Quadratic Probing (Q.P).

Theorem:

Assumption
1. Q.P ✓
2. Table Size is prime ✓

Then a new element can always be inserted if the table is atleast half empty.

Proof: Let TableSize be a prime greater than 3

We show that $\left\lceil \frac{\text{TableSize}}{2} \right\rceil$ alternative locations, including the initial $h_0(x)$ are all distinct.

show this

Proof by contradiction:

Consider 2 alternative locations in the first $\left\lceil \frac{\text{TableSize}}{2} \right\rceil$ set, namely

1 — $h(x) + i^2 \pmod{\text{Table Size}}$

2 — $h(x) + j^2$ (mod TableSize).

where $0 \le i, j \le \left\lfloor \frac{TableSize}{2} \right\rfloor$ 2

Assume that [these locations are the same] but $i \ne j$. 1 (Assume contradiction)

$\Downarrow$

1 == 2

$h(\cancel{x}) + i^2$ (mod TableSize) = $\cancel{h(x)} + j^2$ (mod TableSize)

even though $i \ne j$

$i^2 - j^2 = 0$ (mod TableSize).

$(i+j)(i-j) = 0$ (mod TableSize).

Since TableSize is prime,

either $i+j = 0$ (mod TableSize)

$i-j = 0$ (mod TableSize) $\rightarrow$

But

$i-j \ne 0$ (mod TableSize) since $i \ne j$ by assumption

+

$i+j \ne 0$ because $0 \le i, j \le \left\lceil \frac{TableSize}{2} \right\rceil$

$\Rightarrow$ So $i^2 - j^2$ cannot be $= 0$ (mod TableSize) the 2 alternative locations we chose.

$\Rightarrow$ Our assumption $h(x) - i^2$ (mod TableSize) + $h(x) + j^2$ (mod TableSize) $0 \le i, j \le \lfloor TableSize/2 \rfloor$ are equal is false $\Rightarrow$ they are distinct!

QED.

* In QP example, if we remove 89. almost all other find ops will fail..

Lazy deletion.

Quadratic Probing ← Computing probing sequence.

① Difference between 2 consecutive square #'s is odd number

$f(i+1)$ $f(i)$

$(i+1)^2 - i^2$

$\Rightarrow \cancel{i^2} + 2i + 1 \cancel{- i^2}$

$\Rightarrow 2i + 1$ odd #.

i^2	i
0	0
1	1^2
4	2^2
9	3^2
16	4^2
25	5^2
36.	6^2
49.	7^2
$f(i)$	i

$f(i+1) - f(i) = 2i+1.$

$$\boxed{f(i+1) = f(i) + 2i + 1}.$$

eg $f(7)$, $= f(6) + 2 \times 6 + 1.$

$i = 6$ $= 36 + 12 + 1.$

$= 49.$

The difference between consecutive odd #s is 2!

Check the code for contains, insert,
remove ~~5~~, 17