Neural Lyapunov Control

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NEURAL INFORMATION PROCESSING SYSTEMS

OVERVIEW

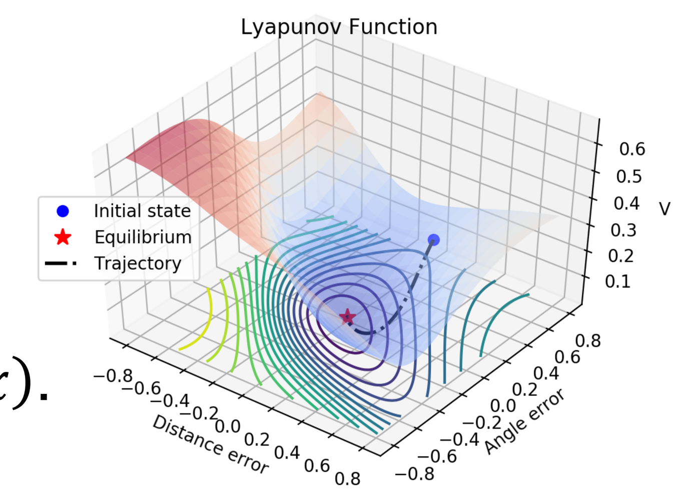
We propose new methods for learning control policies and neural network (NN) Lyapunov functions for nonlinear control problems, with provable guarantee of stability.

- The framework consists of a learner that attempts to find the control and Lyapunov functions, and a falsifier that finds counterexamples to quickly guide the learner towards solutions.
- The approach significantly simplifies the process of Lyapunov control design, provides end-to-end correctness guarantee, and can obtain much larger regions of attraction (ROA) than existing methods such as linear-quadratic regulators (LQR) and sum-of-squares (SOS) and semidefinite programming (SDP).

PRELIMINARIES

Proposition 1 (Lyapunov Functions for Asymptotic Stability)
Consider a controlled system $\frac{dx}{dt} = \mathrm{f_u}(x)$ with equilibrium at the origin, and suppose there exists a continuously differentiable function $\mathrm{V}: \mathcal{D} \to \mathbb{R}$ that satisfies the following conditions:

$$V(0) = 0,$$
$$\forall x \in \mathcal{D} \setminus \{0\}, V(x) > 0,$$
$$\forall x \in \mathcal{D} \setminus \{0\}, \nabla_{f_u} V(x) < 0,$$



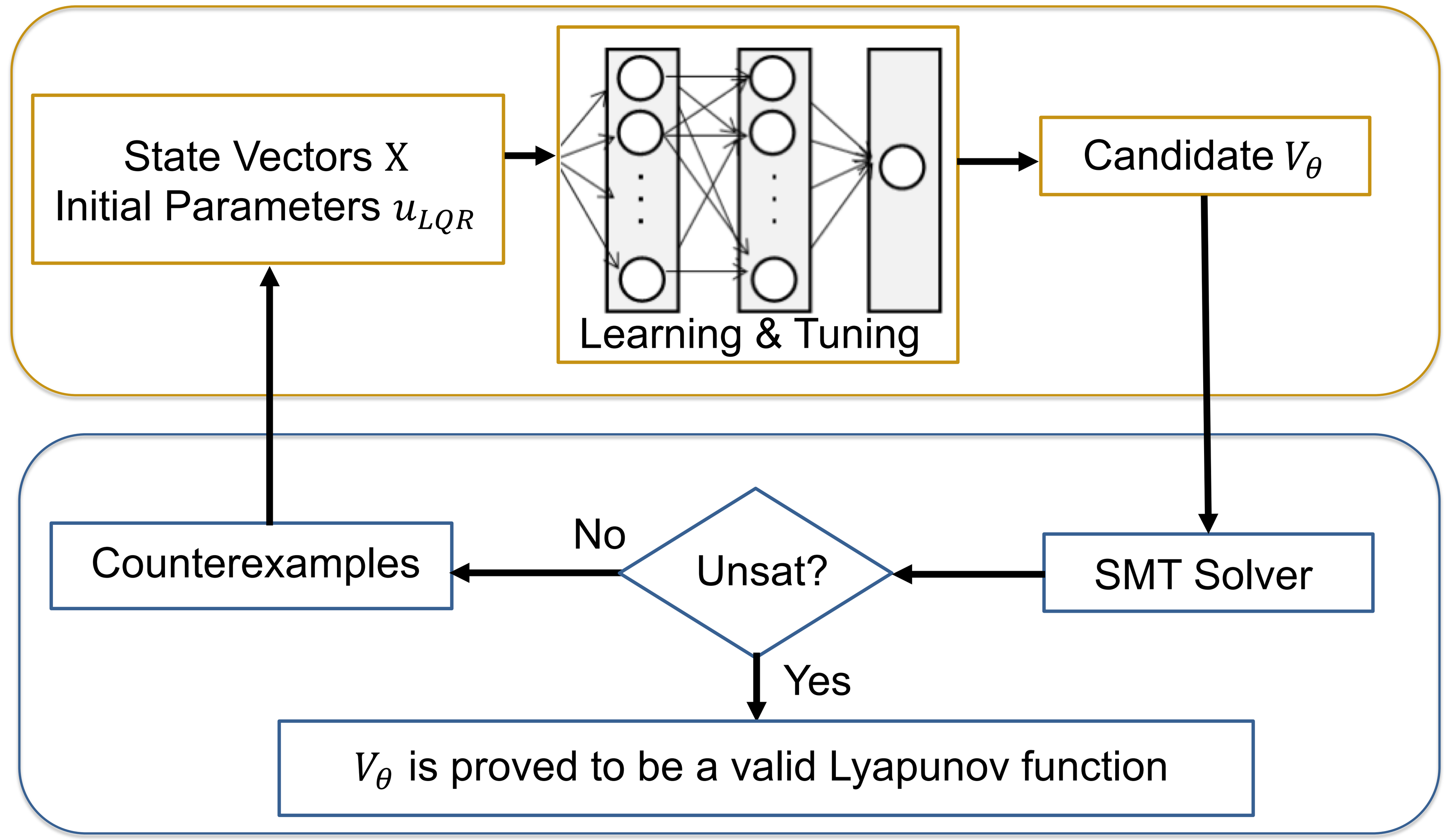
where $\nabla_{f_u} V(x) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \frac{dx_i}{dt} = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} [f_u]_i(x)$.

