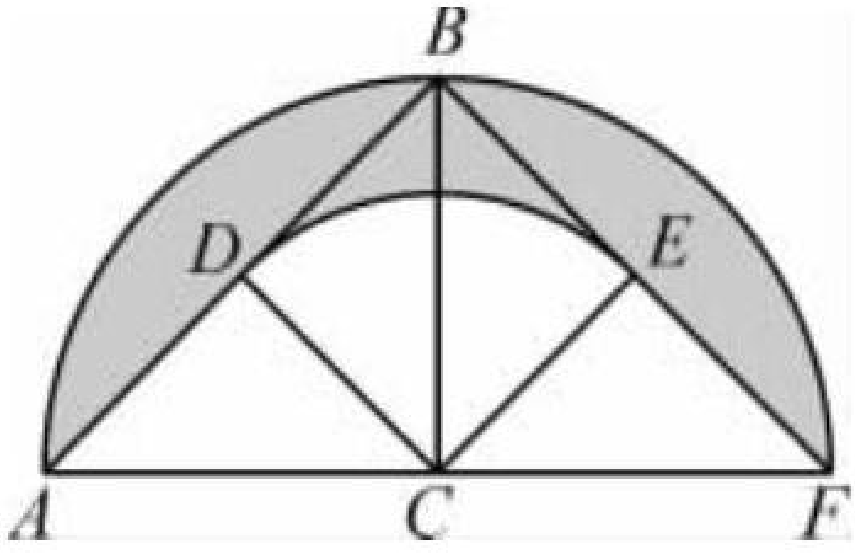
2. ⊠⊠500 ⊠⊠1 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠1 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $S = 500\times1+\pi\times1^2 = (500+\pi)$ (⊠⊠⊠)⊠

3. $S_{\text{⊠}} = S_{\text{⊠⊠}} + 2\left(S_{\text{⊠⊠ABE}} - S_{\triangle ADB} - S_{\text{⊠}AD}\right) = S_{\text{⊠⊠DF}} + 2S_{\text{⊠⊠}ABE} - 2S_{\triangle ADB} - 2\left(S_{\text{⊠⊠}AOD} - S_{\triangle AOD}\right) = S_{\text{⊠⊠}D^2} + 2S_{\text{⊠⊠}ABE} - 2S_{\triangle ADB} - S_{\text{⊠⊠ADB}} + S_{\triangle ADB} = S_{\text{⊠⊠}EDF} + 2S_{\text{⊠⊠}ABE} - S_{\triangle ADB} - S_{\text{⊠⊠}ADB} = \frac{90}{360}\pi(2-\sqrt{2})^2 + \frac{2\times45}{360}\times\pi\times2^2 - \frac{1}{2}\times2\times1 - \frac{1}{2}\pi\times1^2 = (2-\sqrt{2})\pi - 1.$

4. ⊠ AB ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ADB ⊠ BEF ⊠⊠⊠⊠⊠⊠ DBE ⊠⊠, ⊠ $S_{\text{⊠}} = \frac{1}{2}\pi\times2^2 - \frac{1}{2}\times2\times4 + (\sqrt{2})^2 - \frac{1}{4}\pi\times(\sqrt{2})^2 = 2\pi - 4 + 2 - \frac{\pi}{2} = \frac{3\pi}{2} - 2.$



(⊠3 ⊠)

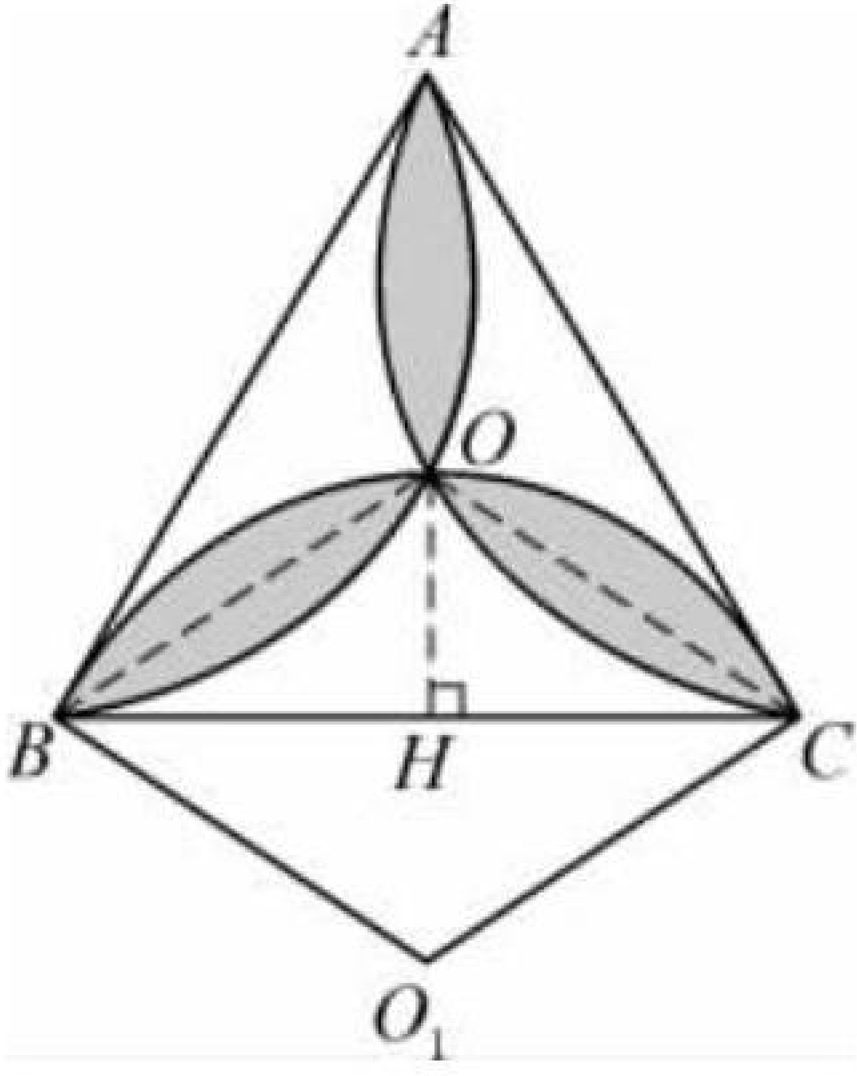


(⊠4⊠)

5. ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠1 cm⊠⊠ n ⊠⊠⊠⊠⊠⊠⊠⊠⊠1

cm ⊠⊠⊠⊠⊠ $r_n = 1$ cm ⊠⊠ $a_n = 2r_n \tan\left(\frac{180}{n}\right)^\circ$ ⊠⊠ $S = \frac{n}{2} \times r_n \times a_n - \pi r_n^2 = \left(n \tan\left(\frac{180}{n}\right)^\circ - \pi\right) r_n^2 = \left(n \tan\left(\frac{180}{n}\right)^\circ - \pi\right)(\text{cm}^2)$.

6. ⊠⊠ $OB\ OC$, ⊠ $OH \perp BC$ ⊠⊠ H, ⊠ $S_{\triangle OBC} = \frac{1}{2} \times \frac{\sqrt{3}}{6}a \times a = \frac{\sqrt{3}}{12}a^2$. ⊠ BOC ⊠⊠⊠⊠ O_1, ⊠⊠⊠⊠⊠⊠⊠ $O_1B = \frac{\sqrt{3}}{3}a$ ⊠⊠⊠⊠⊠ BO_1C ⊠⊠⊠⊠⊠ $BOCO_1$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠" ⊠⊠⊠⊠" ⊠⊠⊠⊠⊠⊠⊠ ⊠⊠ $S_{⊠} = 3\left[\frac{1}{3}\pi \times \left(\frac{\sqrt{3}}{3}a\right)^2 - 2S_{\triangle OBC}\right] = 3\left(\frac{1}{9}\pi a^2 - \frac{\sqrt{3}}{6}a^2\right) = \frac{\pi}{3}a^2 - \frac{\sqrt{3}}{2}a^2$.

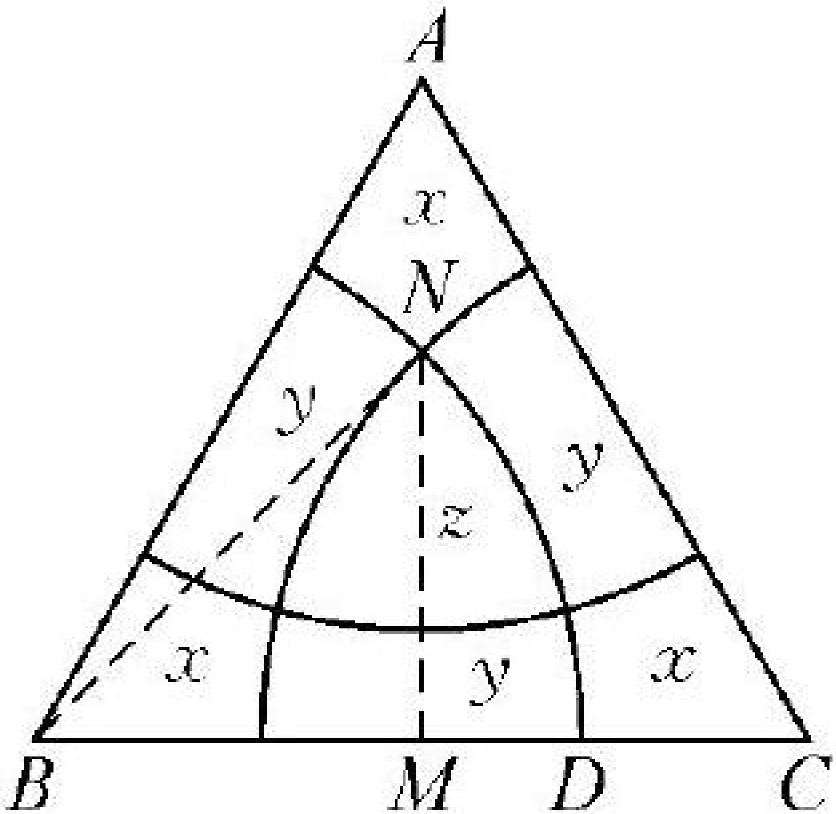


(⊠6⊠)

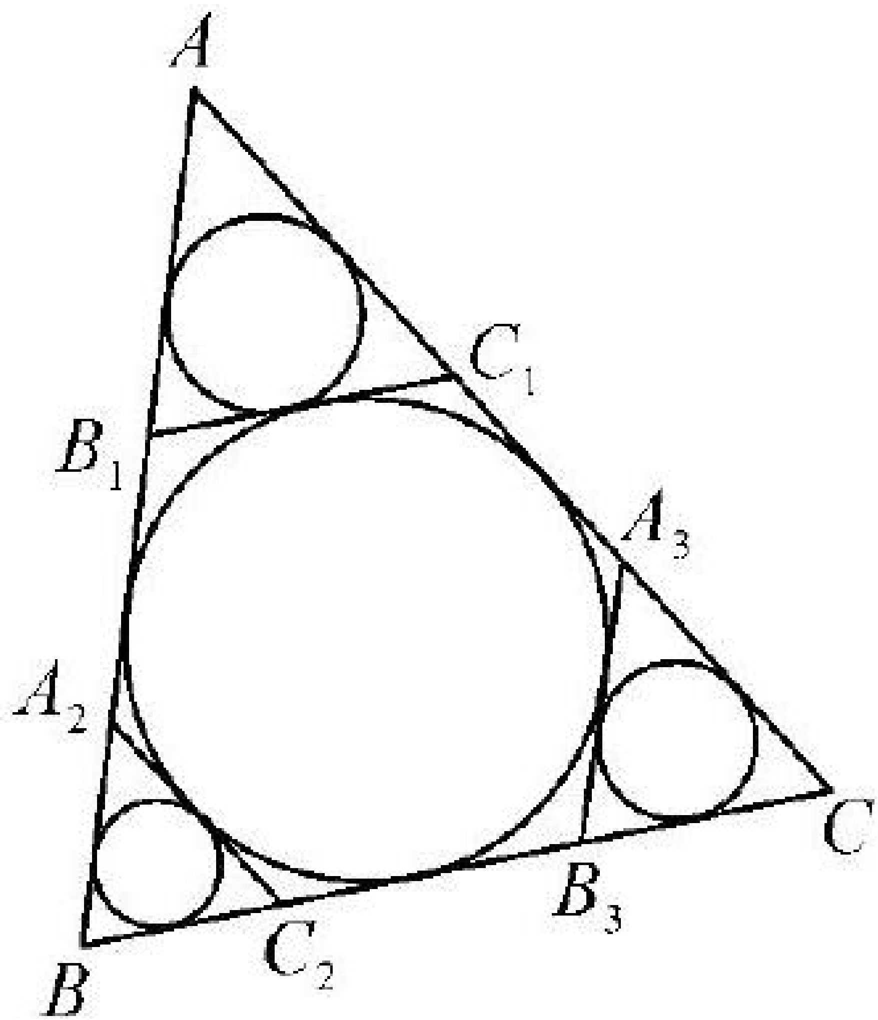
7. ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $x\ y$ ⊠z ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $3x + 3y + z = \sqrt{3}a^2 \cdots$ (1), $x + 2y + z = \frac{1}{6}\pi(\sqrt{2}a)^2 = \frac{\pi}{3}a^2 \cdots$ (2). ⊠⊠⊠

$BN^2 + CN^2 = (\sqrt{2}a)^2 + (\sqrt{2}a)^2 = (2a)^2 = BC^2$ ⊠⊠ $BN \perp NC$ ⊠⊠⊠⊠⊠⊠⊠ NM ⊠⊠⊠⊠ BC ⊠$\angle NBM = \angle BNM = 45^\circ$ ⊠⊠⊠ $y + z = 2\,(S_{⊠⊠} = \angle DD$ $S_{\triangle NBM}) = 2\left[\frac{45}{360}\pi(\sqrt{2}a)^2 - \frac{1}{2}a^2\right] = \frac{\pi}{2}a^2 - a^2 \cdots$ (3). ⊠ (1)⊠(2)⊠(3) ⊠⊠ $z = \frac{a^2}{2}(\pi + 2\sqrt{3} - 6)$ ⊠

8. ⊠ $\triangle ABC$ ⊠⊠⊠⊠ S, ⊠⊠⊠⊠ p, ⊠ $\triangle AB_1C_1$ ⊠⊠⊠⊠⊠ $p-a, \triangle BA_2C_2$ ⊠⊠⊠⊠⊠ $p-b$, $\triangle CA_3B_3$ ⊠⊠⊠⊠⊠ $p-c$, ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠ $r = \frac{S}{p}$, ⊠ $\triangle ABC \backsim \triangle AB_1C_1 \backsim \triangle A_2BC_2 \backsim \triangle A_3B_3C$, ⊠⊠ $S_{AB_1C_1} = \left(\frac{p-a}{p}\right)^2 S, r_1 = \frac{S_{AB_1C_1}}{p-a} = \frac{p-a}{p^2} \cdot S$ ⊠⊠⊠ $r_2 = \frac{S_{A_2BC_2}}{p-b} = \frac{p-b}{p^2} \cdot S, r_3 = \frac{S'_{A_3B_3C}}{p-c} = \frac{p-c}{p^2} \cdot S$, ⊠



(⊠7⊠)



(☒8☒)

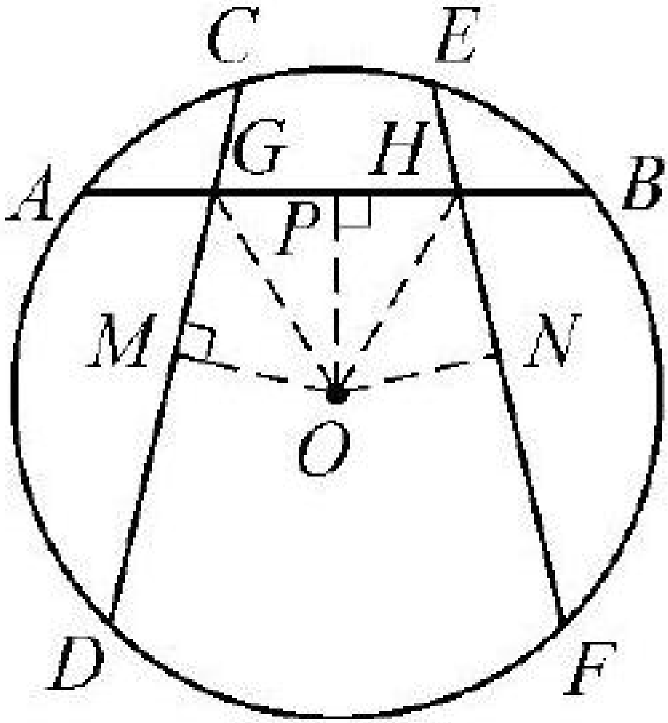
$S_{☒} = \pi \left(r^2 + r_1^2 + r_2^2 + r_3^2\right) = \pi \left[\frac{1}{p^2} + \frac{(p-a)^2}{p^4} + \frac{(p-b)^2}{p^4} + \frac{(p-c)^2}{p^4}\right] S^2 = \pi S^2 \frac{\left(a^2+b^2+c^2\right)}{p^4}$,

☒ $S^2 = p(p-a)(p-b)(p-c)$, ☒ $S_{☒} = \frac{(p-a)(p-b)(p-c)\left(a^2+b^2+c^2\right)}{p^3}\pi$, ☒☒ $p = \frac{a+b+c}{2}$.

2

1. ☒ $CH \perp AD$ ☒☒ H, ☒Rt $\triangle ACB$ ☒ $CH \perp AB$ ☒☒ H, ☒ $AC^2 = AH \cdot AB$, ☒☒ $AH = \frac{AC^2}{AB} = \frac{64}{17}$, ☒☒ $AD = \frac{128}{17}, BD = \frac{161}{17}$.
2. ☒ $OP \perp AB$ ☒☒ P, ☒☒ $OG\,OH$, ☒ $AP = BP$, ☒ $AG = BH$ ☒☒☒ $GP = HP$ ☒☒☒ $\triangle GPO \cong \triangle HPO$ ☒☒ $OG = OH, \angle BGO = \angle AHO$

☒☒☒☒ $\angle DGB = \angle FHA$, ☒☒ $\angle DGO = \angle FHO$, ☒ $OM \perp CD$ ☒☒ $M, ON \perp EF$

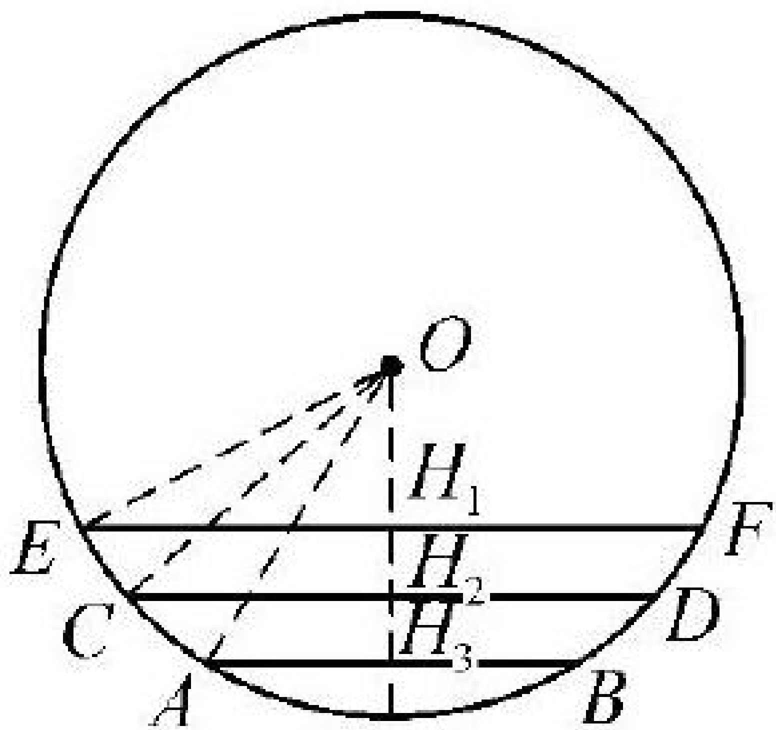


(☒2 ☒)

☒☒ N, ☒☒ $\triangle OGM \cong \triangle OHN$, ☒☒ $OM = ON$, ☒☒ $CD = EF$ ☒

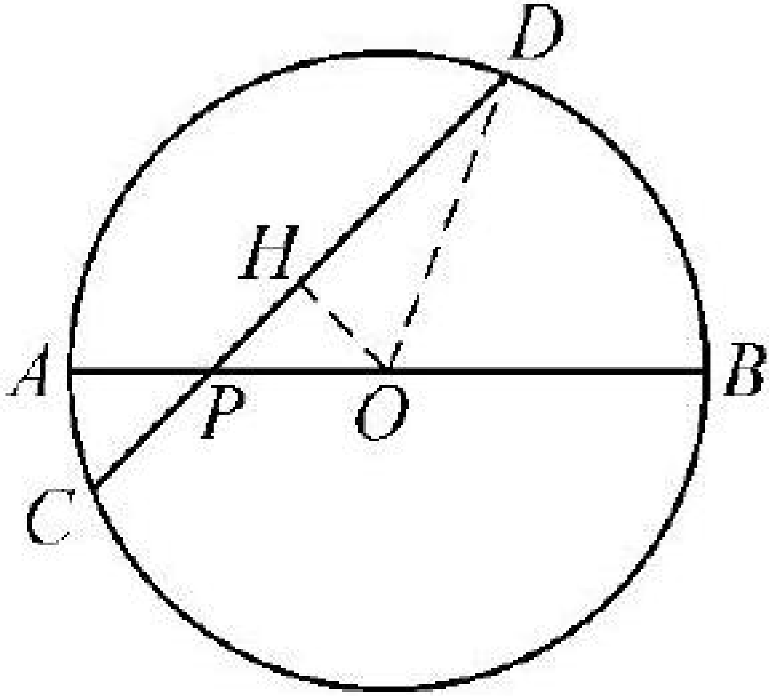
3. ☒ $OH_1 \perp EF$ ☒☒ H_1, ☒☒ OH_1 ☒ CD ☒☒ H_2, ☒ AB ☒☒ H_3. ☒ $OE = OC = OA = R, OH_1 = x, H_1H_2 = H_2H_3 = y$, ☒ $\begin{cases} R^2 - x^2 = 21, \\ R^2 - (x+y)^2 = 16, \\ R^2 - (x+2y)^2 = 9, \end{cases}$

$$\begin{cases} x = 2, \\ y = 1, \\ R = 5. \end{cases}$$

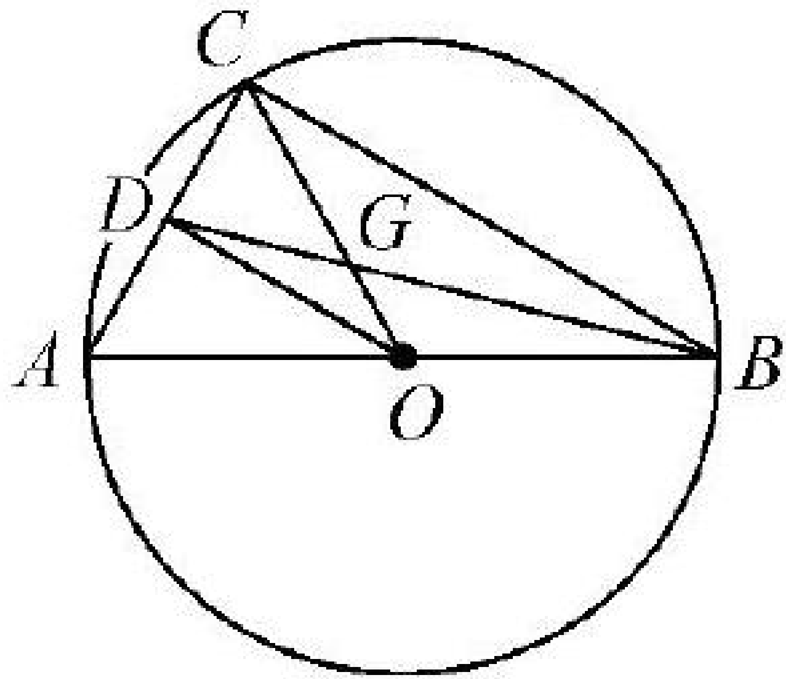


(图3 图)

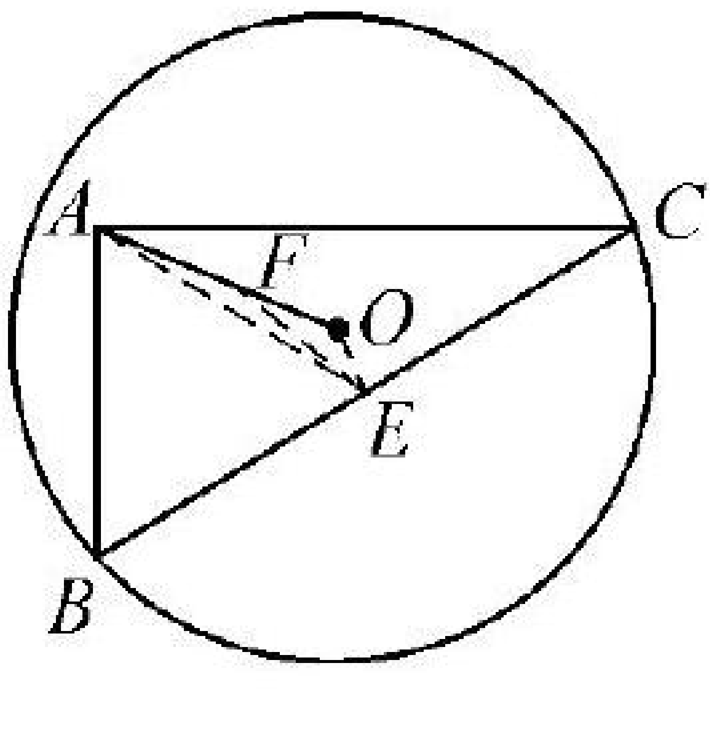
4. 作 $OH \perp CD$ 于点 H, 则 $HC = HD = \frac{1}{2}CD$. 于是 $PC^2 + PD^2 = \left(\frac{1}{2}CD - HP\right)^2 + \left(\frac{1}{2}CD + HP\right)^2 = \frac{1}{2}CD^2 + 2HP^2$. 又因 $\angle DPB = 45°$ 所以 有 $HP = OH$ 从而有 $PC^2 + PD^2 = \frac{1}{2}CD^2 + 2OH^2 = 2\left[\left(\frac{1}{2}CD\right)^2 + OH^2\right] =$



(⊠4⊠)
$2OD^2 = 2OA^2$.
5. ⊠⊠ $OD \perp AC$ ⊠⊠ D, ⊠⊠ D ⊠ AC ⊠⊠, ⊠⊠⊠ O ⊠ AB ⊠⊠, ⊠⊠ G ⊠ $\triangle ACB$ ⊠⊠⊠, ⊠⊠ $DG = \frac{1}{3}BD = 3$ ⊠



(☒5 ☒)

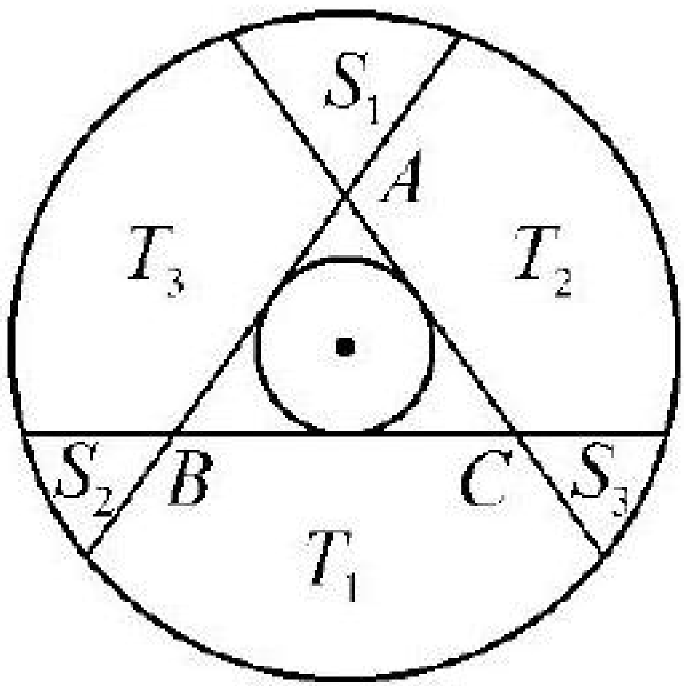


(☒6☒)

