Distributed Model Predictive Control for Nonlinear Networked Systems with Asynchronous Communication

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Introduction

Distributed model predictive control for networked nonlinear systems:

- The large-scale systems with decoupled dynamics
- The large-scale systems with coupled dynamics

This presentation considers the distributed networked nonlinear systems subject to asynchronous communication.

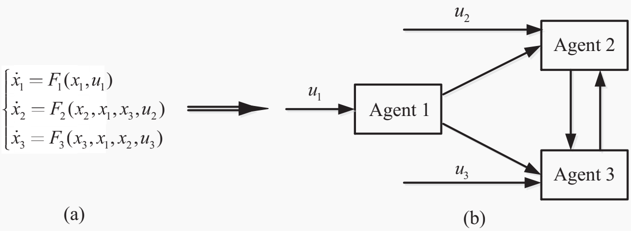


Figure: The concerned control structure

System setup

Nonlinear dynamics of each agent $\mathcal{A}_i$, $i \in \mathcal{I} := I[1, N]$:

$$\dot{x}_i(t) = f_i\left(x_i(t), u_i(t)\right) + g_i\left(\left\{x_j(t)\right\}_{j \in \mathcal{N}_i}\right), \quad t \geq t_0 \tag{1}$$

where $x_i \in \mathcal{X}_i \subseteq \mathbb{R}^n$, $u_i \in \mathcal{U}_i \subseteq \mathbb{R}^m$.
Define a graph: $\mathcal{G} = (\mathcal{V}, \epsilon)$ with $\mathcal{V} = \{1, ..., N\}$,
$\epsilon = \{(i, j) \in \mathcal{V} \times \mathcal{V}\}$.

- By the the connected graph of (7), we get the upstream neighbor set: $\mathcal{N}_i \subseteq \mathcal{I} \setminus \{i\}$;
- Through the communication of agent $\mathcal{A}_i$, we obtain the downstream neighbor set: $\mathcal{D}_i \subseteq \mathcal{I} \setminus \{i\}$;

Then, we have:

- $\mathcal{N}_i := \{j \in \mathcal{V} | (j, i) \in \epsilon\}$, $\mathcal{D}_i := \{j \in \mathcal{V} | (i, j) \in \epsilon\}$;
- $\mathcal{N}_i \cup \mathcal{D}_i \subset \mathcal{I}$, $i \notin \mathcal{N}_i \cap \mathcal{D}_i$;
- $j \in \mathcal{N}_i$ if and only if $i \in \mathcal{D}_j$.

System setup

Let $M = |\epsilon|$ and $d_i = |\mathcal{N}_i|$. There is at least one agent in $\mathcal{I}$ satisfying $\mathcal{N}_i \neq \emptyset$.

$$0 < \max_{i \in \mathcal{I}} d_i \leq M \tag{2}$$

If $d_i \neq 0$, then let $\mathcal{N}_i = \{i_1, i_2, ..., i_{d_i}\}$. Denote the sequences of state trajectories for all neighbors in $\mathcal{N}_i$ as $x_{-i}(t)$, i.e.,

$$x_{-i}(t) = vect\{x_{i_1}(t), x_{i_2}(t), ..., x_{i_{d_i}}(t)\} \tag{3}$$

the whole system can be described as

$$\dot{x} = f(x, u) + g(x) := F(x, u) \tag{4}$$

where $f(x, u) = vect(f_1(x_1, u_1), f_2(x_2, u_2), ..., f_N(x_N, u_N))$ and $g(x) = diag(g_1(x_{-1}), g_2(x_{-2}), ..., g_N(x_{-N}))$.

Preliminary results

linearized (1) around the origin, we get

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \Sigma_{j\in\mathcal{N}_i} A_{ij} x_j(t), \quad t \geq t_0 \qquad (5)$$

where $A_i = \partial f_i/\partial x_i|_{(0,0)}$, $B_i = \partial f_i/\partial u_i|_{(0,0)}$, $A_{ij} = \partial g_i/\partial x_j|_{(0,0)}$.

Assumption 2.

For each agent $\mathcal{A}_i$ in (7), there exists a decoupled static feedback matrix K_i such that $A_{di} := A_i + B_i K_i$ is Hurwitz.

