

IB MATHEMATICS EXTENDED ESSAY

MAP PROJECTIONS

**How do we quantify geometric distortions of different
map projections using differential geometry?**

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1 Introduction

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Definition 1.1. Cauchy Inequality

For any list of reals u_1, u_2, \dots, u_n and v_1, v_2, \dots, v_n ,

$$\left(\sum_{i=1}^n u_i v_i \right)^2 \leq \left(\sum_{i=1}^n u_i^2 \right) \left(\sum_{i=1}^n v_i^2 \right),$$

with equality if and only if there exists a constant t such that $u_i = t v_i$ for all $1 \leq i \leq n$, or if one list consists of only zeroes.

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Example 1.1. Graphing a hyperbola

Sketch the hyperbola given by $\frac{(y-2)^2}{25} - \frac{(x-1)^2}{4} = 1$.

SOLUTION The hyperbola is centered at $(1, 2)$; $a = 5$ and $b = 2$. We draw the prescribed rectangle centered at $(1, 2)$ along with the asymptotes defined by its diagonals. The hyperbola has a vertical transverse axis, so the vertices are located at $(1, 7)$ and $(1, -3)$.

We also find the location of the foci: as $c^2 = a^2 + b^2$, we have $c = \sqrt{29} \approx 5.4$. Thus the foci are located at $(1, 2 \pm 5.4)$ as shown in the figure below.