Network Centrality

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International Trade Network

Figure 1: International trade picture

From Florida State Hispanic Chamber of Commerce

Production Network

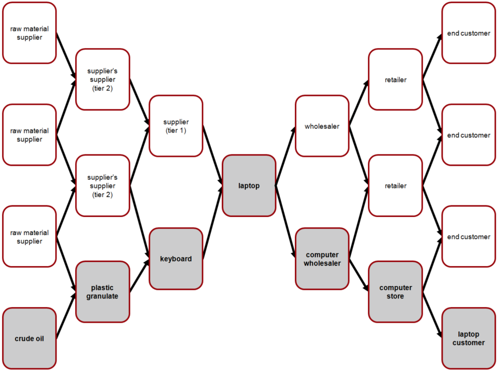


Figure 2: Supply chain

From Wikipedia

Social Network

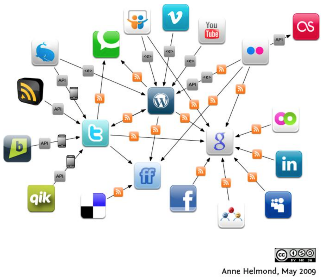


Figure 3: A sample of social network

From Webpage Kikolani

International Financial Network

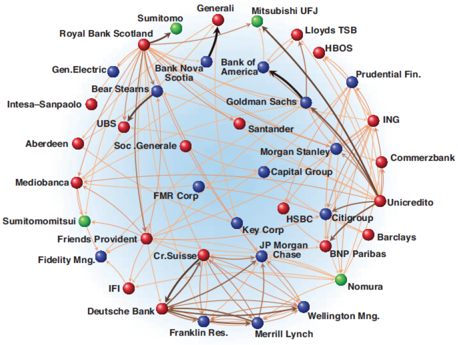


Figure 4: A sample of international financial network

From Frank Schweitzer, et al. (Science 325, 422 (2009))

Network of Coauthorship

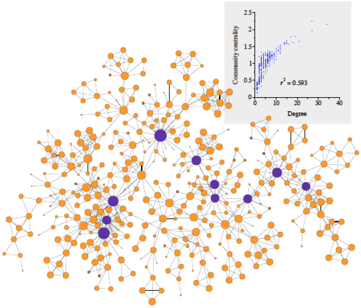


Figure 5: A network of coauthorships between 379 scientists whose research centers on the properties of networks of one kind or another

From M.E.J. Newman (Phys. Rev. E 74, 036104 (2006))

A More Specific Coauthorship Graph

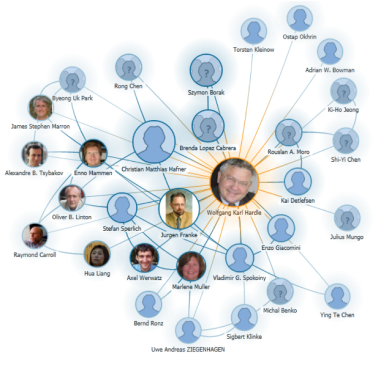


Figure 6: Coauthorship graph of W.K. Härdle

Source: Microsoft Academic

A More Specific Coauthorship Graph

Figure 7: Coauthorship path of W.K. Härdle

Source: Microsoft Academic

A More Specific Coauthorship Graph

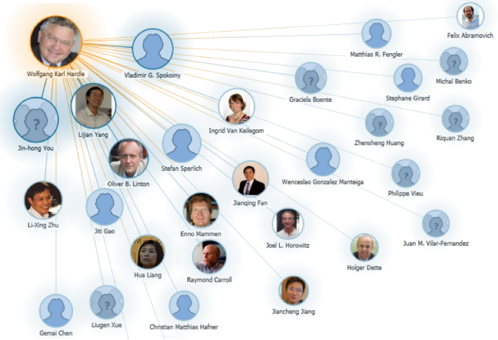


Figure 8: Citation path of W.K. Härdle

Source: Microsoft Academic

Outline

1. Motivation ✓
2. Definition
3. Relation to Adjacency Matrix
4. Network Centrality
5. Comparison of Various Centralities

Network Structure

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$

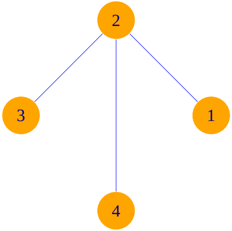
- vertices $\mathcal{V}$: each individual in your context
- edges $\mathcal{E}$: linkages between every pair of vertices

Adjacency Matrix & Network

Example 1: symmetric, unweighted adjacency matrix with 4 nodes

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

A sample symmetric, undirected network of 4 nodes



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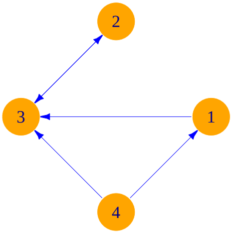
Adjacency Matrix & Network

Example 2: asymmetric, unweighted adjacency matrix with 4 nodes

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

A sample directed, unweighted network of 4 nodes



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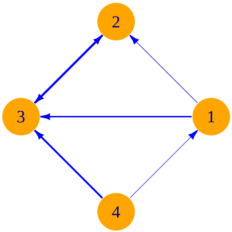
Adjacency Matrix & Network

