UC San Diego

Neural Lyapunov Control

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ABSTRACT

We propose new methods for learning control policies and neural network Lyapunov functions for nonlinear control problems, with provable guarantee of stability. The framework consists of a learner that attempts to find the control and Lyapunov functions, and a falsifier that finds counterexamples to quickly guide the learner towards solutions. The procedure terminates when no counterexample is found by the falsifier, in which case the controlled nonlinear system is provably stable. The approach significantly simplifies the process of Lyapunov control design, provides end-to-end correctness guarantee, and can obtain much larger regions of attraction than existing methods such as LQR and SOS/SDP. We show experiments on how the new methods obtain high-quality solutions for challenging robot control problems such as humanoid robot balancing and wheeled vehicle path-following.

PRELIMINARIES

Proposition 1 (Lyapunov Functions for Asymptotic Stability).

Consider a controlled system $\frac{d_x}{d_t} = f_u(x)$ *with equilibrium at the origin.*

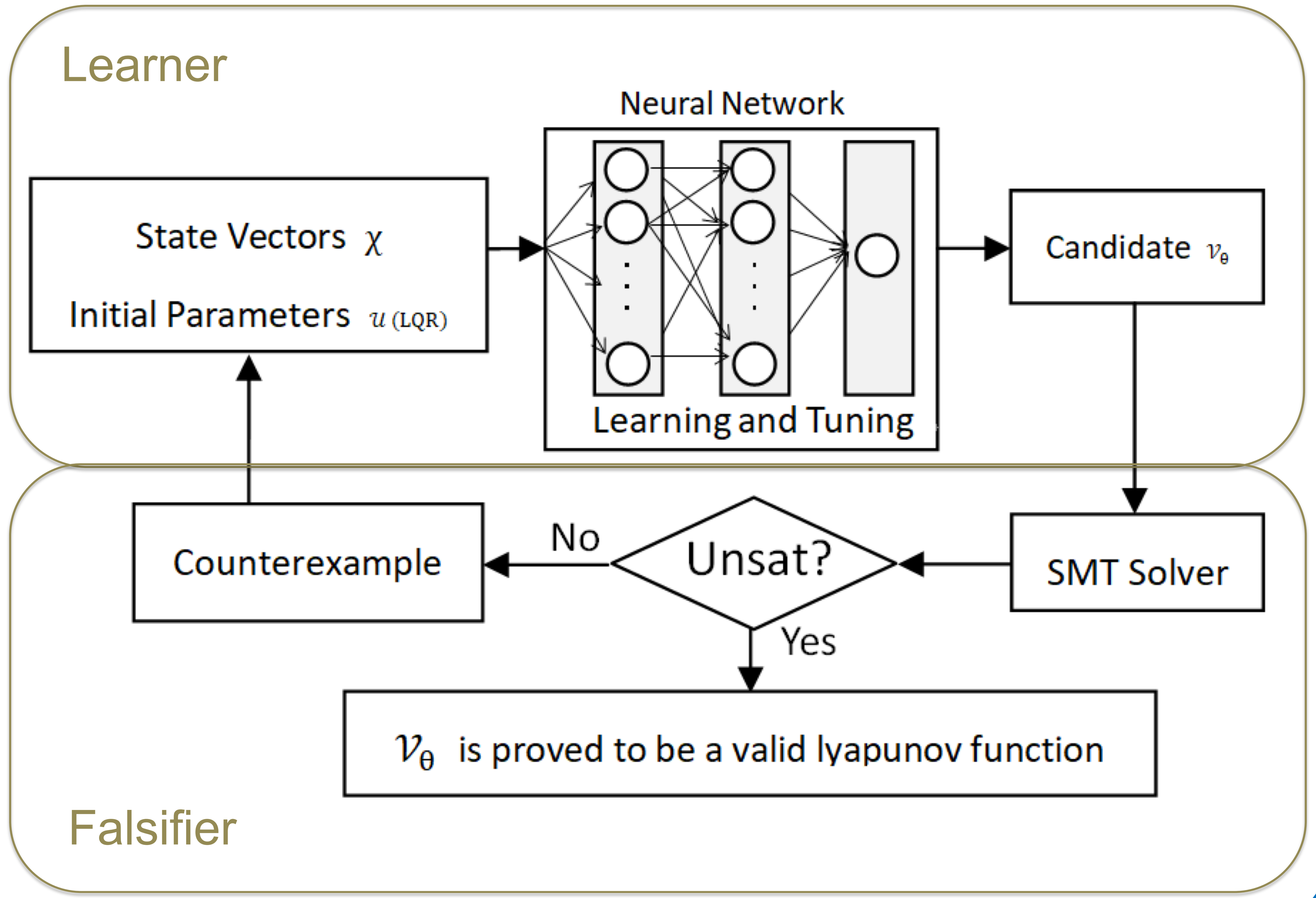
Suppose there exists a continuously differentiable function $V: \mathcal{D} \rightarrow \mathcal{R}$ *that satisfies the following conditions:*