Lemma 1[24]. For each $\mathcal{A}_i$ in (1) with $R_i > 0$, $Q_i > 0$, there exist $P_i > 0$ and $\varepsilon_i > 0$, satisfying: (I) $P_i A_{di} + A_{di}^T P_i = -\bar{Q}_i$; (II) $PA_o + A_o^T P \leq \bar{Q}/2$, where $\bar{Q}_i = Q_i + K_i^T R_i K_i$, $A_o = A_c - A_d$, $\Xi = diag(\Xi_1, ..., \Xi_N)$ for any set $\Xi_i \in \{P_i, K_i, \bar{Q}_i, A_{di}\}$, such that the set $\Omega_i(\varepsilon_i) \overset{\Delta}{=} \{x_i(t) : \|x_i(t)\|_{P_i}^2 \leq \varepsilon_i^2\}$ is a positively invariant region for $\dot{x}_i(t) = f_i(x_i(t), u_i(t)) + g_i(x_{-i}(t))$. Additionally, $x_i \in \mathcal{X}_i$, $K_i x_i \in \mathcal{U}_i$ for all $x_i \in \Omega_i(\varepsilon_i)$.

Problem description

With the different control and communication frequency, define the update time sequence $\{t_k^i, k \in \mathbb{N}\}$. At each t_k^i,

(1) Measure $x_i(t_k^i)$ for $\mathcal{A}_i$ by the sampler;

(2) Obtain $\hat{u}_i^*(\cdot; t_k^i)$ by the designed optimization problem P_i and apply to $\mathcal{A}_i$;

(3) Send out states and controls to all neighbors in $\mathcal{D}_i$;

Since for agents $\mathcal{A}_i$ and $\mathcal{A}_l$, $i, l \in \mathcal{I}$, $l \in \mathcal{N}_i$, $\{t_k^i\}_{k=1}^{\infty} \not\equiv \{t_k^l\}_{k=1}^{\infty}$.

The control target:

To design an effective control optimization problem P_i for agent $\mathcal{A}_i$ which also coordinates the asynchronous communication for all its neighboring agents.

Develop an DMPC to steer the state to $\Omega_i(\varepsilon_i)$

To-be-minimized cost function $J_i := J_i(x_i(t_k^i), u_i(s; t_k^i))$ associated with (7) is defined as

$$J_i = \int_{t_k^i}^{t_k^i+T_i} \|x_i(s; t_k^i)\|_{Q_i}^2 + \|u_i(s; t_k^i)\|_{R_i}^2 ds + \|x_i(t_k^i + T_i; t_k^i)\|_{P_i}^2 \quad (6)$$

For $\mathcal{A}_i$ at t_k^i, design an *OCP* (P_i) as

$$\hat{u}_i^*(s; t_k^i) = \arg \min_{\hat{u}_i(s; t_k^i)} J_i(x_i(t_k^i), \hat{u}_i(s; t_k^i))$$

$$s.t. \quad \dot{\hat{x}}_i(s; t_k^i) = f_i\left(\hat{x}_i(s; t_k^i), \hat{u}_i(s; t_k^i)\right) + g_i\left(\tilde{x}_{-i}(s; t_k^i)\right) \quad (7a)$$

$$\|\hat{x}_i(s; t_k^i)\|_{P_i} \le \frac{T}{s - t_k^i} \frac{\alpha_i}{(q+1)} \varepsilon_i \quad (7b)$$

$$\hat{u}_i(s; t_k^i) \in \mathcal{U}_i \quad (7c)$$

Virtual input design with asynchronous communication

Define

$$[k]_i = \max\{l : t_l^j \leq t_k^i, j \in \mathcal{N}_i, \quad l = 0, 1, 2, \dots\} \tag{8}$$

For $j \in \mathcal{N}_i := \{i_{d_1}, i_{d_2}, \dots, i_{d_i}\}$, $\tilde{x}_j(s; t_k^i)$ for $\mathcal{A}_j$ is generated by following the rules:

r1 : IF $t_{[k]_i}^j + T_i \leq t_k^i$, THEN take

$$\tilde{x}_j(s; t_k^i) = x_j^L(s), \quad s \in [t_{[k]_i}^j + T_i, t_k^i + T_i] \tag{9}$$

