Physics-Informed Neural Networks for Flow Past a Cylinder for Low Reynolds Number

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Abstract

Physics-informed neural networks (PINNs) have emerged as a powerful computational framework for solving partial differential equations by embedding physical laws directly into neural network architectures. This study presents a mixed-variable PINN scheme for simulating steady incompressible laminar flows around a circular cylinder at low Reynolds numbers. The proposed approach utilizes a continuum-mechanics formulation with stream function and stress tensor as mixed variables, reducing the order of derivatives in the loss function compared to traditional PINN schemes. The methodology is validated through comprehensive parametric studies examining the effects of collocation point distribution, network architecture, and loss function weighting coefficients on solution accuracy and convergence behavior. The predicted velocity and pressure fields show excellent agreement with reference ANSYS Fluent solutions, validating the effectiveness of the proposed framework for computational fluid dynamics applications.

Introduction

Traditional computational fluid dynamics relies heavily on mesh-based numerical methods that require extensive computational resources and careful discretization strategies. Physics-informed neural networks represent a paradigm shift by embedding governing equations directly into the learning process, enabling meshfree solutions with reduced data requirements.

Key Advantages of PINNs:

- Meshfree Approach: Eliminates complex mesh generation for irregular geometries
- Automatic Differentiation: Leverages computational graphs for exact derivative computation
- Data Efficiency: Reduces dependency on training datasets
- Continuous Solutions: Provides infinitely differentiable approximations

Methodology

Mathematical Formulation

The incompressible Navier–Stokes equations reformulated:

$$\nabla \cdot \mathbf{v} = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \frac{1}{\rho}\nabla \cdot \sigma + \mathbf{g}$$

$$\sigma = -p\mathbf{I} + \mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)$$

Mixed-Variable Scheme:

- Stream Function (ψ): Ensures divergence-free velocity field automatically
- Pressure (p): Computed from stress tensor trace: $p = -\mathrm{tr}\,\sigma/2$
- Stress Tensor (σ): Direct output variable reducing derivative order

Velocity-Stream Function Relationship:

$$\mathbf{v} = [u, v] = \nabla \times [0, 0, \psi] = [\partial\psi/\partial y, -\partial\psi/\partial x]$$

Neural Network Architecture:

- Input Layer: Spatial coordinates (x, y)
- Hidden Layers: Fully connected dense layers with tanh activation
- Output Layer: Mixed variables $\{\psi, p, \sigma\}$