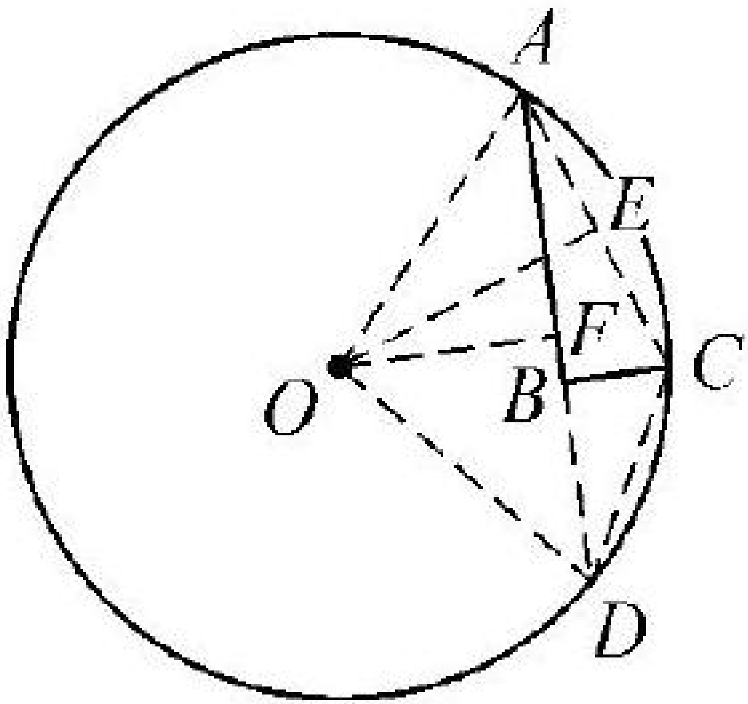
6. ☒☒ AE EF OE ☒☒ OE ☒☒☒ BC, EF ☒ $\triangle AEO$ ☒ AO ☒☒☒☒☒☒☒ $(2EF)^2 + AO^2 = 2\left(AE^2 + OE^2\right)$ (☒☒☒☒☒). ☒☒ $\angle BAC = 90°, E$ ☒ BC ☒☒☒, ☒☒ $AE = \frac{1}{2}BC$, ☒☒ $4EF^2 + AO^2 = 2\left(\frac{1}{4}BC^2 + OE^2\right) = 2OC^2 = 2R^2$ ☒☒ $EF = \frac{1}{2}\sqrt{2R^2 - m^2}$ ☒

7. ☒☒☒☒☒☒, ☒☒☒ K_1 ☒☒☒☒☒☒☒ K_2 ☒☒☒☒☒☒" "" ☒☒☒☒ T_1 ☒T_2 T_3 (☒☒☒☒). ☒ $S_1+T_3+S_2 = S_2+T_1+S_3 = S_3+T_2+S_1 = K_2, T_1+T_2+S+S_3 = T_1+T_3+S_2+S = T_2+T_3+S_1+S = K_1$ ☒☒ $6K_2 - 3K_1 = 3\left(S_1+S_2+S_3-S\right)$ ☒☒ $S_1+S_2+S_3-S = 2K_2 - K_1$ ☒☒☒☒

8. ☒☒ AB ☒ $\odot O$ ☒☒ D, ☒☒ OA AC CD ☒OD, ☒ $OE \perp AC$ ☒☒ $E, OF \perp AD$ ☒☒ F, ☒ $AC = \sqrt{AB^2 + BC^2} = 2\sqrt{10}$, ☒ $AE = \sqrt{10}$, ☒☒ $OE = 2\sqrt{10} \cdot$ $\angle BDC = \frac{1}{2}\angle AOC = \angle AOE, \angle OEA = \angle DBC = 90°$, ☒ $\triangle AOE \backsim \triangle CDB$, ☒☒ $\frac{BC}{BD} = \frac{AE}{OE} =$



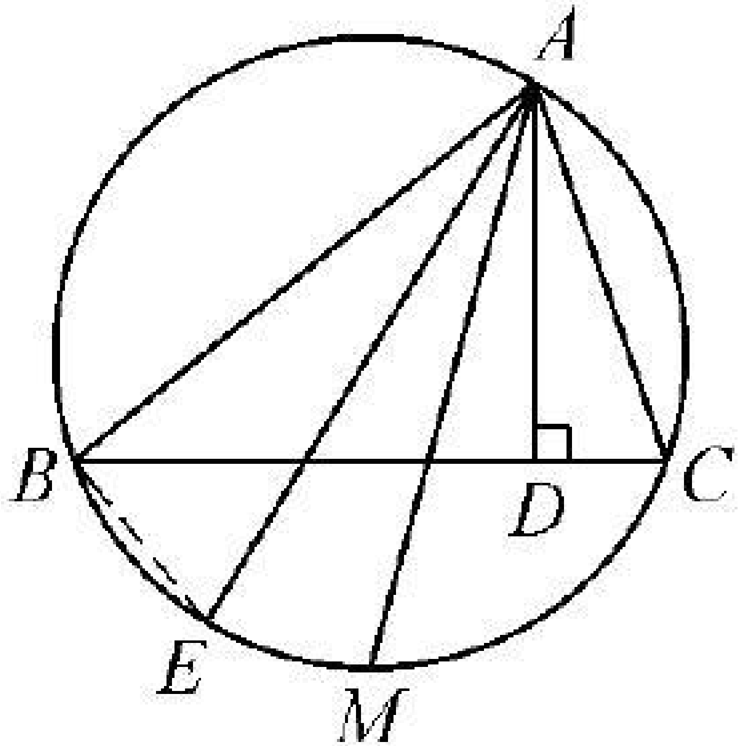
(☒7☒)



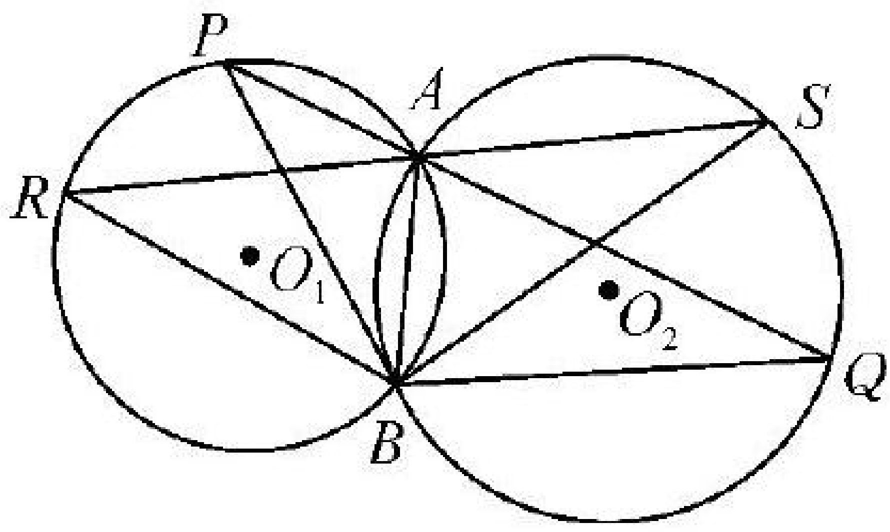
(⊠8 ⊠)
$\frac{1}{2}$. ⊠ $BC = 2$, ⊠⊠ $BD = 4, AD = 10, AF = 5, BF = 1, OF = \sqrt{OA^2 - AF^2} = 5$, ⊠⊠ $OB = \sqrt{OF^2 + BF^2} = \sqrt{26}$.

3

1. ⊠⊠ BE, ⊠ $\angle E = \angle C$. ⊠ AE ⊠⊠⊠, ⊠ $\angle ABE = 90^\circ$.$AD \perp BC$, ⊠ $\angle ADC = 90^\circ$, ⊠⊠ $\angle BAE = \angle DAC$. ⊠⊠ M ⊠ BC ⊠⊠, ⊠⊠ $\angle BAM = \angle CAM$ ⊠⊠⊠ $\angle EAM = \angle DAM$.



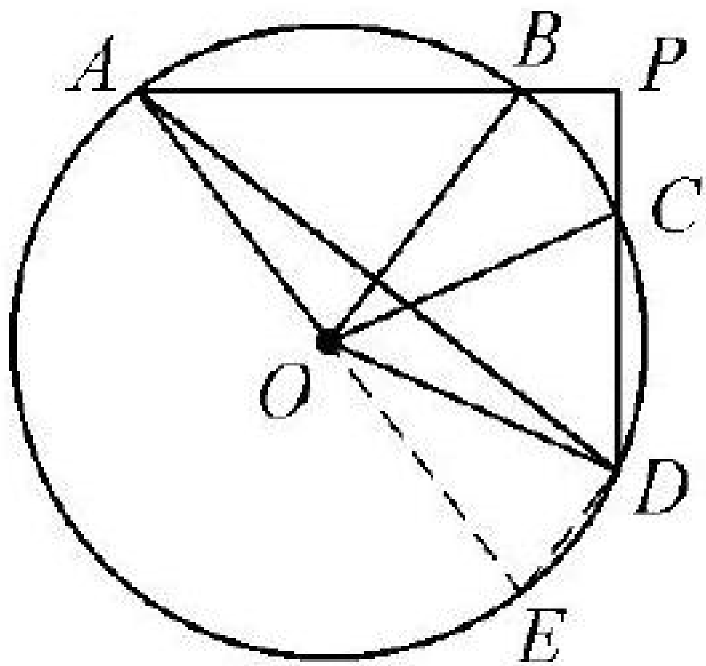
(⊠1 ⊠)



(⊠2⊠)

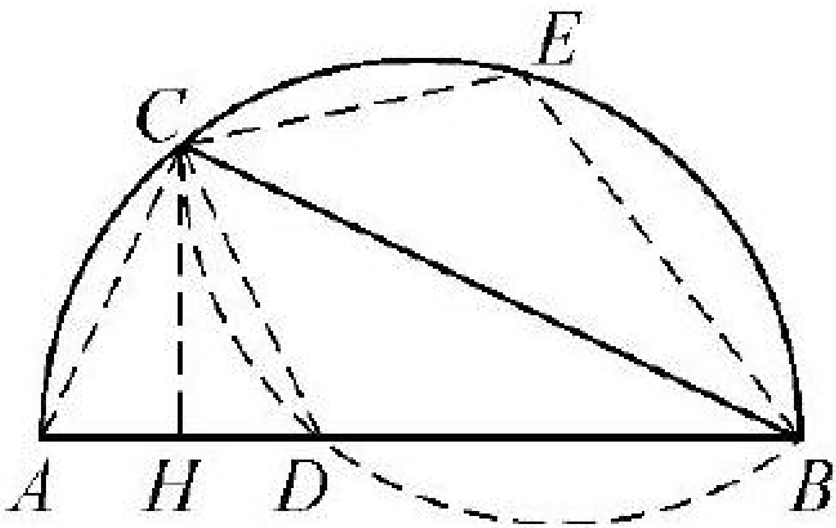
2. $\angle R = \angle P, \angle S = \angle Q$, ⊠⊠ $\triangle PBQ$ ⊠ $\triangle RBS$ ⊠, ⊠ $\angle PBQ = \angle RBS$, ⊠⊠ $\angle PBR = \angle SBQ$.

3. ⊠⊠ AO ⊠ $\odot O$ ⊠⊠ E, ⊠⊠ ED. ⊠ $\angle P = 90°$, ⊠ $90° = \angle PAD + \angle PDA \stackrel{m}{=} \frac{1}{2}BC + \frac{1}{2}CD + \frac{1}{2}AB + \frac{1}{2}BC$, ⊠ $90° \stackrel{m}{=} \frac{1}{2}(AB + BC + CD + DE)$, ⊠⊠ $BC = DE$ ⊠⊠ $BC = DE$ ⊠⊠⊠ $OB = OC = OD = OE$ ⊠⊠⊠ $\triangle OBC \cong \triangle ODE$, ⊠⊠ O ⊠ AE ⊠⊠, ⊠⊠ $S_{\triangle OBC} =$



(⊠3 ⊠)
$S_{\triangle ODE} = S_{\triangle OAD}$.

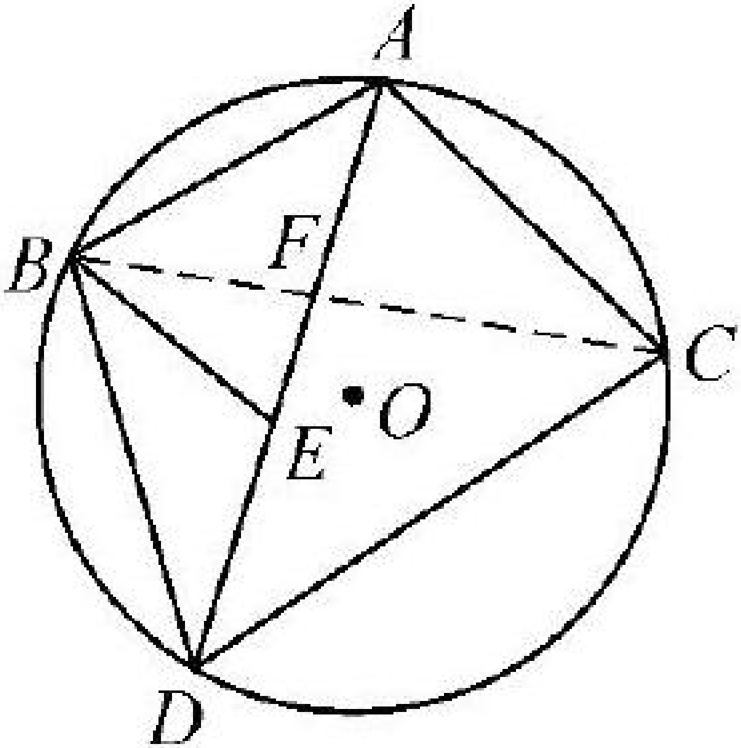
4. ⊠⊠ $AC\,CD$, ⊠ $CH \perp AB$ ⊠⊠ H, ⊠ D ⊠⊠ BC ⊠⊠⊠ E ⊠⊠⊠⊠⊠⊠ $\angle A + \angle CDB = \angle A + \angle E = 180°$ ⊠⊠ $\angle ADC + \angle CDB = 180°$, ⊠ $\angle A = \angle CDA$ ⊠⊠⊠ $AH = HD = 2$ ⊠⊠ $CA = x, BC = y$ ⊠⊠ $x^2 + y^2 = 81 \cdots$ (1). ⊠ $CH \cdot AB = CA \cdot BC$ ⊠⊠



(☒4 ☒)

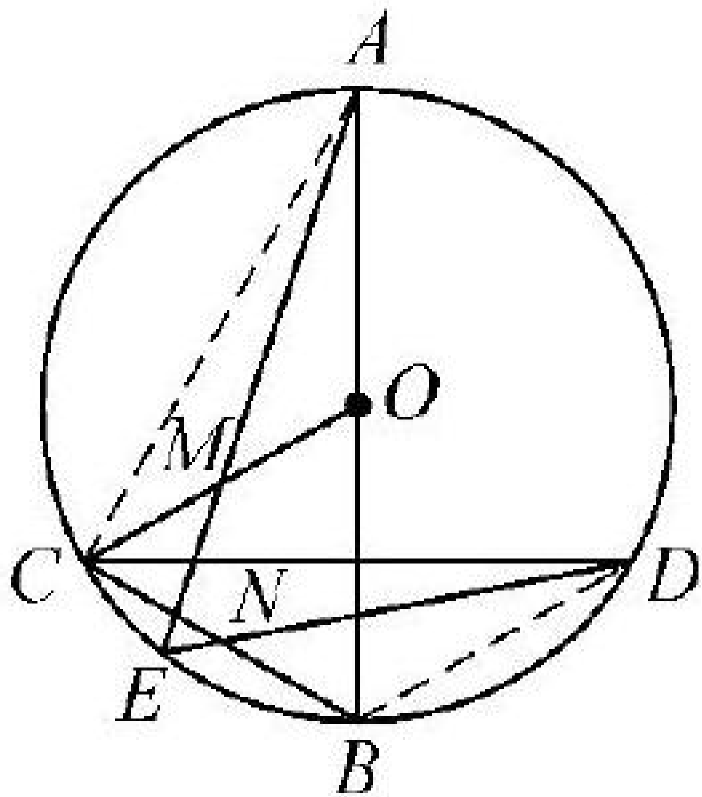
$9\sqrt{x^2-4} = xy\cdots$ (2). ☒ (1)☒(2) ☒☒ $BC = y = \sqrt{63} = 3\sqrt{7}$ (☒☒☒☒☒☒ ☒☒). ☒☒☒ $AC^2 = AH \cdot AB = 18$ ☒☒☒ $BC^2 = AB^2 - AC^2 = 81 - 18 = 63$.

5. (1) ☒☒ BC ☒ AD ☒☒ F, ☒ $\angle CAD = \angle DBC . \angle ACB = \angle ADB$ ☒☒☒☒ $AB = AC = AE$ ☒☒☒ $\angle ABC = \angle ACB = \angle ADB . \angle ABC + \angle EBC =$



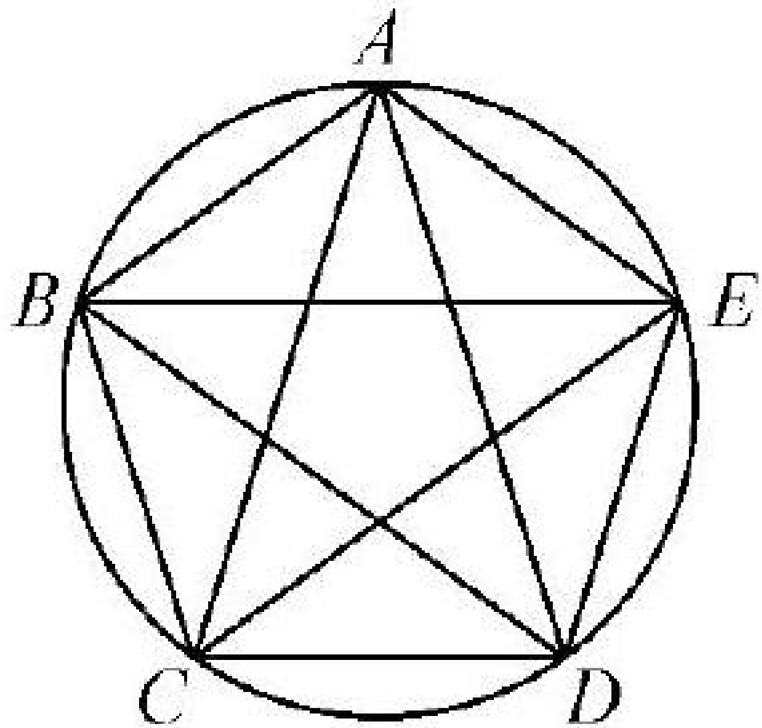
(☒5 ☒)
$\angle ABE = \angle AEB = \angle ADB + \angle DBE$ ☒☒☒ $\angle EBC = \angle DBE$ ☒☒☒ $\angle CAD = \angle DBC = \angle DBE + \angle EBC = 2\angle DBE.$ ☒2☒☒ $\angle FBA = \angle ACF = \angle BDE, \angle BAF = \angle DAB$ ☒☒☒ $\triangle BAF \backsim \triangle DAB$ ☒☒☒ $AB^2 = AD \cdot AF$ ☒☒☒ $AD^2 - AB^2 = AD(AD - AF) = AD \cdot DF \cdots$ (1). ☒ $\angle BDF = \angle ADC$, $\angle DBF = \angle DAC$ ☒☒ $\triangle BDF \backsim \triangle ADC$ ☒☒☒ $AD \cdot DF = BD \cdot DC \cdots (2)$ ☒☒ (1)☒(2) ☒ $AD^2 - AB^2 = BD \cdot DC$ ☒

6. ☒☒ AC BD ☒☒☒ $\angle CAO = \angle CDB$ ☒☒☒☒☒☒☒ $CAO \hookrightarrow$ ☒☒☒☒☒ CDB ☒☒☒ $\frac{CB}{CO} = \frac{CD}{CA} \cdots$ (1). ☒ $\angle CAM = \angle CDN, \angle ACM = \angle DCN$, ☒ ☒ $\triangle ACM \backsim \triangle DCN$, ☒☒ $\frac{CM}{CN} = \frac{AC}{DC} \cdots$ (2). ☒ (1)☒(2) ☒ $\frac{CM}{CN} = \frac{AC}{DC} = \frac{CO}{CB} = \frac{2CM}{CB}$, ☒☒ $CN = \frac{1}{2}CB$, ☒ $CN = BN$.



(⊠6 ⊠)

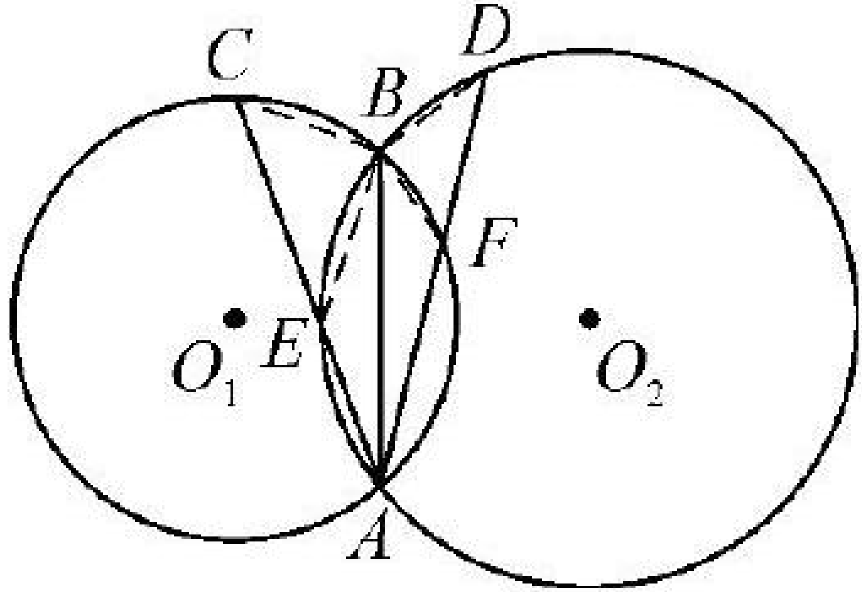
7. ⊠⊠ $AB//CE, AC//DE$, ⊠ $\angle BAC = \angle ACE = \angle CED$, ⊠⊠ $BC = CD$, ⊠⊠ $BC = CD$. ⊠⊠⊠⊠ $CD = DE, DE = EA, EA = AB$ ⊠⊠⊠ $AB = BC = CD = DE = EA \cdots$ (1). ⊠ (1) ⊠ $ED//AC$ ⊠, ⊠⊠⊠ $EDCA$ ⊠⊠⊠⊠⊠⊠⊠⊠ $\angle DEA = \angle CDE$ ⊠⊠⊠⊠⊠ $\angle BCD = \angle ABC, \angle BCD = \angle CDE, \angle DEA = \angle EAB$ ⊠⊠⊠⊠⊠ $ABCDE$ ⊠⊠$\angle ABC = \angle BCD = \angle CDE = \angle DEA =$



(☒7 ☒)

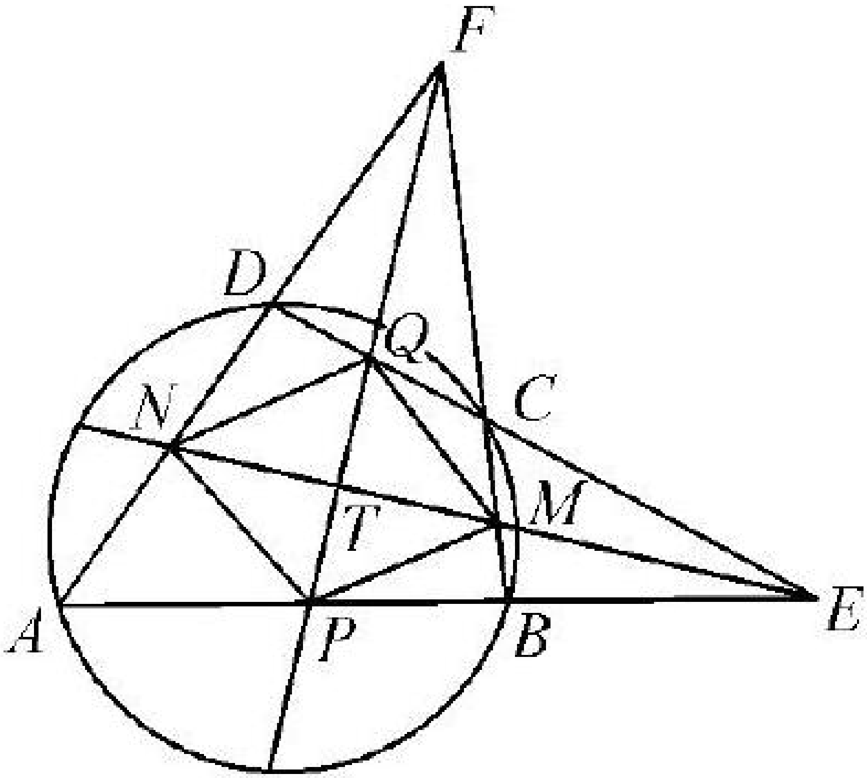
$\angle EAB \cdots$ (2). ☒ (1)☒(2) ☒☒☒☒ $ABCD$ ☒☒☒☒☒☒

8. ☒☒ BC BD BE BF ☒☒☒☒☒☒ $AEBD$ ☒☒☒ $\odot O_2$, ☒ $\angle CEB = \angle FDB$ ☒☒☒ $\angle ECB = \angle DFB$ ☒☒☒ $CE = DF \Leftrightarrow \triangle ECB \cong \triangle DFB \Leftrightarrow CB = FB \Leftrightarrow CB = FB \Leftrightarrow AB$ ☒☒ $\angle CAD$ ☒



(⊠8⊠)

9. ⊠ PQ ⊠ MN ⊠⊠ T, ⊠ $\angle NTP = \angle FNT + \angle NFT = \angle NAP + \angle NEP + \angle NFT$ ⊠$\angle QCM = \angle CFT + \angle FQC = \angle CFT + \angle QTM +$ $\angle CET$ ⊠⊠⊠ $\angle QTM = \angle QCM - \angle CFT -$ $\angle CET$ ⊠⊠⊠⊠ $\angle NFT = \angle QFC$ $\angle CET = \angle NEP, \angle NTP = \angle QTM$, ⊠ $\angle NTP = \frac{\angle NAP + \angle QCM}{2} = 90^\circ$ ⊠⊠⊠ $PQ \perp MN$ ⊠⊠⊠ FT



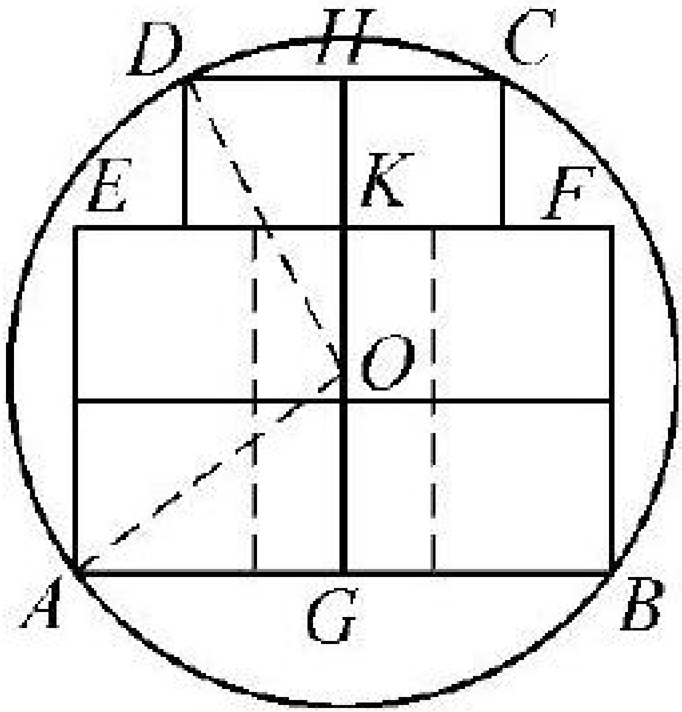
(☒9 ☒)

☒☒ $\angle NFM$ ☒☒☒ $NT = TM$ ☒☒☒ $QT = PT$ ☒☒☒☒☒☒ $MPNQ$ ☒☒☒.

