Fixed Effect

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In [ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
%config InlineBackend.figure_format = 'retina'
import statsmodels.api as sm
import statsmodels.formula.api as smf
```

Exercise 1

Download the data and do some EDA

In [ ]:
```
beerdata = pd.read_csv('https://media.githubusercontent.com/media/nickeubank/'+
                       'MIDS_Data/master/us_driving_fatalities.csv')
beerdata.head()
```

Out[ ]:

	Unnamed: 0	state	year	spirits	unemp	income	emppop	beertax	baptist	mormon	...	nfatal2124	afatal	
0	1	al	1982	1.37	14.4	10544.152344	50.692039	1.539379	30.355700	0.32829	...	32	309.437988	39
1	2	al	1983	1.36	13.7	10732.797852	52.147030	1.788991	30.333599	0.34341	...	35	341.834015	39
2	3	al	1984	1.32	11.1	11108.791016	54.168087	1.714286	30.311501	0.35924	...	34	304.872009	39
3	4	al	1985	1.28	8.9	11332.626953	55.271137	1.652542	30.289499	0.37579	...	45	276.742004	40
4	5	al	1986	1.23	9.8	11661.506836	56.514496	1.609907	30.267401	0.39311	...	29	360.716003	40

5 rows × 35 columns

In [ ]:
```
beerdata.describe()
```

Out[ ]:

	Unnamed: 0	year	spirits	unemp	income	emppop	beertax	baptist	mormon	drinl
count	336.000000	336.000000	336.000000	336.000000	336.000000	336.000000	336.000000	336.000000	336.000000	336.00(
mean	168.500000	1985.000000	1.753690	7.346726	13880.184533	60.805676	0.513256	7.156925	2.801933	20.45!
std	97.139076	2.002983	0.683575	2.533405	2253.046291	4.721656	0.477844	9.762621	9.665279	0.89!
min	1.000000	1982.000000	0.790000	2.400000	9513.761719	42.993198	0.043311	0.000000	0.100000	18.00(
25%	84.750000	1983.000000	1.300000	5.475000	12085.849854	57.691426	0.208849	0.626752	0.272160	20.00(
50%	168.500000	1985.000000	1.670000	7.000000	13763.128906	61.364660	0.352589	1.749250	0.393111	21.00(
75%	252.250000	1987.000000	2.012500	8.900000	15175.124268	64.412504	0.651573	13.127125	0.629320	21.00(
max	336.000000	1988.000000	4.900000	18.000000	22193.455078	71.268654	2.720764	30.355700	65.916496	21.00(

8 rows × 31 columns

In [ ]:

```
#how many states are there?
beerdata['state'].nunique()
```

Out[ ]: 48

- This dataset contains 48 states.
- The time frame of this dataset is from 1982 to 1988.
- A single observation (state-year) consists data of different states in different years from 1982 to 1988, i.e. each row contains data for a single state in a single year.

Exercise 2

Construct dependent variable: fatality rate per 10,000.