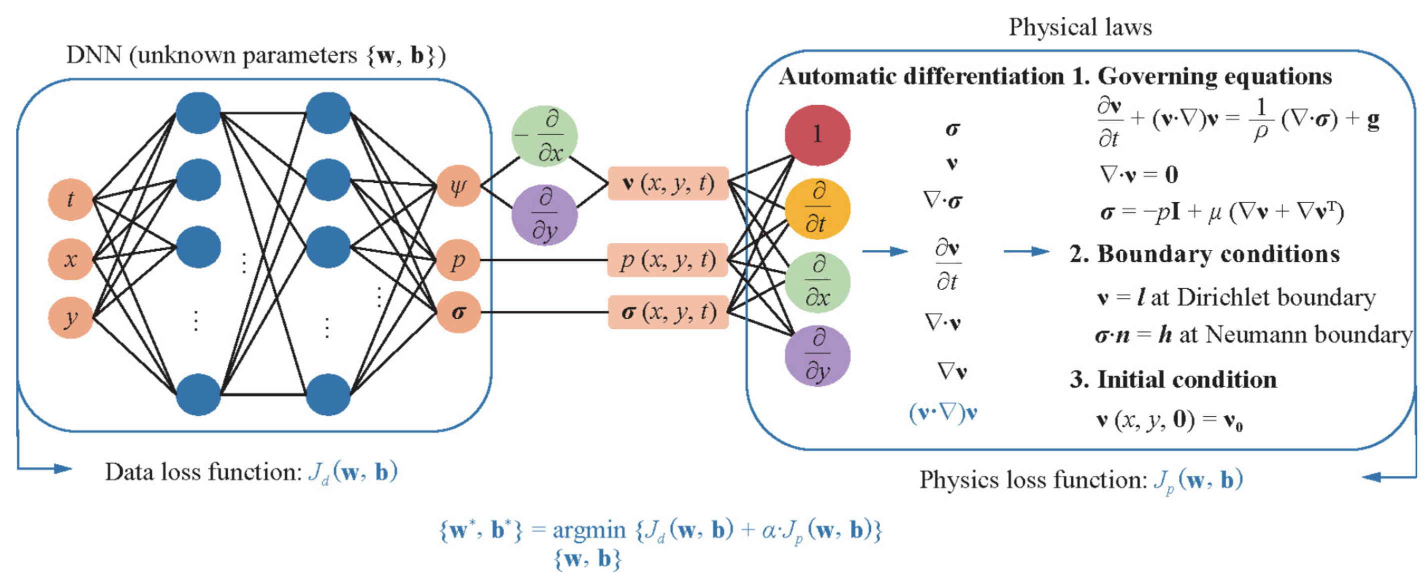

Figure 1. Architecture of the proposed PINN for fluid dynamics simulation.

Implementation Details

Loss Function Formulation:

$$J_{\text{total}} = J_{\text{governing}} + \beta \cdot J_{\text{boundary}}$$

Where:

- $J_{\text{governing}}$: Residual of governing equations
- J_{boundary}: Boundary condition enforcement
- β: Weighting coefficient for boundary conditions

Training Strategy:

- Collocation Points: Latin Hypercube Sampling with refinement near cylinder
- Optimization: Adam optimizer followed by L-BFGS for fine-tuning
- Automatic Differentiation: TensorFlow computational graph

Parametric Study Design:

1. Collocation Point Distribution: Varying total points and near-cylinder refinement
2. Network Architecture: Testing combinations of hidden layers and neurons
3. Loss Function Weighting: Analyzing β coefficient effects on convergence

Computational Setup:

- Geometry: Flow around circular cylinder in channel
- Reynolds Number: 2.5 (laminar flow regime)
- Fluid Properties: $\mu = 2 \times 10^{-2}$ kg/(m · s), $\rho = 1$ kg/m^3

