Exercise 4A

1 An octagonal die is thrown 500 times and the results are noted. It is assumed that the die is unbiased. A test is to be done to see whether the observed results differ from the expected ones. Write down a null hypothesis and an alternative hypothesis that can be used.

2 For 5 degrees of freedom find the critical value of χ^2 which is exceeded with a probability of 5%.

3 Find the values of the following from the table on page 137.

a χ^2_5 (5%) **b** χ^2_8 (1%) **c** χ^2_{10} (10%)

4 With $\nu = 10$ find the value of χ^2 that is exceeded with 0.05 probability.

5 With $\nu = 8$ find the value of χ^2 that is exceeded with 0.10 probability.

6 The random variable Y has a χ^2 distribution with 8 degrees of freedom. Find y such that $P(Y < y) = 0.99$.

7 The random variable X has a χ^2 distribution with 5 degrees of freedom. Find x such that $P(X < x) = 0.95$.

8 The random variable Y has a χ^2 distribution with 12 degrees of freedom. Find:

a y such that $P(Y < y) = 0.05$, **b** y such that $P(Y < y) = 0.95$.

Exercise 4B

1 The following table shows observed values for what is thought to be a discrete uniform distribution.

x	1	2	3	4	5	6	7	8
Frequency of x	12	24	18	20	25	17	21	23

a Calculate the expected frequencies and, using a 5% significance level, conduct a goodness of fit test.

b State your conclusions.

2 The following tables show observed values (O) and expected values (E) for a goodness of fit test of a binomial distribution model. The probability used in calculating the expected values has not been found from the observed values.

O	17	28	32	15	5	3
E	19.69	34.74	27.59	12.98	4.01	0.99

a Conduct the test using a 5% significance level and state your conclusions.

b Suggest how the model might be improved.

3 The following table shows observed values for a distribution which it is thought may be modelled by a Poisson distribution.

x	0	1	2	3	4	5	>5
Frequency of x	12	23	24	24	12	5	0

A possible model is thought to be Po(2). From tables, the expected values are found to be as shown in the following table.

x	0	1	2	3	4	5	>5
Expected frequency of x	13.53	27.07	27.07	18.04	9.02	3.61	1.66

a Conduct a goodness of fit test at the 5% significance level.

b It is suggested that the model could be improved by estimating the value of λ from the observed results. What effect would this have on the number of constraints placed upon the degrees of freedom?

4 A mail order firm receives packets every day through the mail.

They think that their deliveries are uniformly distributed throughout the week. Test this assertion, given that their deliveries over a 4-week period were as follows. Use a 0.05 significance level.

Day	Mon	Tues	Wed	Thurs	Fri	Sat
Frequency	15	23	19	20	14	11

5 Over a period of 50 weeks the number of road accidents reported to a police station were as shown.

Number of accidents	0	1	2	3	4
Number of weeks	15	13	9	13	0

a Find the mean number of accidents per week.

b Using this mean and a 0.10 significance level, test the assertion that these data are from a population with a Poisson distribution.

6 A marksman fires 6 shots at a target and records the number r of bull's-eyes hit. After a series of 100 such trials he analyses his scores, the frequencies being as follows.

r	0	1	2	3	4	5	6
Frequency	0	26	36	20	10	6	2

a Estimate the probability of hitting a bull's-eye.

b Use a test at the 0.05 significance level to see if these results are consistent with the assumption of a binomial distribution.

7 The table below shows the number of employees in thousands at five factories and the number of accidents in 3 years.

Factory	A	B	C	D	E
Employees (thousands)	4	3	5	1	2
Accidents	22	14	25	8	12

Using a 0.05 significance level, test the hypothesis that the number of accidents per 1000 employees is constant at each factory.

8 In a test to determine the red blood cell count in a patient's blood sample, the number of cells in each of 80 squares is counted with the following results.

Number of cells per square, x	0	1	2	3	4	5	6	7	8
Frequency, f	2	8	15	18	14	13	7	3	0

It is assumed that these will fit a Poisson distribution. Test this assertion at the 0.05 significance level.

9 A factory has a machine. The number of times it broke down each week was recorded over 100 weeks with the following results.

Number of times broken down	0	1	2	3	4	5
Frequency	50	24	12	9	5	0

It is thought that the distribution is Poisson.

a Give reasons why this assumption might be made.

b Conduct a test at the 0.05 level of significance to see if the assumption is reasonable.

10 In a lottery there are 505 prizes, and it is assumed that they will be uniformly distributed throughout the numbered tickets. An investigation gave the following:

Ticket number	1–1000	1001–2000	2001–3000	3001–4000	4001–5000	5001–6000	6001–7000	7001–8000	8001–9000	9001–10000
Frequency	56	49	35	47	63	58	44	52	51	50

Using a suitable test with a 0.05 significance level, and stating your null and alternative hypotheses, see if the assumption is reasonable.