In [ ]:

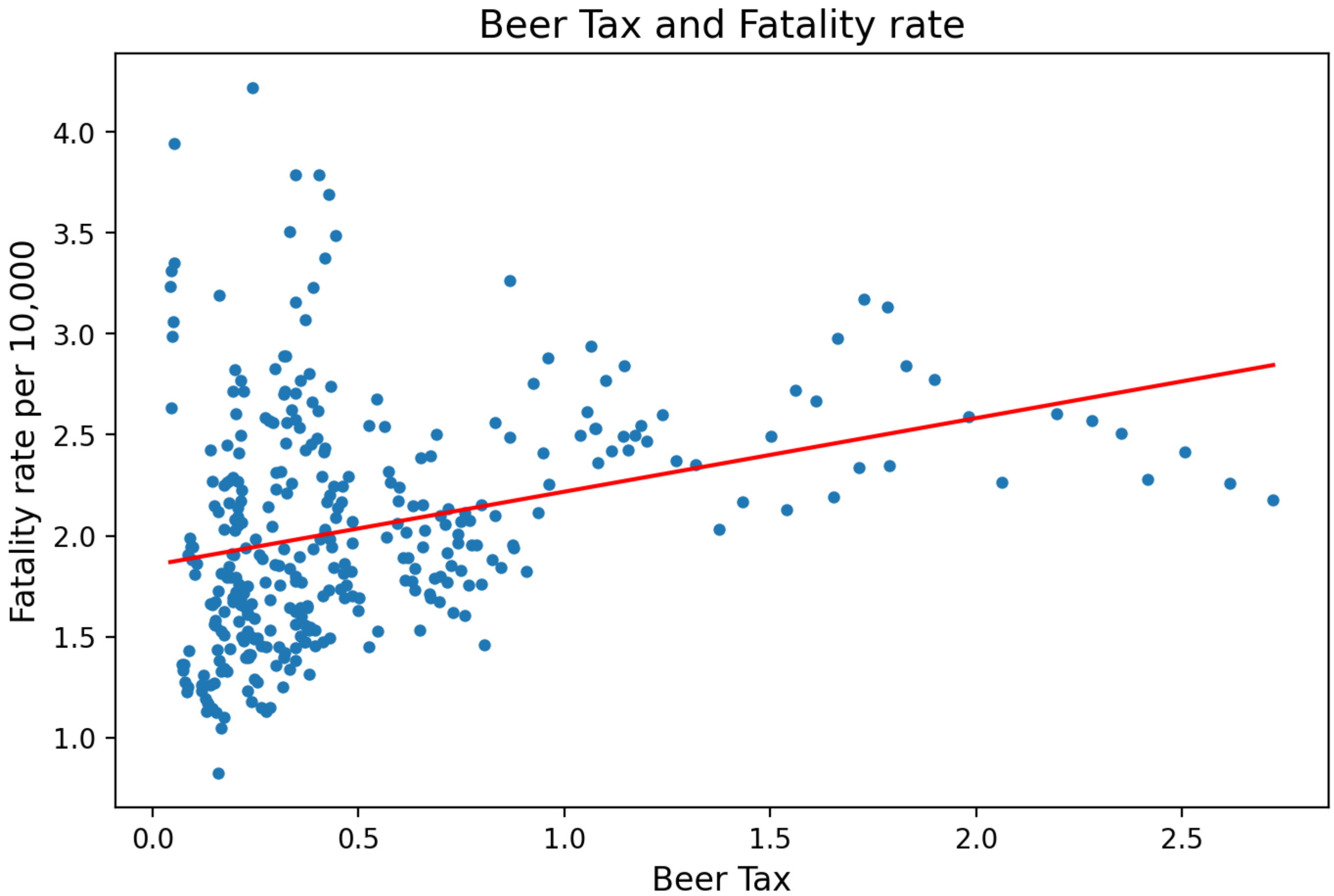
```
# compute fatalities per 10,000 people
beerdata['fat_rate'] = beerdata['fatal'] / beerdata['pop'] * 10000
beerdata['fat_rate'].describe()
```

Out[ ]:
```
count    336.000000
mean       2.040444
std        0.570194
min        0.821210
25%        1.623710
50%        1.955955
75%        2.417888
max        4.217840
Name: fat_rate, dtype: float64
```

Exercise 3

Draw a scatter plot and a fitted line showing the correlation between these two variables

In [ ]:
```
# Draw a scatter plot using beertax as the x-axis, and fat_rate as the y-axis
# and a fitted line
plt.figure(figsize=(8, 5), dpi=100)
plt.scatter(beerdata['beertax'], beerdata['fat_rate'], s = 12)
plt.plot(np.unique(beerdata['beertax']), np.poly1d(
    np.polyfit(beerdata['beertax'], beerdata['fat_rate'], 1))(np.unique(beerdata['beertax'])), color='red')
plt.xlabel('Beer Tax', fontsize=12)
plt.ylabel('Fatality rate per 10,000', fontsize=12)
plt.title('Beer Tax and Fatality rate', fontsize=14)
plt.show()
```



Exercise 4

Fit a simple OLS regression --- 'pooled' regression

In [ ]:
```
pooled_model = smf.ols(formula='fat_rate ~ beertax', data=beerdata).fit()
pooled_model.summary()
```

Out[ ]:

OLS Regression Results

Dep. Variable:	fat_rate	**R-squared:**	0.093
Model:	OLS	**Adj. R-squared:**	0.091
Method:	Least Squares	**F-statistic:**	34.39
Date:	Thu, 23 Feb 2023	**Prob (F-statistic):**	1.08e-08
Time:	20:06:52	**Log-Likelihood:**	-271.04
No. Observations:	336	**AIC:**	546.1
Df Residuals:	334	**BIC:**	553.7
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	1.8533	0.044	42.539	0.000	1.768	1.939
beertax	0.3646	0.062	5.865	0.000	0.242	0.487

Omnibus:	66.653	**Durbin-Watson:**	0.465
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	112.734
Skew:	1.134	**Prob(JB):**	3.31e-25
Kurtosis:	4.707	**Cond. No.**	2.76

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

- There is a positive relationship betwwen beer tax and fatality rate per 10,000.