4

1. ☒☒. ☒ $2n+1$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ O ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒☒☒☒☒☒ $2n+1$ ☒☒☒☒☒☒☒☒ O ☒☒☒ $2n+1$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ $r_1 < r_2 < \cdots < r_{n+1} < \cdots < r_{2n+1}$ ☒☒☒☒ O ☒☒☒☒r_{n+1} ☒☒☒☒☒☒☒☒☒ ☒☒

2. ☒☒. ☒8 ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒ ☒ $ABCD$ ☒☒☒☒☒☒☒☒☒☒☒☒☒ $ABCD$ ☒☒☒☒☒☒☒, ☒☒☒☒ O, ☒ O ☒ $HG \perp AB$ ☒☒ $G, HG \perp CD$ ☒☒ H, ☒ $HD = 1$, $AG = \frac{3}{2}$. ☒ $OG = x$, ☒ $OH = 3 - x$. ☒☒ $DH^2 + OH^2 = OD^2 = OA^2 = AG^2 + OG^2$, ☒ $1 + (3-x)^2 = \frac{9}{4} + x^2$ ☒☒☒



(⊠2⊠)

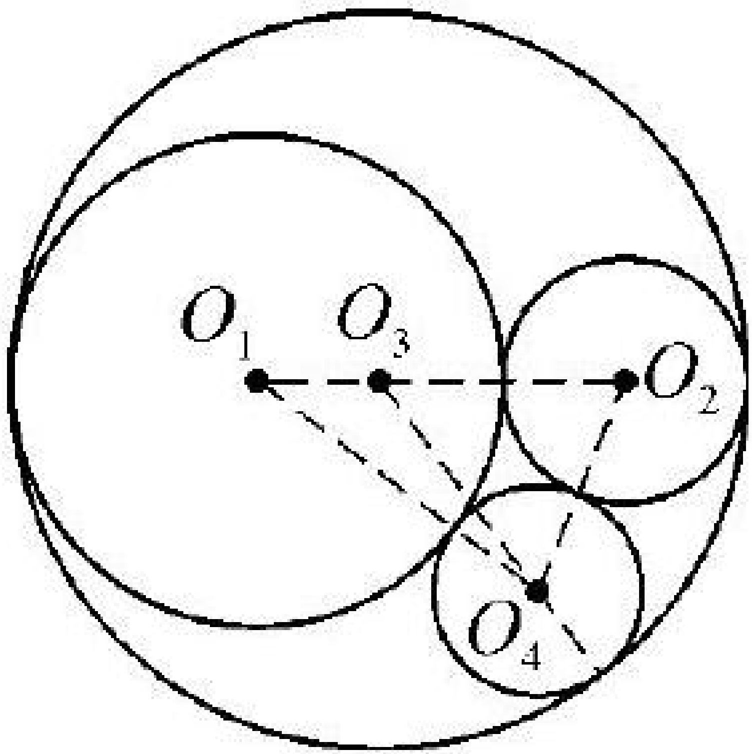
$x = \frac{31}{24}$, ⊠⊠ $OA = \sqrt{\frac{2257}{576}} = \frac{\sqrt{2257}}{24}$, ⊠ $OK = \frac{17}{24}, EK = \frac{3}{2}$, ⊠ $OE = \sqrt{OK^2+EK^2} = \sqrt{\left(\frac{17}{24}\right)^2+\frac{9}{4}} < \sqrt{\left(\frac{31}{24}\right)^2+\frac{9}{4}} = OA$ ⊠⊠⊠ E ⊠ $\odot O$ ⊠⊠⊠ ⊠⊠ $OA = \frac{\sqrt{2257}}{24} < 2$. ⊠⊠8 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠2 ⊠⊠⊠.

3. ⊠ AB ⊠⊠⊠ E, ⊠ $OE = \frac{1}{2}(AD+BC) = 2$. ⊠ $\angle ABC = 90^\circ$ ⊠, $OE \perp AB$ ⊠⊠ $OE = 2, \odot O$ ⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠⊠. ⊠ $\angle ABC < 90^\circ$ ⊠⊠⊠ $OH \perp AB$ ⊠⊠ H ⊠⊠ H ⊠⊠⊠ AE ⊠⊠$EH = OE \cdot \cos\angle AEO = OE\cdot \cos\angle ABC$ ⊠ ⊠⊠ E ⊠ $\odot O$ ⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠⊠ $\odot O$ ⊠⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠ F, ⊠⊠ $2EH = 2OE\cos\angle ABC = 4\cos\angle ABC \leqslant AE = 2$ ⊠, F ⊠⊠ AB ⊠. ⊠⊠ $60^\circ \leqslant \angle ABC < 90^\circ$ ⊠⊠ $2EH > 2$ ⊠⊠⊠ $\cos\angle ABC > \frac{1}{2}$ ⊠⊠⊠ ⊠ $0^\circ < \angle ABC < 60^\circ$ ⊠⊠F ⊠⊠ AB ⊠⊠⊠$\odot O$ ⊠⊠ AB ⊠⊠⊠⊠⊠⊠. ⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $90^\circ < \angle ABC \leqslant 120^\circ$ ⊠⊠$\odot O$ ⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠⊠ $\angle ABC > 120^\circ$ ⊠, $\odot O$ ⊠ AB ⊠⊠⊠⊠⊠⊠. ⊠⊠⊠⊠1⊠$\odot O$ ⊠⊠ AB ⊠⊠⊠⊠⊠ ⊠⊠⊠$60^\circ \leqslant \angle ABC \leqslant 120^\circ$ ⊠⊠ $\angle ABC \neq 90^\circ$ ⊠⊠2⊠$\odot O$ ⊠⊠ AB ⊠1 ⊠⊠⊠ ⊠⊠, $\angle ABC = 90^\circ$, ⊠ $0^\circ < \angle ABC < 60^\circ$ ⊠ $120^\circ < \angle ABC < 180^\circ$ ⊠

4. ⊠⊠⊠ $d = |3-2t|, r_A = 1+2t, r_B = 1$. (1) $|3-2t| = 2+2t$ ⊠ $|3-2t| = |2t|$ ⊠⊠⊠ $t = \frac{1}{4}$ ⊠ $\frac{3}{4}$. ⊠2⊠$|2t| < |3-2t| < 2+2t$ ⊠⊠⊠ $\frac{1}{4} < t < \frac{3}{4}$. ⊠3⊠$|3-2t| > 2+2t$ ⊠ $0 \leqslant |3-2t| < |2t|$ ⊠⊠⊠ $0 \leqslant t < \frac{1}{4}$ ⊠ $t > \frac{3}{4}$. ⊠⊠: ⊠ $t = \frac{1}{4}$ ⊠ $t = \frac{3}{4}$ ⊠, ⊠⊠⊠⊠; ⊠ $\frac{1}{4} < t < \frac{3}{4}$ ⊠, ⊠⊠⊠⊠; ⊠

$0 \leqslant t < \frac{1}{4}$ ☒ $t > \frac{3}{4}$ ☒, ☒☒☒☒.

5. ☒☒ O_1O_2 O_1O_4 O_2O_4, O_3 ☒ O_1O_2 ☒, ☒ $\odot O_3$ ☒☒☒☒21. ☒ $\odot O_4$ ☒☒☒☒ r, ☒ $O_1O_4 = 14+ r, O_3O_4 = 21 - r, O_2O_4 = 7 + r, O_1O_2 = 21, O_1O_3 = 7, O_2O_3 = 14$. ☒☒☒☒☒☒☒☒ $\cos\angle O_1O_3O_4 = \frac{-(14+r)^2+7^2+(21-r)^2}{2\times 7\times(21-r)}$, $\cos \angle O_2O_3O_4 = \frac{-(7+r)^2+14^2+(21-r)^2}{2\times 14\times(21-r)}$. ☒☒ $\angle O_1O_3O_4+$



☒☒ **5** ☒☒

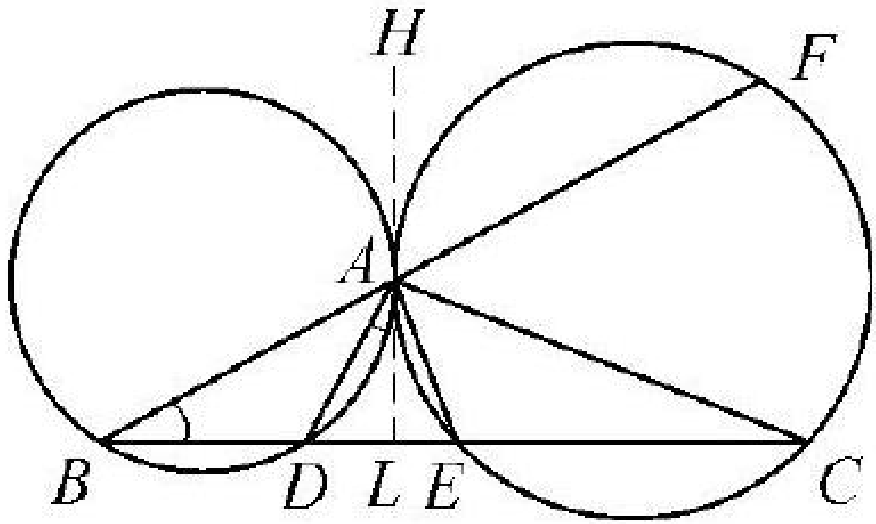
$\angle O_2O_3O_4 = 180^\circ$, ☒☒ $\cos\angle O_1O_3O_4 + \cos\angle O_2O_3O_4 = 0$, ☒ $\frac{-(14+r)^2+7^2+(21-r)^2}{2\times 7\times(21-r)} + \frac{-(7+r)^2+14^2+(21-r)^2}{2\times 14\times(21-r)} = 0$, ☒☒ $r = 6$. ☒ $\odot O_4$ ☒☒☒☒6 .

☒☒☒ O_4 ☒ $O_4H \perp O_1O_2$ ☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒☒.

6. ☒☒ $\frac{1}{2}AB\times(AS+BT) = \frac{1}{2}AB\times AD$ ☒☒ $AS+BT = AD$ ☒☒ $AD = AS+SD$, ☒ $BT = SD$, ☒ $BT//SD$, ☒☒☒☒☒ $SDTB$ ☒☒☒☒☒☒☒. ☒ BD ☒ ST ☒☒ ☒ O, ☒☒ $AB = 1, AD = \sqrt{3}, \angle BAD = 90^\circ$, ☒☒ $BD = 2$, ☒ $BO = 1$, ☒ $BP \perp ST, BP = 1$, ☒ O ☒ P ☒☒. ☒☒☒☒ $\triangle SPD \backsim \triangle BAD$, ☒☒☒ $\frac{SP}{BA} = \frac{PD}{AD}$, ☒☒ $SP = \frac{BA\cdot PD}{AD} = \frac{1}{\sqrt{3}}$, ☒☒ $ST = 2SP = \frac{2}{\sqrt{3}}$.

7. ☒☒ A ☒ $\triangle ABD$ ☒☒☒☒☒☒ HL ☒☒ DE ☒☒ L, ☒ $\angle DAL = \angle B$, ☒ $\angle DAE = \angle DAL + \angle EAL, \angle CAF = \angle B + \angle C, \angle DAE = \angle CAF$ ☒☒ $\angle C = \angle EAL$ ☒☒☒ HL ☒ $\triangle AEC$ ☒☒☒☒☒☒☒

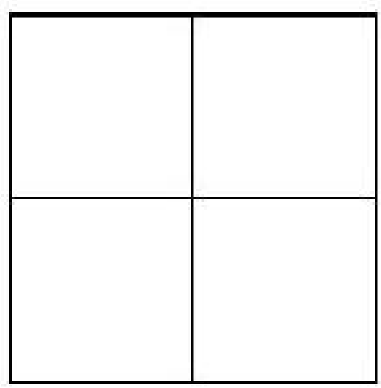
8. ⊠⊠⊠5 ⊠⊠⊠⊠⊠⊠⊠⊠⊠5 ⊠⊠⊠⊠



(⊠7⊠)

⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠ b, ⊠⊠⊠⊠⊠⊠⊠⊠

⊠⊠⊠⊠2 ⊠⊠⊠⊠⊠ b ⊠⊠, ⊠ $\frac{\sqrt{2}}{2}b \geqslant 2$, ⊠⊠ $b \geqslant 2\sqrt{2}$. ⊠ $a \geqslant b+2$, ⊠



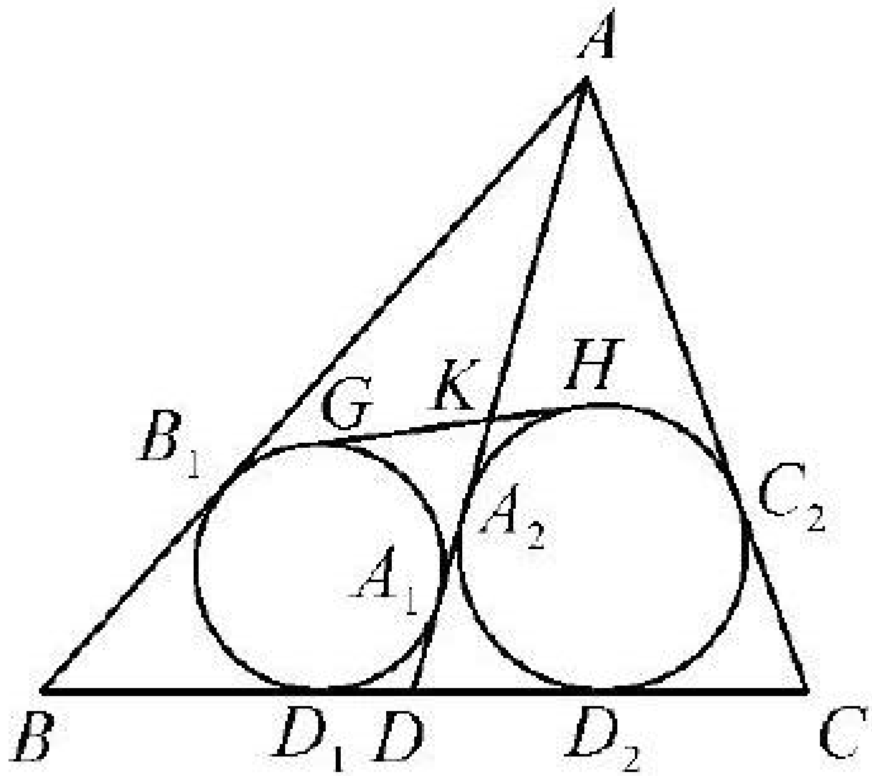
(⊠8 ⊠)

$a \geqslant 2+2\sqrt{2}$, ⊠⊠⊠ a ⊠⊠⊠⊠⊠ $2+2\sqrt{2}$.

5

1. ⊠⊠⊠⊠, $\triangle ABD$ ⊠⊠⊠⊠⊠ AB ⊠⊠ B_1, ⊠ AD ⊠⊠ A_1, ⊠ BD ⊠⊠ D_1; $\triangle ACD$ ⊠⊠⊠⊠⊠ AD ⊠⊠ A_2, ⊠ CD ⊠⊠ D_2, ⊠ AC ⊠⊠ C_2, GH ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠ $GH = D_1D_2$. ⊠⊠ $2AK = AD-(KA_1+A_1D)+AD-(KA_2+A_2D) = 2AD-KG-DD_1-KH-DD_2 = 2AD-GH-$

$D_1D_2 = 2AD - 2D_1D_2 = AA_1 + A_1D + AA_2 +$ $A_2D - 2D_1D_2 = AB - BB_1 + DD_1 + AC - CC_2 +$

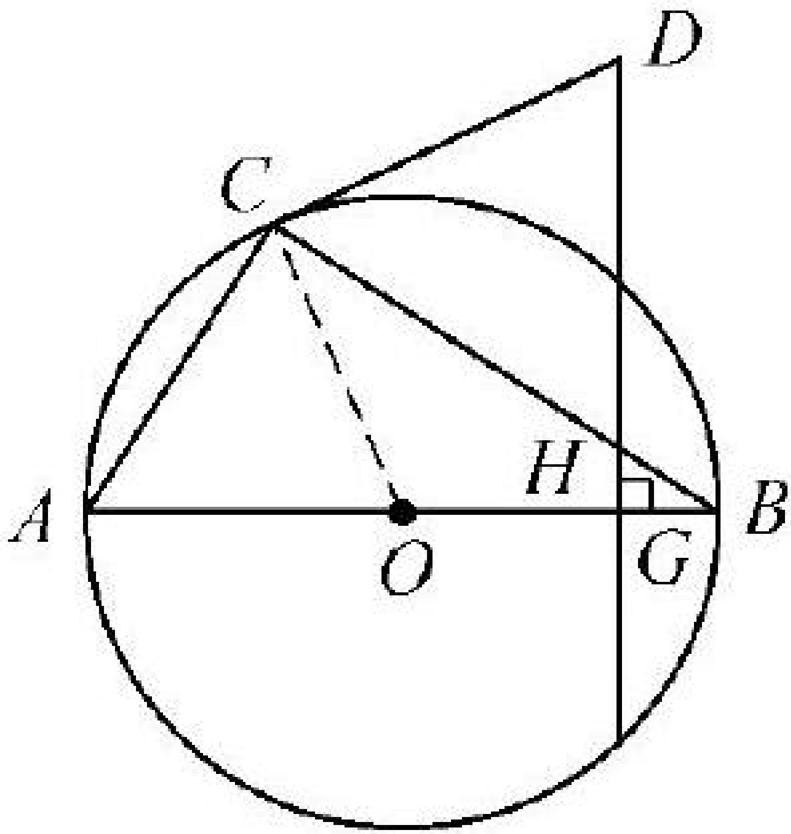


(⊠ **1** ⊠)

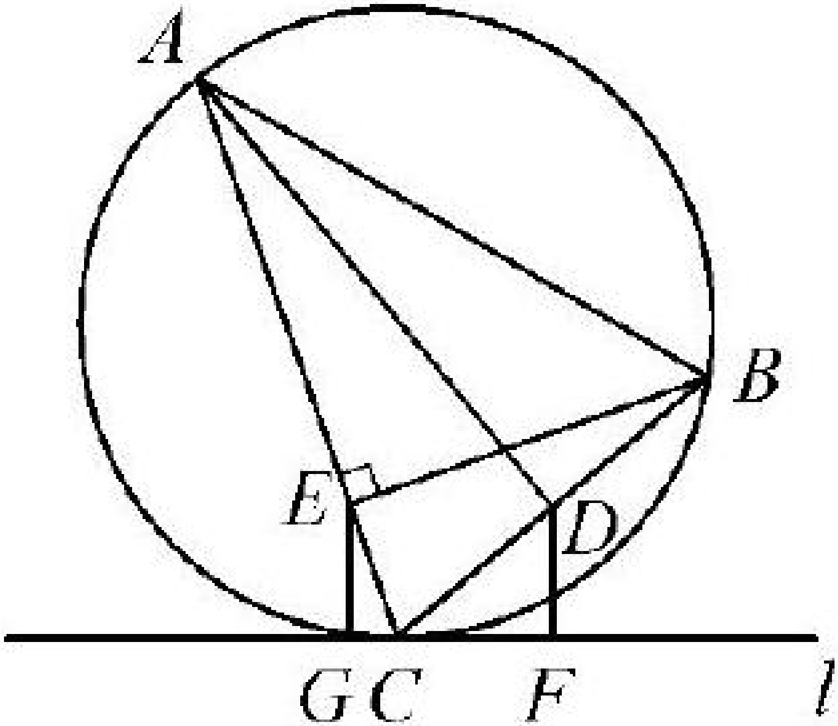
$DD_2 - 2D_1D_2 = AB + AC - BD_1 - CD_2 - D_1D_2 = AB + AC - BC = c + b -$ a. ⊠⊠, $AK = \frac{c+b-a}{2}$.

2. ⊠⊠⊠. ⊠ $AB = c, AC = b, AM = m, BC = a, \triangle ABM$ $\triangle ACM$ ⊠⊠⊠⊠⊠⊠ ⊠⊠ r_B r_C, ⊠⊠⊠ $S_{\triangle ABM} = S_{\triangle ACM}$, ⊠⊠⊠⊠ $S_{\triangle ABM} = \frac{r_B}{2}\left(c + \frac{a}{2} + m\right), S_{\triangle ACM} = \frac{r_C}{2}\left(b + \frac{a}{2} + m\right)$, ⊠ $r_B\left(c + \frac{a}{2} + m\right) = r_C\left(b + \frac{a}{2} + m\right)$. ⊠ $r_B = 2r_C$, ⊠ $2c + a + 2m = b + \frac{a}{2} + m$ ⊠⊠⊠ $2c + \frac{a}{2} + m = b$ ⊠⊠ $\frac{a}{2} + m > b$ ⊠⊠⊠ $c < 0$ ⊠⊠⊠.

3. ⊠⊠ OC, ⊠ $\angle B = \angle OCB$. ⊠⊠ $DC = DH$, ⊠ $\angle DCH = \angle DHC$. ⊠ $HG \perp AB$ ⊠⊠ G, ⊠ $\angle OCD = \angle OCB + \angle HCD = \angle B + \angle DHC = \angle B + \angle GHB = 90^\circ$ ⊠⊠ C ⊠⊠ O ⊠⊠⊠ CD ⊠ $\odot O$ ⊠⊠⊠⊠



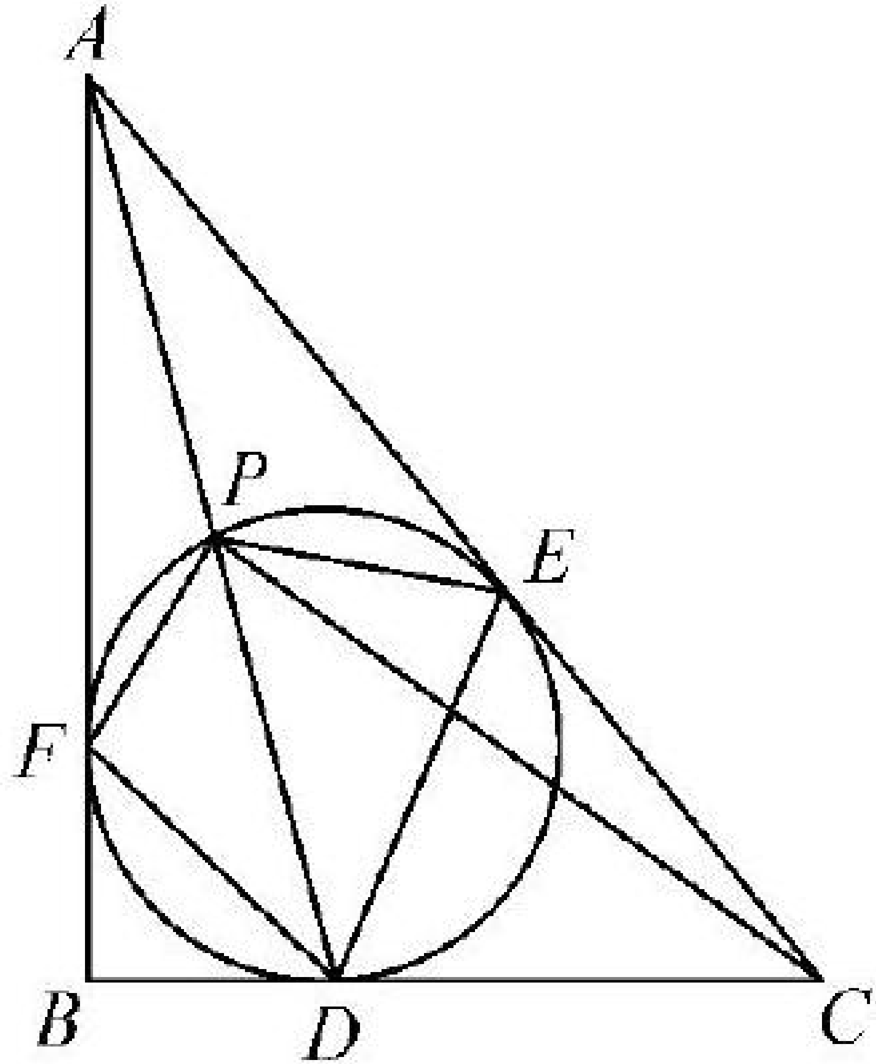
(⊠3⊠)



(⊠4⊠)

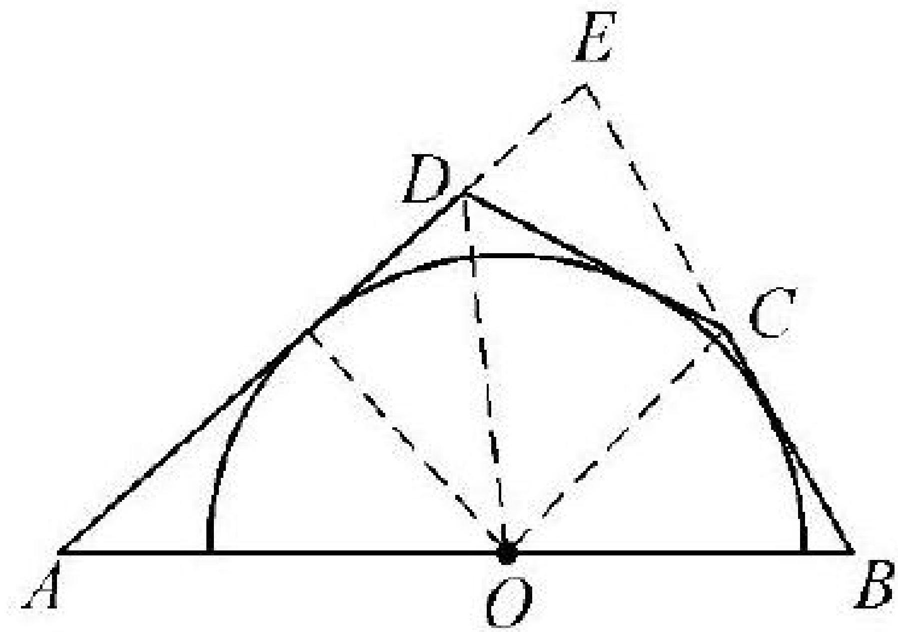
4. ⊠⊠⊠⊠ l ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠ C, ⊠ $\angle FCD = \angle EAB$, ⊠ $\angle AEB = \angle CFD$ ⊠⊠⊠ $\triangle AEB \backsim \triangle CFD$ ⊠⊠⊠ $DF = BE \cdot \frac{CD}{AB}$ ⊠⊠⊠ $EG = AD \cdot \frac{CE}{AB}$. ⊠ $\triangle ACD \backsim \triangle BCE$, ⊠⊠ $\frac{AD}{BE} = \frac{CD}{CE}$, ⊠ $BE \cdot CD = AD \cdot CE$, ⊠⊠ $DF = EG$.

