Combined Iterative Learning and Model Predictive Control Scheme for Nonlinear Batch Processes

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The 4th International Conference on Industrial Artificial Intelligence
(IAI 2022)
August 26, 2022

Injection Molding Applications

Introduce the reciprocating-screw injection molding (IM) machine at HKUST:

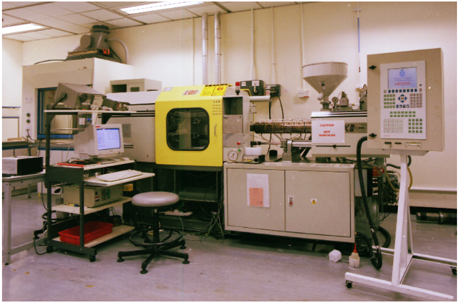


Figure: Photo of the IM machine.

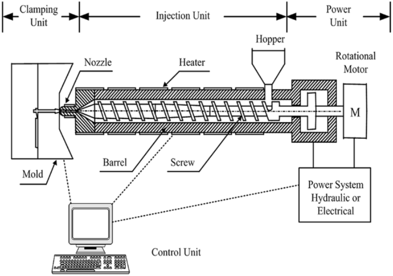


Figure: Simplified schematic diagram.

- IM is a batch process [Wang et al. 2009];
- Methods developed for IM is equally well used for other types of batch processes [Zhou et al. 2022a].

Background

Two effective control methods for IM process:

Iterative learning control (ILC)

1. ILC can learn from historical process data [Wang et al. 2009];
2. ILC can not handle hard constraints;
3. Constrained ILCs limit learning capability.

Model predictive control (MPC)

1. MPC is good at handling hard constraints [Xi & Li, 2019];
2. Uncertain model & disturbances affect performance;
3. Large effort spent on model identification offline (time & money).

Remarks

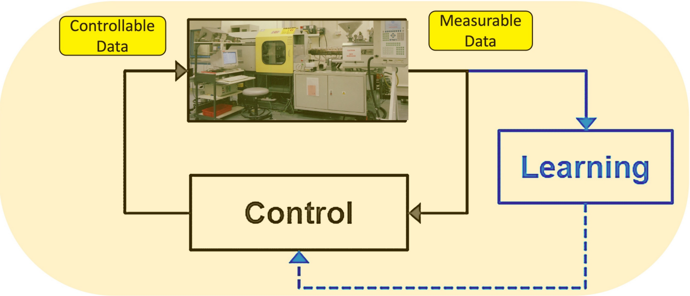
ILC and MPC both have their advantages and limitations.

Motivations

We want to ask:

How to inherit their respective strengths and weaken their limitations ?

To solve the problem, here comes the batch process control learning paradigm:



Our solution in the present work

A combined ILC and MPC scheme through a two-dimensional (2D) framework.

A combined 2D design paradigm:

The ILC part:

- Using optimal run-to-run feedback with the historical batch data;

The MPC part:

- Using real-time feedback with the current sampled measurements within the batch.

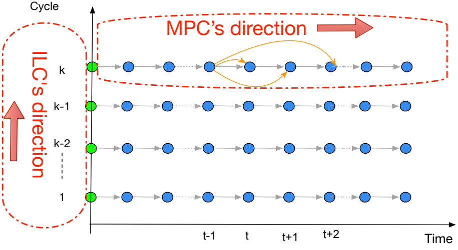


Figure: Illustration of the 2D-ILMPC scheme.

Advantage:

This improves batch and time control performance and allows time-wise constraints to be met.