- The coefficient for beer tax is **0.3646** with $p-value < 0.05$, meaning that with 1 unit increase on beer tax, the fatality rate per 10,000 people will increases by 0.3646.

Exercise 5

Add state fixed effects

In [ ]:
```
# add state fixed effects
fixed_model = smf.ols(formula='fat_rate ~ beertax + C(state)', data=beerdata).fit()
fixed_model.summary()
```

Out[ ]:

OLS Regression Results

Dep. Variable:	fat_rate	**R-squared:**	0.905
Model:	OLS	**Adj. R-squared:**	0.889
Method:	Least Squares	**F-statistic:**	56.97
Date:	Thu, 23 Feb 2023	**Prob (F-statistic):**	1.96e-120
Time:	20:06:53	**Log-Likelihood:**	107.97
No. Observations:	336	**AIC:**	-117.9
Df Residuals:	287	**BIC:**	69.09
Df Model:	48		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	3.4776	0.313	11.098	0.000	2.861	4.094
C(state)[T.ar]	-0.6550	0.219	-2.990	0.003	-1.086	-0.224
C(state)[T.az]	-0.5677	0.267	-2.129	0.034	-1.093	-0.043
C(state)[T.ca]	-1.5095	0.304	-4.960	0.000	-2.109	-0.910
C(state)[T.co]	-1.4843	0.287	-5.165	0.000	-2.050	-0.919
C(state)[T.ct]	-1.8623	0.281	-6.638	0.000	-2.414	-1.310
C(state)[T.de]	-1.3076	0.294	-4.448	0.000	-1.886	-0.729
C(state)[T.fl]	-0.2681	0.139	-1.924	0.055	-0.542	0.006
C(state)[T.ga]	0.5246	0.184	2.852	0.005	0.163	0.887
C(state)[T.ia]	-1.5439	0.253	-6.092	0.000	-2.043	-1.045
C(state)[T.id]	-0.6690	0.258	-2.593	0.010	-1.177	-0.161
C(state)[T.il]	-1.9616	0.291	-6.730	0.000	-2.535	-1.388
C(state)[T.in]	-1.4615	0.273	-5.363	0.000	-1.998	-0.925
C(state)[T.ks]	-1.2232	0.245	-4.984	0.000	-1.706	-0.740
C(state)[T.ky]	-1.2175	0.287	-4.240	0.000	-1.783	-0.652
C(state)[T.la]	-0.8471	0.189	-4.490	0.000	-1.218	-0.476

C(state)[T.ma]	-2.1097	0.276	-7.641	0.000	-2.653	-1.566
C(state)[T.md]	-1.7064	0.283	-6.025	0.000	-2.264	-1.149
C(state)[T.me]	-1.1079	0.191	-5.797	0.000	-1.484	-0.732
C(state)[T.mi]	-1.4845	0.236	-6.290	0.000	-1.949	-1.020
C(state)[T.mn]	-1.8972	0.265	-7.157	0.000	-2.419	-1.375
C(state)[T.mo]	-1.2963	0.267	-4.861	0.000	-1.821	-0.771
C(state)[T.ms]	-0.0291	0.148	-0.196	0.845	-0.321	0.263
C(state)[T.mt]	-0.3604	0.264	-1.365	0.173	-0.880	0.159
C(state)[T.nc]	-0.2905	0.120	-2.424	0.016	-0.526	-0.055
C(state)[T.nd]	-1.6234	0.254	-6.396	0.000	-2.123	-1.124
C(state)[T.ne]	-1.5222	0.249	-6.106	0.000	-2.013	-1.032
C(state)[T.nh]	-1.2545	0.210	-5.983	0.000	-1.667	-0.842