5. (1) ⊠⊠ BF BD ⊠⊠⊠⊠⊠, ⊠ $BF = BD$, ⊠ $\angle B = 90^\circ$ ⊠⊠⊠ $\angle BFD = \angle BDF = 45^\circ$ ⊠⊠ $\angle FPD = \angle FDB, FP \perp PC$ ⊠⊠⊠ $\angle FPD = \angle DPC = 45^\circ$. ⊠ $\angle PFD = \angle PDC$ ⊠⊠⊠ $\triangle PFD \backsim \triangle PDC$ ⊠⊠2⊠⊠⊠ $\triangle AFP \backsim \triangle ADF, \triangle AEP \backsim \triangle ADE$ ⊠⊠ $\frac{PF}{FD} = \frac{AP}{AF}, \frac{PE}{DE} = \frac{AP}{AE}$ ⊠⊠ $AE = AF$ ⊠⊠ ⊠ $\frac{PF}{FD} = \frac{PE}{DE}$. ⊠⊠ $\triangle PFD \backsim \triangle PDC$, ⊠ $\frac{PF}{FD} = \frac{PD}{DC}$, ⊠⊠ $\frac{PD}{DC} = \frac{PE}{DE}$ ⊠⊠ $PE \cdot DC = PD \cdot DE$ ⊠



(☒5 ☒)

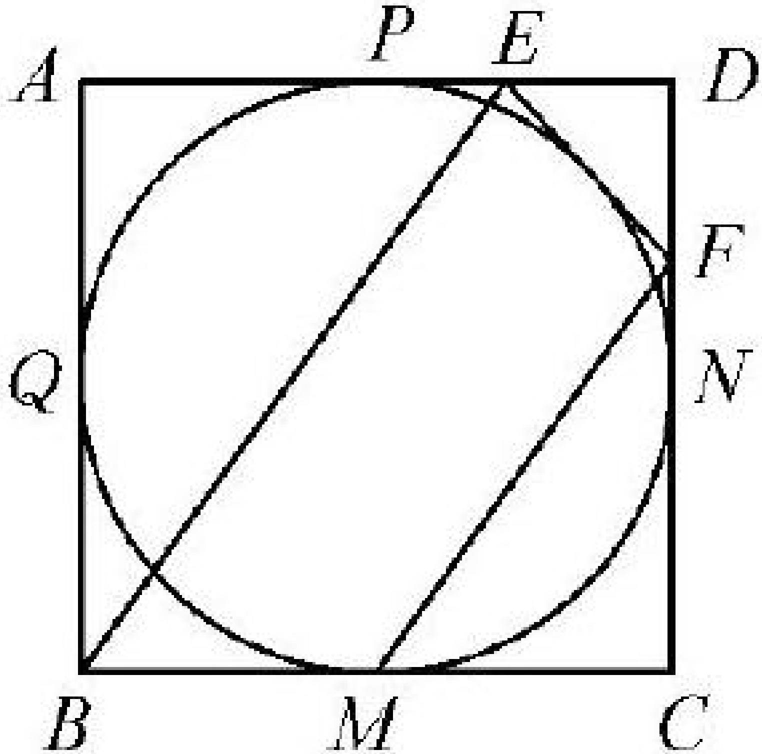
6. ☒ AD BC ☒☒☒ E, ☒ $\odot O$ ☒ $\triangle ECD$ ☒☒☒☒☒☒ $ED = a, EC = b, CD = c, \odot O$ ☒☒☒☒ r, ☒☒ OD OC, ☒ $S_{\triangle EDC} = \frac{r}{2}(a + b - c)$, ☒ $S_{\triangle EAB} = \frac{r}{2}(EA + EB)$, ☒☒ $ABCD$ ☒☒☒☒☒☒☒☒☒ $\triangle ECD \backsim \triangle EAB$ ☒☒ $\frac{EA}{EC} = \frac{EB}{ED} = \frac{AB}{CD}$ ☒☒



(⊠6 ⊠)
$\frac{EA}{b} = \frac{EB}{a} = \frac{AB}{c} = k$ (⊠⊠), ⊠ $EA = kb, EB = ka, AB = kc$. ⊠ $\frac{S_{\triangle EAB}}{S_{\triangle ECD}} = k^2$, ⊠⊠ $\frac{ka+kb}{a+b-c} = k^2$, ⊠⊠ $a + b = k(a + b - c) = EA + EB - AB$, ⊠ $AB = EA - b + EB - a = EA - ED + EB - EC = AD + BC$.

7. ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠ $AB = c, BC = a, CA = b$ ⊠⊠ $c^2 = a^2 + b^2, 10 = \frac{b+c-a}{2}, 3 = \frac{a+c-b}{2}$ ⊠⊠⊠ $a = 5, b = 12, c = 13$ ⊠⊠⊠ $r = \frac{a+b-c}{2} = 2$.

8. ⊠⊠⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠ $1, PE = x, FN = y$, ⊠ $EF = x + y, ED = \frac{1}{2} - x, DF = \frac{1}{2} - y$, ⊠

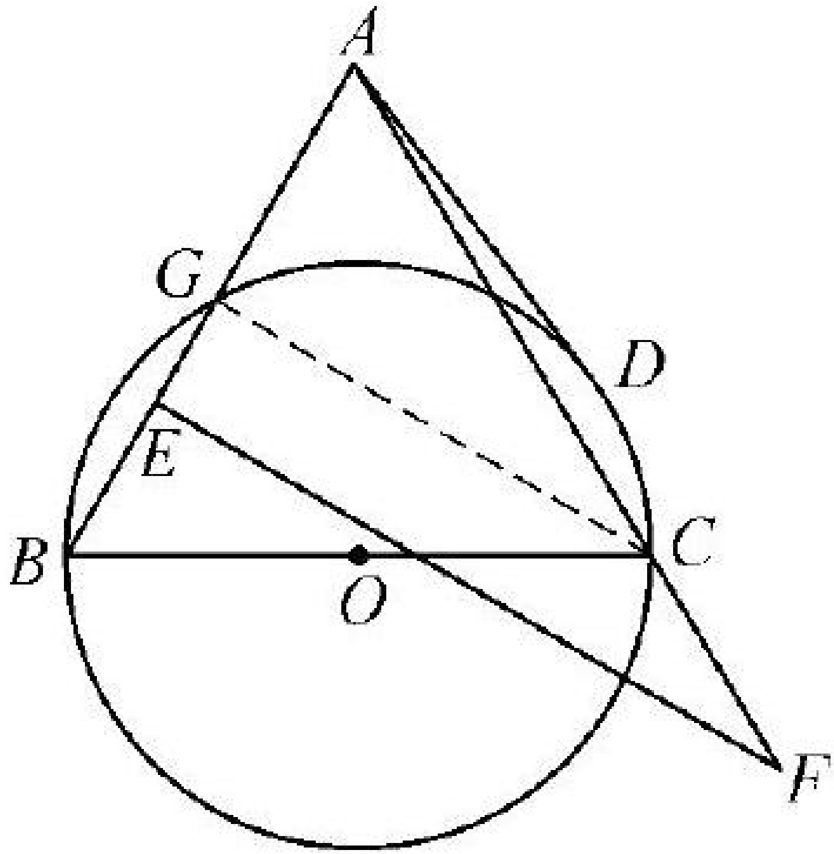


(☒8☒)

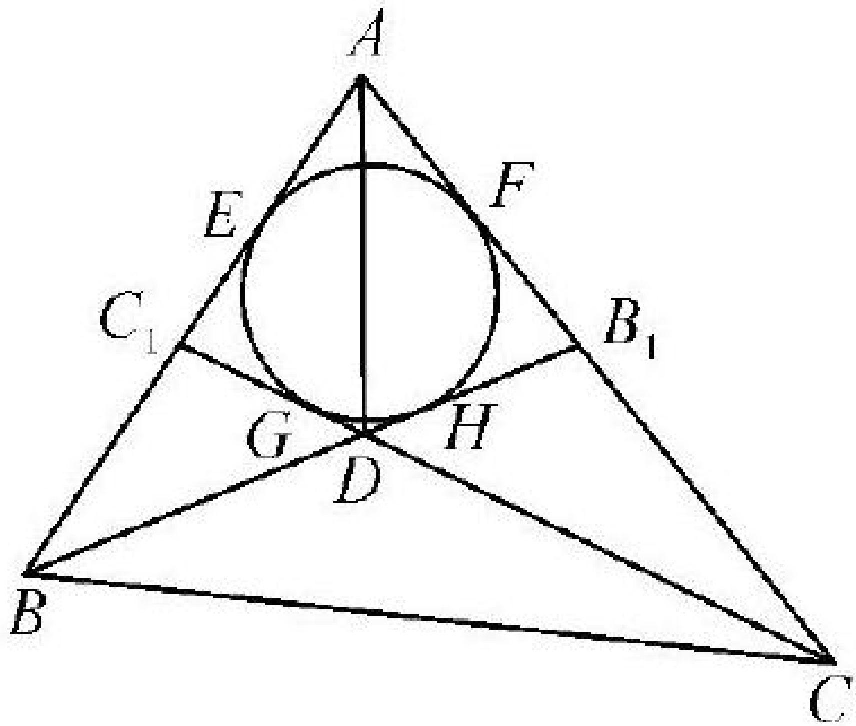
Rt△DEF ☒☒ $(x+y)^2=\left(\frac{1}{2}-x\right)^2+\left(\frac{1}{2}-y\right)^2$, ☒☒☒☒☒☒ $\left(x+\frac{1}{2}\right)\left(y+\frac{1}{2}\right)=\frac{1}{2}$, ☒ $AE\cdot CF=\frac{1}{2}$. ☒ $AB=1, MC=\frac{1}{2}$, ☒ $\frac{AE}{AB}=\frac{MC}{CF}$, ☒☒ $\angle A=\angle C=90^\circ$, ☒ $\triangle ABE\backsim\triangle CFM$, ☒☒ $\angle FMC=\angle AEB=\angle EBC$, ☒ $BE//MF$ ☒

9. ☒ $\odot O$ ☒ AB ☒☒ G, ☒☒ CG, ☒ $CG\perp AB$, ☒ $EF\perp AB$, ☒ $CG//EF$, ☒☒ $\frac{AG}{AE}=\frac{AC}{AF}$, ☒ $AC\cdot AE=AG\cdot AF$, ☒☒ $\frac{AB}{AF}=\frac{AE}{AC}$, ☒ $AE\cdot AF=AB\cdot AC$ ☒☒☒ $AE^2=AG\cdot AB$ ☒☒☒ AD ☒ $\odot O$ ☒☒ D, ☒ $AD^2=AG\cdot AB$, ☒☒ $AE^2=AD^2$ ☒☒ $AE=AD$ ☒

10. ☒ $\triangle ABD$ ☒☒☒☒☒ AD ☒☒ X,



⊠⊠9 ⊠⊠

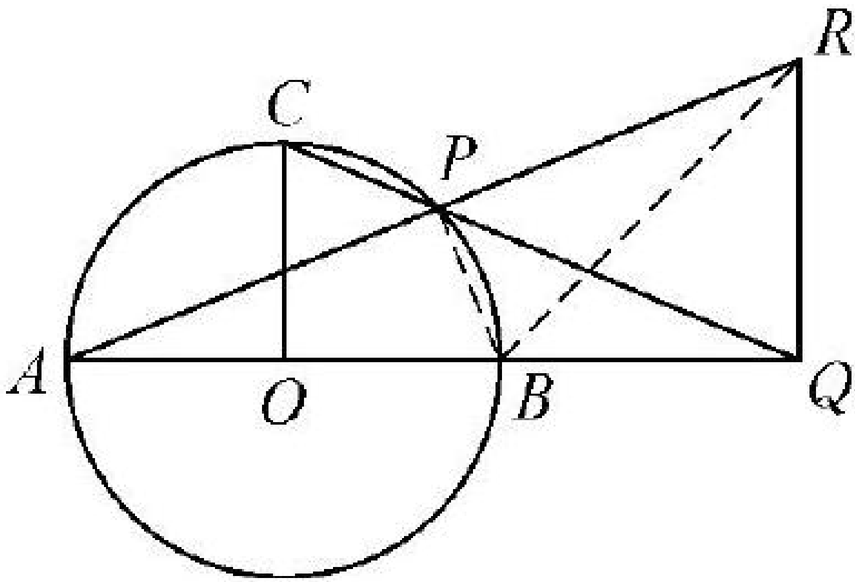


(⊠10 ⊠)
$\triangle ACD$ ⊠⊠⊠⊠⊠ AD ⊠⊠ Y, ⊠ $AX = \frac{1}{2}(AB+ AD-BD), AY = \frac{1}{2}(AC+AD-CD)$. ⊠⊠⊠⊠ AB_1DC_1 ⊠⊠⊠⊠, ⊠ $AC_1 + DB_1 = C_1D + AB_1 \cdots$ (1). ⊠⊠⊠⊠⊠ AB_1DC_1 ⊠⊠⊠⊠⊠ AB ⊠⊠ E, ⊠ AC ⊠⊠ F, ⊠ C_1D ⊠⊠ G, ⊠ DB_1 ⊠⊠ H, ⊠ $AE = AF, AE = AB - BE = AB - BH = AB-(BB_1 - B_1H) = AB - BB_1 + B_1F = AB - BB_1 + AB_1 - AF, AF = AC-CF = AC - CG = AC-(CC_1 - C_1G) = AC - CC_1 + C_1G = AC - CC_1+AC_1 - AE$ ⊠⊠⊠ $AB - BB_1 + AB_1 = AC - CC_1 + AC_1 \cdots$ (2). (1) + (2) ⊠ $AB - BD = AC - CD$, ⊠ $AX = AY, X$ ⊠ Y ⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠.

6

1. ⊠⊠ DF ⊠⊠⊠ $D\ C\ E\ F$ ⊠⊠⊠⊠⊠⊠⊠ $\angle BFD = \angle C = 45^\circ$, ⊠ $\angle BAD = 45^\circ$, ⊠⊠ $\angle BFD = \angle BAD$ ⊠⊠⊠ $A\ B\ D\ F$ ⊠⊠⊠⊠⊠⊠⊠ $\angle AFB = \angle ADB = 90^\circ$ ⊠⊠ $AF \perp BE$ ⊠

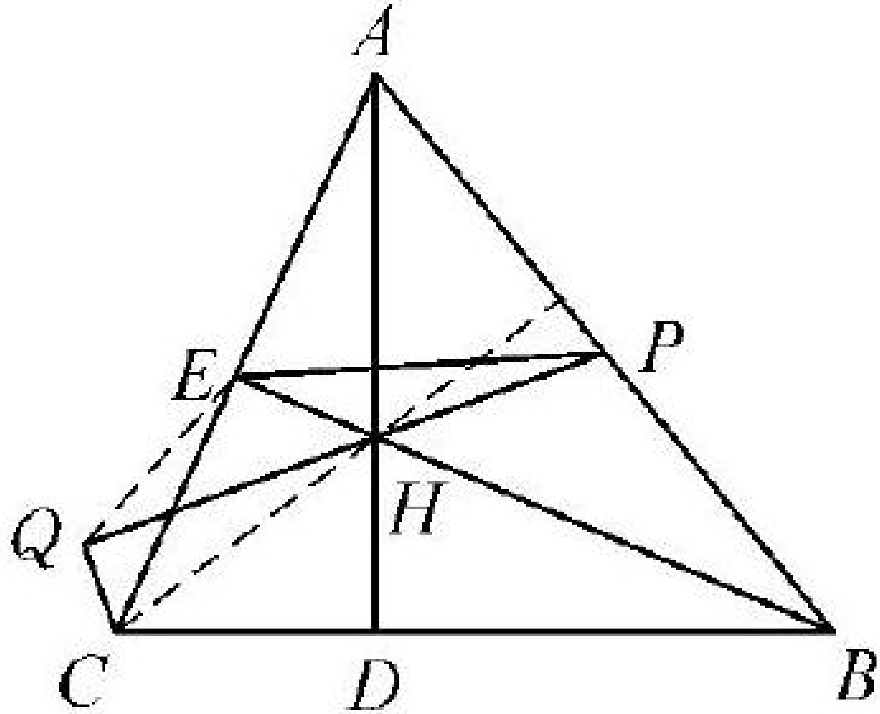
2. ⊠⊠ PB ⊠ BR, ⊠ $\angle APC = 45^\circ, \angle APB = 90^\circ$ ⊠⊠⊠ $\angle BPQ = 45^\circ$ ⊠⊠⊠ $B\ Q\ R\ P$ ⊠⊠⊠



(⊠2 ⊠)

⊠, ⊠⊠ $\angle BRQ = \angle BPQ = 45^\circ$, ⊠⊠⊠⊠ BQR ⊠⊠⊠⊠⊠⊠⊠⊠, $BQ = QR$ ⊠

3. ⊠⊠ $QE\,CH$, ⊠⊠ $AD \perp BC, BE \perp CA$, ⊠⊠ $CH \perp AB$. ⊠ $\angle ABE = \angle ACH$. ⊠⊠ P ⊠ AB ⊠⊠⊠, ⊠⊠ $AP = BP = EP$ ⊠⊠⊠ $\angle ABE = \angle PEB$ ⊠⊠ ⊠ $\angle PEB = \angle ACH \cdots$ (1). ⊠⊠ $CQ \perp PQ, BE \perp CA$ ⊠⊠⊠⊠$C\ H\ E\ Q$ ⊠⊠⊠ ⊠. ⊠⊠ $\angle EQH = \angle ACH \cdots$ (2). ⊠ (1)⊠(2) ⊠⊠$\angle EQH = \angle BEP = \angle PEH$. ⊠ $\angle QPE = \angle EPH$ ⊠⊠⊠



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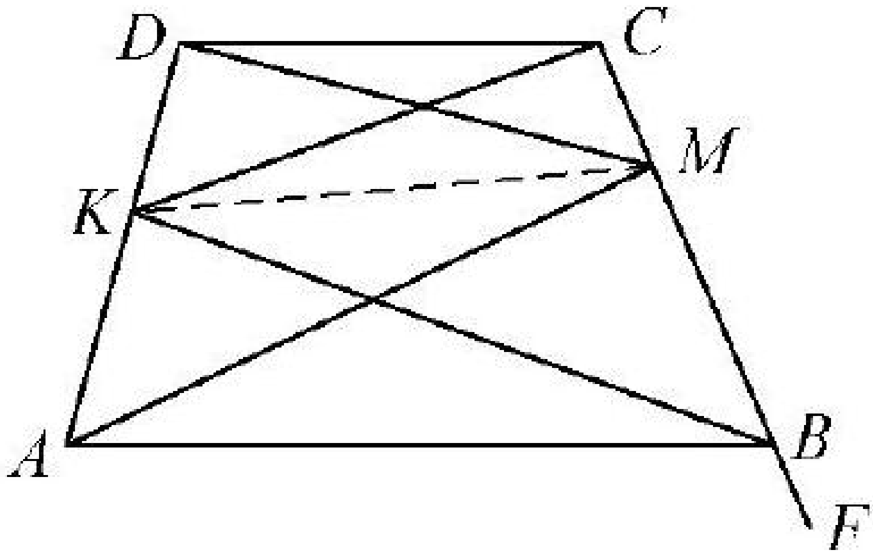
$\triangle EPH \backsim \triangle QPE$. ⊠⊠ $\frac{EP}{HP} = \frac{QP}{EP}$, ⊠ $PE^2 = PH\cdot PQ$.

4. ⊠⊠ KM ⊠⊠⊠ $\angle DAM = \angle CBK$ ⊠⊠⊠ A B M K ⊠⊠⊠⊠⊠⊠ $\angle ABF = \angle AKM$ ⊠⊠⊠⊠ $AB//CD$ ⊠⊠⊠ $\angle ABF = \angle DCM$ ⊠⊠⊠ $\angle AKM = \angle DCM$ ⊠⊠ K M C D ⊠⊠⊠⊠⊠⊠⊠ $\angle DKC = \angle DMC$. ⊠⊠⊠ $\angle AMB = \angle AKB$ ⊠⊠⊠ $\angle CKB = 180^\circ - \angle AKB - \angle DKC =$



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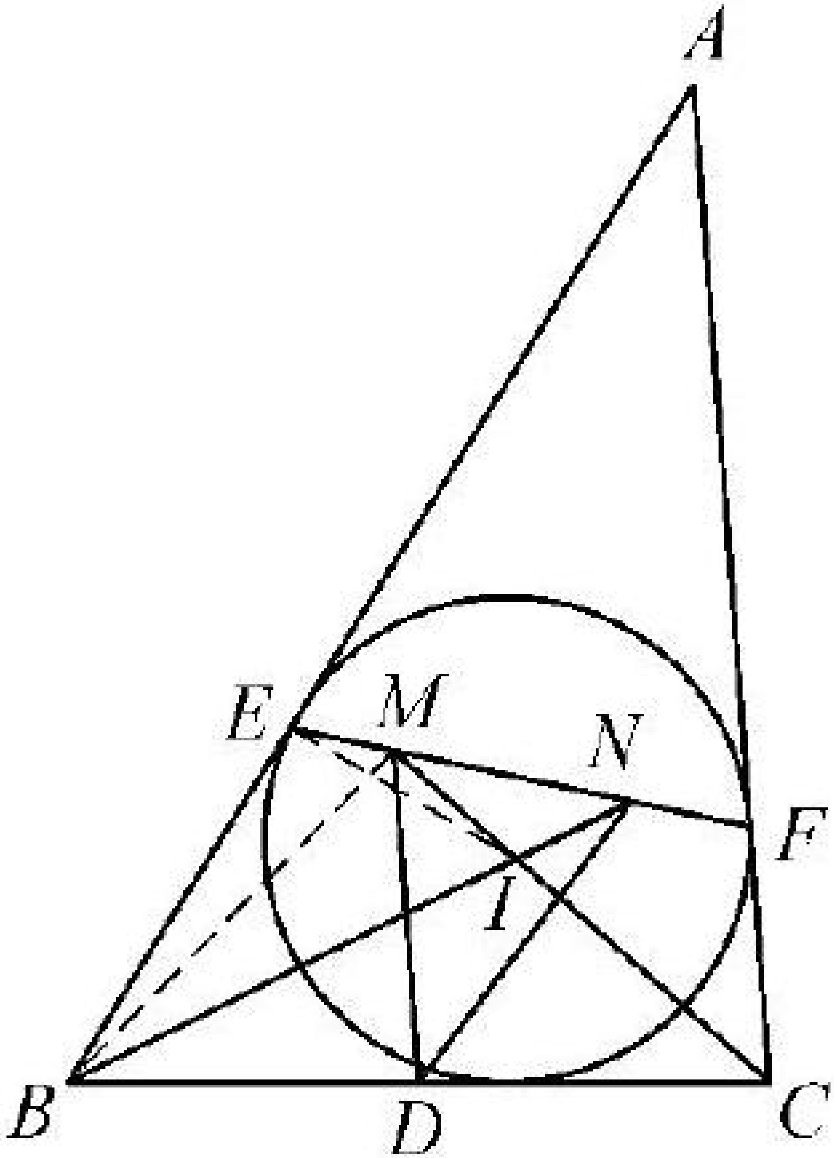

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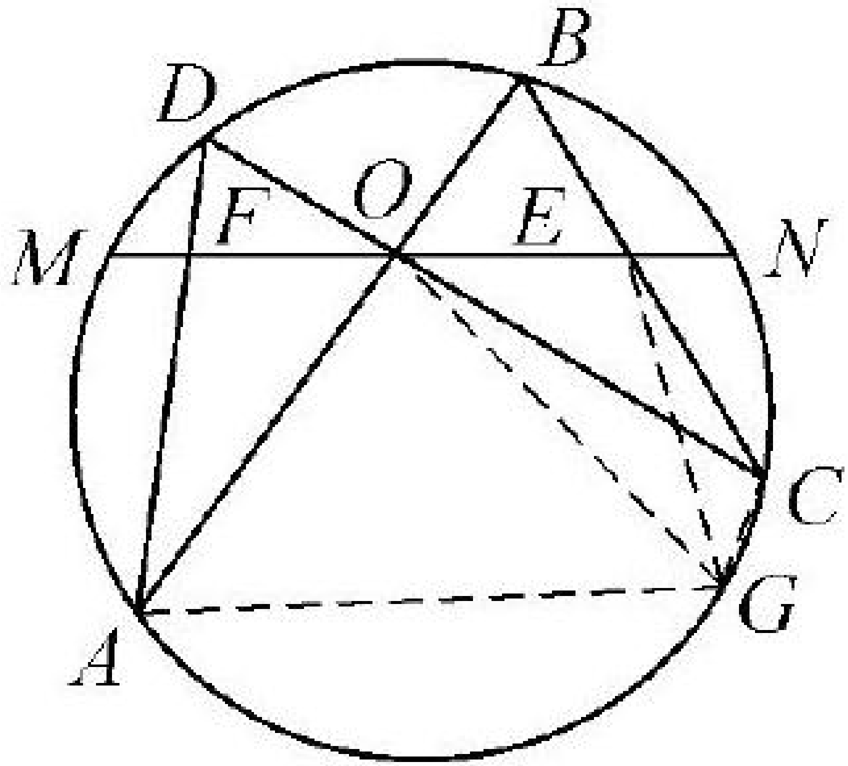
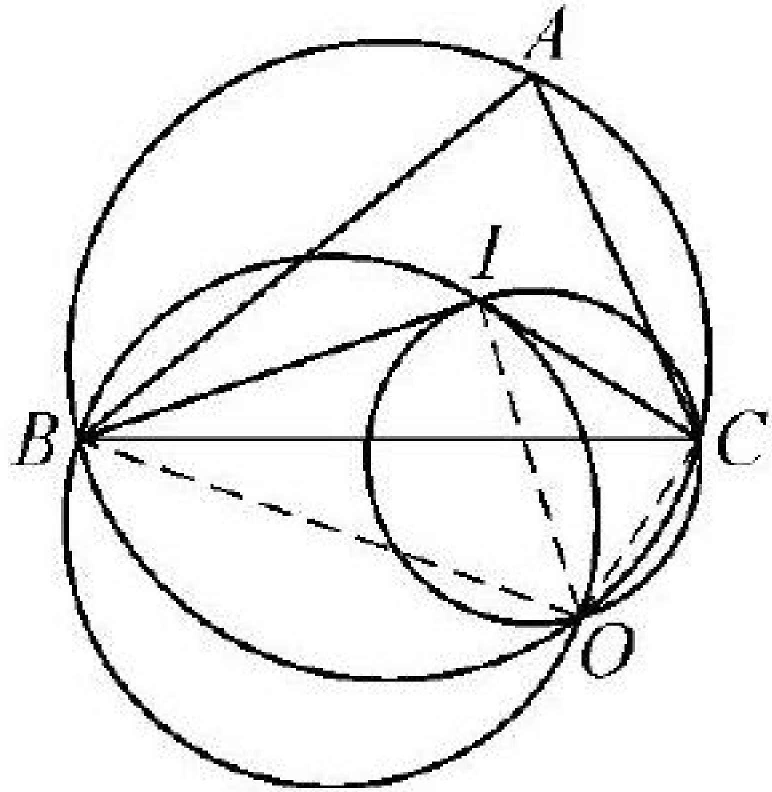


(⊠6 ⊠)

7. ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ I, ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $I, BI\ CI$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠ $BI\ CI$ ⊠⊠ I ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $B\ I\ C$ ⊠⊠⊠⊠⊠⊠ I ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ O ⊠⊠⊠ $BO\ OC\ OI$ ⊠⊠ $\angle CIO = \angle IBO, \angle BIO = \angle ICO$, $\angle ABO + \angle ACO = \angle ABI + \angle IBO + \angle ACI +$ $\angle$ICO $= \angle CIO + \angle BIO + \frac{1}{2}(\angle ABC + \angle ACB) =$

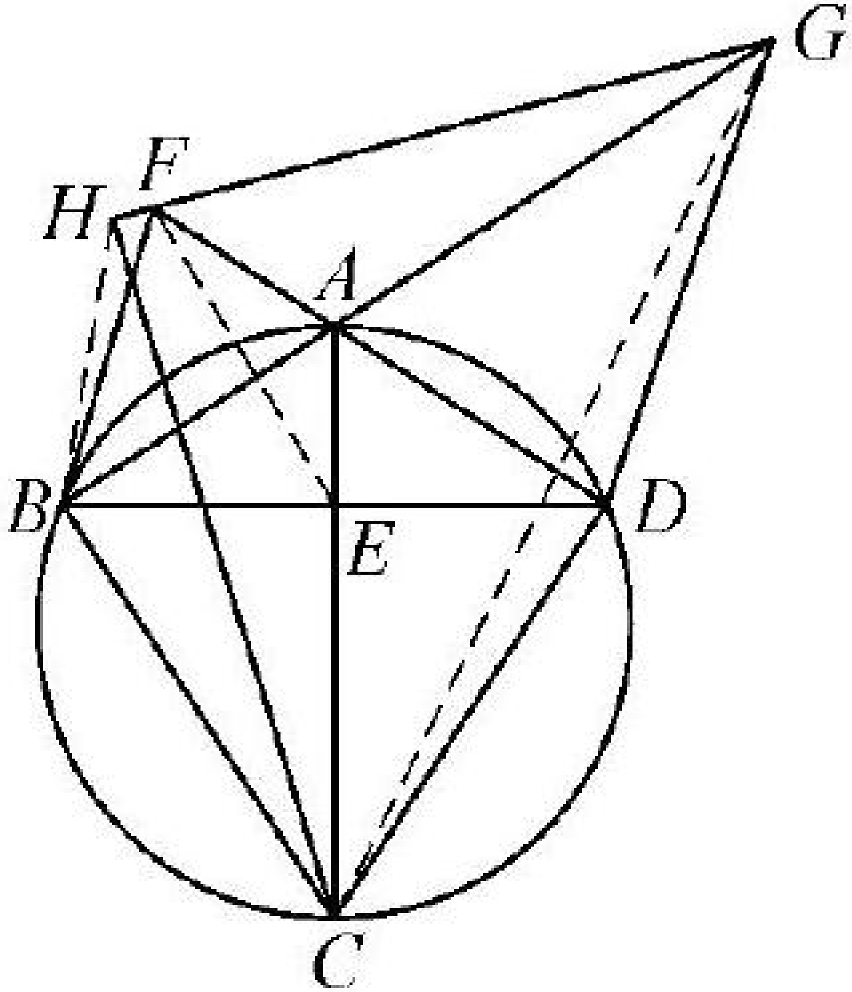


(⊠7 ⊠)

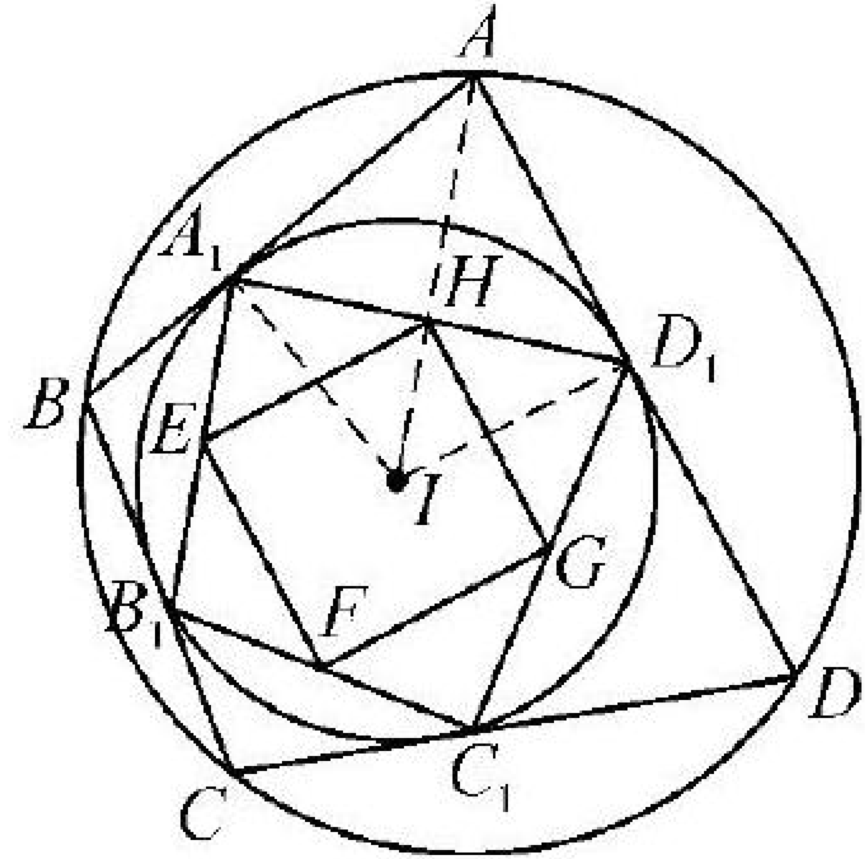
$\angle BIC + \frac{1}{2}(\angle ABC + \angle ACB) = 90^\circ + \frac{1}{2}\angle BAC + \frac{1}{2}(\angle ABC + \angle ACB) = 180^\circ$, ⊠⊠ A B O C ⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠.

