RSM-based Generalized LL Parsing Algorithm

All You Need is Generalized GLL

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Recursive State Machine

Definition (Recursive State Machine)

Recursive state machine (RSM) is a tuple $\mathcal{R} = \langle \mathcal{N}, \Sigma, B, B_S, Q, Q_S \rangle$

- N is a set of nonterminal symbols
- Σ is a set of terminal symbols
- Q is a set of states of the RSM
- Q_S is a set of the start states of all *boxes*
- $B = \{B_{N_i} \mid N_i \in \mathcal{N}\}$ is a set of **boxes**, where each $B_{N_i} = \langle Q_{N_i}, q_S, Q_F^{N_i}, \delta \rangle$ is a deterministic finite automaton without ε-transitions
  - $Q_{N_i} \subseteq Q$ is a set of states of the box
  - $q_S \in Q_{N_i} \cap Q_S$ is the start state of the box
  - $Q_F^{N_i} \subseteq Q_{N_i}$ is a set of final states of the box
  - $\delta \subseteq Q_{N_i} \times (\Sigma \cup Q_S) \times Q_{N_i}$ is a transition function of the box
- $B_S \in B$ is the start box of the RSM

Configuration of RSM

Definition (Configuration of RSM)

A *Configuration* $C_{\mathcal{R}}$ of the computation of the RSM $\mathcal{R} = \langle \mathcal{N}, \Sigma, B, B_S, Q, Q_S \rangle$ over the graph $D = \langle V, E, L \rangle$ is a tuple $(q, v, \mathcal{S})$

- $q \in Q$ is a current state of RSM
- $\mathcal{S}$ is the current stack, which frames have one of two types
  - Return addresses frame (elements of Q) to specify states to continue computation after the call is finished
  - Parsing tree node to store fragments of a parsing tree
- $v \in V$ is the current vertex (current position in the input)

Transition Step

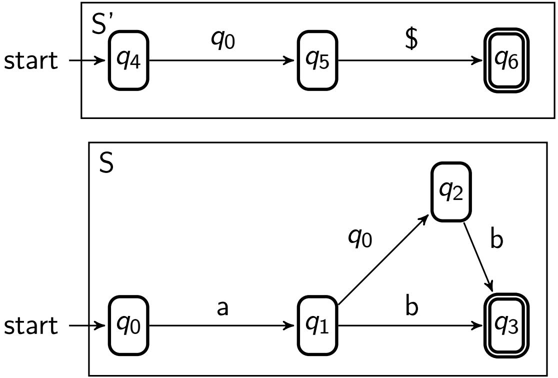
Definition (Step of RSM Computation)

A *transition step* of the RSM specifies how to get new configurations of RSM, given the current configuration. $C_{\mathcal{R}} \vdash W$ denotes that $\mathcal{R}$ can go to each configuration in W from the configuration $C_{\mathcal{R}}$

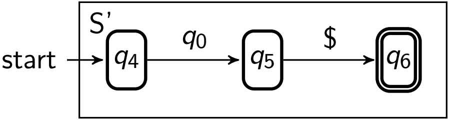
$$\begin{aligned}(q, v, w_0 :: s :: \mathcal{S}) \vdash & \{(q', v', t :: w_0 :: s :: \mathcal{S}) \mid (q, t, q') \in \delta, (v, t, v') \in E\} \\ & \cup \{(s', v, q' :: w_0 :: s :: \mathcal{S}) \mid (q, s', q') \in \delta, s' \in Q_S\} \\ & \cup \{(s, v, Node(N_i, w_0) :: \mathcal{S}) \mid q \in Q_F^{N_i}\}\end{aligned}$$

, where w_0 is a possibly empty sequence of terminals and nonterminal nodes.

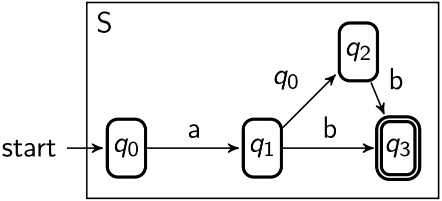
Extended RSM for grammar $S \rightarrow a\ b \mid a\ S\ b$

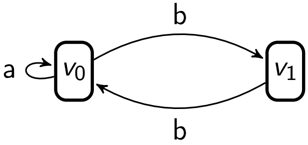


Example



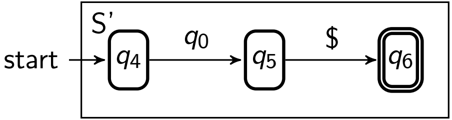
$$(q_4, v_0, []) \vdash \{(q_0, v_0, [q_5])\} \tag{1}$$