Example 3: asymmetric, weighted adjacency matrix with 4 nodes

$$A = \begin{bmatrix} 0 & 0.3 & 0.7 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0.1 & 0 & 0.9 & 0 \end{bmatrix}$$

A sample directed, weighted network of 4 nodes



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Network Centrality

- Degree centrality
- Closeness centrality
- Betweenness centrality
- Eigenvector centrality
- Katz centrality
- PageRank centrality
- Percolation centrality
- Cross-clique centrality
- Freeman Centrality

Degree Centrality

For a network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, degree centrality equals

$$C_D(v) = \deg(v)$$

where $\deg(v)$ denotes the total number of edges vertice v has.

Closeness Centrality

For a network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, closeness centrality equals

$$C_C(v_1) = \frac{N-1}{\sum_{v_2} d(v_1, v_2)}$$

- N: the total number of vertices
- $d(v_1, v_2)$: distance between v_1 and v_2

Betweenness Centrality

For a network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, betweenness centrality equals

$$C_B(v) = \sum_{s \neq v \neq t \in \mathcal{V}} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

- σ_{st}: total amount of shortest paths from vertice s to vertice t
- $\sigma_{st}(v)$: total amount of shortest paths from vertice s to vertice t that passes through vertice v

More about 'distance'

- related to C_C and C_B
- the number of edges in the shortest connecting path (in the sense of edge number) – unweighted
- the real length of shortest connecting path (in the sense of real length)– weighted
- considered as a 'cost'

Example is given at the end of this talk : Minnesota Road Networks

Eigenvector Centrality

For a network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, eigenvector centrality equals

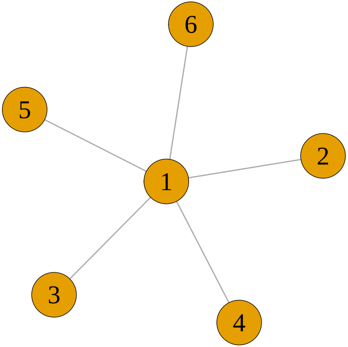
$$C_E(v) = \frac{1}{\lambda} \sum_{t \in M(v)} C_E(t) = \frac{1}{\lambda} \sum_{t \in \mathcal{G}} a_{v,t} C_E(t)$$

$$\lambda C_E = A C_E$$

- $A = \{a_{v,t}\}_{v,t=1}^{N}$ is a 0-1 adjacency matrix
- λ: maximum eigenvalue of A
- $M(v)$: set of neighbors of v
- $a_{v,t}$: the vt_{th} element of A

which is the central node?

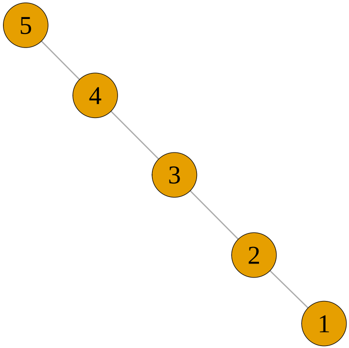
Example 4:



- $\mathbf{C_D(1) = 5}$, $C_D(2) = C_D(3) = C_D(4) = C_D(5) = C_D(6) = 1$
- $\mathbf{C_C(1) = 1}$, $C_C(2) = C_C(3) = C_C(4) = C_C(5) = C_C(6) = 0.56$
- $\mathbf{C_B(1) = 1}$, $C_B(2) = C_B(3) = C_B(4) = C_B(5) = C_B(6) = 0$
- $\mathbf{C_E(1) = 0.71}$, $C_E(2) = C_E(3) = C_E(4) = C_E(5) = C_E(6) = 0.32$

which is the central node?

Example 5:



- $C_D(1) = C_D(5) = 1, \mathbf{C_D(2) = C_D(3) = C_D(4) = 2}$
- $C_C(1) = C_C(5) = 0.4, C_C(2) = C_C(4) = 0.57, \mathbf{C_C(3) = 0.67}$
- $C_B(1) = C_B(5) = 0, C_B(2) = C_B(4) = 0.50, \mathbf{C_B(3) = 0.67}$
- $C_E(1) = C_E(5) = 0.29, C_E(2) = C_E(4) = 0.50, \mathbf{C_B(3) = 0.58}$

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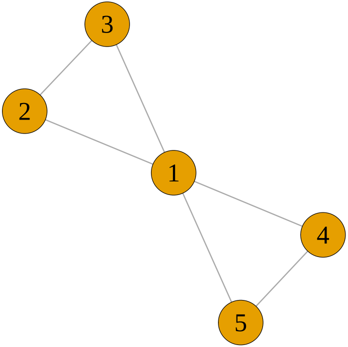
which is the central node?