8. ⊠⊠ BH EF CG, ⊠⊠ $\triangle BAF \backsim \triangle GAD$, ⊠⊠ $\frac{FA}{AB} = \frac{DA}{AG} \cdots$ (1), ⊠⊠⊠ $\triangle ABE \backsim \triangle ACD$, ⊠⊠ $\frac{AB}{EA} = \frac{AC}{DA} \cdots$ (2), (1) $\times$ (2) ⊠ $\frac{FA}{EA} = \frac{AC}{AG}$. ⊠⊠ $\angle FAE = \angle CAG$ ⊠⊠⊠ $\triangle FAE \backsim \triangle CAG$ ⊠⊠⊠ $\angle FEA = \angle CGA$. ⊠⊠⊠⊠, $\angle CBG = \angle CHG = 90^\circ$ ⊠⊠⊠ B ⊠C G H ⊠⊠⊠⊠⊠⊠ $\angle BHC = \angle BGC$ ⊠⊠⊠ $\angle BHF + \angle BEF = \angle BHC + 90^\circ + \angle BEF = \angle BGC + 90^\circ + \angle BEF = \angle FEA + 90^\circ + \angle BEF = 180^\circ$ ⊠⊠⊠⊠B E F H ⊠⊠⊠⊠⊠

9. ⊠⊠⊠⊠, ⊠ I ⊠⊠⊠⊠ $ABCD$ ⊠⊠⊠⊠⊠⊠⊠. ⊠⊠ H ⊠ D_1A_1 ⊠⊠⊠⊠⊠ AA_1 ⊠ AD_1 ⊠⊠⊠ A ⊠⊠⊠⊠ I ⊠⊠⊠⊠⊠ H ⊠ AI ⊠⊠⊠ $AI \perp A_1D_1$. ⊠ $ID_1 \perp AD_1$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $IH \cdot IA = ID_1^2 = r^2$, ⊠⊠ r ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, E ⊠ BI ⊠⊠⊠ $IE \cdot IB = r^2$ ⊠⊠⊠⊠$IE \cdot IB = IH \cdot IA$ ⊠⊠⊠ A H E B ⊠⊠⊠⊠⊠⊠⊠⊠ $\angle EHI = \angle ABE$ ⊠⊠⊠⊠⊠⊠⊠ $\angle IHG = \angle ADG$ $\angle IFE = \angle CBE$

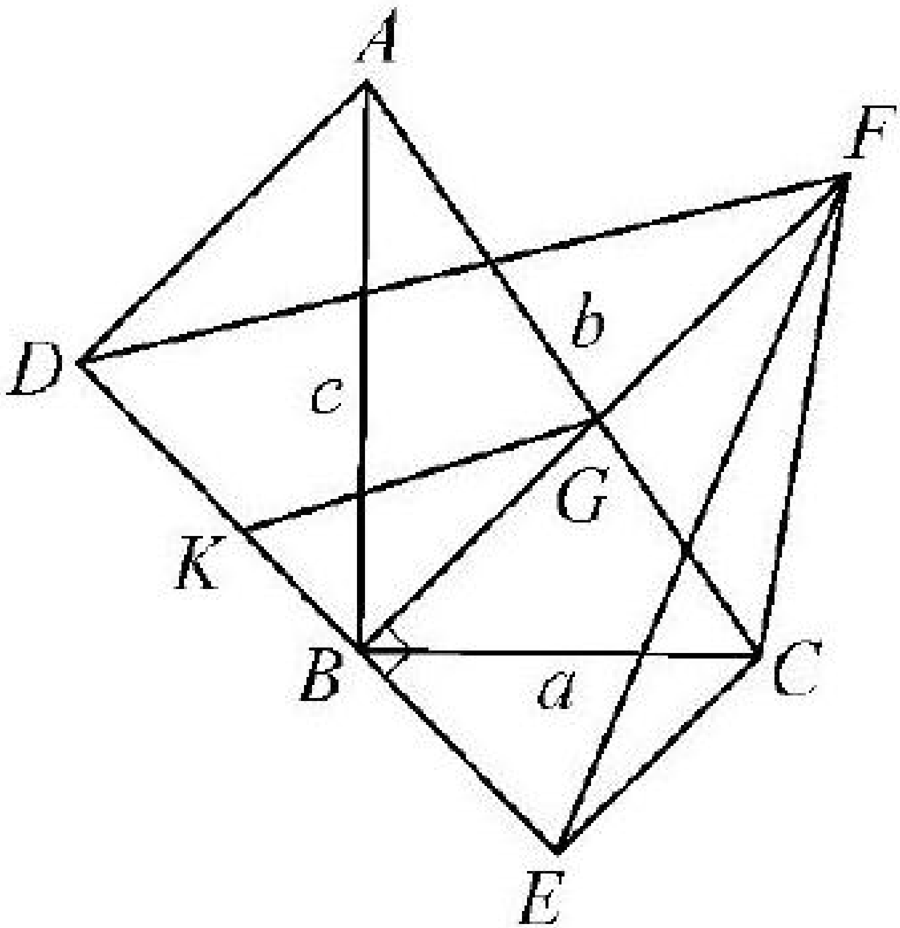


(⊠8 ⊠)



(⊠9 ⊠)
$\angle IFG = \angle CDG$. ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\angle EHG+ \angle EFG = \angle ABC + \angle ADC$ ⊠⊠⊠⊠ $A\ B\ C\ D$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ E ⊠$F\ G\ H$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠ $EFGH$ ⊠⊠⊠ ⊠⊠⊠⊠⊠⊠ $A\ B\ C\ D$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

10. ⊠⊠⊠ AB ⊠⊠⊠ c, ⊠⊠ BC ⊠⊠⊠ a, ⊠⊠ AC ⊠⊠⊠ b ⊠⊠ AC ⊠ BF ⊠⊠⊠ ⊠ G ⊠⊠⊠ $\angle EBC = \angle DBA = 45^\circ$ ⊠⊠ $D\ B\ E$ ⊠⊠⊠⊠⊠⊠⊠ $\angle ABC = \angle AFC = 90^\circ$ ⊠⊠⊠ $A\ F\ C\ B$ ⊠⊠⊠⊠, ⊠⊠ $\angle FBC = \angle FAC = 45^\circ$. ⊠⊠⊠ $\angle FBE = \angle FBC + \angle CBE = 90^\circ$ ⊠⊠⊠ $FB \perp DE$ ⊠⊠⊠⊠⊠⊠ $BF\ AD$ ⊠ CE ⊠⊠⊠ ⊠. (1) ⊠⊠ $BF > BG$ ⊠⊠⊠ $S_{\triangle DBF} > S_{\triangle ABG}$ ⊠ $S_{\triangle EBF} > S_{\triangle CBG}$ ⊠⊠ $S_{\triangle DEF} > S_{\triangle ABC}$, ⊠⊠⊠⊠ DEF ⊠⊠⊠⊠ ABC ⊠⊠⊠⊠⊠⊠⊠⊠⊠

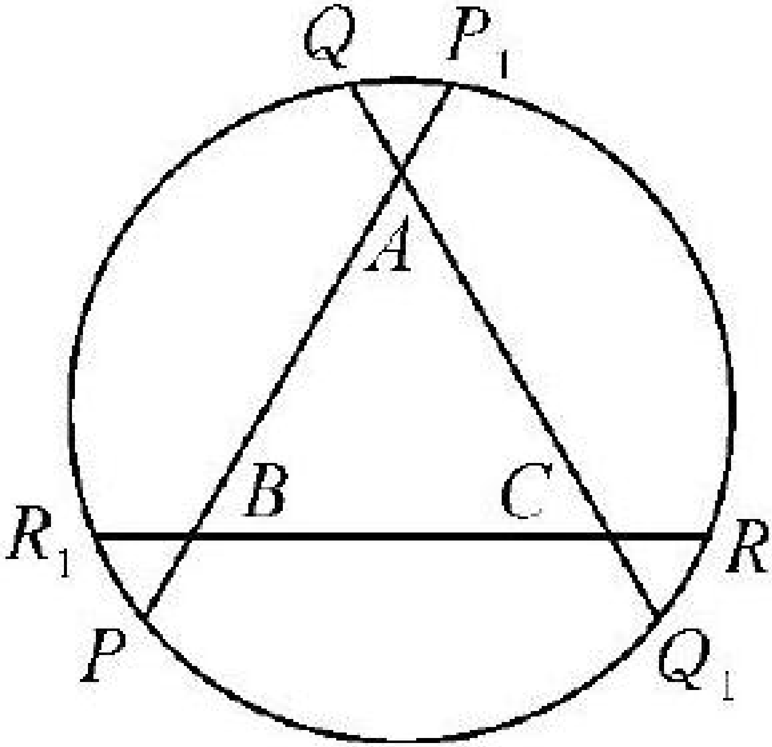


(⊠10 ⊠)

1. (2) ⊠ K ⊠⊠⊠⊠ ABC ⊠⊠⊠⊠⊠ DE ⊠⊠⊠⊠⊠, ⊠ FK ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $FG = GK$ ⊠⊠ K ⊠ B ⊠⊠⊠⊠ $FB = FG + GB = GK + GB = 2GB$ ⊠⊠ B ⊠ K ⊠⊠⊠⊠⊠ $\angle KBG = 90^\circ, FB = FG + GB = GK + GB > 2GB$ ⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $FB \geqslant 2GB$ ⊠⊠ $S_{\triangle DEF} \geqslant 2S_{\triangle ABC}$ ⊠

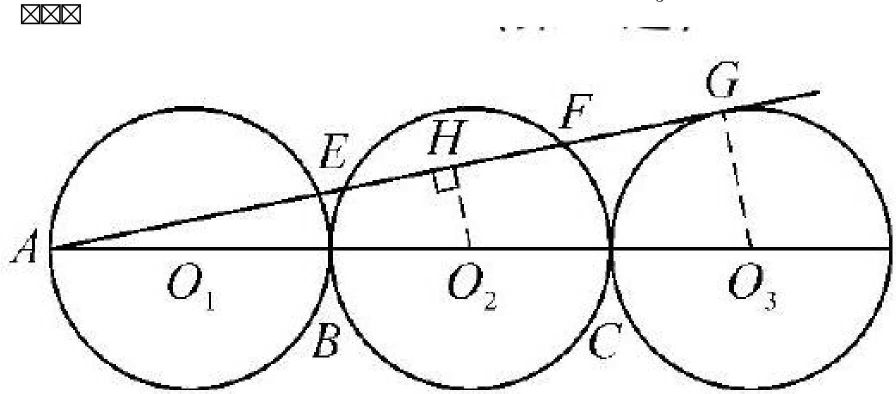
7

1. ⊠ $AP_1 = BR_1 = CQ_1 = a, AQ = BP = CR = b, AB = x, BC = y, CA = z$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\begin{cases} a(z+b) = b(y+a), \\ a(x+b) = b(z+a), \\ a(y+b) = b(x+a), \end{cases}$ ⊠⊠ $\begin{cases} az = by, \\ ax = bz, \\ ay = bx, \end{cases}$ ⊠⊠⊠⊠⊠⊠ $a = b$ ⊠⊠⊠ $x = y = z$ ⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠



(☒1 ☒)

2. ☒ $AE = x, EF = y$, ☒ $O_2H \perp EF$ ☒☒ H, ☒☒ O_3G, ☒☒ $AG^2 = AO_3^2 - O_3G^2$, ☒ $AG = 2\sqrt{6}$, ☒☒☒ $O_2H//O_3G$, ☒☒ $\frac{AH}{AG} = \frac{AO_2}{AO_3} = \frac{3}{5}, AH = \frac{3}{5}AG$ ☒☒☒

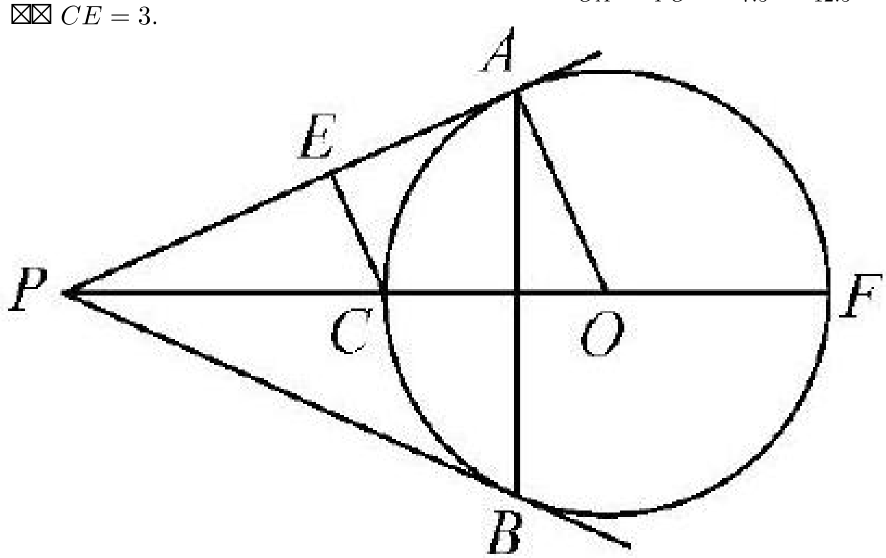


(☒2 ☒)

$AE \cdot AF = AB \cdot AC = 8$, ☒☒ $\left\{ \begin{array}{l} x + \frac{y}{2} = \frac{6\sqrt{6}}{5}, \\ x(x+y) = 8, \end{array} \right.$ ☒☒ $y = \frac{8}{5}$, ☒ EF ☒☒☒☒

$\frac{8}{5}$.

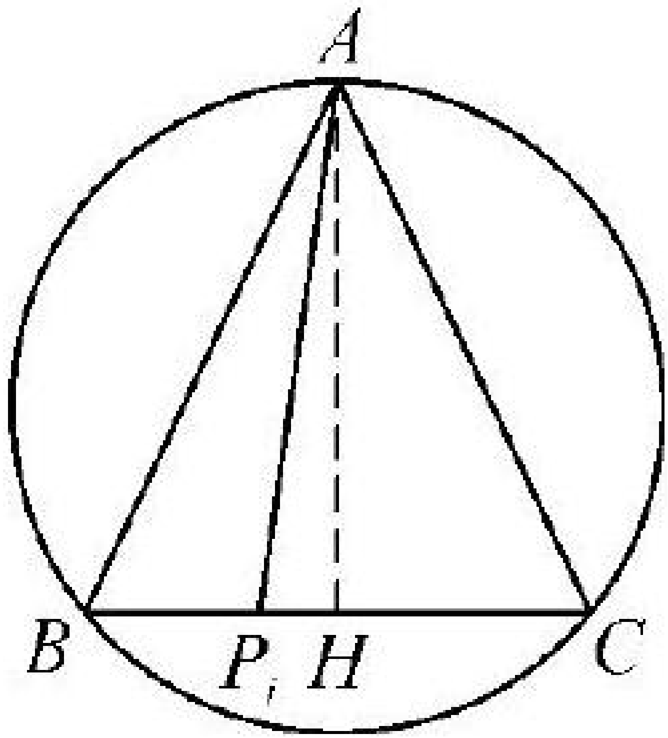
3. ⊠⊠ PO ⊠ $\odot O$ ⊠⊠ F, ⊠⊠ $\odot O$ ⊠⊠⊠⊠ r, ⊠⊠⊠⊠⊠⊠⊠ $PA^2 = PC \cdot PF$, ⊠ $10^2 = 5(5+2r)$ ⊠⊠⊠ $r = 7.5$ ⊠⊠ $CE//OA$ ⊠⊠ $\frac{CE}{OA} = \frac{PC}{PO}$, ⊠ $\frac{CE}{7.5} = \frac{5}{12.5}$, ⊠⊠ $CE = 3$.



(⊠3⊠)

4. ⊠ BC ⊠⊠⊠⊠ AH ⊠ BC ⊠⊠ H, ⊠ $BH = HC$. ⊠ BC ⊠⊠⊠ P_i, ⊠ $BP_i \cdot P_iC = (BH - P_iH)(CH + P_iH) = (BH - P_iH)(BH + P_iH) = BH^2 - P_iH^2$, ⊠ $AP_i^2 = P_iH^2 + AH^2$, ⊠⊠ $m_i = AP_i^2 + BP_i \cdot P_iC = BH^2 + AH^2 = 4$ ⊠⊠⊠ $m_1 + m_2 + \cdots + m_{100} = 400$ ⊠

5. ⊠ $AH \cdot AJ = AG \cdot AF$, ⊠⊠ $AH = 3$, ⊠⊠ $BJ =$

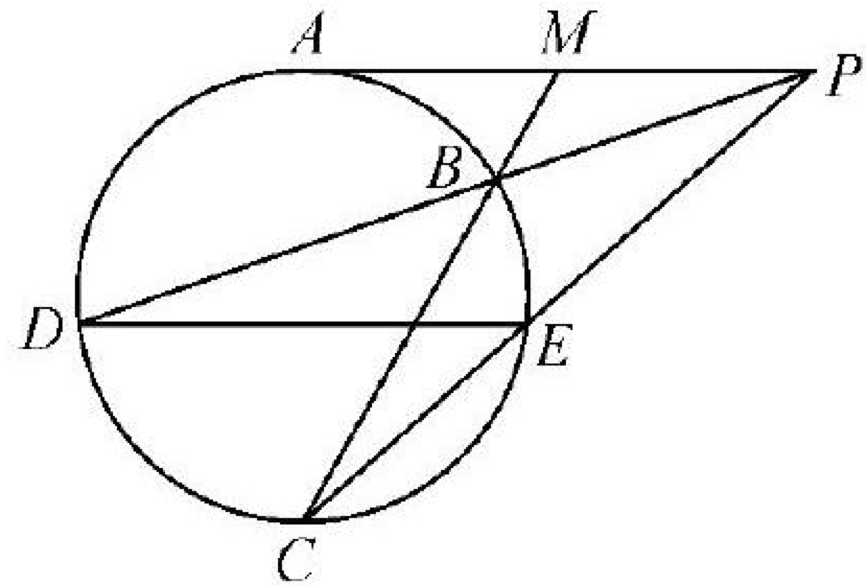


(⊠4 ⊠)

6 , ⊠ $BJ \cdot BH = BD \cdot BE, CE \cdot CD = CF \cdot CG$ ⊠⊠ $BD+ DE + EC = 16$ ⊠⊠⊠ $DE = 2\sqrt{22}$ ⊠

6. ⊠⊠⊠⊠⊠⊠⊠ $MA^2 = MB \cdot MC$, ⊠ $MA = MP$ ⊠⊠⊠ $MP^2 = MB \cdot MC$ ⊠⊠ $\frac{MP}{MB} = \frac{MC}{MP}$ ⊠⊠ $\angle BMP = \angle PMC$ ⊠⊠⊠ $\triangle PMB \backsim \triangle CMP$ ⊠⊠⊠ $\angle MPB = \angle MCP$ ⊠⊠⊠⊠ $\angle MCP = \angle PDE$ ⊠⊠⊠ $\angle MPB = \angle PDE$ ⊠⊠⊠ $DE//PA$ ⊠

7. ⊠ A ⊠⊠⊠, AB ⊠⊠⊠⊠ $\odot A$, ⊠⊠ DA



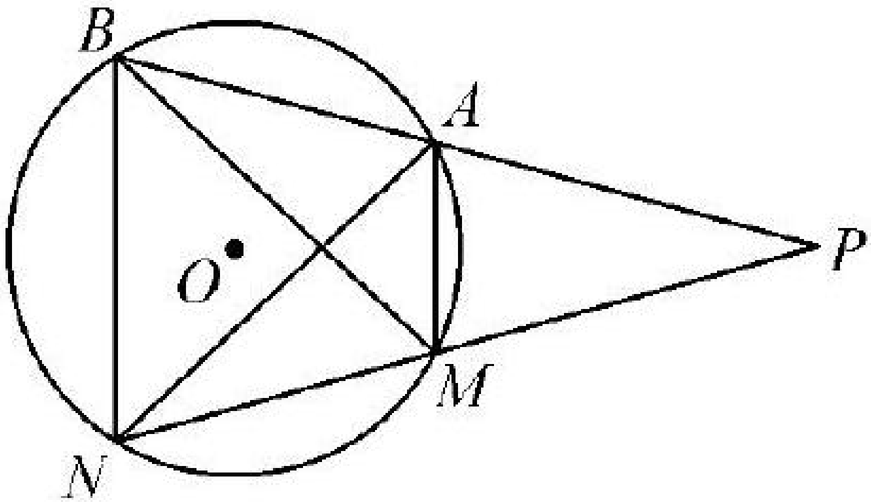
(☒6 ☒)

☒ $\odot A$ ☒☒ E F, ☒ $BD\cdot CD = DE\cdot DF = (AD+R)(AD-R) = AD^2-R^2 = AD^2-AB^2$ (☒☒ $AB=AC=R$) ☒

☒☒☒☒ D ☒☒☒☒☒☒ ABC ☒☒☒ BC ☒☒☒☒☒, ☒ $AB^2-AD^2 = BD\cdot CD$ ☒

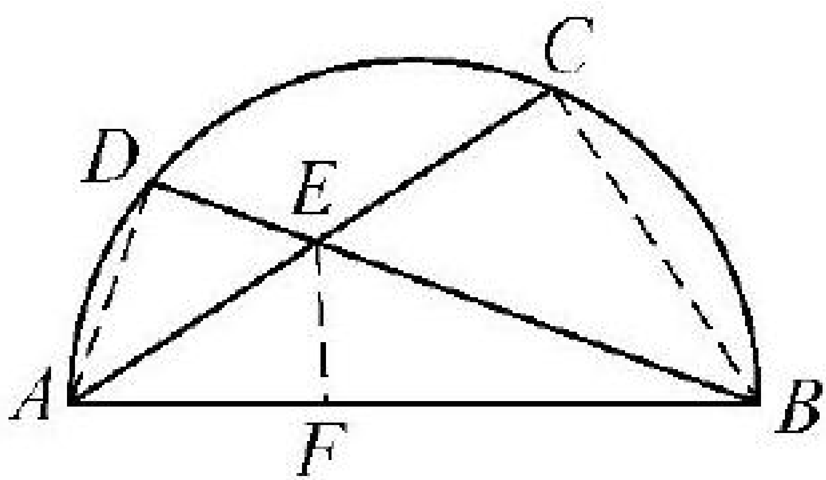
8. ☒ $IE=ED=x$, ☒ $BD=CD=ID=2x$ (☒☒☒☒☒), ☒ $AE=y$, ☒☒☒☒☒☒: $BD^2-DE^2=BE\cdot EC$, ☒ $BE\cdot EC=AE\cdot ED$ ☒☒☒ $BD^2-DE^2=AE\cdot ED$ ☒☒☒ $y=3x$ ☒☒ $AI=2IE$ ☒☒☒☒ I ☒☒☒☒☒☒ $\frac{AB}{BE}=\frac{AI}{IE}=\frac{AC}{EC}$ ☒☒☒ $\frac{AB+AC}{BC}=\frac{AI}{IE}=2$, ☒☒ $BC=\frac{7}{2}$.

9. ☒ $\triangle PAM \backsim \triangle PNB$, ☒ $\frac{AM}{NB}=\frac{PA}{PN}$, ☒ $\triangle PMB \backsim \triangle PAN$, ☒ $\frac{BM}{NA}=\frac{PM}{PA}$, ☒☒ $\frac{AM\cdot BM}{NB\cdot NA}=\frac{PA\cdot PM}{PN\cdot PA}=\frac{PM}{PN}$, ☒☒ $AM\cdot BM\cdot PN = AN\cdot$



(⊠9 ⊠)
$BN \cdot PM$.

10. ⊠ $EF \perp AB$ ⊠⊠ F, ⊠⊠ AD_1 BC, ⊠⊠⊠ AB ⊠⊠⊠, ⊠ $\angle ADB = \angle EFA = 90^\circ$, ⊠⊠ A D ⊠E F ⊠⊠⊠⊠⊠⊠ $BE \cdot BD = BF \cdot BA$ ⊠⊠⊠ C B ⊠F E ⊠⊠⊠ ⊠⊠⊠ $AE \cdot AC = AF \cdot AB$ ⊠⊠⊠ $AE\cdot$

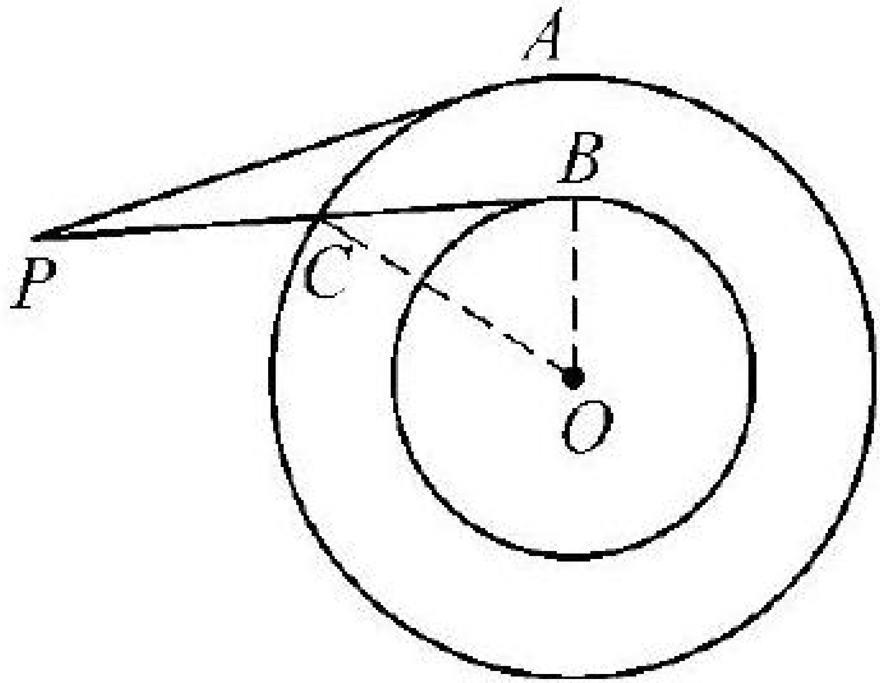


(⊠10 ⊠)
$AC + BE \cdot BD = AF \cdot AB + BF \cdot BA = AB \cdot (AF + FB) = AB^2$.