Problem Formulation

Consider a nonlinear batch process

$$x_k(t+1) = f(x_k(t), u_k(t), t), \tag{1}$$
$$y_k(t) = Cx_k(t), \tag{2}$$

subject to the constraints

$$u_{i,k}(t) \in \mathbb{U}_i \triangleq \left[\underline{u}_i, \overline{u}_i\right] \tag{3}$$
$$u_{i,k}(t) - u_{i,k}(t-1) \in \delta\mathbb{U}_i \triangleq \left[\overline{\delta u}_i, \underline{\delta u}_i\right] \tag{4}$$
$$u_{i,k}(t) - u_{i,k-1}(t) \in \Delta\mathbb{U}_i \triangleq \left[\underline{\mathrm{d}u}_i, \overline{\mathrm{d}u}_i\right] \tag{5}$$

Remarks

In the case of an unknown f, design an optimal input such that (1)–(2) can track the desired trajectory $y_r(t)$, where $y_r(t)$ is assumed to be realizable for (1)–(2).

Problem Formulation

Assume that linear model (A, B) is available:

$$\bar{x}_k(t+1) = A\bar{x}_k(t) + Bu_k(t), \tag{6}$$

$$\bar{y}_k(t+1) = C\bar{x}_k(t+1), \tag{7}$$

then the induced model discrepancy can be quantified as

$$\zeta_k(t) \triangleq f(x_k(t), u_k(t)) - Ax_k(t) - Bu_k(t), \tag{8}$$

The tracking error is given as

$$e_k(t) = y_r(t) - y_k(t), \tag{9}$$

Then, the actual tracking error system is formulated as

$$\begin{aligned} e_k(t+1) &= y_r(t+1) - Cx_k(t+1) \\ &= y_r(t+1) - CAx_k(t) - CBu_k(t) - C\zeta_k(t) \end{aligned} \tag{10}$$

Problem Formulation

Control objective

The control objective for the system (1) and (2) is to drive the tracking error $e_k(t)$ to the minimum along time direction and convergent to the origin at every time over several successive batches, which operates in two dimensions.

Design of combined ILC-MPC Scheme

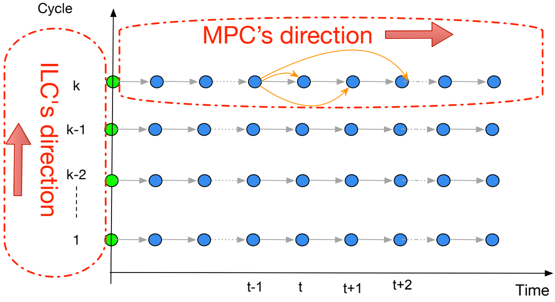


Figure: Illustration of the 2D-ILMPC scheme.

Advantage:

This improves batch and time control performance and allows time-wise constraints to be met.

First, using (1) and (8), we obtain

$$x_k(t+1) = Ax_k(t) + Bu_k(t) + \zeta_k(t), \tag{11}$$

Note that the linear prediction error from the previous cycle is

$$z_{k-1}(t+1) = y_{k-1}(t+1) - \bar{y}_{k-1}(t+1). \tag{12}$$

Hence, using (6) and (7), the single-step forward prediction of the output $y_k(t+1)$ can be designed as

$$\hat{y}_k(t+1) = CAx_k(t) + CBu_k(t) + z_{k-1}(t+1), \tag{13}$$

where $t \in \mathbb{I}_{[0,T]}$.

Design of combined ILC-MPC Scheme

Next, we design the input signal $u_k(t)$ as

$$u_k(t) = u_{k,\mathrm{ILC}}(t) + u_{k,\mathrm{MPC}}(t) \tag{14}$$

with

$$u_{k,\mathrm{ILC}}(t) = u_{k-1}(t) + \Delta u_{k,\mathrm{ILC}}(t). \tag{15}$$

where $u_{k,\mathrm{ILC}}(t)$ denotes the ILC control law and $u_{k,\mathrm{MPC}}(t)$ denotes the MPC control law.

To satisfy the time-wise constraints given by (3)-(5), the cycle- wise optimal ILC law will be split into time-wise components for further optimal revisions in the subsequent MPC design part.

