FINAL EXAM (U1-16)

NAME ______________________

ANSWERS ALL QUESTIONS ON THIS EXAM.
This exam is worth a total of 160 points. The points for each section & question are noted in parenthesis.

I. Units 2—6 (54 points)

1. Give an example of a non truth-functional operator and demonstrate that it is not truth-functional. (4)

2. If p, q are inconsistent statements, can p, q, r be consistent? Explain your answer. (4)

3. Can an argument form with inconsistent premises be invalid? (4)

4. Are the following true or false? (2 off for each wrong, up to 4)

(a) $(A \supset (\sim B \vee D)) \equiv \sim A$ is a substitution instance of $(p \supset (q \vee r)) \equiv \sim s$

(b) The biconditional of two contradictions is a tautology.

(c) Any contingent form logically implies any contradiction.

(d) A truth table can have 2 rows.

Symbolize the following, using the indicated abbreviations.

L ≡ Moon letters can be seen
S ≡ The moon shines behind the moon letters
M ≡ The moon is of the same shape as the day that the moon letters were written
A ≡ The moon is of the same season as the day that the moon letters were written
C ≡ You can go by the cave
G ≡ You stand by the greystone
D ≡ It is midnight
E ≡ It is a total eclipse of the sun

5. Only if the moon shines behind the moon letters they can be seen, provided that the moon is of the same shape and season as the day when they were written. (4)

6. You can't go in the cave unless you stand by the greystone and it is either midnight or a total eclipse of the sun. (4)

Truth Table Problems.

7. Use the <u>full</u> truth table method to determine whether the following argument form is valid or not. Show your work, be explicit in your answer and <u>explain</u> your answer. (8)

$(p \vee r) \supset \sim q$, $(p \bullet q) \vee (p \bullet r)$ $/ \therefore$ $(p \supset r) \supset q$

8. Use the full truth table method to determine whether the following is a tautology, contradiction, or contingent form. Show your work, be explicit in your answer and explain your answer. (7)

$\sim(\sim p \supset (\sim p \vee q)) \supset q$

9. Use the full truth table method to determine whether or not (a) and (b) are equivalent. Show your work, be explicit in your answer and explain your answer. (8)

(a) $(p \vee q) \supset (p \bullet q)$ (b) $\sim(p \equiv q)$

Equivalent or not? Why?

10. Use the full truth table method to determine whether or not (a) logically implies (b). Show your work, be explicit in your answer and explain your answer. (7)

(a) $\sim p \equiv q$ (b) $p \supset q$

II. Covering Units 7—9 (53 points)

Definitions and questions.

11. What are the two ways in which the application of the replacement rules differs from that of the basic inference rules? (4)

12. State the following rules of inference symbolically. (4)

    (a) Addition

    (b) Hypothetical Syllogism

13. What is a theorem? (4)

14. What is a derivation? (4)

15. What rule has been applied? (4)

    $A \supset C \quad / \therefore \ \sim A \vee C$

Construct proofs for the following arguments. Use IP in at least one of these problems (Qs 16-18). (11 each)

16. $(A \supset C)$, $C \supset (E \vee D)$, $\sim E$ / $\therefore$ $A \supset D$

17. $(P \supset S)$, $S \equiv \sim (B \vee D)$, $(P \supset D)$ / $\therefore$ $\sim P$

Prove the following theorem. (11)

18. $(\sim p \supset q) \supset (r \supset (q \vee p))$

III. Covering Units 10—16 (53 points)

Symbolize the following, using the indicated abbreviations.

$Ax \equiv$ x is an animal
$Lx \equiv$ x has three legs
$Dx \equiv$ x is a dog
$Mx \equiv$ x is a mammal

19. All animals have three legs only if some dogs have three legs. (7)

20. No dog has three legs. (7)

21. Not every three-legged animal is a dog unless all dogs are mammals. (7)

22. (a) Identify the form of the following as A, E, I, or O; and (b) represent it by a Venn Diagram. (6)

All dogs are mammals.

23. What is a prepositional function? Give an example of one. (4)

24. Prove the following theorem. Be careful about quantifier scope. (14)

$$(x)\,(Dx \supset Gx) \;\supset\; ((\exists x)\,Dx \supset (\exists x)\,Gx)$$

25. Use the model universe method to show that the following argument is invalid. (8)

$(\exists x)(Fx \bullet Gx)$, $(x)(Gx \supset {\sim}Hx)$ / $\therefore$ $(x)(Ax \supset {\sim}Fx)$

END OF EXAM