$$V(0) = 0,$$
$$\forall x \in \mathcal{D} \setminus \{0\}, V(x) > 0,$$
$$\forall x \in \mathcal{D} \setminus \{0\}, \nabla_{f_u} V(x) < 0.$$

Then, the system is asymptotically stable at the origin and V is called a Lyapunov function.

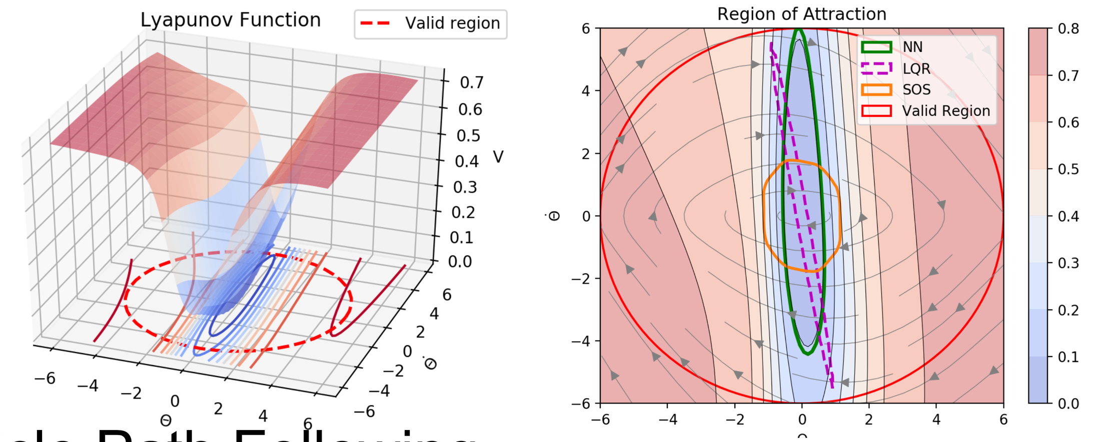
PROCEDURE

The framework consists of a learner and a falsifier. The learner uses stochastic gradient descent to find parameters in both a control function and a neural Lyapunov function, by iteratively minimizing the *Lyapunov risk* which measures the violation of the Lyapunov conditions. The falsifier takes a control function and Lyapunov function from the learner, and searches for *counterexample* state vectors that violate the Lyapunov conditions. The counterexamples are added to the training set for the next iteration of learning, generating an effective curriculum. The falsifier uses delta-complete constraint solving, which guarantees that when no violation is found, the Lyapunov conditions are guaranteed to hold for all states in the verified domain.