C(state)[T.nj]	-2.1057	0.307	-6.855	0.000	-2.710	-1.501
C(state)[T.nm]	0.4264	0.254	1.677	0.095	-0.074	0.927
C(state)[T.nv]	-0.6008	0.286	-2.101	0.037	-1.164	-0.038
C(state)[T.ny]	-2.1867	0.299	-7.316	0.000	-2.775	-1.598
C(state)[T.oh]	-1.6744	0.254	-6.597	0.000	-2.174	-1.175
C(state)[T.ok]	-0.5451	0.169	-3.223	0.001	-0.878	-0.212
C(state)[T.or]	-1.1680	0.286	-4.088	0.000	-1.730	-0.606
C(state)[T.pa]	-1.7675	0.276	-6.402	0.000	-2.311	-1.224
C(state)[T.ri]	-2.2651	0.294	-7.711	0.000	-2.843	-1.687
C(state)[T.sc]	0.5572	0.110	5.065	0.000	0.341	0.774
C(state)[T.sd]	-1.0037	0.210	-4.788	0.000	-1.416	-0.591
C(state)[T.tn]	-0.8757	0.268	-3.267	0.001	-1.403	-0.348
C(state)[T.tx]	-0.9175	0.246	-3.736	0.000	-1.401	-0.434
C(state)[T.ut]	-1.1640	0.196	-5.926	0.000	-1.551	-0.777
C(state)[T.va]	-1.2902	0.204	-6.320	0.000	-1.692	-0.888

C(state)[T.vt]	-0.9660	0.211	-4.576	0.000	-1.382	-0.550
C(state)[T.wa]	-1.6595	0.283	-5.854	0.000	-2.217	-1.102
C(state)[T.wi]	-1.7593	0.294	-5.985	0.000	-2.338	-1.181
C(state)[T.wv]	-0.8968	0.247	-3.636	0.000	-1.382	-0.411
C(state)[T.wy]	-0.2285	0.313	-0.730	0.466	-0.844	0.387
beertax	-0.6559	0.188	-3.491	0.001	-1.026	-0.286

Omnibus:	53.045	**Durbin-Watson:**	1.517
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	219.863
Skew:	0.585	**Prob(JB):**	1.81e-48
Kurtosis:	6.786	**Cond. No.**	187.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

- With state fixed effects, the relationship between beer tax and fatality rate per 10,000 people varies by state. Overall, there is a negative relationship betwwen beertax and fatality rate per 10,000.
- The coefficient for beer tax is **-0.6559** with $p-value < 0.05$, meaning that with 1 unit increase on beer tax, the fatality rate per 10,000 people will decreases by 0.6559 compared with the reference state(al).

Exercise 6

- Without fixed effects, we did not take baseline differences(fatality rate) into consideration, which happened to be correlated with treatment assignment. The positive association between beer tax and fatality rate per 10,000 people may be driven by other state-specific factors, such as the state's education level, demographics, initial fatality rate before implementing the beer tax, which are not included in the model.
- This imply that states with high beer taxes set high beer taxes due to the fact of high fatality rate.