Then, the system is asymptotically stable at the origin, and V is called a Lyapunov function.

PROCEDURE

Learner :
Updates parameters in both a control function u and a neural Lyapunov function V_θ by iteratively minimizing the following Lyapunov risk:

$$L_\rho(\theta, u) =$$
$$\mathbb{E}_{x\sim\rho(\mathcal{D})}\left(\max\big(0, -V_\theta(x)\big) + \max\left(0, \nabla_{f_u} V_\theta(x)\right) + V_\theta^2(0)\right).$$



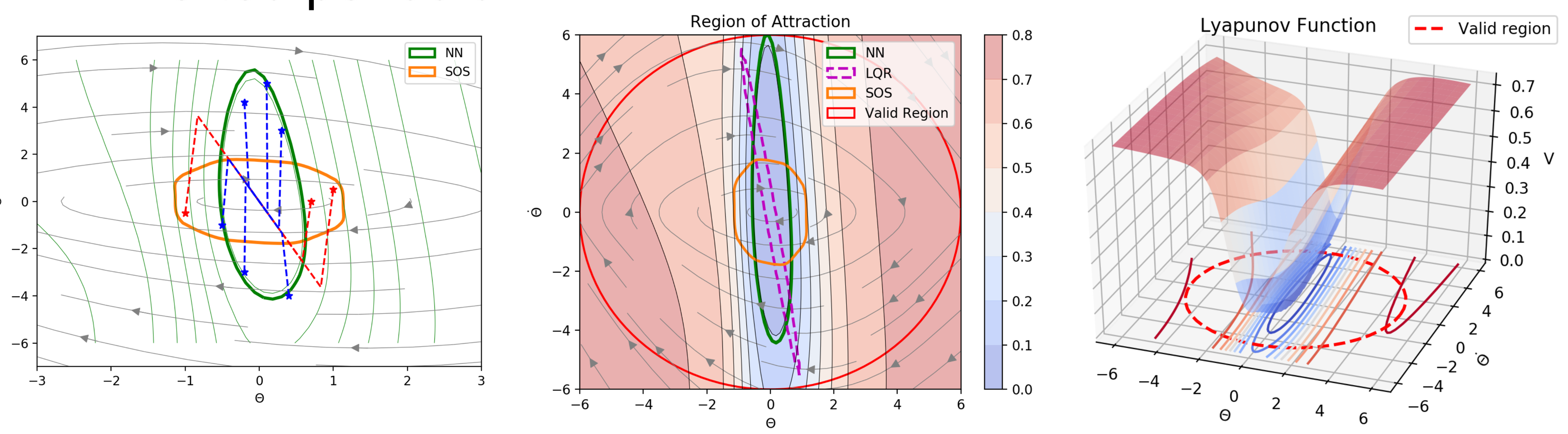
Falsifier :
Solves nonlinear constraint problems over real numbers to search for counterexample state vectors that violate the Lyapunov conditions. The falsification constraints are defined as follows:

$$\Phi_\varepsilon(x) = \left(\sum_{i=1}^{n} x_i^2 \geq \varepsilon\right) \wedge \left(V(x) \leq 0 \vee \nabla_{f_u} V(x) \geq 0\right).$$