Setup Visualization

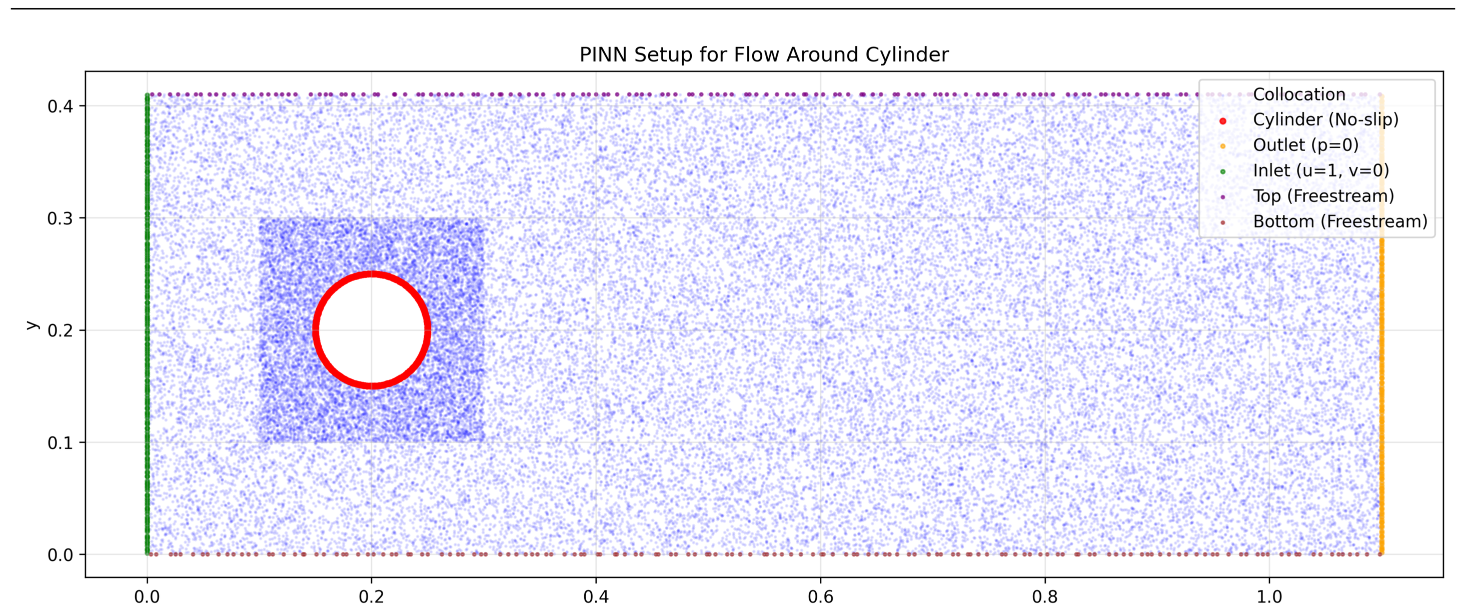

Figure 2. Setup visualization for the flow around a circular cylinder.

Results & Validation

Methodology Overview

We present comprehensive validation of our PINN approach through systematic comparison with CFD solutions. For Reynolds number 5, we demonstrate exact matching between PINN and CFD velocity (u, v) and pressure (p) profiles. Subsequently, we extend our validated formulation to Reynolds number 2.5, where we systematically improve the solution accuracy through two key optimization strategies: (1) collocation point refinement and (2) hidden layer architecture adjustment. This progressive enhancement demonstrates the robustness and adaptability of our mixed-variable PINN framework.