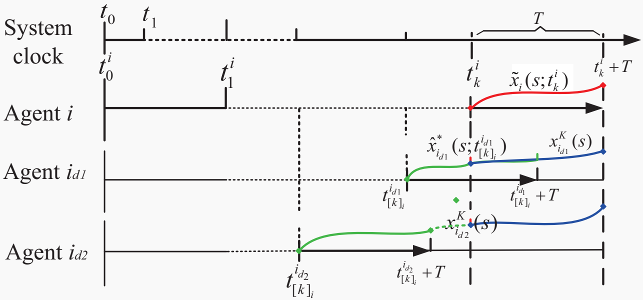
r2 : IF $t_{[k]_i}^j + T_i > t_k^i$, THEN take

$$\tilde{x}_j(s; t_k^i) = \begin{cases} \hat{x}_j^*(s; t_{[k]_i}^j), & s \in [t_k^i, t_{[k]_i}^j + T_i) \\ x_j^L(s), & s \in [t_{[k]_i}^j + T_i, t_k^i + T_i] \end{cases} \tag{10}$$

where $x_j^L(s)$ follows the solution of linearization dynamics

$$\begin{cases} \dot{x}_j^L(s) = A_{dj} x_j^L(s) \\ x_j^L(t_{[k]_i}^j + T_i) = \hat{x}_j^*(t_{[k]_i}^j + T_i; t_{[k]_i}^j) \end{cases} \tag{11}$$

Virtual input design with asynchronous communication



Consider $\mathcal{N}_i = \{i_{d_1}, i_{d_2}\}$,

- When $t^j_{[k]_i} + T_i \leq t^i_k$, then $\tilde{x}_j(s;t^i_k)$ is taken over by the closed-loop linearization response which ignores coupling;
- For $t^j_{[k]_i} + T_i > t^i_k$, $\tilde{x}_j(s;t^i_k)$ is constructed by the remainder of the previously predicted trajectory $\hat{x}^*_j(s;t^j_{[k]_i})$, concatenated with $x^L_j(s)$ in (11)

Asynchronous DMPC with dual-mode strategy

The developed asynchronous DMPC approach will be improved by the dual-mode strategy.
The terminal region for $\mathcal{A}_i$: $\mathcal{X}_i^f := \{x_i : W_i(\hat{x}_i(t_k^i + T_i; t_k^i)) \leq \varepsilon_i^2\}$.
For agent $\mathcal{A}_i$ at time t_k^i,

- when $x_i(t_k^i) \notin \Omega_i(\alpha_i \varepsilon_i)$, then the optimization problem of DMPC P_i is solved and the optimized control input $u_i(s; t_k^i) = \hat{u}_i^*(s; t_k^i)$ is applied during $[t_k^i, t_{k+1}^i)$;
- when $x_i(t_k^i) \in \Omega_i(\alpha_i \varepsilon_i)$, then the control input is switched into the static feedback control law as

$$\hat{u}_i(s; t_k^i) = K_i x_i(s; t_k^i) \tag{12}$$

.

Algorithm 1:

0) *Initialization*: $x_i(t_0)$, $\hat{u}_i^*(s;t_0)$ (or $u_i(s;t_0)$) at time $t_0^i = t_0$, $\tilde{x}_i(s;t_0^i)$, $s \in [t_0^i, t_0^i + T_i]$, and $T_i \in (0,\infty)$, $q \in \{1,2,3,\dots\}$.

1) Apply $\hat{u}_i^*(s;t_k^i)$ for $s \in [t_k^i, t_k^i + T_i]$ and measure $x_i(s;t_k^i)$, if $x_i(t) \in \Omega_i(\alpha_i\varepsilon_i)$, then go to step 4).

2) Obtain $(t^j_{[k+1]_i}, x_j(t^j_{[k+1]_i}), \hat{x}_j^*(s;t^j_{[k+1]_i}))$ for all $\mathcal{A}_j$ where $j \in \mathcal{N}_i$, generate $\tilde{x}_j(s;t_{k+1}^i)$ according to (9) or (10) and stack them together into $\tilde{u}_{-i}(s;t_k^i)$.