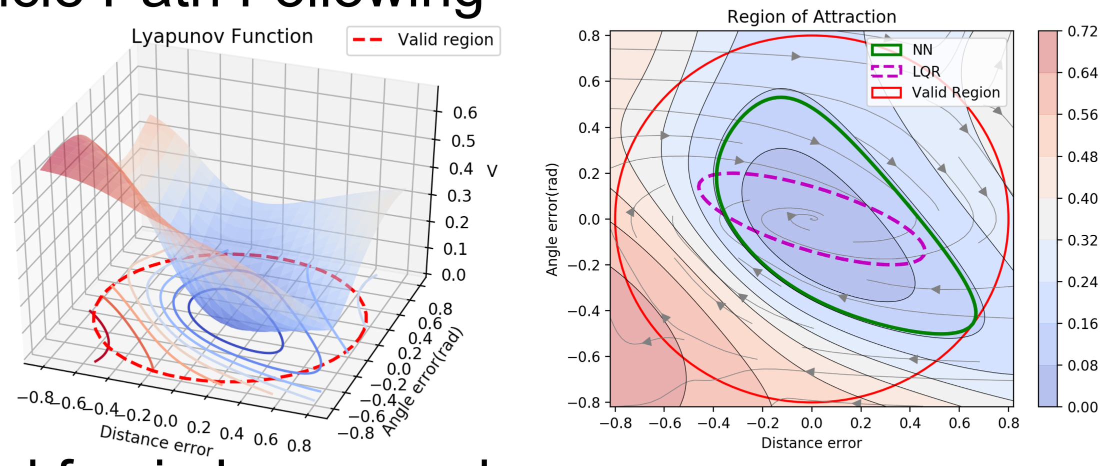


RESULTS

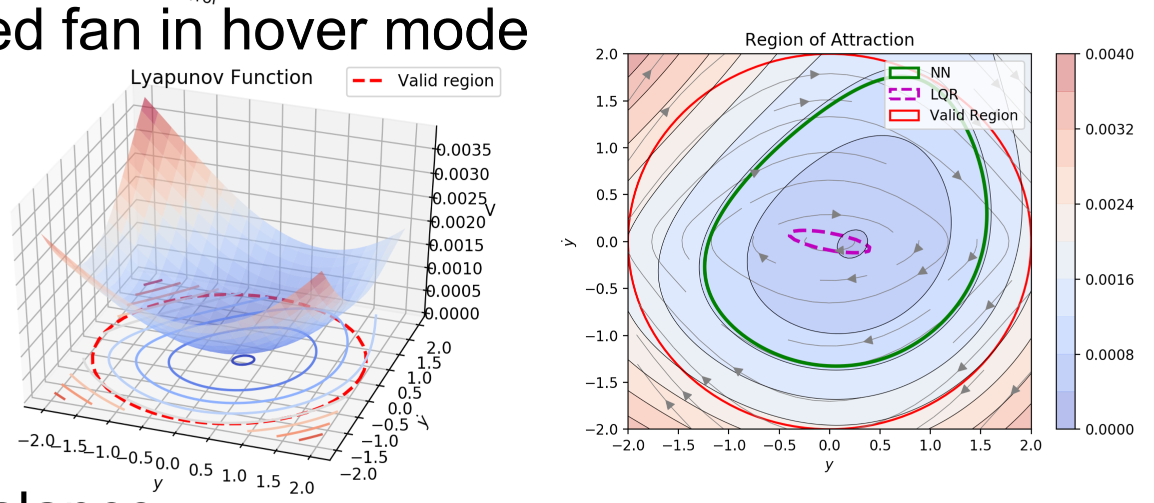
- Inverted pendulum



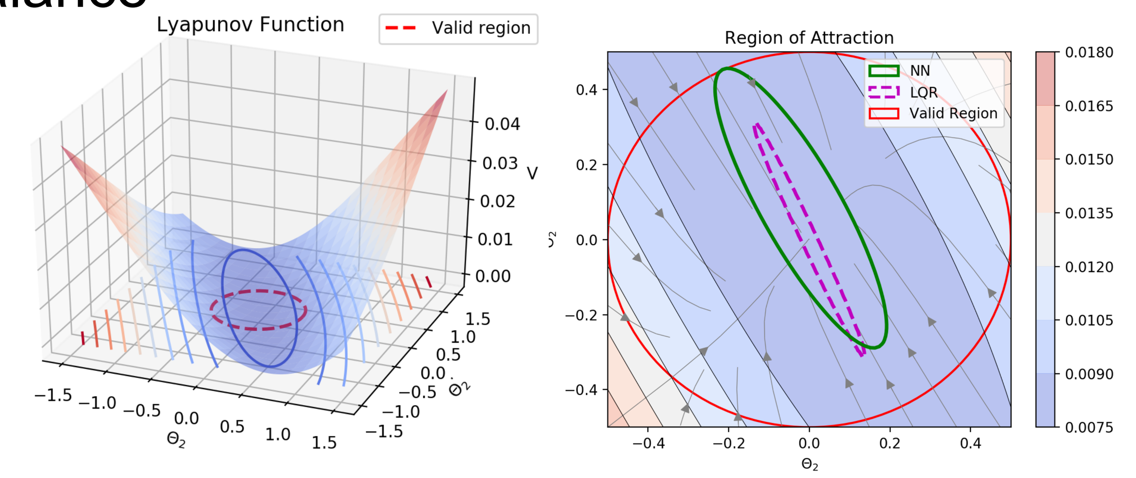
- Wheeled Vehicle Path Following



- Caltech ducted fan in hover mode



- Humanoid balance



Project page available at