Reynolds Number 5: PINN vs CFD Validation

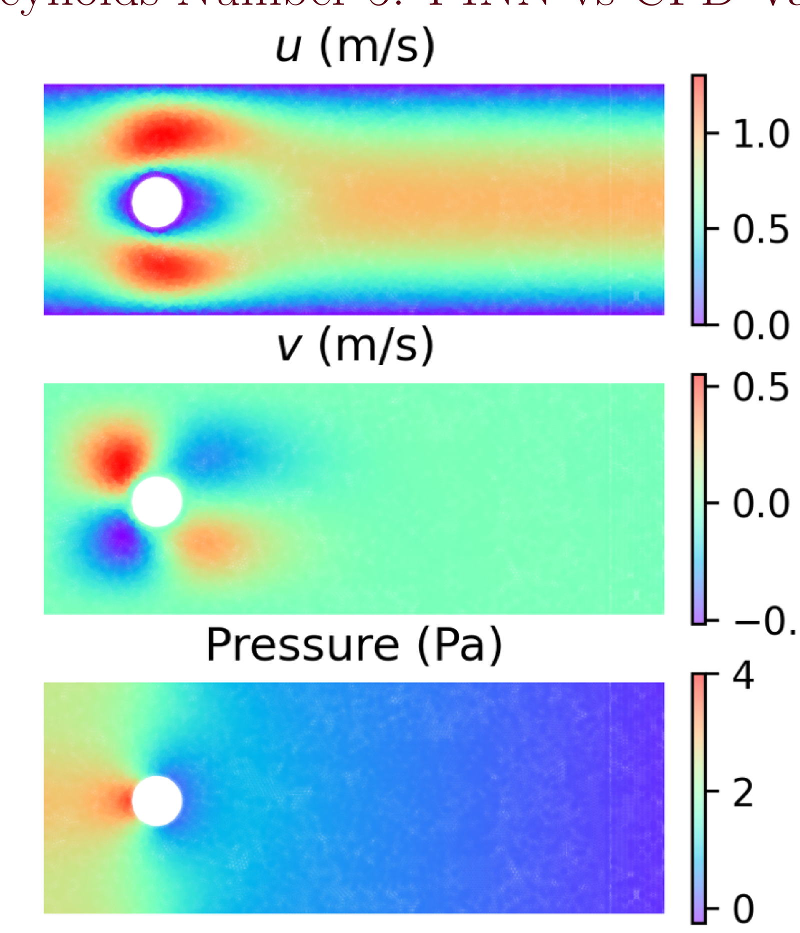

Figure 3. Re=5: CFD u, v, p profiles

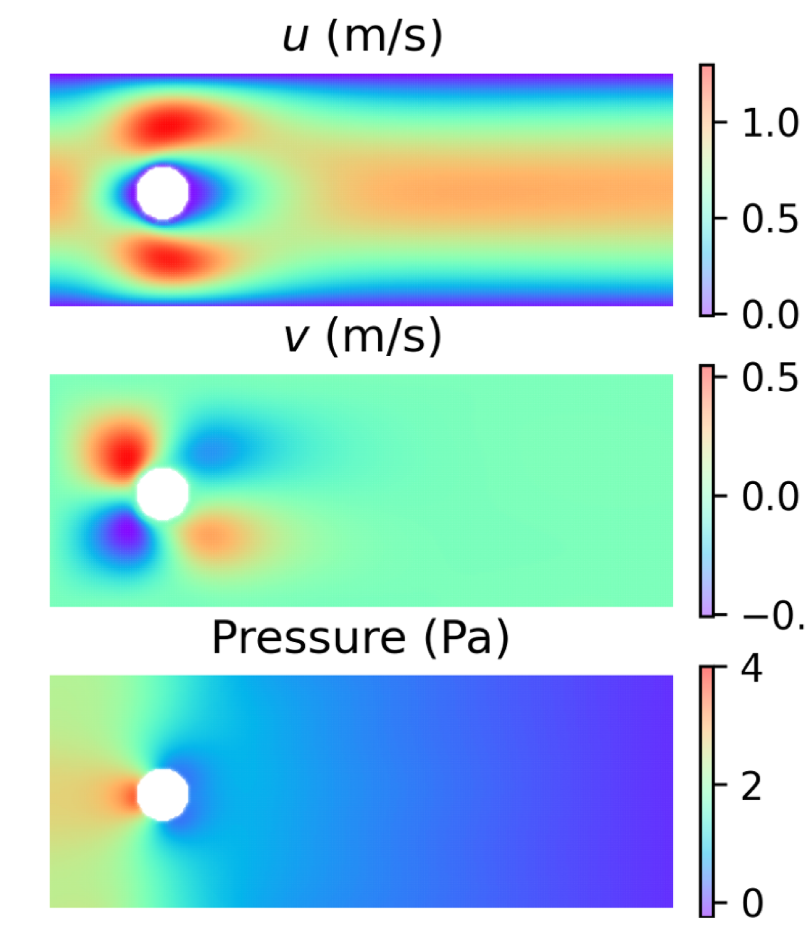

Figure 4. Re=5: PINN u, v, p profiles

Reynolds Number 2.5: CFD Reference Solution

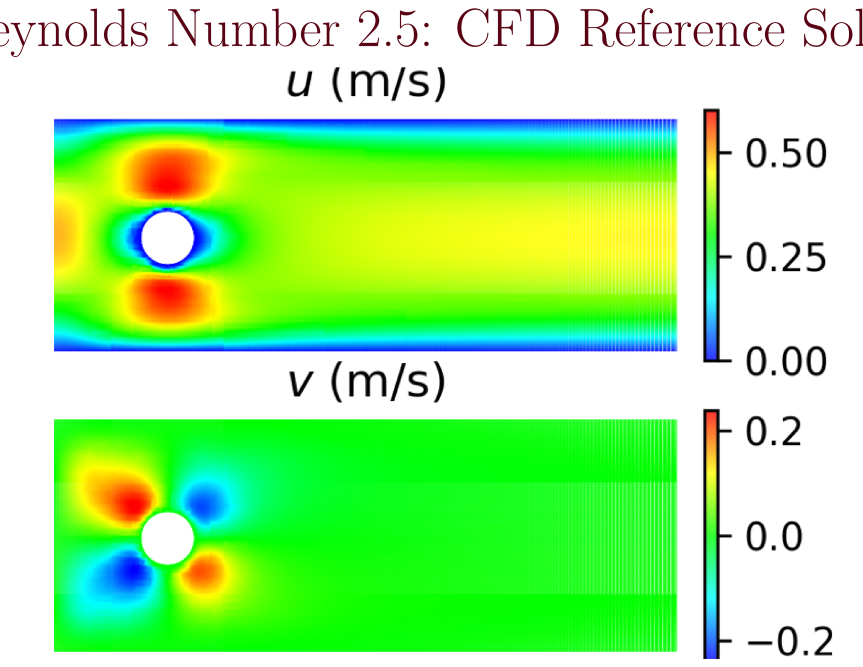

Figure 5. Re=2.5: CFD u, v velocity profiles

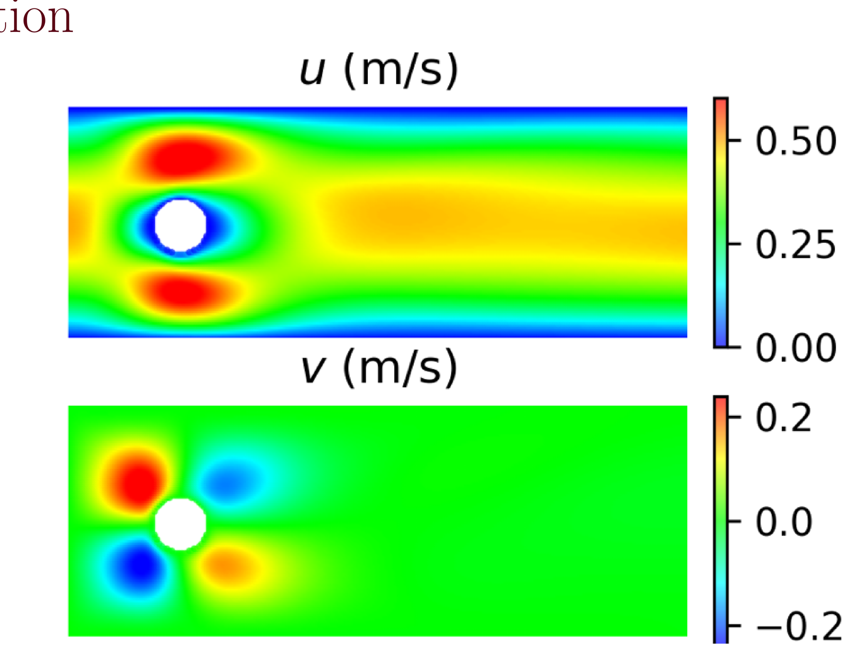