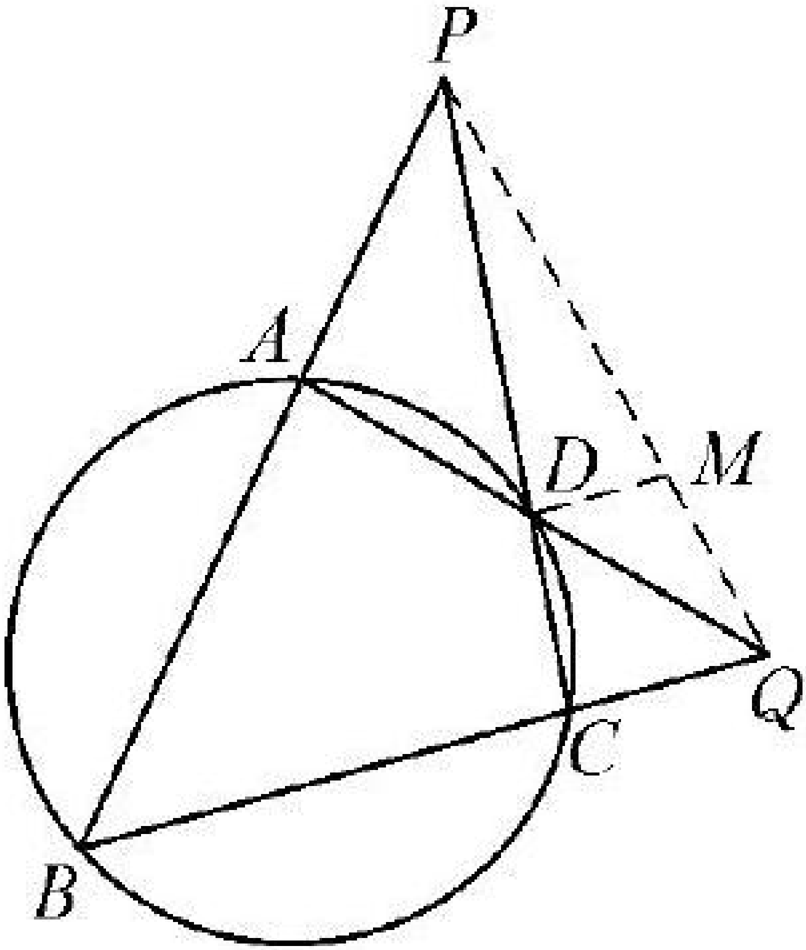
8

1. ⊠ $DE = x$, ⊠ $\frac{2}{PD} = \frac{1}{PE} + \frac{1}{PC}$, ⊠⊠ $\frac{2}{2+x} = \frac{1}{2} + \frac{1}{x+3}$, ⊠⊠ $x = \frac{-3+\sqrt{17}}{2}$.

2. ☒☒☒☒☒☒☒☒☒☒☒ R r, ☒ $PB^2 = PO^2 - r^2, PA^2 = PO^2 - R^2$, ☒ $PB^2 - PA^2 = R^2 - r^2 = OC^2 - OB^2 = CB^2$ ☒



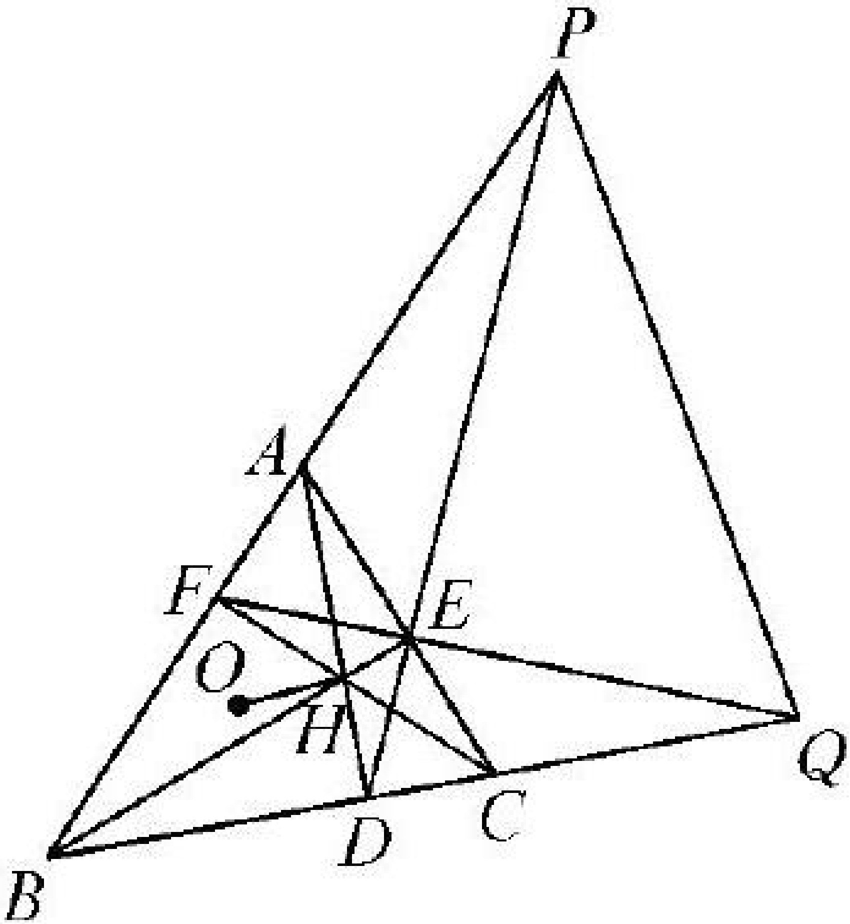
(☒2 ☒)



⊠⊠3⊠⊠

3. ⊠ PQ ⊠⊠⊠⊠ M, ⊠ $\angle PMD = \angle PCQ$, ⊠ $D\ M\ Q\ C$ ⊠⊠⊠⊠, ⊠ $PD \cdot PC = PM \cdot PQ$. ⊠ $\angle QMD = 180° - \angle PMD = 180° - \angle DCQ = \angle DCB = \angle PAD$. ⊠ $D\ M\ P\ A$ ⊠⊠⊠⊠⊠ $QM \cdot QP = QD \cdot QA$ ⊠⊠ $PQ^2 = PQ(PM+QM) = PQ \cdot PM + PQ \cdot QM = PD \cdot PC + QD \cdot QA = P$ ⊠⊠ $+Q$ ⊠⊠⊠

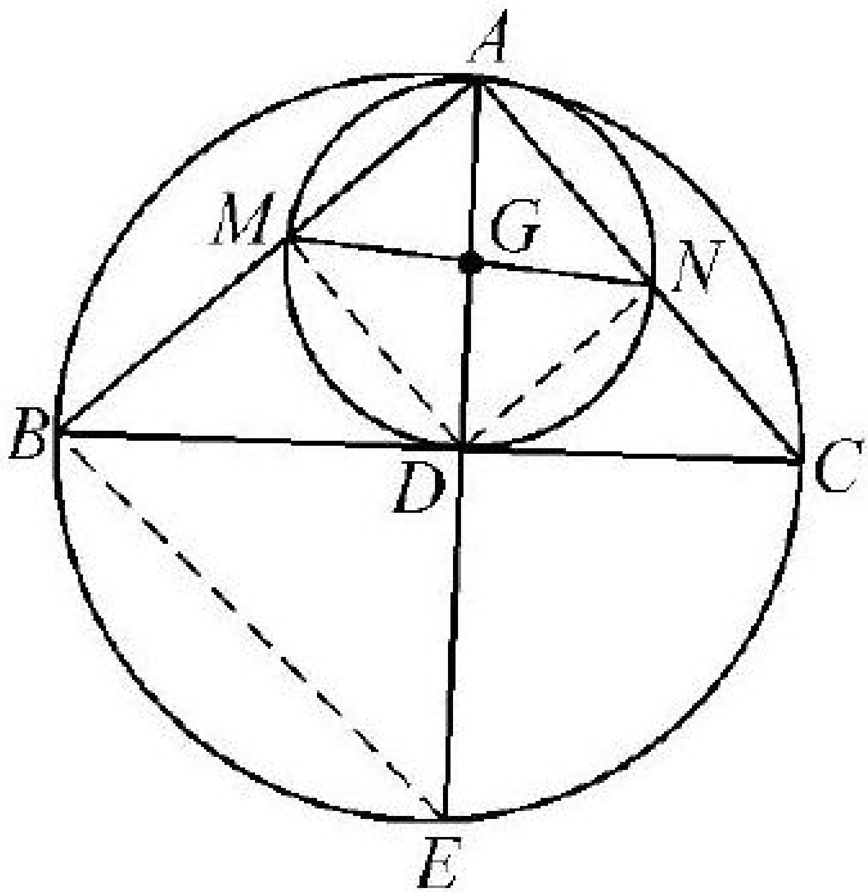
4. ⊠⊠ $A\ B\ D\ E$ ⊠⊠⊠⊠, ⊠⊠⊠ AB, ⊠ $PH^2 = P$ ⊠⊠ $+H$ ⊠⊠ $= PA \cdot PB + BH \cdot HE$, ⊠⊠ AB ⊠⊠⊠ G ⊠⊠ $OG \perp AB, AO = R(R$ ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠ $PH^2 = PG^2 - \frac{1}{4}AB^2 + BH \cdot HE = PO^2 - OG^2 - \frac{1}{4}AB^2 + BH \cdot HE = PO^2 - OA^2 + BH \cdot HE = PO^2 - R^2 + BH \cdot HE$ ⊠⊠⊠ $QH^2 = Q$ ⊠⊠ $+H$ ⊠⊠ $= QO^2 - R^2 + BH \cdot HE$ ⊠⊠⊠ $PH^2 - QH^2 = PO^2 - QO^2$ ⊠⊠



(☒4☒)
$PQ \perp OH$.

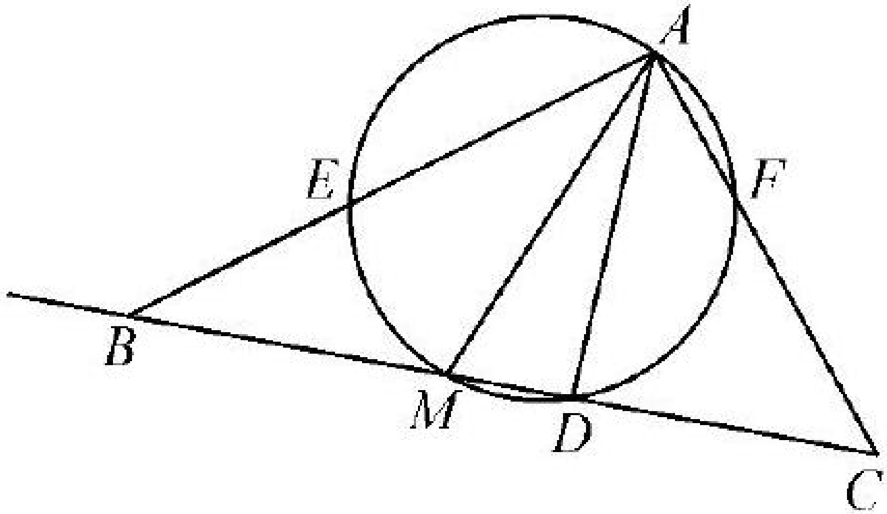
5. ☒1☒☒☒ MD ND ☒☒ $DM \perp AB$ ☒$DN \perp AC$ ☒☒ $AD \perp BC$ ☒☒☒ $AD^2 = AM \cdot AB$ ☒$AD^2 = AN \cdot AC$ ☒☒ $AM \cdot AB = AN \cdot AC$ ☒☒☒ M ☒N C B ☒☒☒☒. ☒2☒☒☒ BE ☒☒ $\angle E = \angle C = \angle ADN = \angle AMN$ ☒☒ M G E B ☒☒☒☒☒☒☒ $AM \cdot AB = AG \cdot AE$ ☒☒ $AD^2 = AM \cdot AB$ ☒☒☒ $AD^2 = AG \cdot AE$ ☒

6. $EF^2 = E$ ☒☒ $+F$ ☒☒ $= EO^2 - r^2 + FO^2 -$



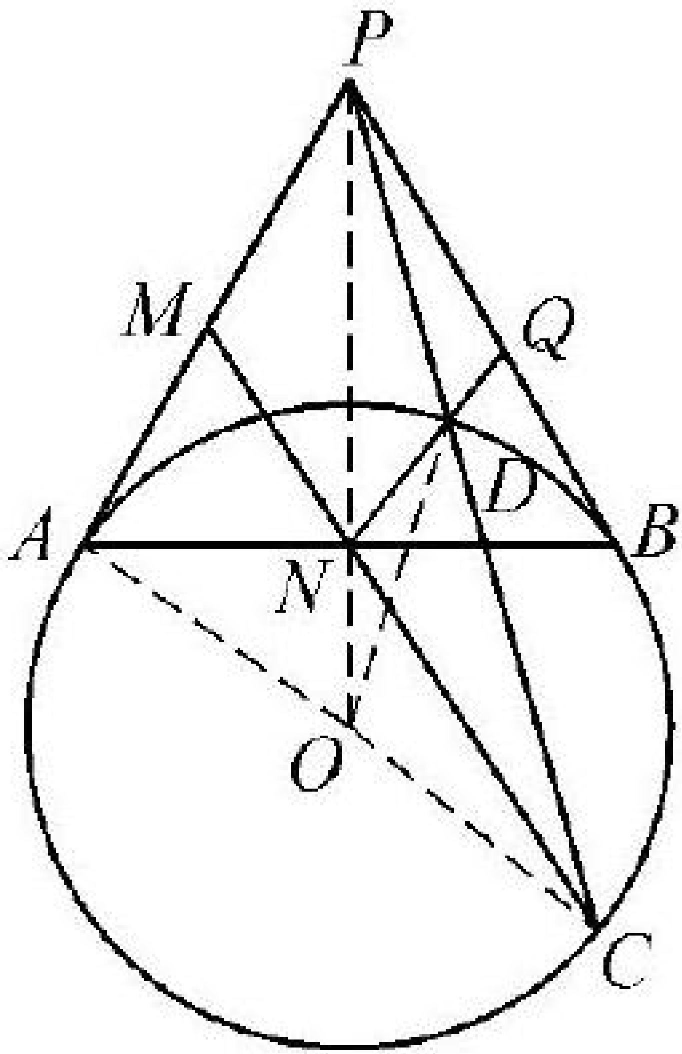
(⊠5 ⊠)
$r^2 = 23^2 + 20^2 - 10^2 - 10^2 = 729$ ⊠⊠ $EF = 27$ ⊠

7. ⊠⊠⊠⊠⊠⊠ $BE \cdot BA = BM \cdot BD, CF \cdot CA = CD \cdot CM$ ⊠⊠⊠ $\frac{BE}{CF} = \frac{BD}{CD} \cdot \frac{CA}{BA} \cdot \frac{BM}{CM}$, ⊠ M ⊠ BC ⊠⊠, ⊠ $BM = CM$ ⊠⊠⊠ $\frac{BM}{CM} = 1, AD$ ⊠⊠ $\angle BAC$ ⊠⊠ $\frac{BD}{DC} = \frac{BA}{CA}$ ⊠⊠⊠ $\frac{BD \cdot CA}{DC \cdot AB} = 1$ ⊠⊠⊠ $\frac{BE}{CF} = 1$ ⊠⊠ $BE = CF$ ⊠



(⊠7⊠)

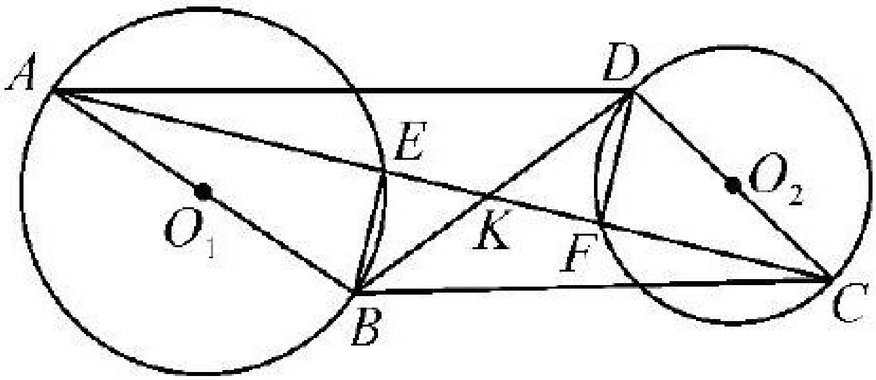
8. ⊠⊠ PN NO, ⊠⊠ P N O ⊠⊠⊠⊠, ⊠⊠ OA OC OD ⊠⊠⊠ $OA \perp PA$ $AN \perp PO$ ⊠⊠ $PA^2 = PN \cdot PO$ ⊠⊠ $PA^2 = PD \cdot PC$ ⊠⊠⊠ $PN \cdot PO = PD \cdot PC$ ⊠⊠ N D C O ⊠⊠⊠⊠⊠⊠⊠ $\angle PNQ = \angle OCD \cdots$ (1). $\angle DOC = \angle DNC$ ⊠⊠⊠⊠ $MN//PB$ ⊠⊠ $\angle PQN = \angle DNC$ ⊠⊠⊠ $\angle DOC = \angle PQN \cdots$ (2). ⊠ (1)⊠(2) ⊠ $\triangle PQN \backsim \triangle DOC$ ⊠⊠⊠⊠ $OD = OC$ ⊠⊠⊠ $PQ = QN$ ⊠⊠⊠ ⊠ $\angle MPN = \angle QPN$ ⊠⊠ $PM = MN$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $PMNQ$ ⊠⊠⊠⊠



(⊠8 ⊠)

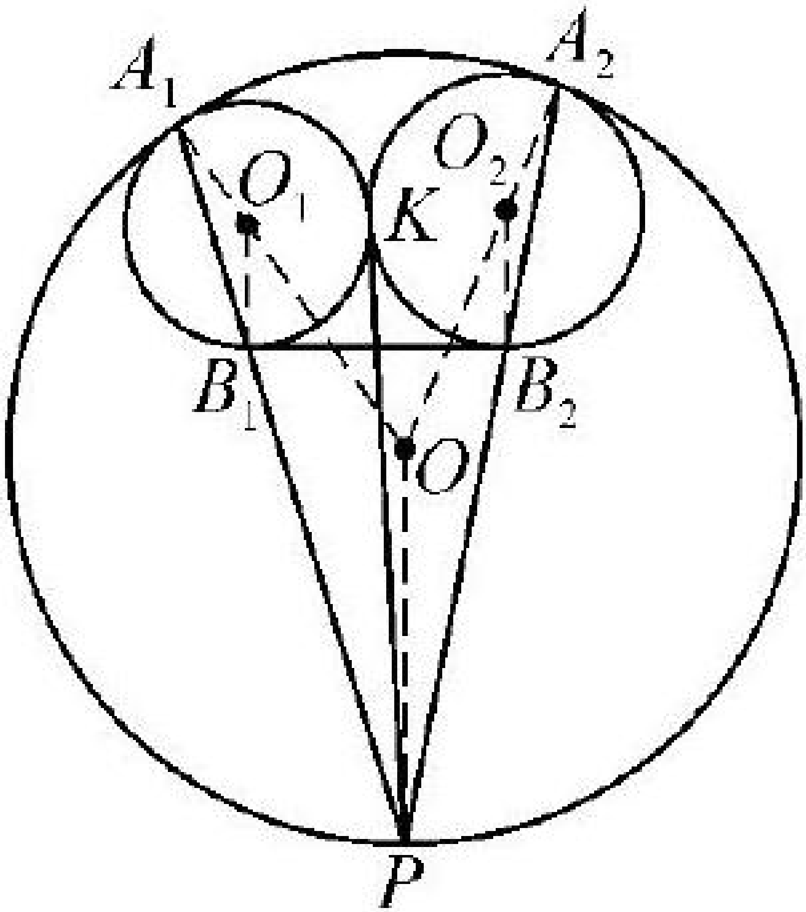
9. ⊠⊠⊠⊠⊠⊠⊠⊠⊠ K ⊠⊠⊠⊠⊠⊠⊠.

⊠⊠⊠ AC ⊠ $\odot O_1$ ⊠⊠⊠⊠ E ⊠⊠ $\odot O_2$ ⊠⊠⊠⊠ F. ⊠⊠ $AB\,CD$ ⊠⊠⊠, ⊠ $BE \perp AC, DF \perp AC$ ⊠⊠⊠ $BE//DF$ ⊠⊠ $\frac{KB}{KD} = \frac{KE}{KF} \cdots$ (1). ⊠ $AD//BC$, ⊠⊠ $\frac{KA}{KC} = \frac{KD}{KB} \cdots$ (2). ⊠ (1)⊠(2) ⊠ $KE \cdot KA =$



(☒9 ☒)
$\frac{KF\cdot KB}{KD}\cdot\frac{KC\cdot KD}{KB}=KF\cdot KC$ ☒☒ K ☒☒☒☒☒☒☒☒☒☒ K ☒☒☒☒☒☒☒☒☒☒☒.
☒☒☒ K ☒☒☒☒☒☒☒☒

10. ☒☒ O O_1 A_1 ☒☒☒O O_2 A_2 ☒☒☒☒ $PB_1\cdot PA_1 = PK^2 = PB_2\cdot PA_2$ ☒☒ A_1 A_2 B_2 B_1 ☒☒☒☒☒☒☒ $\angle PB_1B_2 = \angle PA_2A_1 = \frac{1}{2}\angle POA_1$ ☒☒☒ O_1A_1 ☒O_1B_1 O_1O OO_2 O_2A_2 O_2B_2 PO, ☒ $A_2O = OP = A_1O, A_1O_1 = B_1O_1$, ☒ $\angle POA_1 = 180^\circ - 2\angle PA_1O = \angle A_1O_1B_1$, ☒☒ $\angle O_1B_1B_2 = 180^\circ - \angle PB_1B_2 - \angle AB_1O_1 = 180^\circ - \frac{1}{2}\angle POA_1 - \frac{1}{2}(180^\circ - \angle A_1O_1B_1) =$



(☒10 ☒)
$180° - \frac{1}{2}\angle A_1O_1B_1 - 90° + \frac{1}{2}\angle A_1O_1B_1 = 90°$ ☒☒☒ $\angle O_2B_2B_1 = 90°$ ☒☒ B_1B_2 ☒ $\odot O_1$ $\odot O_2$ ☒☒☒☒.

9

1. ☒Rt $\triangle ABC$ ☒, $\angle B$ ☒☒☒. ☒ AC ☒☒☒☒Rt $\triangle ABC$ ☒☒☒☒. ☒☒☒ B ☒☒☒ ☒☒☒☒☒☒☒ D, ☒☒☒☒ $ABCD$ ☒☒☒. ☒☒☒☒☒☒, $AB \cdot CD + AD \cdot BC = AC \cdot BD$ ☒☒ $AB^2 + BC^2 = AC^2$ ☒

2. (1) ☒☒ O ☒☒☒☒ $ABCD$ ☒☒☒, $AP \perp PB$, ☒☒ $\angle APB = \angle AOB = 90°$, ☒☒ $A\ P\ O\ B$ ☒☒☒☒, ☒ $AP \cdot OB + PO \cdot AB = AO \cdot PB$, ☒☒☒☒☒ ☒☒ $AO = OB = \frac{\sqrt{2}}{2}AB$, ☒☒ $PB = 16$, ☒☒ $AB = \sqrt{AP^2 + PB^2} = \sqrt{4^2 + 16^2} = 4\sqrt{17}$. ☒2☒☒☒☒☒☒ $PABC$ ☒☒☒☒☒☒ $PA \cdot BC + PC \cdot AB = PB \cdot AC$, ☒☒☒ $PDAB$ ☒☒☒☒, ☒ $PB \cdot AD + PD \cdot AB = PA \cdot BD$,

⊠⊠⊠ $\sqrt{2}AB=\sqrt{2}BC=\sqrt{2}AD=AC=BD$ ⊠⊠ $PA=\sqrt{2}, PC=\frac{\sqrt{2}}{2}$ ⊠ ⊠⊠ $PB=\frac{3}{2}, PD=\frac{1}{2}$, ⊠⊠ $PB\cdot PD=\frac{3}{4}$.

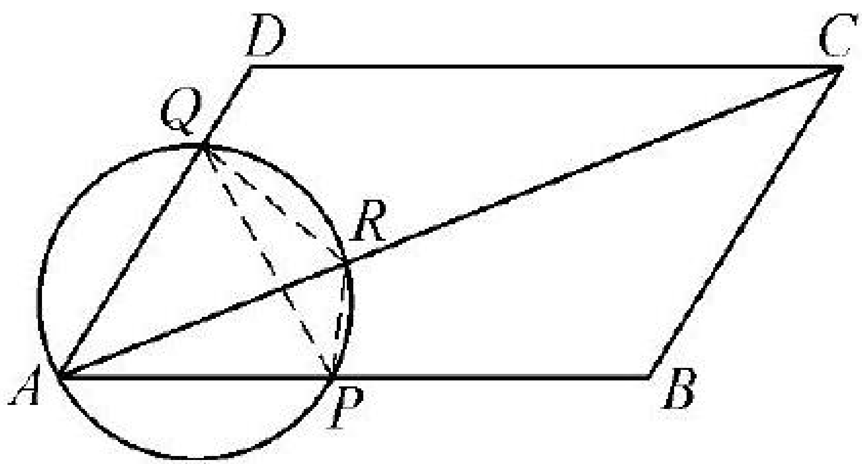
3. $\angle EGA \overset{m}{=} \frac{1}{2}(ADE+BC)=\frac{1}{2}(ADE+AC)=\frac{1}{2}EDAC \overset{m}{=} 180^\circ-\angle EDC$ ⊠⊠⊠ $EDFG$ ⊠⊠. ⊠⊠ $FD\cdot GE+DE\cdot FG=DG\cdot EF$ ⊠

4. ⊠⊠ EF DF, ⊠ $\angle FCA=\angle FBA=\angle FDE, \angle DEF=\angle DBF=\angle FAC$, ⊠ $\triangle AFC\backsim\triangle EFD$, ⊠⊠ $\frac{EF}{AF}=\frac{DE}{CA}=\frac{DF}{CF}=k$, ⊠⊠⊠⊠⊠ $BDFE$

⊠⊠⊠⊠⊠⊠ $BF\cdot DE=EF\cdot BD+BE\cdot DF$ ⊠⊠⊠ $kCA\cdot BF=kAF\cdot BD+kCF\cdot BE$ ⊠⊠⊠ $kAC=kBE=kBD\neq 0$ ⊠⊠⊠ $BF=AF+CF$ ⊠

5. ⊠⊠⊠⊠⊠⊠⊠ $BDEA$ ⊠, $BD\cdot AE+AB\cdot DE=AD\cdot BE$, ⊠⊠⊠⊠⊠⊠⊠ $ABDG$ ⊠⊠$AB\cdot DG+AG\cdot BD=AD\cdot BG$ ⊠⊠⊠⊠ $BE=AE=DG=AD=a, BD=BG=b, AB=DE=AG$ ⊠⊠⊠ $a\cdot b+AB^2=a^2$ ⊠$AB\cdot a+AB\cdot b=a\cdot b$ ⊠⊠⊠ AB ⊠⊠⊠⊠ $(a+b)^2(a-b)=ab^2$ ⊠

6. ⊠⊠ QR RP QP ⊠⊠⊠⊠ $\angle PQR=\angle PAR=\angle ACD, \angle RPQ=\angle RAQ$ ⊠ ⊠⊠ $\triangle PQR\backsim\triangle ACD$ ⊠⊠⊠ $\frac{PQ}{AC}=\frac{PR}{AD}=\frac{QR}{CD}=k$ ⊠ $PQ=kAC, PR=kAD, QR=kCD$ ⊠⊠⊠⊠⊠⊠ $APRQ$ ⊠⊠⊠⊠⊠⊠ $AR\cdot PQ=QR\cdot$



(⊠6 ⊠)
$AP+PR\cdot AQ$ ⊠⊠⊠ $AR\cdot kAC=kCD\cdot AP+kAD\cdot AQ$, ⊠ $ABCD$ ⊠⊠⊠⊠⊠ ⊠, ⊠ $AB=CD$, ⊠ $k\neq 0$, ⊠⊠ $AR\cdot AC=AB\cdot AP+AD\cdot AQ$ ⊠

7. ⊠⊠⊠⊠⊠⊠⊠⊠ a, ⊠⊠⊠⊠⊠ b, ⊠⊠⊠⊠⊠⊠⊠ $PBCD$ ⊠, $PC\cdot BD=BC\cdot PD+PB\cdot CD$ ⊠⊠⊠⊠⊠⊠⊠⊠ $PAED$ ⊠⊠$PE\cdot AD=PD\cdot AE+PA\cdot DE$ ⊠⊠⊠⊠⊠⊠⊠ ⊠ $PCDE$ ⊠⊠$PD\cdot CE=PC\cdot DE+PE\cdot CD$ ⊠⊠ $bPC=a(PD+PB); bPE=a(PD+PA); bPD=a(PC+PE)$ ⊠⊠⊠ $b(PC+PE)=a(PA+PB+PD)+aPD=a(PA+PB+PD)+\frac{a^2}{b}(PC+PE)$ ⊠⊠⊠ $\left(b^2-a^2\right)(PC+PE)=ab(PA+PB+PD)$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ ACD ⊠, ⊠⊠⊠ 36°, ⊠⊠⊠ 72°, ⊠ $\angle ACD$ ⊠⊠⊠⊠ CF ⊠ AD ⊠⊠ F, ⊠⊠⊠ $AF=CF=CD=a$ ⊠⊠ $FD=b-a$ ⊠⊠ $\triangle CFD\backsim\triangle ACD$ ⊠⊠⊠ $\frac{FD}{CD}=\frac{CD}{AD}$ ⊠⊠ $\frac{b-a}{a}=\frac{a}{b}$, ⊠⊠ $b^2-a^2=ab\neq 0$, ⊠⊠ $PC+PE=PA+PB+PD$.

