In[39]:=

```
(*Assumptions*)
$Assumptions = Element[{r, s, x, y}, Reals] && r > 0 && s > 0 && x ≥ 0 && x ≤ s && y ≥ 0 && y ≤ s;

(*Definitions*)
star[x_, y_, r_, s_] :=
   Max[0, r^2 ArcCos[(x + s/2) / r] - (x + s/2) Sqrt[r^2 - (x + s/2)^2]] +
   Max[0, r^2 ArcCos[(y + s/2) / r] - (y + s/2) Sqrt[r^2 - (y + s/2)^2]];
fn[x_, y_, r_, s_] := Piecewise[{
     {star[x, y, r, s], 0 ≤ s ≤ 2 r},
     {0, 2 r < s}
   }];
p[x_, y_] := 1/s^2;
FN[r_, s_] = Integrate[fn[x, y, r, s] p[x, y], {x, 0, s}, {y, 0, s}]
(*Prints*)
star[x, y, r, s] // TraditionalForm
PiecewiseExpand[star[x, y, r, s]] // Simplify // TraditionalForm
fn[x, y, r, s] // TraditionalForm
PiecewiseExpand[fn[x, y, r, s]] // Simplify // TraditionalForm
FN[r, s] // Simplify // TraditionalForm
```

Out[43]=

$$\int_0^s \int_0^s \frac{\begin{cases} \mathrm{Max}\left[0, -\left(\frac{s}{2}+x\right)\sqrt{r^2-\left(\frac{s}{2}+x\right)^2}+r^2\,\mathrm{ArcCos}\left[\frac{\frac{s}{2}+x}{r}\right]\right] + \mathrm{Max}\left[0, -\left(\frac{s}{2}+y\right)\sqrt{r^2-\left(\frac{s}{2}+y\right)^2}+r^2\,\mathrm{ArcCos}\left[\frac{\frac{s}{2}+y}{r}\right]\right] & 0 \le s \le 2r \\ 0 & \mathrm{True} \end{cases}}{s^2}\, dy\, dx$$

Out[44]//TraditionalForm=

$$\max\left(0, r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right)-\left(\frac{s}{2}+x\right)\sqrt{r^2-\left(\frac{s}{2}+x\right)^2}\right)+\max\left(0, r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right)-\left(\frac{s}{2}+y\right)\sqrt{r^2-\left(\frac{s}{2}+y\right)^2}\right)$$

Out[45]//TraditionalForm=

$$\begin{cases} r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right)-\frac{1}{4}(s+2x)\sqrt{4r^2-(s+2x)^2} & (s+2x)\sqrt{4r^2-(s+2x)^2} < 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right) \land \\ & (s+2y)\sqrt{4r^2-(s+2y)^2} \ge 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right) \\ r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right)-\frac{1}{4}(s+2y)\sqrt{4r^2-(s+2y)^2} & (s+2x)\sqrt{4r^2-(s+2x)^2} \ge 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right) \land \\ & (s+2y)\sqrt{4r^2-(s+2y)^2} < 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right) \\ \frac{1}{4}\Big(-s\sqrt{4r^2-(s+2x)^2}-2x\sqrt{4r^2-(s+2x)^2}+ & (s+2x)\sqrt{4r^2-(s+2x)^2} < 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right) \land \\ \quad 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right)-s\sqrt{4r^2-(s+2y)^2}- & (s+2y)\sqrt{4r^2-(s+2y)^2} < 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right) \\ \quad 2y\sqrt{4r^2-(s+2y)^2}+4r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right)\Big) & \end{cases}$$

Out[46]//TraditionalForm=

$$\begin{cases} \max\left(0, r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right)-\left(\frac{s}{2}+x\right)\sqrt{r^2-\left(\frac{s}{2}+x\right)^2}\right)+\max\left(0, r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right)-\left(\frac{s}{2}+y\right)\sqrt{r^2-\left(\frac{s}{2}+y\right)^2}\right) & 0 \le s \le 2r \\ 0 & \mathrm{True} \end{cases}$$

Out[47]//TraditionalForm=

$$\begin{cases} 0 & \left((s+2x)\sqrt{4r^2-(s+2x)^2} \geq 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right) \land (s+2y)\sqrt{4r^2-(s+2y)^2} \geq 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right)\right) \lor 2r<s \\ r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right) - \frac{1}{4}(s+2y)\sqrt{4r^2-(s+2y)^2} & 2r \geq s \land (s+2x)\sqrt{4r^2-(s+2x)^2} \geq 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right) \\ r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right) - \frac{1}{4}(s+2x)\sqrt{4r^2-(s+2x)^2} & 2r \geq s \land (s+2y)\sqrt{4r^2-(s+2y)^2} \geq 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right) \\ \frac{1}{4}\left(-s\sqrt{4r^2-(s+2x)^2} - 2x\sqrt{4r^2-(s+2x)^2} + 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right) - s\sqrt{4r^2-(s+2y)^2} - 2y\sqrt{4r^2-(s+2y)^2} + 4r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right)\right) & \text{True} \end{cases}$$

Out[48]//TraditionalForm=

$$\int_0^s \int_0^s \frac{1}{s^2} \begin{cases} \max\left(0, r^2\cos^{-1}\left(\frac{\frac{s}{2}+x}{r}\right) - \left(\frac{s}{2}+x\right)\sqrt{r^2-\left(\frac{s}{2}+x\right)^2}\right) + \max\left(0, r^2\cos^{-1}\left(\frac{\frac{s}{2}+y}{r}\right) - \left(\frac{s}{2}+y\right)\sqrt{r^2-\left(\frac{s}{2}+y\right)^2}\right) & 0 \leq s \leq 2r \\ 0 & \text{True} \end{cases} dy\, dx$$