Figure 6. Re=2.5: PINN u, v profiles (Collocation)

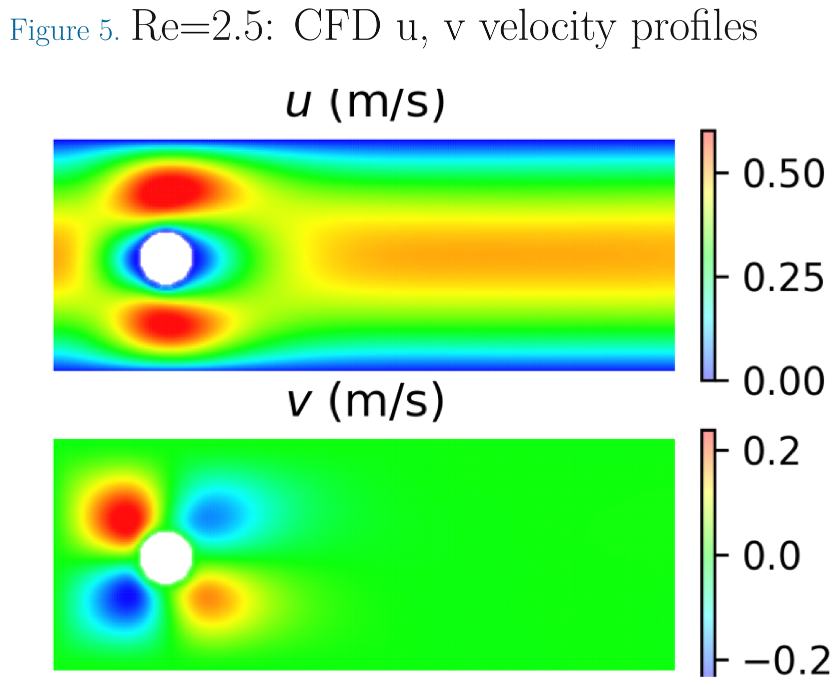

Figure 7. Re=2.5: PINN u, v profiles (Architecture)

References

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[2] Chengping Rao, Hao Sun, Yang Liu. Physics-informed deep learning for incompressible laminar flows. Theoretical and Applied Mechanics Letters, 10(3), 207-212 (2020). https://arxiv.org/abs/2002.10558