Estimating Assortment Transference from Stockouts

Goal. Quantify *transference* (diversion) within a CDT: when a focal SKU i is unavailable, what fraction of customers who previously bought i purchase each alternative j, and what fraction walk (buy nothing in the CDT)?

Natural-experiment lens. Temporary stockouts (OOS) of i act as quasi-exogenous removals from the local choice set. Conditioning on customers whose *last CDT purchase* was i, and restricting to visits during i's OOS episodes, we observe their next choice among available j's and the outside option (walk). With adequate controls (price/promo of j, seasonality, store×week fixed effects), we interpret the observed reallocation as the causal transference pattern for i.

Units of observation and features. We work at the visit (user–store–week) level. Alternatives carry rich attributes (price, promo depth, size, brand, SKU2Vec/text embeddings). Customer taste is represented by a time-decayed profile derived from their purchase history in the CDT; store context includes location, format, traffic, and operational fixed effects.

Choice set and availability. For a store s and week t, the feasible set is the locally carried, sellable SKUs $C_{st} \subseteq S$ plus an outside option 0 (walk). We use only visits where i is OOS (thus $i \notin C_{st}$). To scale across 7,000+ stores and large catalogs, we optionally *sample* a subset of non-chosen alternatives per visit and apply a consistent correction (see Sec. : Choice-set sampling).

Outputs. For each focal i we produce a transference row $T_{i \to j}$ over all $j \in S \setminus \{i\}$ and $T_{i \to 0}$ (walk). We also compute store- (or region-) specific rows $T^{(s)}$ by conditioning on local assortments, with rules to redistribute probability mass from non-carried items.

Scale, store size, and heterogeneity. We absorb broad store-level differences with store×week fixed effects and optionally allow coefficients to vary with store-size proxies (traffic, selling space) via interactions. Final T can be produced nationally, by region, or per store, with traffic-based weighting to reflect target deployment.

Non-overlapping assortments. When two items i and j are rarely co-carried in the same store, we still infer $T_{i \to j}$ from other stores where j is available by leveraging feature-driven substitution structure (brand/size proximity, embeddings) and hierarchical shrinkage. For stores where j is not carried at scoring time, we re-normalize $T_{i \to j}$ over the locally available set (see Sec. : Deployment).

Limitations (reset vs. temporary OOS). Although OOS provides a clean natural experiment, it may *understate or overstate* long-run behavior under a planogram *reset*. In OOS episodes, some customers may defer purchase until i returns, whereas permanent resets force immediate re-optimization of preferences. This introduces bias relative to steady-state behavior; we flag this when applying T to long-horizon decisions and complement with robustness checks (event studies, longer OOS spells).

Blueprint with Math and Options

Notation and sample

CDT S; stores s; weeks t; users u. Availability indicator $A_{kst} \in \{0,1\}$. Define the visit set for focal i:

$$\mathcal{V}_i = \{(u,s,t): \text{ last CDT purchase} = i,\ A_{ist} = 0,\ \text{user visits CDT at } (s,t)\}.$$

Local choice set $C_{st} = \{j \in S: \ A_{jst} = 1\} \cup \{0\}$ (outside option 0).

Model (multinomial, with taste & context)

For each $(u,s,t) \in \mathcal{V}_i$ and $j \in C_{st}$,

$$U_{ujst} = \alpha_j + \beta^\top X_{ujst} + \gamma^\top Z_{uij} + \eta_{st} + \varepsilon_{ujst}, \tag{1}$$

$$P_{ujst} = \Pr(\text{choose } j \mid C_{st}) = \frac{\exp(U_{ujst})}{\sum_{\ell \in C_{st}} \exp(U_{u\ell st})}. \tag{2}$$

where

X_{ujst}: alternative-specific covariates (price, promo, size/brand, embeddings, store attributes).
Z_{uij}: pairwise similarity between i and j (embedding distances, format match, brand lineage).
η_{st}: store$\times$week fixed effects.
Options: (a) Mixed logit with random coefficients; (b) segment mixture on taste.

Choice-set sampling for scale

When $|C_{st}|$ is large, sample R non-chosen alternatives with inclusion probabilities π_j and add a correction to maintain consistency (weighted/exogenous sampling):

$$U^{\text{corr}}_{ujst} = U_{ujst} - \log \pi_j \quad \text{for sampled } j.$$

Train on the corrected utilities; at scoring, use the full C_{st} (or a high-coverage candidate set).