Implement the entity-demeaned approach by hand

In [ ]:
```
avg_fata_state = beerdata.groupby('state')['fat_rate'].mean()
avg_beertax_state = beerdata.groupby('state')['beertax'].mean()
# for each state, compute the difference between the fatality rate and the average fatality rate
beerdata['fat_rate_diff'] = beerdata['fat_rate'] - avg_fata_state[beerdata['state']].values
beerdata['beertax_diff'] = beerdata['beertax'] - avg_beertax_state[beerdata['state']].values
```

In [ ]:
```
fixed_model = smf.ols(formula='fat_rate_diff ~ beertax_diff', data=beerdata).fit()
fixed_model.summary()
```

Out[ ]:

OLS Regression Results

Dep. Variable:	fat_rate_diff	**R-squared:**	0.041
Model:	OLS	**Adj. R-squared:**	0.038
Method:	Least Squares	**F-statistic:**	14.19
Date:	Thu, 23 Feb 2023	**Prob (F-statistic):**	0.000196
Time:	20:06:53	**Log-Likelihood:**	107.97
No. Observations:	336	**AIC:**	-211.9
Df Residuals:	334	**BIC:**	-204.3
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-2.168e-17	0.010	-2.26e-15	1.000	-0.019	0.019
beertax_diff	-0.6559	0.174	-3.767	0.000	-0.998	-0.313

Omnibus:	53.045	**Durbin-Watson:**	1.517
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	219.863
Skew:	0.585	**Prob(JB):**	1.81e-48
Kurtosis:	6.786	**Cond. No.**	18.1

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Exercise 7

Fit the model with state fixed-effect using PanelOLS / lfe

In [ ]:
```
# set index to state-year
beerdata = beerdata.set_index(['state', 'year'])
```

In [ ]:
```
beerdata.head()
```

Out[ ]:

state	year	Unnamed: 0	spirits	unemp	income	emppop	beertax	baptist	mormon	drinkage	dry	...	pc
al	1982	1	1.37	14.4	10544.152344	50.692039	1.539379	30.355700	0.32829	19.00	25.006300	...	208999.5
	1983	2	1.36	13.7	10732.797852	52.147030	1.788991	30.333599	0.34341	19.00	22.994200	...	202000.(
	1984	3	1.32	11.1	11108.791016	54.168087	1.714286	30.311501	0.35924	19.00	24.042601	...	196999.9
	1985	4	1.28	8.9	11332.626953	55.271137	1.652542	30.289499	0.37579	19.67	23.633900	...	194999.7
	1986	5	1.23	9.8	11661.506836	56.514496	1.609907	30.267401	0.39311	21.00	23.464701	...	203999.8

5 rows × 36 columns

In [ ]:
```
# using panelOLS with state fixed effects
from linearmodels.panel import PanelOLS
panel_model = PanelOLS(beerdata['fat_rate'], beerdata['beertax'], entity_effects=True).fit()
panel_model.summary
```

Out[ ]:

PanelOLS Estimation Summary

Dep. Variable:	fat_rate	**R-squared:**	0.0407
Estimator:	PanelOLS	**R-squared (Between):**	-0.3805
No. Observations:	336	**R-squared (Within):**	0.0407
Date:	Thu, Feb 23 2023	**R-squared (Overall):**	-0.3775
Time:	20:06:53	**Log-likelihood**	107.97
Cov. Estimator:	Unadjusted		
		F-statistic:	12.190
Entities:	48	**P-value**	0.0006
Avg Obs:	7.0000	**Distribution:**	F(1,287)
Min Obs:	7.0000		
Max Obs:	7.0000	**F-statistic (robust):**	12.190
		P-value	0.0006
Time periods:	7	**Distribution:**	F(1,287)
Avg Obs:	48.000		
Min Obs:	48.000		
Max Obs:	48.000		

Parameter Estimates

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
beertax	-0.6559	0.1878	-3.4915	0.0006	-1.0256	-0.2861

F-test for Poolability: 52.179
P-value: 0.0000
Distribution: F(47,287)

Included effects: Entity

- The coefficient for beertax is the same as the result in Exercise 6 but the standard deviation varies a little bit.
- The coefficient for beertax is -0.6559 with $p - value < 0.05$, meaning that on average, within each state, with 1 unit increase on beer tax, the fatality rate per 10,000 people will decreases by 0.6559.