8. (1) ⊠⊠ $\angle ABC = \angle FED = 120^\circ$, ⊠⊠⊠⊠⊠⊠⊠ $AC = DF = \sqrt{a^2+ab+b^2}$ ⊠⊠⊠⊠⊠⊠⊠⊠ $ACDF$ ⊠⊠$CF^2 = CF \cdot AD = AF \cdot CD +$ $AC \cdot FD$ ⊠⊠⊠ $CF = a+b$ ⊠⊠⊠⊠⊠⊠⊠⊠ $ABCF$ ⊠⊠$BF \cdot AC = AB \cdot CF + AF \cdot BC$ ⊠⊠⊠ $BF = \frac{a^2+2ab}{\sqrt{a^2+ab+b^2}}$. ⊠2⊠⊠⊠⊠⊠⊠⊠⊠ $ABDE$ ⊠⊠$AD \cdot BE = AB \cdot DE + BD \cdot AE$ ⊠⊠⊠ $AD \cdot BE \cdot CF = AB \cdot DE \cdot CF +$ $BD \cdot AE \cdot CF$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $BCDF$ ⊠⊠$BD \cdot CF = BF \cdot CD + BC \cdot DF$ ⊠⊠⊠⊠⊠⊠ $AD \cdot BE \cdot CF = AB \cdot DE \cdot CF + AE \cdot BF \cdot CD + AE \cdot BC \cdot$
$DF \cdots$ (1), ⊠⊠⊠⊠⊠⊠⊠ $ABEF$ ⊠, $AE \cdot BF = AB \cdot EF + BE \cdot AF$ ⊠⊠⊠⊠ ⊠⊠⊠⊠ $ADEF$ ⊠, $AE \cdot DF = AD \cdot EF + DE \cdot AF$ ⊠⊠⊠ (1) ⊠⊠ $AD \cdot BE \cdot CF = AB \cdot DE \cdot CF + CD \cdot AB \cdot EF + CD \cdot BE \cdot AF + BC \cdot AD \cdot EF +$ $BC \cdot DE \cdot AF$.

9. ⊠⊠ PA PB PC ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $ABPC$ ⊠⊠⊠⊠⊠⊠⊠ $BC \cdot AP = AC \cdot BP + AB \cdot CP$ ⊠⊠⊠ $\frac{BC}{PK} \cdot AP \cdot PK = \frac{AC}{PL} \cdot BP \cdot PL + \frac{AB}{PM} \cdot CP \cdot PM$ ⊠⊠⊠⊠ $\triangle KBP \backsim \triangle LAP, \triangle BPM \backsim \triangle CPL$ ⊠⊠⊠ $AP \cdot PK = BP \cdot PL$ $CP \cdot PM = BP \cdot PL$ ⊠⊠ $AP \cdot PK = BP \cdot PL = CP \cdot PM$ ⊠⊠⊠ $\frac{BC}{PK} = \frac{AC}{PL} + \frac{AB}{PM}$.

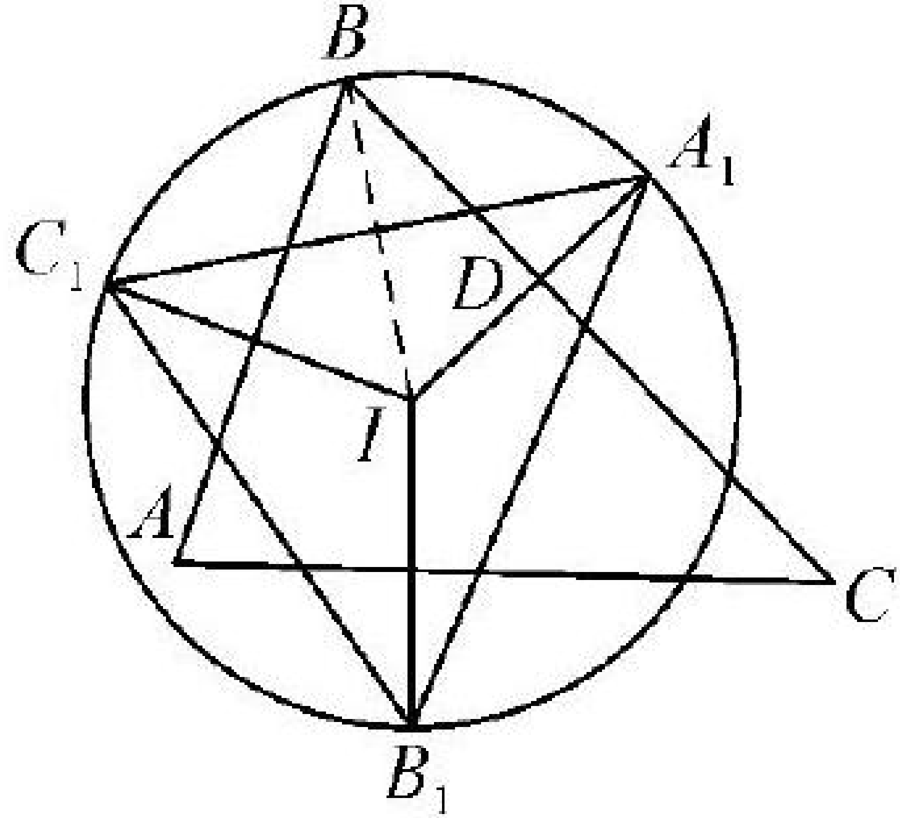
10. ⊠⊠⊠⊠ $ABEC$ ⊠, $AB \cdot EC + BE \cdot AC \geqslant AE \cdot BC$, ⊠⊠⊠⊠ $ACDB$ ⊠⊠ $AB \cdot CD + BD \cdot AC \geqslant AD \cdot BC$ ⊠⊠⊠⊠ $\sqrt{2}BC = \sqrt{2}BE = \sqrt{2}CD = BD = EC, AB = 1, AC = 2$, ⊠⊠ $AD + AE \leqslant 3\sqrt{2} + 3$, ⊠⊠⊠ $\angle BAC = 135^\circ$ ⊠⊠⊠.

10

1. BE ⊠⊠ $\angle ABC, DE$ ⊠⊠ $\angle ADC$, ⊠⊠ E ⊠ $\triangle ADB$ ⊠⊠⊠, ⊠⊠ AE ⊠⊠ $\angle BAD$ ⊠⊠⊠, ⊠ AD ⊠⊠ $\angle BAC$, ⊠⊠ $\angle BAC = \frac{2}{3} \times 180^\circ = 120^\circ$.

2. ⊠⊠ $IA_1 = IB_1 = IC_1 = 2r$ ⊠r ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠), ⊠⊠, I ⊠ $\triangle A_1B_1C_1$ ⊠⊠⊠⊠⊠⊠⊠. ⊠ IA_1 ⊠ BC ⊠⊠⊠⊠ D ⊠⊠ $IB = IA_1 = 2ID$ ⊠⊠⊠, $\angle IBD = 30^\circ$ ⊠⊠⊠, $\angle IBA = 30^\circ$, ⊠⊠, $\angle ABC = 60^\circ$ ⊠

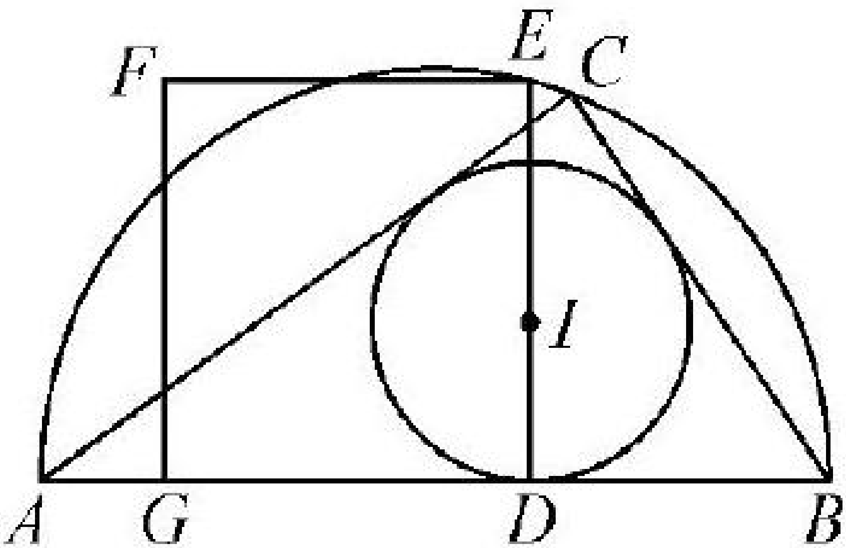
3. ⊠⊠⊠ DE ⊠⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠, D ⊠E ⊠⊠⊠⊠ AB ⊠ AC ⊠. ⊠ $\angle A$ ⊠ ⊠⊠⊠⊠ DE ⊠⊠ P, ⊠ P ⊠ AB AC ⊠⊠⊠⊠ r, P ⊠ BC ⊠⊠⊠



(☒2 ☒)

☒ r_1, ☒☒☒☒☒☒ $\begin{cases} AD+AE=DB+BC+CE, \\ \frac{r}{2}(AD+AE)=\frac{r}{2}(DB+CE)+\frac{r_1}{2}BC, \end{cases}$ ☒☒ $r_1=r$, ☒ P ☒ $\triangle ABC$ ☒☒☒☒

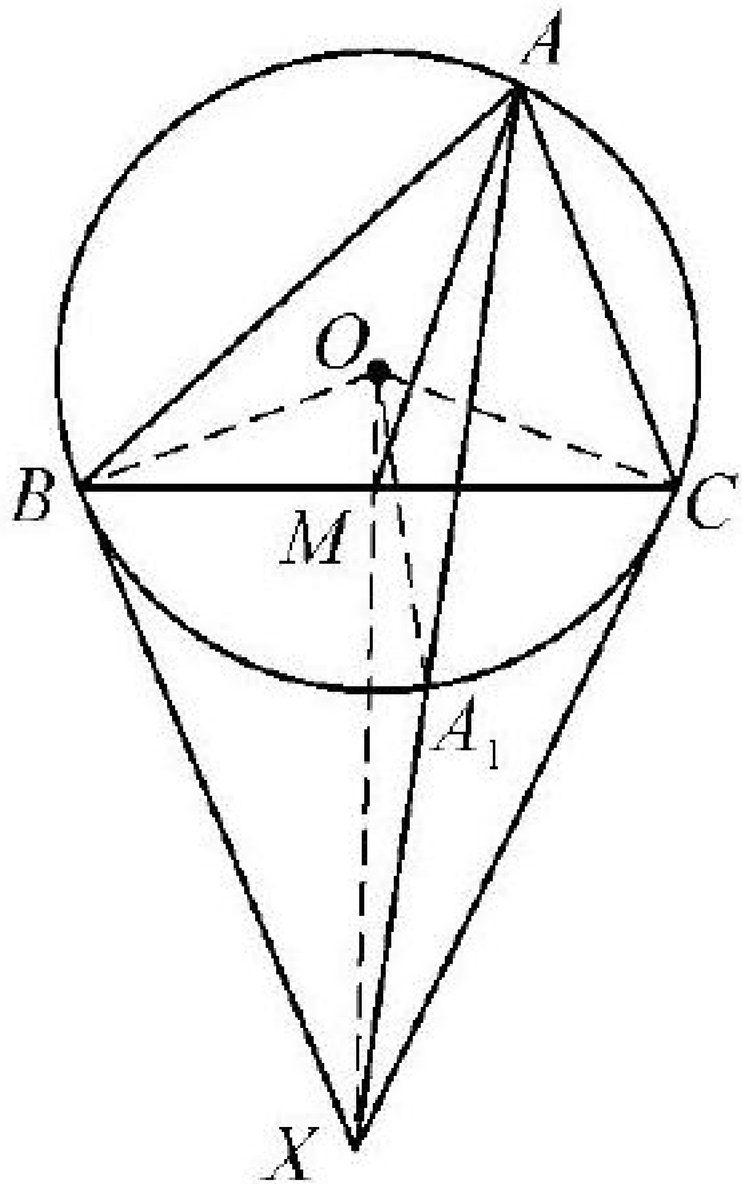
4. ☒☒☒☒☒☒☒ $AD=\frac{b+c-a}{2}, BD=\frac{c+a-b}{2}$, ☒☒ a b c ☒ $\triangle ABC$ ☒☒☒☒. ☒ DE ☒Rt $\triangle ABE$ ☒☒ AB ☒☒☒, ☒☒☒☒☒☒



(⊠4⊠)

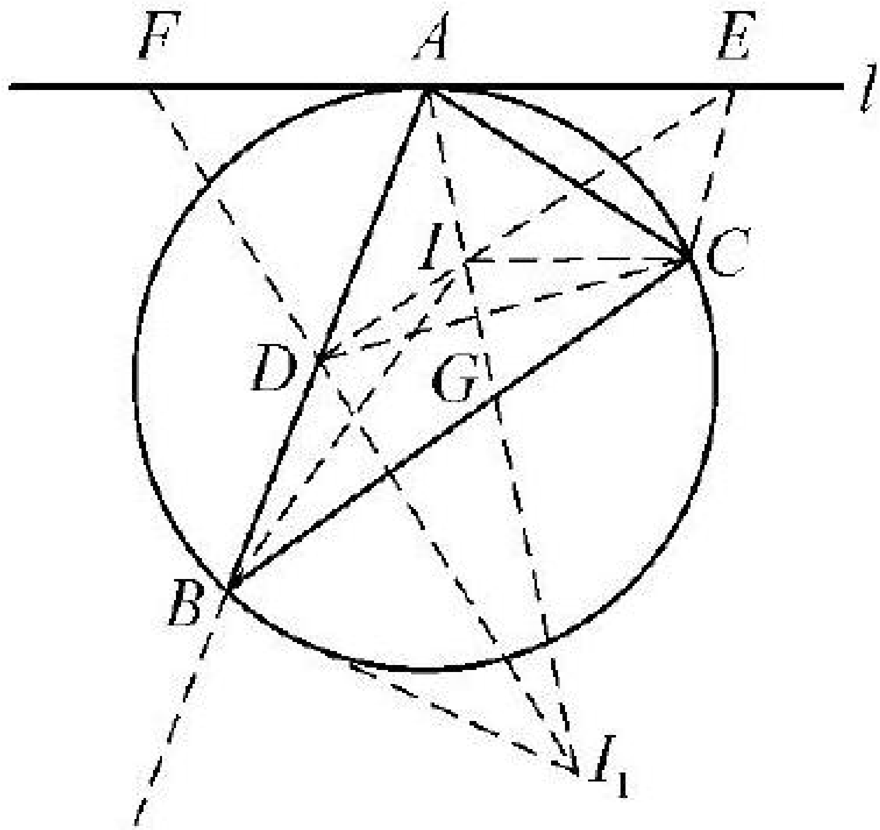
$DE^2 = AD \cdot DB$, ⊠ $100 = \frac{b+c-a}{2} \cdot \frac{c+a-b}{2} = \frac{c^2-(a-b)^2}{4} = \frac{\left(a^2+b^2\right)-\left(a^2-2ab+b^2\right)}{4} = \frac{ab}{2}$, ⊠⊠, $S_{\triangle ABC} = \frac{ab}{2} = 100$.

5. ⊠ AX ⊠⊠ O ⊠⊠⊠⊠ A_1, ⊠⊠ $OB\, OC$ ⊠OA_1. ⊠ M ⊠ BC ⊠⊠⊠, ⊠⊠, ⊠⊠ OX, ⊠⊠⊠ M. ⊠⊠ $OB \perp BX, OX \perp BC$ ⊠⊠⊠ $XB^2 = XM \cdot XO \cdots$ (1)⊠⊠⊠⊠⊠⊠⊠⊠⊠ $XB^2 = XA_1 \cdot XA \cdots$ (2). ⊠ (1)⊠(2) ⊠ $\frac{XM}{XA} = \frac{XA_1}{XO}$, ⊠⊠ $\triangle XMA \backsim \triangle XA_1O$, ⊠⊠ $\frac{AM}{AX} = \frac{OA_1}{OX} = \frac{OB}{OX}$. ⊠ $\angle BOC = 2\angle BAC$, ⊠⊠, $\angle BOX = \angle BAC$ ⊠⊠⊠ $\frac{AM}{AX} = \frac{OB}{OX} = \cos\angle BAC$.



(⊠5 ⊠)

6. ⊠⊠, ⊠⊠ DE DC ⊠⊠ $\angle BAC$ ⊠⊠⊠⊠⊠⊠⊠ DE ⊠⊠ I, ⊠ DC ⊠⊠ G, ⊠ ⊠ IC, ⊠⊠ $AD = AC$ ⊠⊠ $AG \perp DC, ID = IC$ ⊠⊠ $D\ C\ E$ ⊠⊠ A ⊠, ⊠⊠ $\angle IAC = \frac{1}{2}\angle DAC = \angle IEC$ ⊠⊠⊠ $A\ I\ C\ E$ ⊠⊠⊠⊠, ⊠⊠ $\angle CIE = \angle CAE = \angle ABC$ ⊠⊠ $\angle CIE = 2\angle ICD$ ⊠⊠⊠ $\angle ICD = \frac{1}{2}\angle ABC$. ⊠⊠ $\angle AIC = \angle IGC + \angle ICG = 90^\circ +$



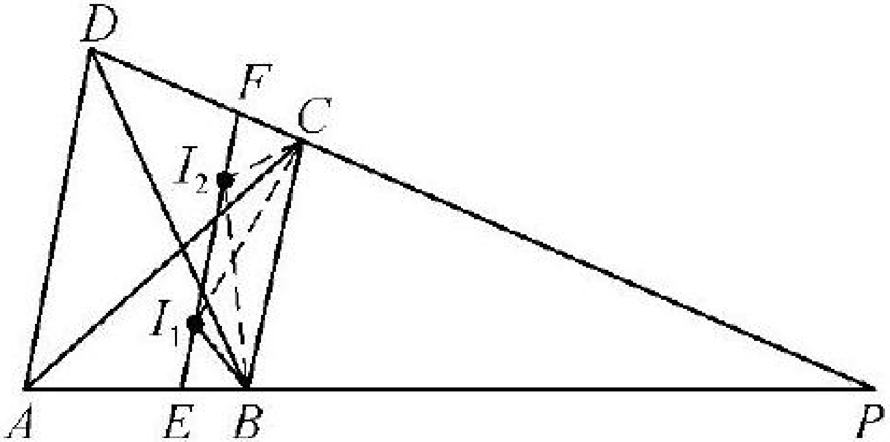
(⊠6 ⊠)

$\frac{1}{2}\angle ABC, \angle ACI = 180^\circ - \angle IAC - \angle AIC = \frac{1}{2}\angle ACB$, ⊠ I ⊠ $\triangle ABC$ ⊠⊠⊠⊠

⊠⊠ FD ⊠⊠⊠⊠ AI ⊠⊠⊠⊠⊠⊠ I_1, ⊠⊠ BI_1 BI, ⊠ $\angle DI_1I = 180^\circ - \angle DFA - \angle FAI_1 = 180^\circ - \left(\frac{180^\circ - \angle FAD}{2}\right) - \angle FAD - \frac{1}{2}\angle BAC = 90^\circ - \frac{1}{2}\angle FAD - \frac{1}{2}\angle BAC = 90^\circ - \frac{1}{2}\angle ACB - \frac{1}{2}\angle BAC = \frac{1}{2}\angle ABC = \angle DBI$, ⊠ B D I I_1 ⊠⊠⊠⊠. ⊠⊠ $\angle I_1BC + \angle CBI = \angle I_1BI = \angle I_1DI = 180^\circ - \angle FDA - \angle ADI = 180^\circ - \frac{1}{2}(180^\circ - \angle FAD) - \angle ACI = 90^\circ + \frac{1}{2}\angle FAD - \frac{1}{2}\angle ACB = 90^\circ$, ⊠⊠ $\angle I_1BC = 90^\circ - \angle CBI = 90^\circ - \frac{1}{2}\angle ABC = \frac{1}{2}(180^\circ - \angle ABC)$, ⊠⊠ I_1B ⊠⊠ $\angle ABC$ ⊠⊠⊠, ⊠ AI_1 ⊠⊠ $\angle BAC$, ⊠⊠ I_1 ⊠ $\triangle ABC$

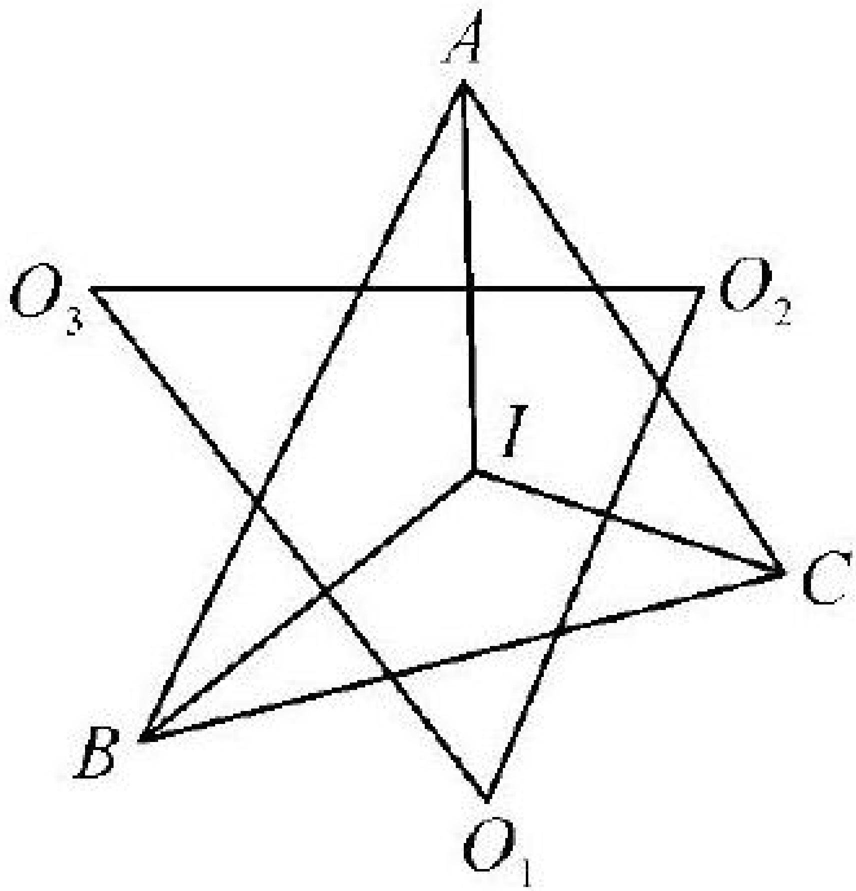
⊠⊠⊠⊠

7. ⊠⊠, ⊠⊠ I_1B I_1C I_2B I_2C, ⊠⊠ $PE = PF$ ⊠⊠⊠ $\angle PEF = \angle PFE$ ⊠⊠ $\angle PEF = \angle I_2I_1B - \angle EBI_1 = \angle I_2I_1B - \angle I_1BC, \angle PFE = \angle I_1I_2C - \angle FCI_2 = \angle I_1I_2C - \angle I_2CB$, ⊠⊠ $\angle I_2I_1B - \angle I_1BC = \angle I_1I_2C - \angle I_2CB$ ⊠⊠ $\angle I_2I_1B + \angle I_2CB =$



(⊠7⊠)
$\angle I_1I_2C + \angle I_1BC$. ⊠ $\angle I_2I_1B + \angle I_2CB + \angle I_1I_2C + \angle I_1BC = 2\pi$, ⊠⊠ $\angle I_2I_1B + \angle I_2CB = \pi$ ⊠⊠⊠⊠I_1 I_2 C B ⊠⊠⊠⊠⊠⊠ $\angle BI_1C = \angle BI_2C$ ⊠⊠⊠ $\angle I_1BC + \angle I_1CB = \angle I_2BC + \angle I_2CB$ ⊠$\angle ABC + \angle ACB = \angle DBC + \angle DCB$ ⊠⊠⊠ $\angle BAC = \angle BDC$ ⊠⊠ A B C ⊠D ⊠⊠⊠⊠⊠

8. ⊠⊠ AI ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠ D, ⊠⊠⊠⊠⊠⊠⊠⊠⊠ $DB = DI = DC$ ⊠⊠⊠ D ⊠ $\triangle IBC$ ⊠⊠⊠⊠⊠⊠ D ⊠ O_1 ⊠⊠⊠⊠ O_1 ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠ O_2 ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠O_3 ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠⊠ $\triangle O_1O_2O_3$ ⊠ $\triangle ABC$ ⊠⊠⊠⊠⊠⊠⊠



