

Theorem: Ave # of inversions in an array of n distinct elements is $\frac{n(n-1)}{4}$. $O(n^2)$

Proof:

- Consider a list L of n integers
- L_r reverse list
- For every pair of indices (i, j) either i, j is in L or it is in L_r

Assumptions

- ① No duplicate elements
- ② All elements are equally likely.

Total # of inversions for both L & L_r is $\binom{n}{2} = \frac{n(n-1)}{2}$. ✓

✓ Average # of inversions.

$$\frac{\frac{n!}{2n!} \binom{n}{2}}{n(n-1)} = \frac{1}{4}$$

Any algorithm that sorts adjacent elements is $\Omega(n^2)$ on average.

$\Omega(n^2)$. Insertion, Bubble, Selection, } exchange adjacent elements.