Mathematical formulas for QM(02/16)

In order to do QIP(quantum information process), we need to know how to represent

1. Quantum state
2. Transformation of a quantum state
3. Measurement of a quantum state
4. Composition systems: 2 qubits (quantum entanglement)
5. $1-4$ in an imperfect world

Quantum state

The most complete characterization of a quantum variable at a given time is "Quantum states are states of (lack of my)knowledge".

Bit		Qubit
state 0	$\rightarrow$	state $\|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
state 1	$\rightarrow$	state $\|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
no knowledge of classical computing	$\rightarrow$	$\|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha\|0\rangle + \beta\|1\rangle$ $\langle\psi\| = [\,\alpha^* \;\; \beta^*\,] = \alpha^*\langle 0\| + \beta^*\langle 1\|$

Table 1. Classical state vs quantum state

$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ where $|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Qubit can be also represented as a density matrix

$$\begin{aligned} \rho &= |\psi\rangle\langle\psi| \\ &= \begin{bmatrix} \alpha \\ \beta \end{bmatrix} [\,\alpha^* \;\; \beta^*\,] \\ &= \begin{bmatrix} \alpha\alpha^* & \alpha\beta^* \\ \beta\alpha^* & \beta\beta^* \end{bmatrix} \end{aligned}$$

A diagonal density matrix implies a classical system.

$$|0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } |1\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$