Optimal Design of the ILC Part

Define the updating ILC law

$$\Delta \mathbf{u}_{k,\mathrm{ILC}} \triangleq \left[\Delta u_{k,\mathrm{ILC}}^\top(0), \ldots, \Delta u_{k,\mathrm{ILC}}^\top(T-1)\right]^\top. \tag{16}$$

Introduce

$$\mathbf{u}_{k,\mathrm{ILC}} = \mathbf{u}_{k-1} + \Delta \mathbf{u}_{k,\mathrm{ILC}} \tag{17}$$

$$\mathbf{z}_{k-1} = \mathbf{y}_{k-1} - \mathbf{S}x_{k-1}(0) - \mathbf{H}\mathbf{u}_{k-1} \tag{18}$$

$$\hat{\mathbf{y}}_k = \mathbf{S}x_k(0) + \mathbf{H}\mathbf{u}_{k,\mathrm{ILC}} + \mathbf{z}_{k-1} \tag{19}$$

$$\hat{\mathbf{e}}_k = \mathbf{y}_r - \hat{\mathbf{y}}_k \tag{20}$$

where

$$\hat{\mathbf{y}}_k = \left[\hat{y}_k^\top(1), \ldots, \hat{y}_k^\top(T)\right]^\top, \quad \mathbf{y}_r = \left[y_r^\top(1), \ldots, y_r^\top(T)\right]^\top$$

$$\hat{\mathbf{e}}_k = \left[\hat{e}_k^\top(1), \ldots, \hat{e}_k^\top(T)\right]^\top, \quad \mathbf{u}_k = \left[u_k^\top(0), \ldots, u_k^\top(T-1)\right]^\top$$

$$\mathbf{u}_{k,\mathrm{ILC}} = \left[u_{k,\mathrm{ILC}}^\top(0), \ldots, u_{k,\mathrm{ILC}}^\top(T-1)\right]^\top$$

$$\mathbf{S} \triangleq \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^T \end{bmatrix}, \quad \mathbf{H} \triangleq \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ CA^{T-1}B & CA^{T-2}B & \cdots & CB \end{bmatrix}.$$

Optimal Design of the ILC Part

The ILC control part is obtained by optimizing the following performance index over the duration of cycle k as

$$\min_{\Delta \mathbf{u}_{k,\mathrm{ILC}}} J_{k,\mathrm{ILC}} = \left\|\hat{\mathbf{e}}_k\right\|_{\mathbf{Q}}^2 + \left\|\Delta \mathbf{u}_{k,\mathrm{ILC}}\right\|_{\mathbf{R}_\Delta}^2 \tag{21a}$$

$$\text{s.t.} \quad (17), (18), (19), \text{and } (20) \tag{21b}$$

Since constrained ILC limits its learning capability, here we do not consider the constraints in order to adequately learn the information from the previous cycles.

Optimal Design of the ILC Part

While no constraint is considered in the ILC part, an analytical solution for the optimization problem (23) can be derived.
First, we have

$$\begin{aligned}\hat{\mathbf{e}}_k &= \mathbf{y}_r - \hat{\mathbf{y}}_k = \mathbf{y}_r - \mathbf{y}_{k-1} - \mathbf{H}\mathbf{u}_{k,\mathrm{ILC}} + \mathbf{H}\mathbf{u}_{k-1} \\ &= \mathbf{e}_{k-1} - \mathbf{H}\Delta\mathbf{u}_{k,\mathrm{ILC}}.\end{aligned} \tag{22}$$

and the performance index (21a)

$$J_{k,\mathrm{ILC}} = \left\|\mathbf{e}_{k-1} - \mathbf{H}\Delta\mathbf{u}_{k,\mathrm{ILC}}\right\|_{\mathbf{Q}}^2 + \left\|\Delta\mathbf{u}_{k,\mathrm{ILC}}\right\|_{\mathbf{R}_\Delta}^2. \tag{23}$$

Using the stationary condition for optimality $\partial J_{k,\mathrm{ILC}}/\partial\Delta\mathbf{u}_{k,\mathrm{ILC}} = 0$ yields

$$\Delta\mathbf{u}_{k,\mathrm{ILC}}^\star = \left(\mathbf{H}^\top\mathbf{Q}\mathbf{H} + \mathbf{R}_\Delta\right)^{-1}\mathbf{H}^\top\mathbf{e}_{k-1}. \tag{24}$$