Visit-level probabilities and aggregation

Transference (cluster/national) for focal i:

$$T_{i\to j} = \frac{\sum_{(u,s,t)\in\mathcal{V}_i} w_{ust}\, P_{ujst}}{\sum_{(u,s,t)\in\mathcal{V}_i} w_{ust}}, \qquad \sum_{j\in S\setminus\{i\}} T_{i\to j} + T_{i\to 0} = 1.$$

Weights w_{ust} can reflect target deployment (e.g., traffic of store s). Store-specific rows: $T^{(s)}_{i\to j}$ computed by restricting the average to visits in store s (or a region) or by post-processing national T to local assortments (below).

Deployment to Local Assortments

Let $A_s \subseteq S$ be the set of SKUs carried in store s. For a focal i with national row $T_{i\to\cdot}$, define

$$S = \sum_{k\in A_s} T_{i\to k}, \quad U = \sum_{k\notin A_s} T_{i\to k}, \quad W = T_{i\to 0}, \qquad S + U + W = 1.$$

We map the national row to store s by choosing a policy for the missing mass U:

$$\textbf{R1 (Pure reallocation):}\quad W' = W, \quad T^{(s)}_{i\to j} = \begin{cases} \dfrac{T_{i\to j}}{S}\,(1-W') & j \in A_s, \\ 0 & j \notin A_s, \end{cases} \tag{3}$$

$$\textbf{R2 (Spill to walk):}\quad W' = W + U, \quad T^{(s)}_{i\to j} = \begin{cases} T_{i\to j} & j \in A_s, \\ 0 & j \notin A_s, \end{cases} \tag{4}$$

so that in both cases $\sum_{j\in A_s} T^{(s)}_{i\to j} + T^{(s)}_{i\to 0} = 1$ with $T^{(s)}_{i\to 0} = W'$. A hybrid policy with parameter $\alpha \in [0,1]$ sets $W' = W + \alpha U$ and $T^{(s)}_{i\to j} = \frac{T_{i\to j}}{S}(1-W')$ for $j \in A_s$. If $S = 0$, set $T^{(s)}_{i\to 0} = 1$ and all other probabilities to 0.

Appendix:Tiny Example (Numerical)

National transference for focal i:

$$T_{i\to j_1} = 0.30, \quad T_{i\to j_2} = 0.20, \quad T_{i\to j_3} = 0.15, \quad T_{i\to j_4} = 0.10, \quad T_{i\to 0} = 0.25.$$

Store s carries $A_s = \{j_1, j_4\}$. Then

$$S = \sum_{k\in A_s} T_{i\to k} = 0.30 + 0.10 = 0.40, \quad U = \sum_{k\notin A_s} T_{i\to k} = 0.20 + 0.15 = 0.35, \quad W = T_{i\to 0} = 0.25.$$

R2 (Spill missing mass to walk).

$$W' = W + U = 0.25 + 0.35 = 0.60, \qquad T^{(s)}_{i\to j_1} = 0.30, \;\; T^{(s)}_{i\to j_4} = 0.10, \;\; T^{(s)}_{i\to 0} = 0.60.$$

R1 (Pure reallocation among available). Scale available items by $\dfrac{1-W'}{S} = \dfrac{1-0.25}{0.40} = \dfrac{0.75}{0.40} = 1.875$ and keep walk unchanged:

$$T^{(s)}_{i\to j_1} = 0.30 \times 1.875 = 0.5625, \quad T^{(s)}_{i\to j_4} = 0.10 \times 1.875 = 0.1875, \quad T^{(s)}_{i\to 0} = 0.25.$$

Alternative	**National** $T_{i\to\cdot}$	$T^{(s)}$ **(R2)**	$T^{(s)}$ **(R1)**
$j_1 \in A_s$	0.30	0.30	0.5625
$j_2 \notin A_s$	0.20	0	0
$j_3 \notin A_s$	0.15	0	0
$j_4 \in A_s$	0.10	0.10	0.1875
walk (0)	0.25	0.60	0.25

In both policies, the probabilities sum to 1 for the local assortment.

Appendix:Diagnostics and identification checks

Event-study on OOS onsets with pre-trend tests; placebo OOS weeks; sensitivity to excluding multi-SKU outages; instrument OOS with upstream disruptions when available. Monitor Brier/log-loss on holdouts and calibration curves for walk vs. purchase.

Appendix: Modeling Multi-Removal Effects in Transference

Motivation. The transference matrix $T_{i \to j}$ is estimated under single-SKU removals: it reflects how demand for SKU i reallocates across alternatives j when i is absent but all other SKUs remain. In an optimization context, multiple SKUs may be removed simultaneously. To quantify the bias this induces, we compute the *effective retained demand* under recursive substitution.

