IB Mathematics Internal Assessment

MAP PROJECTIONS

Noise

June 6, 2025

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1 Introduction

Mathematics does not merely describe nature - it architects worlds. Within the intricate algorithm, virtual worlds take shape as jagged mountains rise, valleys unfurl, and rivers etch serpentine paths. This is not magic; it is mathematics in motion, bending stochasticity into artistry.

Traditionally, the primary technique used in developing terrain has been handcraft, using modelling software to precisely describe the creations. However, this approach suffers from four main drawbacks (Roden & Parberry, 2004):

- **Operability:** handcrafted terrains are slow and resource-intensive, requiring manual adjustments for every detail.
- **Inflexibility:** modifying completed designs can dramatically alter technical requirements.
- **Incompatibility:** handcrafted terrains often lack coherence with physics engines and other computational models.
- **Unscalability:** as virtual worlds grow in complexity with their expansive, dynamic environment, handcrafted terrains become impractical.

In the late 1960s, computer graphics pioneers confronted these setbacks, seeking ways to simulate natural patterns without the efficiencies of manual design (Autodesk, 2024). The answer manifested in the form of noise algorithms, mathematical functions capable of producing structured randomness, often termed "pseudo-randomness" (Perlin, 2001). Over the decades, they have evolved beyond gaming applications to enable hyper-realistic CGIs in films (Pegg, 2010), creating texture (Perlin, 1985), and advancing fluid dynamics simulations (Kim et al., 2008).

My fascination with noise algorithms began in the blocky worlds of the game Minecraft. As a teenager, I spent countless hours exploring its vast biomes, awestruck by the seamless blend of towering cliffs, dense forests, and sprawling cave systems. The terrain felt organic, but I knew it was generated algorithmically. While I appreciate the aesthetics of the game, I lacked insight into the technical requirements underpinning it. Years later, I discovered the role of noise in shaping these landscapes and became curious about which aspects of noise contribute to terrain realism.

There are two main types of noise: Value noise and Perlin noise. Despite the popularity, their ability to replicate real-world terrain has not been rigorously studied under controlled conditions. This IA will determine which algorithm produces more realistic terrain by statistically comparing

- Elevation distribution using **Histograms of normalised height values**.
- Spatial autocorrelation using **Moran's I to measure globl clustering trends**.
- Spectral characteristics using **Power Spectral Density to classify frequency dominance**.

with real-world elevation data from the Global Multi-resolution Terrain Elevation Data (GMTED2010), a model developed through a collaborative effort between the U.S. Geological Survey (USGS) and the National Geospatial-Intelligence Agency (NGA).

Terrain maps will be generated on a fixed 256×256 grid with normalised height values $z \in [0, 1]$. Both algorithms are tested under identical spectral parameters, including amplitude, frequency, persistence, lacunarity, and number of octaves. Cubic smoothstep functions will be applied to bilinear interpolations to enforce continuity. A fixed seed (30042603) will be employed to ensure deterministic comparisons by removing the randomness of the height values out of the effects.

Lastly, all implementations will be done using the Python programming language. All code will be available in the Appendix.