Finally, the updating ILC law $\Delta u_{k,\mathrm{ILC}}(t)$ in (15) is given by

$$\Delta u_{k,\mathrm{ILC}}^\star(t) = \mathbf{1}_t\Delta\mathbf{u}_{k,\mathrm{ILC}}^\star = \mathbf{1}_t\left(\mathbf{H}^\top\mathbf{Q}\mathbf{H} + \mathbf{R}_\Delta\right)^{-1}\mathbf{H}^\top\mathbf{e}_{k-1}(t). \tag{25}$$

where $\mathbf{1}_t \in \mathbb{R}^{1\times T}$ denotes the vector $[0\cdots 0\ 1\ 0\cdots 0]$ with tth component 1 and other components 0, $t \in \mathbb{I}_{[1,T]}$.

First, using (14), (15) and (25), we obtain that

$$\begin{aligned} u_k(t) &= u_{k-1}(t) + \Delta u^{\star}_{k,\mathrm{ILC}}(t) + u_{k,\mathrm{MPC}}(t) \\ &= u_{k-1}(t) + \mathbf{1}_t \left(\mathbf{H}^\top \mathbf{Q}\mathbf{H} + \mathbf{R}_\Delta\right)^{-1} \mathbf{H}^\top \mathbf{e}_{k-1}(t) \\ &\quad + u_{k,\mathrm{MPC}}(t). \end{aligned} \tag{26}$$

For the MPC input $u_{k,\mathrm{MPC}}(t)$ in (26), it seeks to revise the current control input $u_k(t)$ so that the constraints given by (3)–(5) can be satisfied and the input variation defined as

$$\begin{aligned} \Delta u_k(t) &\triangleq u_k(t) - u_{k-1}(t) \\ &= u_{k,\mathrm{MPC}}(t) + \mathbf{1}_t \left(\mathbf{H}^\top \mathbf{Q}\mathbf{H} + \mathbf{R}_\Delta\right)^{-1} \mathbf{H}^\top \mathbf{e}_{k-1}(t) \end{aligned} \tag{27}$$

can be minimized.

At each sampling time t, we define

$$\mathbf{u}_{k,\mathrm{MPC}}(t) \triangleq \left[u_{k,\mathrm{MPC}}^{\top}(t|t), \ldots, u_{k,\mathrm{MPC}}^{\top}(N_p - 1|t)\right]^{\top} \tag{28}$$

Then we have the following iterative predictions:

$$u_k(t) = u_{k-1}(t) + \Delta u_{k,\mathrm{ILC}}^{\star}(t) + u_{k,\mathrm{MPC}}(t) \tag{29}$$

$$z_{k-1}(t) = y_{k-1}(t) - CAx_{k-1}(t) - CBu_{k-1}(t) \tag{30}$$

$$\hat{y}_k(t+1) = CAx_k(t) + CBu_k(t) + z_{k-1}(t) \tag{31}$$

$$\hat{e}_k(t+1) = y_r(t+1) - \hat{y}_k(t+1) \tag{32}$$

where $x_k(t)$ is the real-time measurement obtained from the system (1).

Optimal Design of the MPC Part

Thus, the to-be-optimized $\mathbf{u}_{k,\mathrm{MPC}}(t)$ given by (28) can be determined by solving the optimization problem as

$$\min_{\mathbf{u}_{k,\mathrm{MPC}}} J_{k,\mathrm{MPC}} = \left\| \hat{\mathbf{e}}_k \big|_{t+1}^{N_p} \right\|_{\mathbf{Q}}^2 + \left\| \Delta \mathbf{u}_k \big|_{t}^{N_p-1} \right\|_{\mathbf{R}_\delta}^2 \quad (33a)$$

$$\text{s.t.} \quad (29), (30), (31), \text{and } (32) \quad (33b)$$

$$\Delta u_k(t) = \Delta u_{k,\mathrm{ILC}}^{\star}(t) + u_{k,\mathrm{MPC}}(t) \in \Delta\mathbb{U} \quad (33c)$$