Markov absorption formulation. Let S denote the set of all SKUs and let T be the row-stochastic transference matrix over $S \cup \{0\}$, where index 0 denotes the *walk* (outside) option. Each row $T_{i \to \cdot}$ satisfies $\sum_{j \in S \cup \{0\}} T_{i \to j} = 1$.

Partition S into the chosen (kept) and removed sets:

$$S_{\text{keep}} = \{i : x_i = 1\}, \quad S_{\text{drop}} = \{i : x_i = 0\}.$$

Reorder T as

$$T = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}, \qquad Q \in \mathbb{R}^{|S_{\text{drop}}| \times |S_{\text{drop}}|},\ R \in \mathbb{R}^{|S_{\text{drop}}| \times (|S_{\text{keep}}|+1)}.$$

Here Q captures transitions among removed SKUs, while R links removed SKUs to either kept SKUs or the walk state.

Under standard absorbing Markov chain theory, the *fundamental matrix* is

$$N = (I - Q)^{-1} = I + Q + Q^2 + Q^3 + \cdots,$$

and the corresponding absorption probabilities are

$$B = NR.$$

Entry $B_{i,j}$ gives the total probability that demand originating at removed SKU i ultimately reaches state j (a kept SKU or walk) after any number of recursive transfers.

Hence, the effective retained productivity of removed SKU i is

$$\hat{p}_i^{(\text{keep})} = p_i \sum_{j \in S_{\text{keep}}} B_{i,j}, \qquad \hat{p}_i^{(\text{walk})} = p_i \, B_{i,0}.$$

The total corrected category productivity for assortment x is

$$P_{\text{corr}}(x) = \sum_{i \in S_{\text{keep}}} p_i + \sum_{i \in S_{\text{drop}}} \hat{p}_i^{(\text{keep})}.$$

Comparing $P_{\text{corr}}(x)$ with the modeled first-order objective

$$P_{\text{model}}(x) = \sum_i p_i x_i + \sum_{i,j} T_{i \to j}(1 - x_i) x_j p_j$$

quantifies the magnitude of neglected higher-order transference effects.

Second-order embedded objective. If the gap $\Delta = P_{\text{corr}}(x^*) - P_{\text{model}}(x^*)$ is empirically large (e.g., exceeding 10% of category sales), the optimization objective can be extended to include explicit second-order terms:

$$\max_x \quad \underbrace{\sum_i p_i x_i}_{\text{own productivity}} + \underbrace{\sum_{i,j} T_{i\to j}(1-x_i)x_j p_i}_{\text{first-order transfer}} + \underbrace{\sum_{i,j,k} T_{i\to j}T_{j\to k}(1-x_i)(1-x_j)x_k p_k}_{\text{second-order transfer}}.$$

The third term captures demand that would have been redirected from i to j (had j remained) but must now flow onward to k when both i and j are excluded. It introduces cubic interactions in x. To maintain tractability, we truncate at second order and linearize each triple product $(1-x_i)(1-x_j)x_k$ using auxiliary binary variables and McCormick envelopes. This formulation restores fidelity in categories with deep or hierarchical substitution chains, while keeping the problem solvable as a mixed-integer quadratic (or piecewise linear) program.

Rationale and interpretation. This approach aligns naturally with how the optimization already behaves. Even in its first-order form, the model implicitly prioritizes SKUs that act as strong *absorbers* of demand, i.e., those that receive high inbound transference from neighboring items. Such SKUs tend to remain selected across all model variants because they capture diverted demand and stabilize total productivity when similar items are removed.

Evaluating the optimized assortment through the Markov absorption framework therefore serves two purposes: it quantifies how completely the chosen set retains demand under realistic multi-removal scenarios, and it provides a transparent, data-grounded check on model fidelity.

Comparing the modeled productivity with its recursively adjusted value is an intuitive diagnostic: when the two are close, the current optimization already internalizes most substitution behavior; when they diverge, the difference points precisely to where higher-order dynamics matter. This makes the diagnostic both analytically rigorous and operationally actionable, making a direct bridge between model structure and business interpretability.

Summary. The Markov absorption step provides an ex-post diagnostic of how much lost demand re-enters the assortment through recursive substitution. When this correction is minor, the first-order model suffices. When it is material, the second-order embedded objective above offers a principled extension that internalizes multi-removal effects directly in the optimization.