3) Solve P_i in (7) to obtain $\hat{u}_i^*(s;t_{k+1}^i)$, $s \in [t_{k+1}^i, t_{k+1}^i + T_i]$, compute $\hat{x}_i^*(s;t_{k+1}^i)$, $s \in [t_{k+1}^i, t_{k+1}^i + T_i]$ and send it to all neighbors $\mathcal{A}_j$ in $\mathcal{D}_i$, then go to step 1).

4) Generate the terminal control policy as (12) at instant time t_k^i and apply it to agent $\mathcal{A}_i$ for all $[t_k^i, t_{k+1}^i)$.

Theorem 1 (*Boundedness*). For each agent $\mathcal{A}_i$ with dynamics described in (7), at update time instant t_k^i, if the trajectory $\tilde{x}_j(s; t_k^i)$ is generated by following the rules (9) or (10), then we have

$$\max_{j \in \mathcal{N}_i} \|\hat{x}_j^*(s; t_k^i) - \tilde{x}_j(s; t_k^i)\|_{P_i} \leq \frac{2}{q+1} \bar{\alpha}_i \bar{\varepsilon}_i \tag{13}$$

where $s \in [t_k^i, t_k^i + T_i]$ and $\bar{\alpha}_i \bar{\varepsilon}_i = \max_{j \in \mathcal{N}_i \cup \{i\}} \{\alpha_j \varepsilon_j\}$.

Theorem 2 (*Feasibility*). For each agent $\mathcal{A}_i$ in (7), suppose that Assumption 1-2 hold true and $x_i(t_0) \in \mathcal{X}_i^0$. If the prediction horizon T_i, the minimum update period $\Delta t_i = \min_{k\in\mathbb{N}}\{t_{k+1}^i - t_k^i\}$ and the global constant q are designed to satisfy the following inequalities

$$e^{L_{f_i} T_i} T_i \leq \frac{\alpha_i \varepsilon_i}{4 d_i L_{g_i} \bar{\alpha}_i \bar{\varepsilon}_i} \tag{14a}$$

$$\Delta t_i \geq \frac{2 d_i L_{g_i} \bar{\alpha}_i \bar{\varepsilon}_i e^{L_{f_i} T} {T_i}^2}{\alpha_i \varepsilon_i - 2 d_i L_{g_i} \bar{\alpha}_i \bar{\varepsilon}_i e^{L_{f_i} T_i} T_i} \tag{14b}$$

$$q \geq \frac{(\alpha_i - 1) L_{g_i} \bar{\alpha}_i \bar{\varepsilon}_i + (4 - 2\alpha_i) d_i L_{g_i} \bar{\alpha}_i \bar{\varepsilon}_i e^{L_{f_i} T_i} T_i}{\alpha_i \varepsilon_i - 4 d_i L_{g_i} \bar{\alpha}_i \bar{\varepsilon}_i e^{L_{f_i} T_i} T_i} \tag{14c}$$

then the designed Algorithm 1 for $\mathcal{A}_i$ is recursively feasible to steer the actual trajectory to the positively invariant set $\Omega_i(\varepsilon_i)$.

Remark 4: Theorem 2 indicates that the update time interval sequence $\{t^i_{k+1} - t^i_k, k \in \mathbb{N}\}$ for each $\mathcal{A}_i$ is required to satisfy the allowable lower bound. Since $T_i \geq t^i_{k+1} - t^i_k \geq \Delta t_i = \min_{k\in\mathbb{N}}\{t^i_{k+1} - t^i_k\}$, then the predictive horizon T is bounded as

$$e^{L_{f_i}\Delta t_i}\Delta t_i \leq e^{L_{f_i}T_i}T_i \leq \frac{\alpha_i\varepsilon_i}{4d_i L_{g_i}\bar{\alpha}_i\bar{\varepsilon}_i} \tag{15}$$

Since q plays an important role in guaranteing the feasibility of the proposed algorithm, α_i can be used to adjust the contracting effect of the constraint for the predicted state variables.