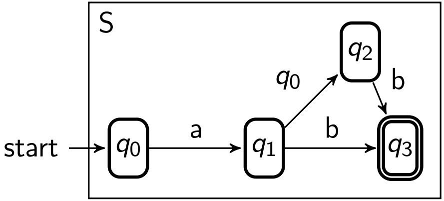


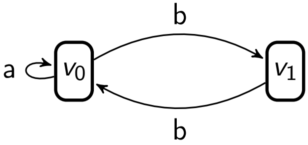


$$\tag{9}$$

Example







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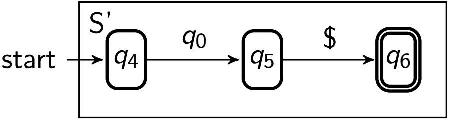
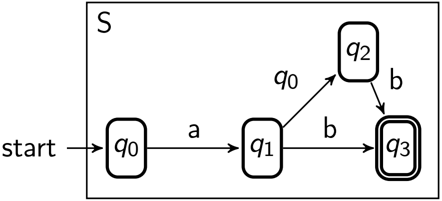
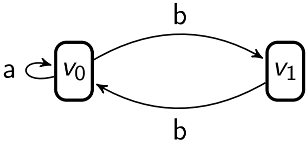
$$(q_0, v_0, [q_5]) \vdash \{(q_1, v_0, [a, q_5])\} \tag{2}$$

(9)

Example







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$$\begin{aligned}(q_1, v_0, [a, q_5]) \vdash \{&(q_0, v_0, [q_2, a, q_5]) \\ &, (q_3, v_1, [b, a, q_5])\}\end{aligned} \tag{3}$$

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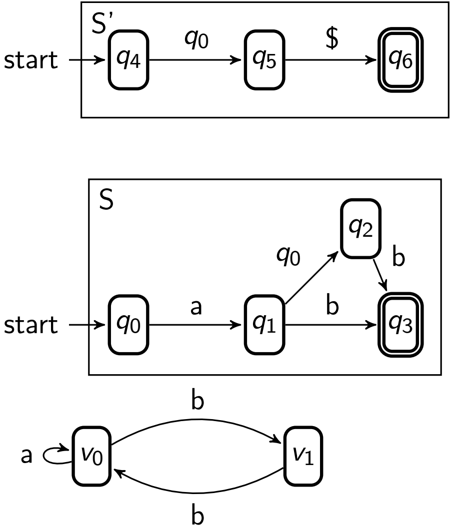
Example



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\begin{aligned}
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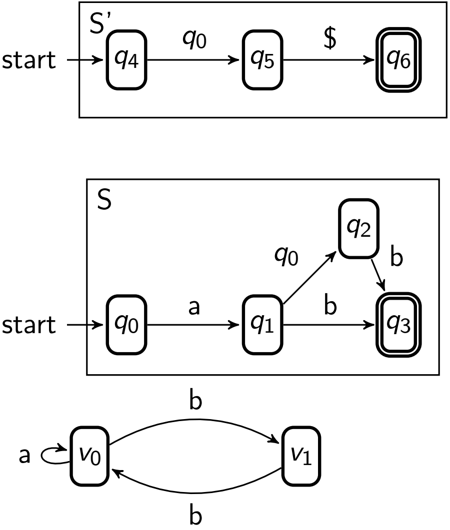

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$$(q_3, v_1, [b, a, q_5]) \vdash \boxed{\{(q_5, v_1, [S(a, b)])\}} \tag{5}$$

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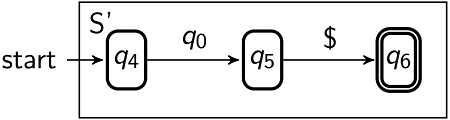

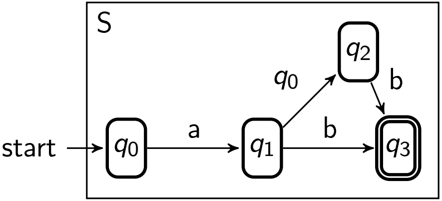

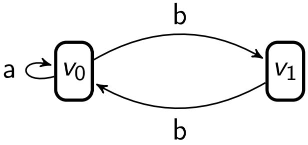

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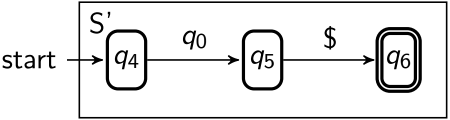