Exercise 8

Add fixed effects for both the state and the year, as well as the other covariates

In [ ]:
```
beerdata.columns
```

Out[ ]:
```
Index(['Unnamed: 0', 'spirits', 'unemp', 'income', 'emppop', 'beertax',
       'baptist', 'mormon', 'drinkage', 'dry', 'youngdrivers', 'miles',
       'breath', 'jail', 'service', 'fatal', 'nfatal', 'sfatal', 'fatal1517',
       'nfatal1517', 'fatal1820', 'nfatal1820', 'fatal2124', 'nfatal2124',
       'afatal', 'pop', 'pop1517', 'pop1820', 'pop2124', 'milestot', 'unempus',
       'emppopus', 'gsp', 'fat_rate', 'fat_rate_diff', 'beertax_diff'],
      dtype='object')
```

In [ ]:
```
# add youngdrivers to the model
# no categorical variables ---> no need to add an intercept to aid in interpretation of controls
pandel_model_2 = PanelOLS(beerdata['fat_rate'], beerdata[[
    'beertax','spirits','youngdrivers','miles']], entity_effects=True, time_effects=True)
#clusered standard errors
pandel_model_2 = pandel_model_2.fit(cov_type='clustered', cluster_entity=True)
pandel_model_2.summary
```

Out[ ]:

PanelOLS Estimation Summary

Dep. Variable:	fat_rate	**R-squared:**	0.2443
Estimator:	PanelOLS	**R-squared (Between):**	0.7924
No. Observations:	336	**R-squared (Within):**	-0.4508
Date:	Thu, Feb 23 2023	**R-squared (Overall):**	0.7835
Time:	20:06:53	**Log-likelihood**	155.93
Cov. Estimator:	Clustered		
		F-statistic:	22.471
Entities:	48	**P-value**	0.0000
Avg Obs:	7.0000	**Distribution:**	F(4,278)
Min Obs:	7.0000		
Max Obs:	7.0000	**F-statistic (robust):**	8.1440
		P-value	0.0000
Time periods:	7	**Distribution:**	F(4,278)
Avg Obs:	48.000		
Min Obs:	48.000		
Max Obs:	48.000		

Parameter Estimates

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
beertax	-0.4561	0.3043	-1.4991	0.1350	-1.0551	0.1428
spirits	1.0128	0.1847	5.4821	0.0000	0.6491	1.3764
youngdrivers	1.6682	1.4104	1.1827	0.2379	-1.1083	4.4447
miles	1.88e-05	1.147e-05	1.6394	0.1023	-3.775e-06	4.138e-05

F-test for Poolability: 49.084

P-value: 0.0000

Distribution: F(53,278)

Included effects: Entity, Time

- By adding year-fixed effects, we are able to control for variables that are constant across entities(states) but vary over time. For example, there may be some nationwide policies and laws on transportation or macroeconomic conditions. This model eliminates omitted variable bias caused by excluding unobserved variables that evolve over time but are constant across entities.

- We added 'spirits','youngdrivers', and 'miles' as important covariates since more consumption of spirits may lead to more traffic deaths; young drivers are less cautious on the road and may cause more traffic accidents and traffic deaths compared to other drivers; and longer vehicle miles may also cause some safety issues on the vehicle and thus have a positive relationship with traffic deaths.

- The coefficient for beertax is -0.4561, meaning that after controlling for states and time, with 1 unit increase on beer tax, the fatality rate per 10,000 people will decreases by 0.4561, holding all other variables constant. With $p - value > 0.05$, this difference is not statistically significant.

- The coefficient for spirits is 1.0128, meaning that after controlling for states and time, with 1 unit increase on spirits consumption, the fatality rate per 10,000 people will increases by 1.0128, holding all other variables constant. With $p - value < 0.05$, this difference is statistically significant.

- The coefficient for youngdrivers is 1.6682, meaning that after controlling for states and time, with 1 unit increase on youngdrivers, the fatality rate per 10,000 people will increase by 1.6682, holding all other variables constant. With $p - value > 0.05$, this difference is not statistically significant.

- The coefficient for miles is 1.88e-05, meaning that after controlling for states and time, with 1 unit increase on vehicle miles, the fatality rate per 10,000 people will increase by 1.88e-052, holding all other variables constant. With $p - value > 0.05$, this difference is not statistically significant.