Real NVP

Let $z = (z_1, z_2, z_3, z_4) \sim \mathcal{N}(0, 1)$ and $x = (x_1, x_2, x_3, x_4) \sim P_x$. The goal is to design an invertible network such that it can transform normal distribution $\mathcal{N}(0, 1)$ to be data distribution P_x.

Give the boolean vector $b = (1, 1, 0, 0)$ and two mappings

$$\begin{aligned} s &: \mathbb{R}^4 \to \mathbb{R}^4 \\ t &: \mathbb{R}^4 \to \mathbb{R}^4, \end{aligned}$$

the coupling layer $x = c(z)$ is defined as the transformation

$$\begin{aligned} c &: \mathbb{R}^4 \to \mathbb{R}^4 \\ &\quad z \to x = b \odot z + (1 - b) \odot [z \odot \exp(s(b \odot z)) + t(b \odot z)] \end{aligned}$$

where exp is component-wise, and $\odot$ is the Hadamard component-wise product. Particularly,

$$\begin{aligned} x_1 &= z_1 \\ x_2 &= z_2 \\ x_3 &= z_3 \cdot \exp([s(b \odot z)]_3) + [t(b \odot z)]_3 \\ x_4 &= z_4 \cdot \exp([s(b \odot z)]_4) + [t(b \odot z)]_4 \end{aligned}$$

where $[s(b \odot z)]_i$ and $[t(b \odot z)]_i$ is the i-th component of $s(b \odot z)$ and $t(b \odot z)$, respectively. By arranging terms,

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= x_2 \\ z_3 &= (x_3 - [t(b \odot z)]_3) \cdot \exp(-[s(b \odot z)]_3) \\ z_4 &= (x_4 - [t(b \odot z)]_4) \cdot \exp(-[s(b \odot z)]_4) \end{aligned}$$

Note that $b \odot z = (1, 1, 0, 0) \odot (z_1, z_2, z_3, z_4) = (z_1, z_2, 0, 0) = (x_1, x_2, 0, 0) = b \odot x$, we have the inverse mapping $z = c^{-1}(x)$

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= x_2 \\ z_3 &= (x_3 - [t(b \odot x)]_3) \cdot \exp(-[s(b \odot x)]_3) \\ z_4 &= (x_4 - [t(b \odot x)]_4) \cdot \exp(-[s(b \odot x)]_4) \end{aligned}$$

or

$$c^{-1}(x) = b \odot x + (1 - b)(x - t(b \odot x)) \odot \exp(s(b \odot x))$$