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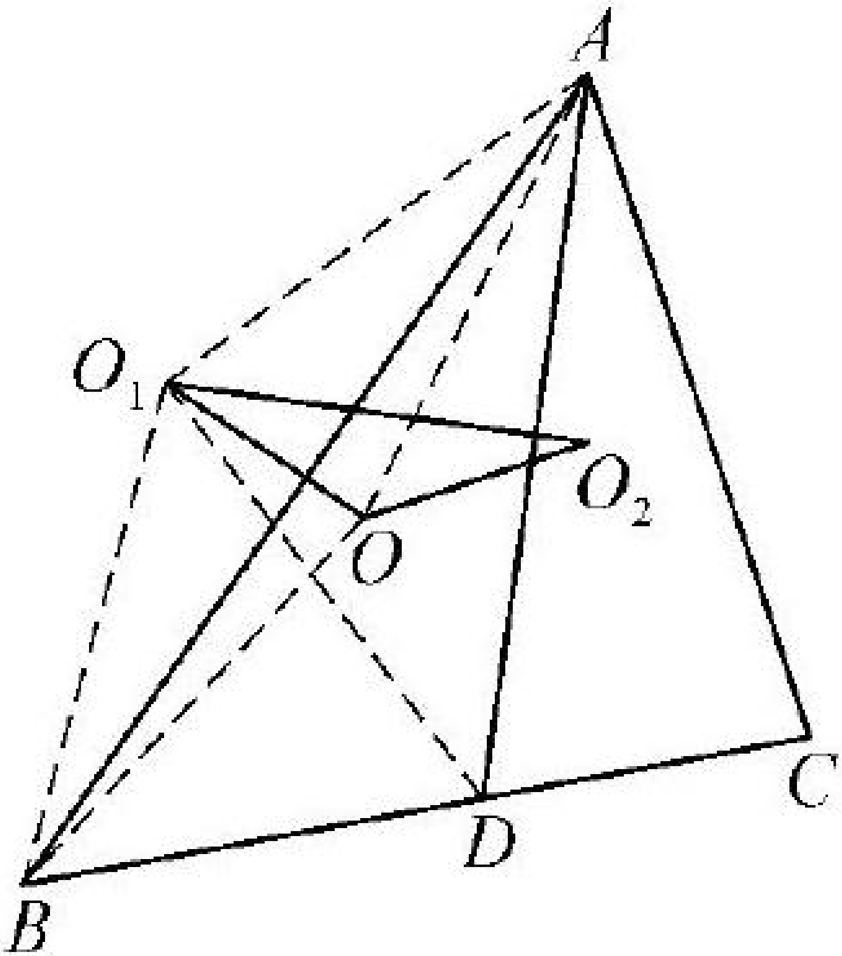
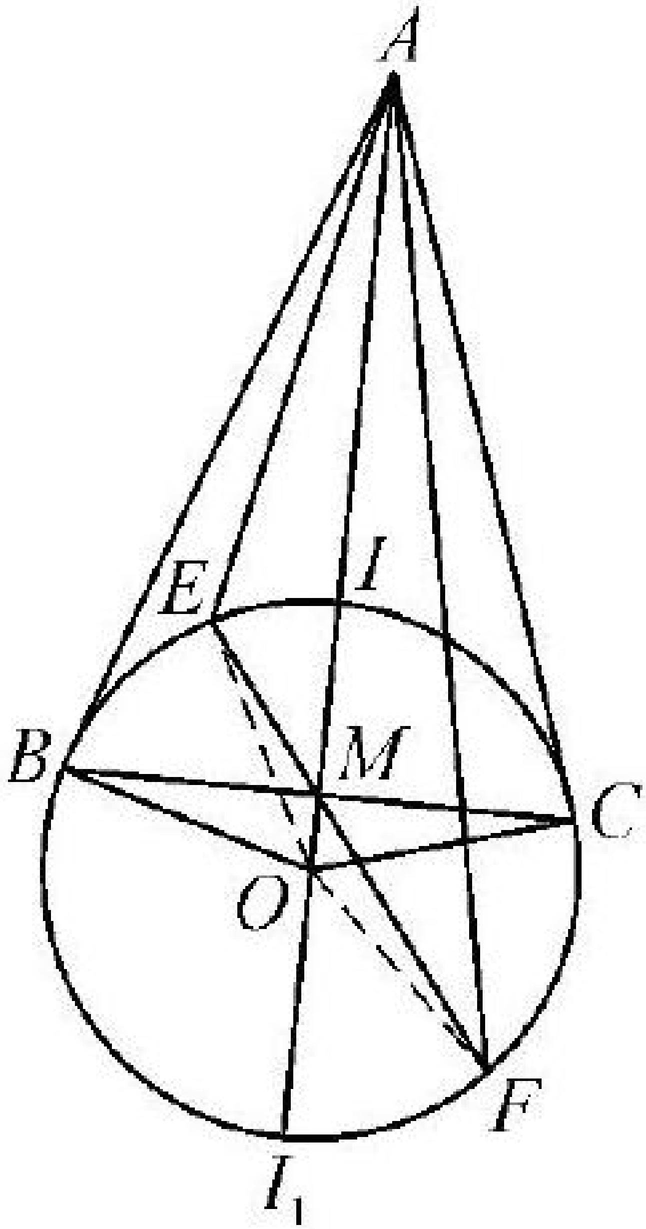


⊠⊠9 ⊠⊠

9. ⊠⊠ $OA\ OB\ O_1A\ O_1B\ O_1D$ ⊠⊠ $AO_1B = 360^\circ - 2\angle ADB$ ⊠OO_1 ⊠⊠⊠⊠ AB, OO_2 ⊠⊠⊠⊠ AC, O_1O_2 ⊠⊠⊠⊠ AD, ⊠⊠ $\angle O_1OO_2 = 180^\circ - \angle BAC, \angle AO_1O = \frac{1}{2}\angle AO_1B = \frac{1}{2}(360^\circ - 2\angle ADB) = 180^\circ - \angle ADB, \angle O_1AB = \frac{1}{2}(180^\circ - \angle AO_1B) = \angle ADB - 90^\circ$, ⊠⊠ $\angle O_1AD = \angle ADB - 90^\circ + \frac{1}{2}\angle BAC$, ⊠⊠ $\angle AO_1O_2 = 90^\circ - \angle O_1AD = 180^\circ - \angle ADB - \frac{1}{2}\angle BAC$, ⊠⊠ $\angle O_2O_1O = \angle AO_1O - \angle AO_1O_2 = \frac{1}{2}\angle BAC$, ⊠⊠ $\angle O_1O_2O = 180^\circ - \angle O_2O_1O - \angle O_1OO_2 = \frac{1}{2}\angle BAC$, ⊠⊠ $\angle O_2O_1O = \angle O_1O_2O$, ⊠ $\triangle OO_1O_2$ ⊠⊠⊠ ⊠⊠⊠⊠

10. ⊠⊠, ⊠⊠, ⊠ I ⊠ $\triangle ABC$ ⊠⊠⊠, ⊠⊠⊠ I ⊠ AM ⊠, ⊠⊠ $A\ B\ O\ C$ ⊠⊠⊠⊠, ⊠⊠⊠⊠⊠3 ⊠ $OB = OC = OI$ ⊠⊠ I ⊠⊠ O ⊠⊠⊠ I_1 ⊠ $\triangle ABC$ ⊠⊠ BC ⊠⊠ ⊠⊠⊠, ⊠⊠⊠ I_1 ⊠⊠⊠ AM ⊠, ⊠ $BI \perp BI_1$, ⊠⊠⊠ AO ⊠⊠ O ⊠⊠⊠⊠ I_2, ⊠ $BI \perp BI_2$, ⊠⊠ I_1 ⊠ I_2 ⊠⊠. ⊠⊠, ⊠⊠ OE ⊠OF ⊠⊠ $OF^2 = OC^2 = OM \cdot OA$ ⊠⊠ $\angle MOF = \angle FOA$ ⊠⊠⊠ $\triangle MOF \backsim \triangle FOA$. ⊠⊠ $\frac{MF}{FA} = \frac{OF}{OA}$, ⊠⊠Rt $\triangle OCA$

⊠ MC ⊠⊠⊠⊠ OA, ⊠⊠ IC ⊠⊠ $\angle ACB$ ⊠ $\frac{MF}{FA} = \frac{OF}{OA} = \frac{OC}{OA} =$



(⊠10⊠)

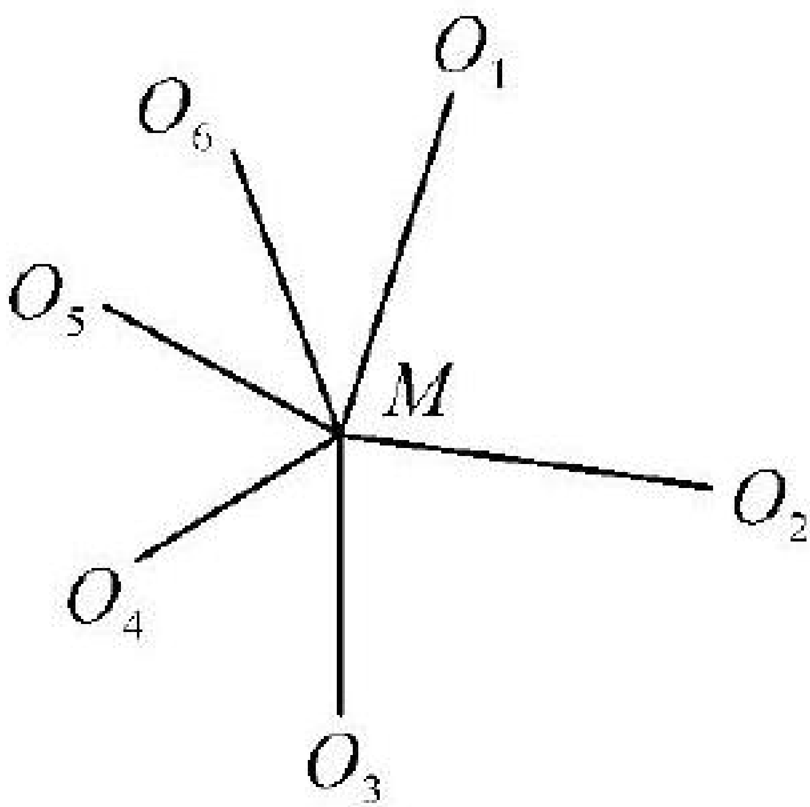
$\frac{MC}{AC} = \frac{MI}{IA}$, ⊠⊠ IF ⊠⊠ $\angle MFA$, ⊠⊠, $OE^2 = OB^2 = OM \cdot OA$, ⊠ $\angle MOE = \angle EOA$, ⊠⊠ $\triangle MOE \backsim \triangle EOA$, ⊠⊠ $\frac{ME}{EA} = \frac{OE}{OA} = \frac{OB}{OA} = \frac{MB}{AB} = \frac{MI}{IA}$, ⊠⊠ IE ⊠⊠ $\angle MEA$ ⊠⊠⊠ I ⊠ $\triangle AEF$ ⊠⊠⊠⊠⊠⊠⊠ I_3 ⊠ $\triangle AEF$ ⊠⊠ EF ⊠⊠⊠⊠⊠⊠

⊠⊠ I_3 ⊠⊠⊠ AM ⊠⊠⊠ $EI \perp EI_3$ ⊠⊠ $EI \perp EI_1$ ⊠⊠⊠ I_1 ⊠ I_3 ⊠⊠⊠⊠⊠ I_1 ⊠⊠ $\triangle AEF$ ⊠⊠⊠⊠

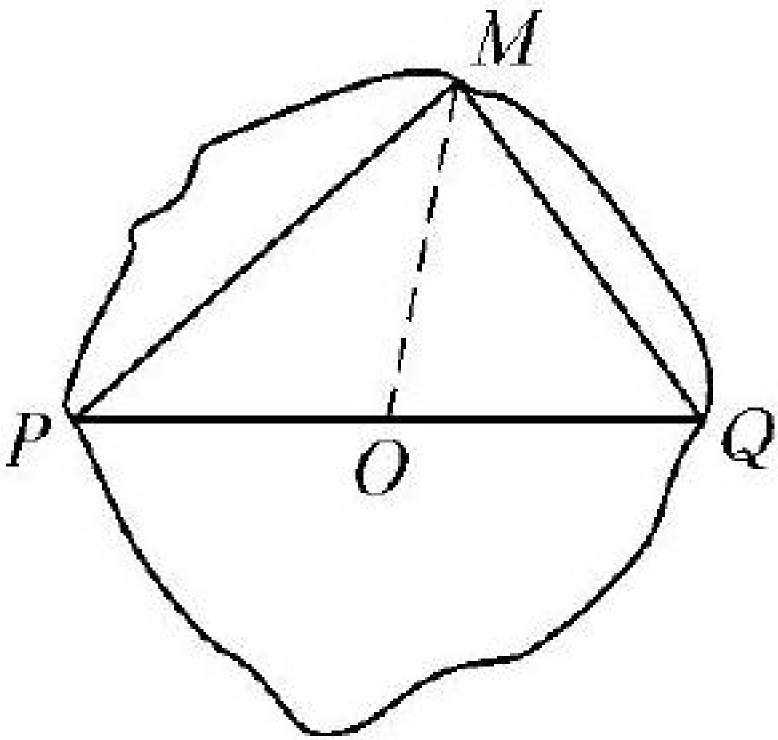
11

1. ⊠⊠⊠⊠ M ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠, ⊠⊠⊠⊠⊠⊠⊠ $O_1, O_2, \cdots, O_6$, ⊠⊠ $MO_1, MO_2, \cdots, MO_6$ ⊠⊠⊠ $\angle O_1MO_2 + \angle O_2MO_3 + \cdots + \angle O_6MO_1 = 360^\circ$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ 60° ⊠⊠⊠⊠ $\angle \mathrm{O_2MO_3} \leqslant 60^\circ$ ⊠⊠⊠ $\triangle O_2MO_3$ ⊠ ⊠⊠⊠⊠⊠⊠⊠⊠ 60° ⊠⊠⊠⊠ $\angle MO_3O_2 \geqslant 60^\circ$ ⊠⊠ $O_2O_3 \leqslant O_2M < r_2$ $\odot O_2$ ⊠⊠⊠⊠⊠⊠ O_3 ⊠ $\odot O_2$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠⊠

2. ⊠⊠⊠⊠⊠⊠⊠⊠⊠ P ⊠ Q ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $2l$ ⊠⊠ O ⊠ PQ ⊠⊠⊠⊠ ⊠⊠⊠⊠⊠⊠⊠ M ⊠⊠⊠ MO ⊠ $MO \leqslant \frac{1}{2}(MP + MQ) \leqslant \frac{1}{2}$ ⊠⊠⊠ $MP+$ ⊠ ⊠ $MQ) = \frac{1}{2} \cdot 2l = l$, ⊠⊠⊠ O ⊠⊠⊠, l ⊠⊠⊠⊠



(⊠1⊠)



(⊠2⊠)

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

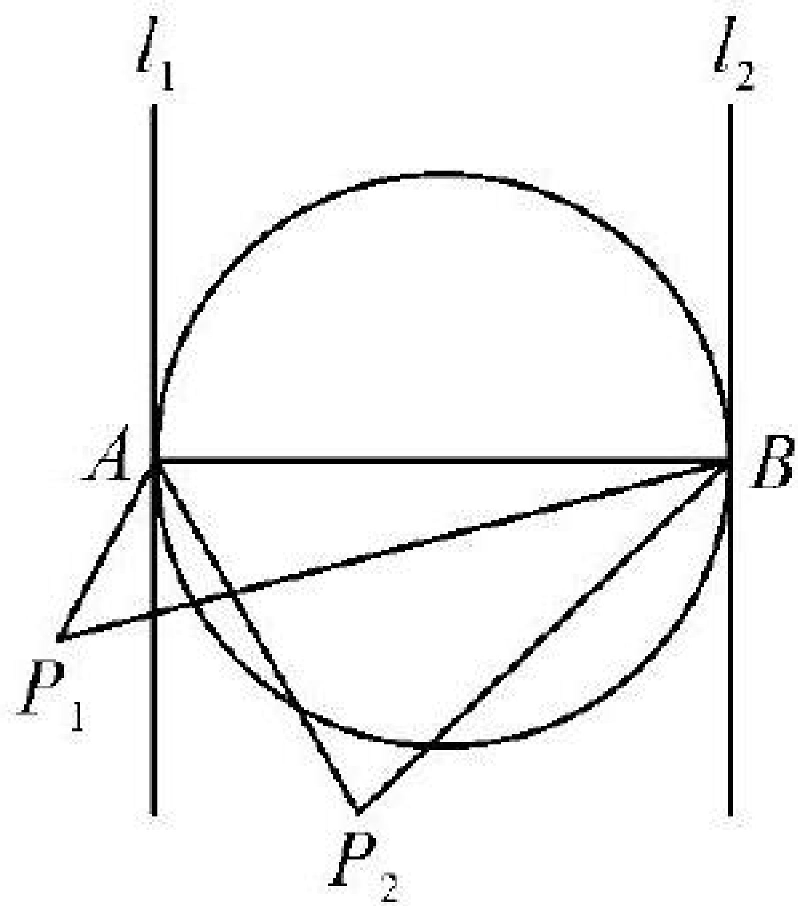
3. 7 ⊠. ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠. ⊠⊠: ⊠⊠7⊠⊠⊠⊠1 ⊠⊠⊠⊠⊠⊠⊠⊠ ⊠2 ⊠⊠. ⊠⊠⊠, ⊠⊠⊠1 ⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠2 , ⊠⊠⊠⊠2 ⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠ $\frac{1}{6}$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠2 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\frac{1}{6}$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠6 ⊠⊠⊠, ⊠⊠, ⊠6 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\frac{1}{6}$, ⊠⊠⊠⊠7 ⊠⊠⊠⊠1 ⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠2 ⊠⊠.

4. 7 ⊠⊠⊠⊠⊠ $\mathrm{C}_7^2 = 21$ ⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\frac{1}{7}$ ⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠⊠ $\frac{1}{7} \times 21 = 3$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $7 - 3 = 4$, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠4 ⊠⊠. ⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\frac{1}{7}$ ⊠

5. ⊠⊠⊠⊠6 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠5 ⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠7 ⊠⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠6 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ A B ⊠⊠, $AB \leqslant 2$, ⊠ A B ⊠⊠ ⊠6 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠6 ⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠2 ⊠, ⊠ AB ⊠⊠ ⊠⊠⊠⊠⊠6 ⊠⊠⊠⊠2 ⊠⊠⊠⊠⊠. ⊠⊠⊠⊠⊠⊠5 ⊠⊠⊠⊠⊠⊠6 ⊠⊠⊠⊠2 ⊠⊠⊠⊠, ⊠⊠ $6 \times \pi \times 2^2 = 24\pi < 25\pi = \pi \times 5^2$, ⊠⊠. ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠1 ⊠⊠.

6. ⊠⊠⊠25 ⊠⊠⊠ A B ⊠⊠⊠⊠⊠⊠, ⊠ $AB \leqslant 1$, ⊠⊠⊠⊠⊠⊠⊠ A ⊠⊠⊠, ⊠⊠⊠1 ⊠⊠ ⊠, ⊠⊠. ⊠⊠, ⊠⊠⊠ A B ⊠⊠⊠, ⊠⊠⊠1 ⊠ $\odot A$ ⊠$\odot B$ ⊠⊠⊠⊠⊠⊠⊠ C ⊠⊠⊠ $CA \leqslant 1$ ⊠ $CB \leqslant 1$ ⊠⊠⊠⊠23 ⊠⊠⊠ $\odot A$ ⊠⊠⊠ $\odot B$ ⊠, ⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠13 ⊠⊠, ⊠⊠.

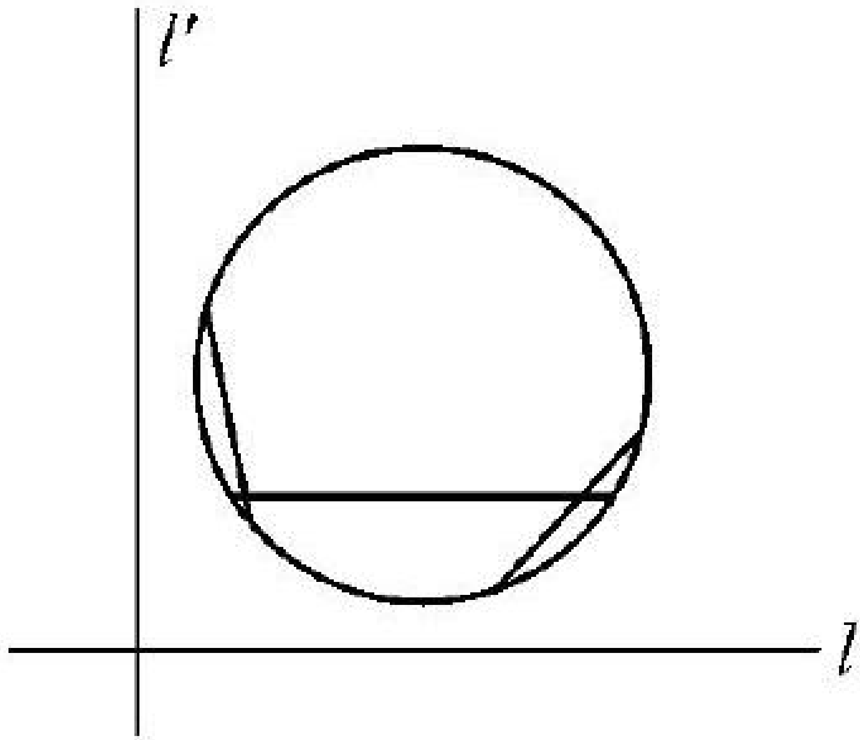
7. ⊠⊠⊠ A B ⊠⊠2012 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠ $AB < 2012$, ⊠⊠ AB ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠2012 ⊠⊠. ⊠⊠⊠, ⊠⊠⊠ A B ⊠ l_1 l_2 ⊠⊠⊠ AB ⊠⊠⊠⊠⊠)⊠⊠⊠⊠ P_1 ⊠ l_1 l_2 ⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠ $\angle P_1AB > 90°$, ⊠ $P_1B > AB$, ⊠ AB ⊠⊠⊠⊠⊠⊠. ⊠⊠⊠ P_2 ⊠ l_1 l_2 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ AB ⊠⊠⊠⊠⊠⊠, ⊠ $\angle AP_2B < 90°$, ⊠ $\angle P_2AB < 90°$, $\angle P_2BA < 90°$ ⊠⊠⊠⊠" ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠



(⊠7 ⊠)

⊠" ⊠⊠⊠. ⊠⊠⊠⊠⊠.

8. ⊠ l' ⊠⊠⊠⊠⊠ l ⊠⊠⊠⊠4 ⊠⊠⊠⊠ l l' ⊠⊠⊠⊠⊠⊠⊠ $a_i, b_i (i = 1, 2, 3, 4)$ ⊠⊠ $a_i + b_i \geqslant 1$. ⊠⊠ $a_1 + b_1 + a_2 + b_2 + a_3 + b_3 + a_4 + b_4 = (a_1 + a_2 + a_3 + a_4) + (b_1 + b_2 + b_3 + b_4) \geqslant 4$ ⊠⊠ $a_1 + a_2 + a_3 + a_4$ ⊠ $b_1 + b_2 + b_3 + b_4$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠2 ⊠⊠⊠⊠ $b_1 + b_2 + b_3 + b_4 \geqslant 2$ ⊠⊠⊠⊠⊠⊠⊠2 ⊠⊠ l' ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠



(⊠8 ⊠)

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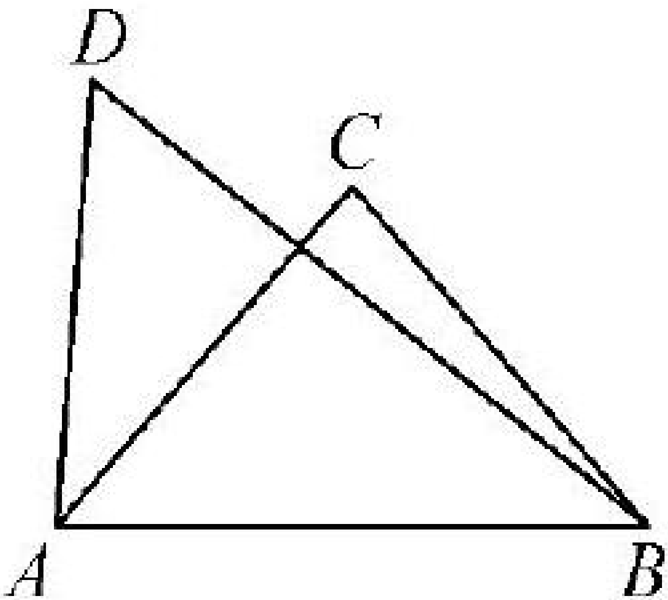
9. ⊠⊠⊠⊠ C D ⊠⊠⊠ AB ⊠⊠⊠. ⊠⊠⊠ A B C ⊠D ⊠⊠⊠⊠⊠ $\angle ADB \neq \angle ACB$ ⊠⊠⊠⊠ $\angle ADB < \angle ACB$ ⊠⊠⊠ A B D ⊠⊠⊠⊠⊠ $\angle ADB < \angle ACB$ ⊠⊠⊠ C ⊠ $\triangle ABD$ ⊠⊠⊠⊠⊠⊠⊠

10. ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠10 ⊠⊠⊠⊠⊠10 ⊠⊠⊠⊠⊠

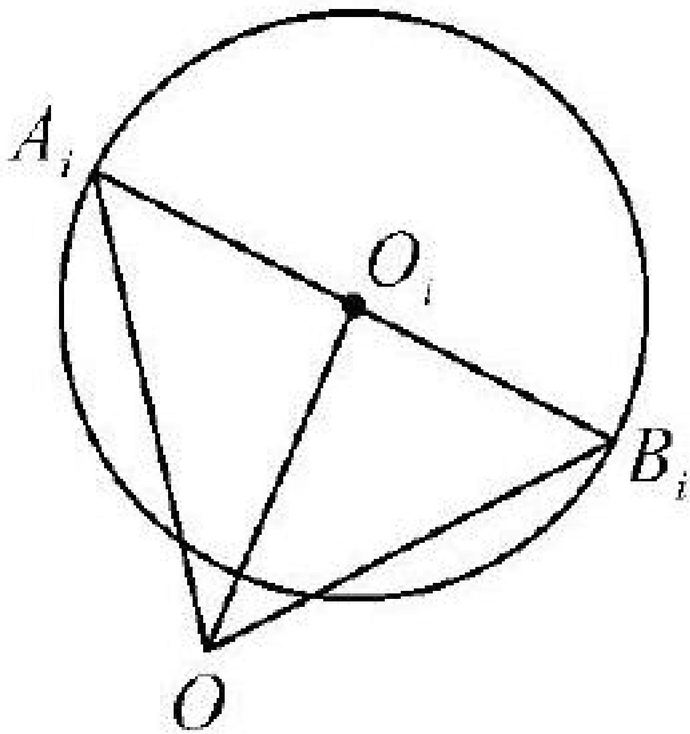


(⊠9 ⊠)

⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ O ⊠⊠⊠10 ⊠⊠⊠⊠⊠. ⊠⊠ O ⊠⊠5 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠ $r_1 < r_2 < r_3 < r_4 < r_5$ ⊠⊠⊠ O ⊠⊠⊠⊠r_3 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠

11. ⊠⊠⊠0.001 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠: ⊠⊠, ⊠⊠, ⊠⊠. ⊠ $\odot O_1$ ⊠ $\odot O_2$ ⊠⊠⊠ A B ⊠ ⊠⊠⊠⊠⊠ AO_1 ⊠ $\odot O_1$ ⊠⊠ C ⊠⊠ $AC = 0.001$ ⊠⊠⊠⊠⊠⊠⊠ $\odot O_1$ ⊠ $\odot O_2$ ⊠⊠⊠⊠ A ⊠⊠⊠ AO_1 ⊠ $\odot O_1$ ⊠⊠ C ⊠⊠ $AC = 0.001$ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠0.002 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ F ⊠⊠ F ⊠⊠⊠⊠⊠⊠0.001 ⊠⊠⊠⊠⊠⊠⊠ F_1 ⊠⊠ F ⊠⊠⊠⊠⊠⊠0.001 ⊠⊠⊠⊠⊠ F_2 ⊠⊠ F, F_1, F_2 ⊠⊠⊠⊠⊠⊠⊠⊠⊠1.001 ⊠⊠⊠⊠ ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ F_1, F_2, F ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ 1.001^2 ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠ F ⊠⊠⊠ $\leqslant \frac{1.001^2}{3} < 0.34$ ⊠

12. ⊠⊠ i ⊠⊠⊠⊠⊠ t ⊠ $t + 30$ ⊠⊠⊠⊠⊠⊠⊠ A_i, B_i ⊠⊠ i ⊠⊠⊠⊠⊠⊠ O_i, ⊠⊠⊠⊠⊠ O, ⊠ $OO_i \leqslant \frac{1}{2}\left(OA_i + OB_i\right)$ ⊠⊠⊠ $2\sum_{i=1}^{50} OO_i \leqslant \sum_{i=1}^{50} OA_i + \sum_{i=1}^{50} OB_i$ ⊠⊠⊠⊠ ⊠⊠⊠⊠, A_i ⊠ B_i ⊠⊠⊠⊠ OO_i ⊠, ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠ $\sum_{i=1}^{50} OO_i < \sum_{i=1}^{50} OA_i$ ⊠ $\sum_{i=1}^{50} OO_i < \sum_{i=1}^{50} OB_i$ ⊠⊠



(⊠12 ⊠)
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