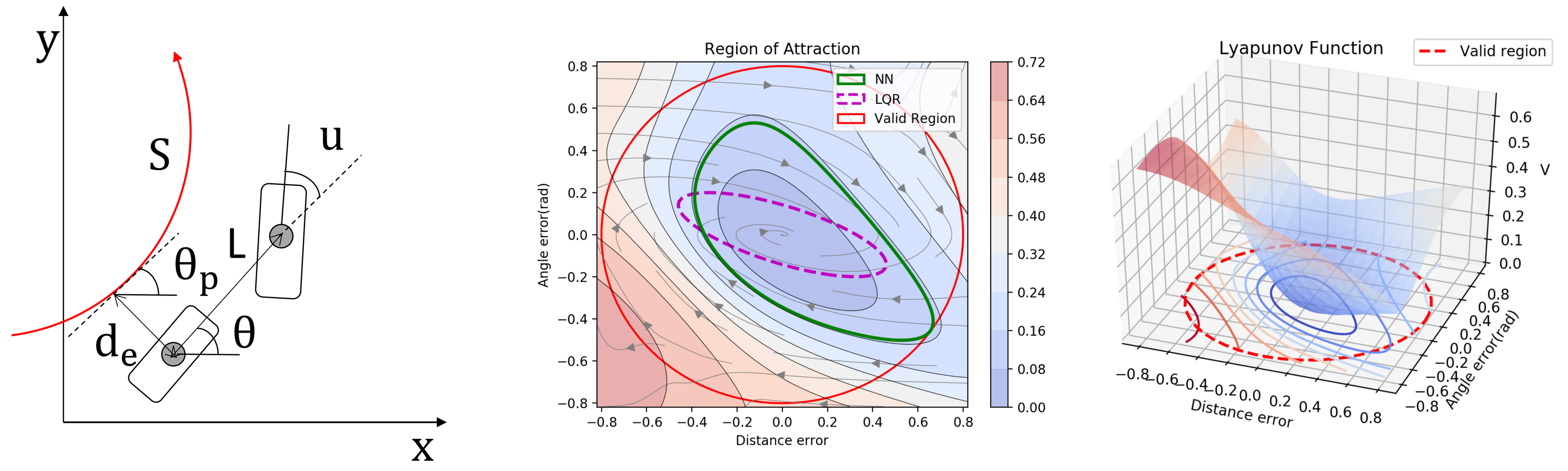
RESULTS

- In all examples, the regions of attraction are enlarged by more than three times compared with LQR results solved by defining both Q and R as identity matrices.
- In an inverted pendulum, we show trajectories (blue) are safely bounded within ROA defined by NN Lyapunov function. On the contrary, many trajectories (red) start inside the SOS region can escape.

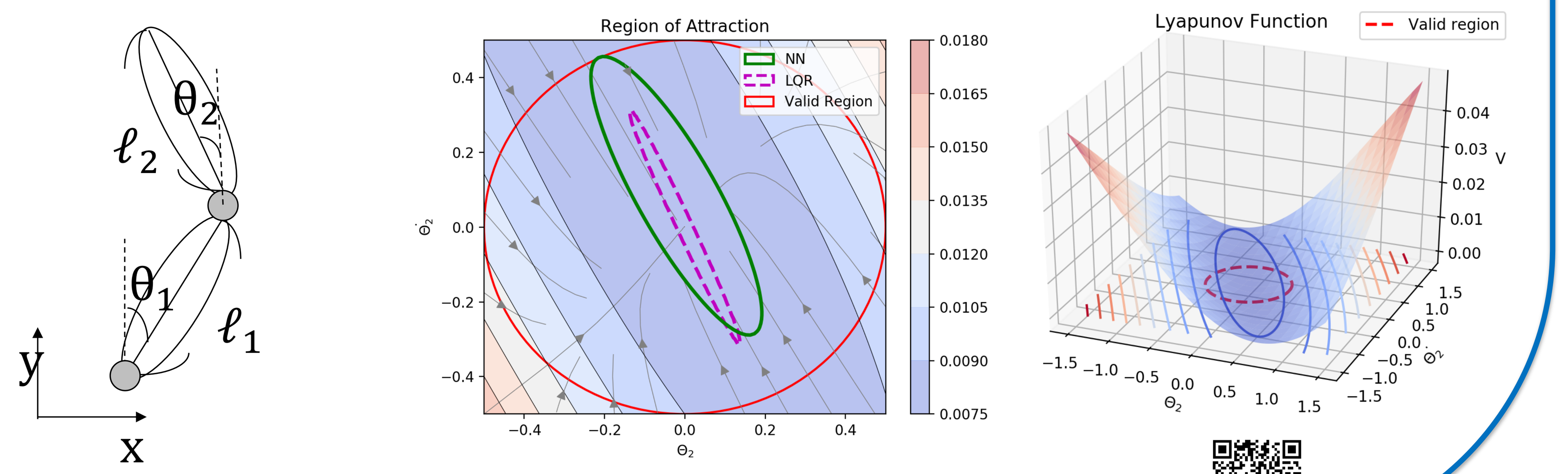
❖ Inverted pendulum



❖ Vehicle path following



❖ Humanoid balance



Project page available at