Theorem 3 (*Stability*). For agent $\mathcal{A}_i$ in (7), suppose that Assumption 1-2 hold, $x_i(t_0) \in \mathcal{X}_i^0$. If the conditions in Theorem 1 are satisfied and the prediction horizon T_i, the constant integer q, the shrinkage rate α_i and the minimum update time interval Δt_i are designed to make the following hold:

$$\begin{aligned} &\lambda_M \frac{4d_i L_{g_i} T_i^2 \bar{\alpha}_i \bar{\varepsilon}_i}{(q+1)^2} \left\{ d_i L_{g_i} \bar{\alpha}_i \bar{\varepsilon}_i \frac{e^{2L_{f_i}(T_i-\Delta t_i)}-1}{2L_{f_i}} \right. \\ &\left. +\alpha_i \varepsilon_i \left[\frac{e^{2L_{f_i}(T_i-\Delta t_i)}-1}{4L_{f_i}} + \frac{(T_i-\Delta t_i)^2}{2T_i\Delta t_i} - \frac{2}{T_i-\Delta t_i} \right] \right\} \\ &< \left[\lambda_m \Delta t_i + 1 - \alpha_i^2\right] \varepsilon_i^2 \end{aligned} \tag{16}$$

where $\lambda_M = \lambda_{\max}(P_i^{-1/2} Q_i P_i^{-1/2})$, $\lambda_m = \lambda_{\min}(P_i^{-1/2} Q_i P_i^{-1/2})$. Then by application of the proposed algorithm for each $\mathcal{A}_i$, the closed-loop networked system in (4) is asymptotically stable.

Remark 5: Based on Theorem 2, Theorem 3 and their proofs, we have:

Principles of choosing parameters

- **S1** Calculate $\mathcal{N}_i, d_i, \mathcal{D}_i, \varepsilon_i, P_i, K_i, L_{f_i}, L_{g_i}$ for each agent $\mathcal{A}_i$;
- **S2** Choose α_i, $\{t_k^i\}, k \in \mathbb{N}$ and T_i, calculate $\bar{\alpha}_i \bar{\varepsilon}_i$, to satisfy the condition in Theorem 1;
- **S3** Choose q to make the inequality in Theorem 2 hold;
- **S4** Examine whether the condition in Theorem 3 could be satisfied. Otherwise, choose a smaller α_i and go back to Step S2, until (16) is satisfied.

Application to Van der Pol oscillators

Consider a walking bipedal locomotor:

$$\begin{cases} \ddot{\theta}_1(t) = 0.1[1-5.25\theta_1^2(t)]\dot{\theta}_1(t) - \theta_1(t) + u_1(t) \\ \ddot{\theta}_2(t) = 0.01[1-6070(\theta_2(t)-\theta_{2e})^2]\dot{\theta}_2(t) - 4(\theta_2(t)-\theta_{2e}) \\ \qquad + 0.057\theta_1(t)\dot{\theta}_1(t) + 0.1(\dot{\theta}_2(t)-\dot{\theta}_3(t)) + u_2(t) \\ \ddot{\theta}_3(t) = 0.01[1-192(\theta_3(t)-\theta_{3e})^2]\dot{\theta}_3(t) - 4(\theta_3(t)-\theta_{3e}) \\ \qquad + 0.057\theta_1(t)\dot{\theta}_1(t) + 0.1(\dot{\theta}_3(t)-\dot{\theta}_2(t)) + u_3(t) \end{cases}$$

with The constraints for $\theta_i(t)$ and $u_i(t)$ are

$$-\frac{\pi}{2} \le \theta_i(t) - \theta_{ie} \le \frac{\pi}{2}, \quad |u_i(t)| \le 1, \quad \forall t \ge 0$$

To perform the control strategy, we define the states $x_i = [x_{i1} \quad x_{i2}]^T$ for dynamics in (7), where $x_{i2} = \dot{\theta}_i$, $x_{i1} = \theta_i - \theta_{ie}$ with θ_{ie} the desired constant angle for $i \in \{1, 2, 3\}$. Then, the system can be converted to

$$\mathcal{A}_1 : \begin{cases} \dot{x}_{11}(t) = x_{12}(t) \\ \dot{x}_{12}(t) = 0.1\left[1 - 5.25x_{11}^2(t)\right] x_{12}(t) - x_{11}(t) + u_1(t) \end{cases}$$