$$u_k(t) - u_k(t-1) \in \delta\mathbb{U} \quad (33d)$$

$$u_k(t) \in \mathbb{U} \quad (33e)$$

where $\Delta\mathbb{U} \triangleq \prod_{i=1}^{m} \Delta\mathbb{U}_i$, $\delta\mathbb{U} \triangleq \prod_{i=1}^{m} \delta\mathbb{U}_i$, and

$$\hat{\mathbf{e}}_k \big|_{t+1}^{N_p} = \begin{bmatrix} \hat{e}_k(t+1) \\ \vdots \\ \hat{e}_k(t+N_p) \end{bmatrix}, \Delta \mathbf{u}_k \big|_{t}^{N_p-1} = \begin{bmatrix} \Delta u_k(t) \\ \vdots \\ \Delta u_k(t+N_p-1) \end{bmatrix}$$

Note that at the initialization step, we have $k = 1$, $u_0(t) = 0$, and $\mathbf{e}_0(t) = \mathbf{y}_r$.
To guarantee the satisfaction of constraints, we have to solve

$$\min_{\mathbf{u}_{1,\mathrm{MPC}}} J_{1,\mathrm{MPC}} = \left\|\hat{\mathbf{e}}_1\right\|_{\mathbf{Q}}^2 + \left\|\mathbf{u}_{1,\mathrm{MPC}}\right\|_{\mathbf{R}}^2 \tag{34a}$$

$$\text{s.t. } \hat{y}_1(t+1) = CAx_1(t) + CBu_{1,\mathrm{MPC}}(t) \tag{34b}$$

$$\hat{e}_1(t+1) = y_r(t+1) - \hat{y}_1(t+1) \tag{34c}$$

$$u_{1,\mathrm{MPC}}(t) \in \mathbb{U} \tag{34d}$$

$$u_{1,\mathrm{MPC}}(t) - u_{1,\mathrm{MPC}}(t-1) \in \Delta\mathbb{U} \tag{34e}$$

where

$$\mathbf{u}_{1,\mathrm{MPC}} = \begin{bmatrix} u_{1,\mathrm{MPC}}(0) \\ \vdots \\ u_{1,\mathrm{MPC}}(T-1) \end{bmatrix}, \quad \hat{\mathbf{e}}_1 = \begin{bmatrix} \hat{e}_1(1) \\ \vdots \\ \hat{e}_1(T) \end{bmatrix}$$

Assumption 2

The optimization problem given by (34a)–(34e) for the first batch is solvable.

The Proposed Control Strategy

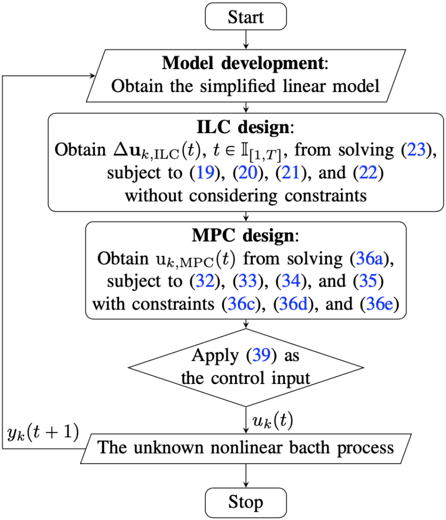


Figure: Combined control scheme for batch processes.

Theorem (Stability)

Consider the batch processes controlled under the proposed scheme given by Figure 5. Then, the control input sequence $\{u_k(t)\}$, $t \in \mathbb{I}_{[0,T]}$ converges to the desired input $u_r(t)$, $t \in \mathbb{I}_{[0,T]}$, as k goes to infinity; that is,

$$\lim_{k\to\infty} u_k(t) = u_r(t), \quad t \in \mathbb{I}_{[0,T-1]}. \tag{35}$$

In addition, $\lim_{k\to\infty} e_k(t) = 0$, $\forall t \in \mathbb{I}_{[0,T-1]}$.