Example 6:



- $\mathbf{C_D(1) = 4}$, $C_D(2) = C_D(3) = C_D(4) = C_D(5) = 2$
- $\mathbf{C_C(1) = 1}$, $C_C(2) = C_C(3) = C_C(4) = C_C(5) = 0.67$
- $\mathbf{C_B(1) = 0.67}$, $C_B(2) = C_B(3) = C_B(4) = C_B(5) = 0$
- $\mathbf{C_E(1) = 0.62}$, $C_E(2) = C_E(3) = C_E(4) = C_E(5) = 0.39$

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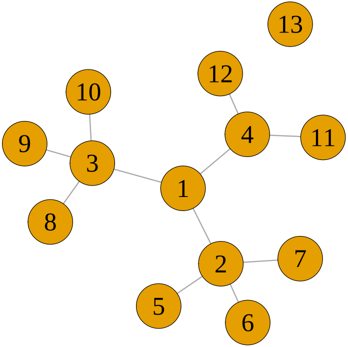
which is the central node?

Example 7:



- $C_D(1) = C_D(4) = 3$, $\mathbf{C_D(2) = C_D(3) = 4}$
- $\mathbf{C_C(1) = 0.63}$, $C_C(2) = C_C(3) = 0.52$, $C_C(4) = 0.48$
- $\mathbf{C_B(1) = 0.61}$, $C_B(2) = C_B(3) = 0.36$, $C_B(4) = 0.27$
- $\mathbf{C_E(1) = 0.51}$, $C_E(2) = C_E(3) = 0.44$, $C_E(4) = 0.33$

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Minnesota Road Networks

- Minnesota Road Net data minnesota.mat is avaiable in Matlab R2016b and later versions
- Minnesota State of United States
- The data contains a G object which is a combination of Nodes (Coordinates of locations) and Edges (roads between each pair of nodes)

Minnesota Road Networks

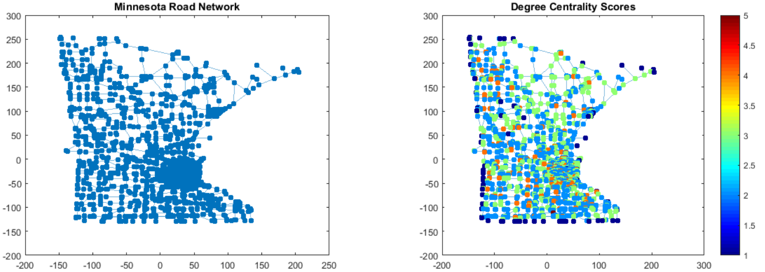

Figure 9: Display of Minnesota Road Net – no centrality displayed (left), degree centrality C_D (right)

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Minnesota Road Networks

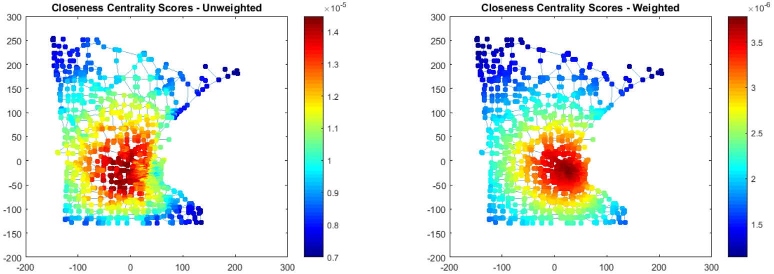

Figure 10: Display of Minnesota Road Net – closeness centrality unweighted (left), closeness centrality weighted (right)

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Minnesota Road Networks

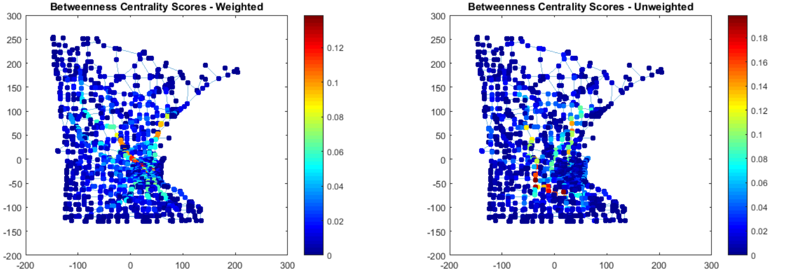

Figure 11: Display of Minnesota Road Net – betweenness centrality unweighted (left), betweenness centrality weighted (right)

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Minnesota Road Networks

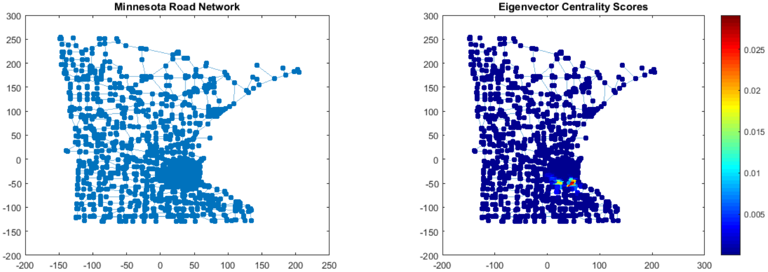

Figure 12: Display of Minnesota Road Net – no centrality displayed (left), eigenvector centrality C_E(right)

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Network Centrality

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