$$\mathcal{A}_2 : \begin{cases} \dot{x}_{21}(t) = x_{22}(t) \\ \dot{x}_{22}(t) = 0.01\left[1 - 6070x_{21}(t)^2\right] x_{22}(t) - 4x_{21}(t) + u_2(t) \\ \qquad\qquad +0.057x_{11}(t)x_{12}(t) + 0.1\left(x_{22}(t) - x_{32}(t)\right) \end{cases}$$

$$\mathcal{A}_3 : \begin{cases} \dot{x}_{31}(t) = x_{32}(t) \\ \dot{x}_{32}(t) = 0.01\left[1 - 192x_{31}(t)^2\right] x_{32}(t) - 4x_{31}(t) + u_3(t) \\ \qquad\qquad +0.057x_{11}(t)x_{12}(t) + 0.1\left(x_{32}(t) - x_{22}(t)\right) \end{cases}$$

$$K_1=\begin{bmatrix} 0.25 \\ -2.1 \end{bmatrix},\ K_i=\begin{bmatrix} 1.5 \\ -3.61 \end{bmatrix},\quad i=2,3$$

$$A_c=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -4 & 0.11 & 0 & -0.1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.1 & -4 & 0.11 \end{bmatrix}$$

$$J_i(x_i(t),u_i(t))=\int_t^{t+T_i}(30x_{i1}^2+30x_{i2}^2+0.1u_i^2)d\tau+\|x_i(t+T_i)\|_{P_i}^2$$

$$P_1=\begin{bmatrix} 53.3 & 20 \\ 20 & 17.6 \end{bmatrix},\quad P_2=P_3=\begin{bmatrix} 37.2 & 6.0 \\ 6.0 & 6.2 \end{bmatrix}$$

$$\begin{cases} L_{f_1}=4.1 \\ L_{g_1}=0 \end{cases};\quad \begin{cases} L_{f_i}=4 \\ L_{g_i}=0.1 \end{cases},\quad i=2,3$$

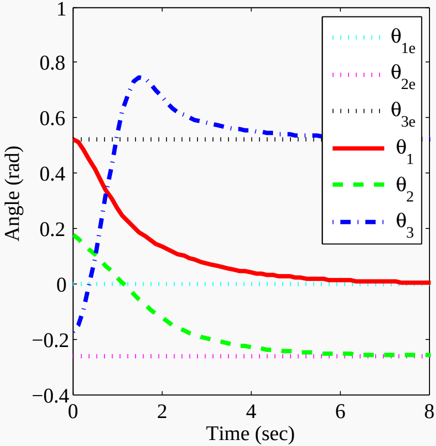


Figure: The closed-loop response of state one

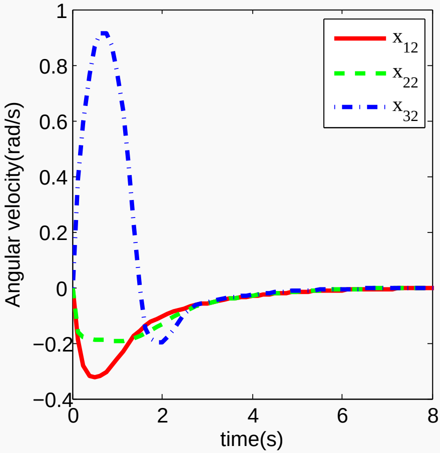


Figure: The closed-loop response of state two

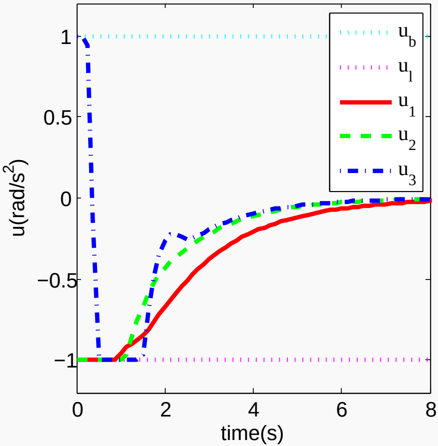


Figure: The inputs of closed-loop system

Conclusion

The basic idea behind this algorithm is to utilize the information associated with the interconnected upstream neighbors of complex networked systems to a distributed design.

Questions ?

Thank you!