Proof.

Please refer to the coming extension of this conference work for details. □

Further Discussions

We give some further discussions for this conference work:

- we do not assume that the accurate linearization model is available to us, but a more accurate prediction model will improve the control performance.
- When compared to the other results, where only one optimization problem needs to solved at the current time, this means $\Delta u^{\star}_{k,\mathrm{ILC}}(t) = 0$ in our proposed framework, and the subsequent control law is given by

$$u_k(t) = u_{k-1}(t) + u^{\star}_{k,\mathrm{MPC}}(t). \tag{36}$$

- This paper discusses nonlinear systems with two separate designs in which the control parts are determined separately by solving their own optimization problems. The ILC part is formulated and solved for every cycle, whereas the MPC part is formulated and solved for each sampling time at each cycle.
- There will be many illustration examples throughout this work for the journal submission.

Comparison to 2D ILMPC

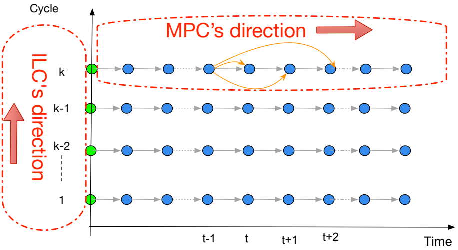


Figure: Our combined control scheme.

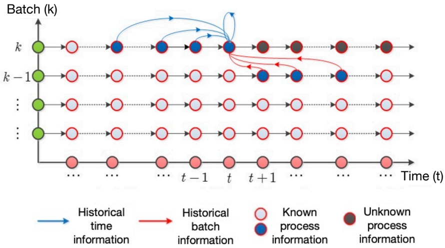


Figure: The 2D-ILMPC scheme.

Conclusions

1. Optimal ILC using optimal run-to-run feedback with the historical batch data.
2. MPC using real-time feed- back with the current sampled measurements within the batch.
3. Combined design of run-to-run ILC and real-time feedback- based MPC.
4. Closed loop tracking stability and monotonic stability.

Future works

1. Would the resulting controller be equivalent to a 2D-ILMPC ? If not, which one is better, computationally or performance-wise ?
2. How can the robust design be shifted to an adaptive design ?
3. How to design model-free (data-driven) controllers ?

Extension

Yuanqiang Zhou, Dewei Li, Xin Lai, and Furong Gao, "Combined iterative learning and model predictive control scheme for nonlinear constrained systems," in preparation for journal submission to *IEEE Transactions on Automation Science and Engineering* (an extension of this conference work), 2022.

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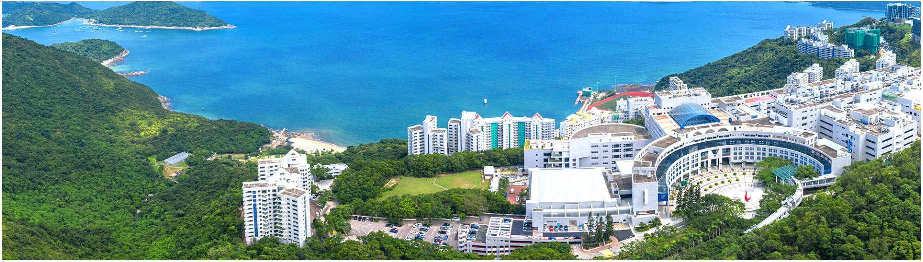
Acknowledgements

Fundings

- Ministry of Science and Technology of China under grant 2019YFB1704905 and SQ2019YFB170029;
- Hong Kong Research Grant Council under grant 16208520;
- Foshan-HKUST Project under grant FSUST19-FYTRI01;
- Guangdong scientific and technological project (202002030323 and 2014B050505002);
- SJTU Global Strategic Partnership Fund (2021 SJTU-HKUST).

Thanks to: Prof. Dewei Li, Prof. Xin Lai, Prof. Furong Gao.



Questions ?