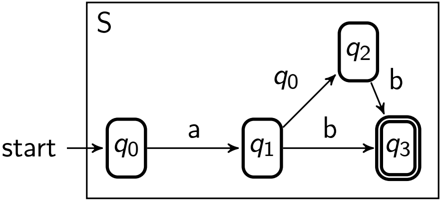

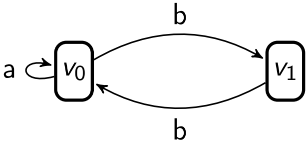

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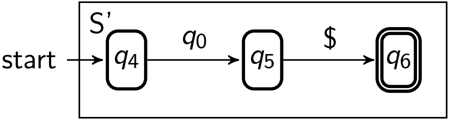

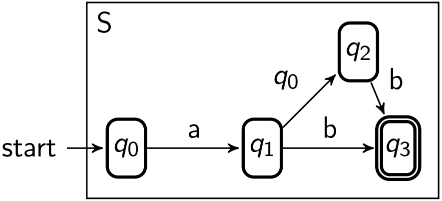

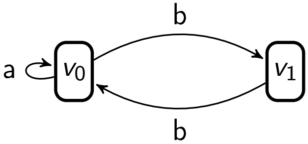

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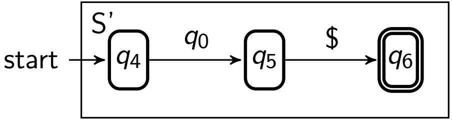

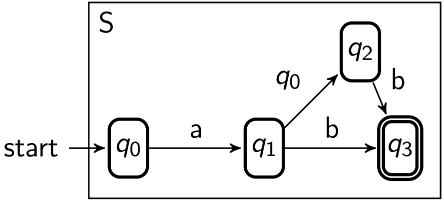

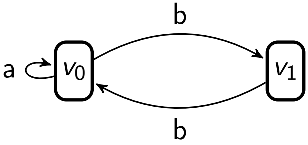

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(q_3, v_0, [b, S(a, b), a, q_5]) &\vdash \boxed{\{(q_5, v_0, [S(a, S(a, b), b)])\}} && (9)
\end{aligned}
$$

Paths Representation

Definition (Matched Range)

Range or a *matched range* $R^{q_i,v_i}_{q_j,v_j}$ corresponds to the fact that there is a chain of transitions from the configuration $(q_i, v_i, \mathcal{S}_i)$ to the configuration $(q_j, v_j, \mathcal{S}_j)$, or $(q_i, v_i, \mathcal{S}_i) \vdash^* (q_j, v_j, \mathcal{S}_j)$. The symbol ϵ denotes the empty range: an empty sequence of configurations.

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Definition (Path Index)

Path index for the graph $D = \langle V, E, L\rangle$ and the query $\mathcal{R} = \langle \mathcal{N}, \Sigma, B, B_S, Q, Q_S\rangle$ is a $\mathcal{K} \times \mathcal{K}$ square matrix $\mathcal{I}$, $\mathcal{K} = |Q| \cdot |V|$. Columns and rows are indexed by tuples $(q_i, v_j), q_i \in Q, v_j \in V$. $\mathcal{I}[(q_i, v_j), (q_l, v_k)]$ is a set which represents information about a $R^{q_i,v_j}_{q_l,v_k}$:

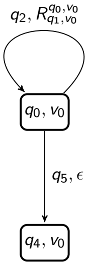
- $t \in \Sigma$ means that $(q_i, v_j, \mathcal{S}') \vdash (q_l, v_k, t :: \mathcal{S}')$;
- $N_m \in \mathcal{N}$ means that $(q_i, v_j, \mathcal{S}') \vdash^* (q_l, v_k, N_m :: \mathcal{S}')$;
- I_{q_a,v_b} is an *intermediate point*: $R^{q_i,v_j}_{q_l,v_k}$ was combined from $R^{q_i,v_j}_{q_a,v_b}$ and $R^{q_a,v_b}_{q_l,v_k}$

$$(q_i, v_j, \mathcal{S}') \vdash^* (q_a, v_b, \mathcal{S}'') \vdash^* (q_l, v_k, \mathcal{S}''')$$

	Configuration	Path	Stack
1	$\vdash (q_4, v_0, (q_4, v_0), \epsilon)$		$\boxed{q_4, v_0}$

	Configuration	Path	Stack
1	$\vdash (q_4, v_0, (q_4, v_0), \epsilon)$		$\boxed{q_4, v_0}$
2	$(q_4, v_0, (q_4, v_0), \epsilon) \vdash \{(q_0, v_0, (q_0, v_0), \epsilon)\}$		$\boxed{q_4, v_0} \xleftarrow[q_5, \epsilon]{} \boxed{q_0, v_0}$

	Configuration	Path	Stack
1	$\vdash (q_4, v_0, (q_4, v_0), \epsilon)$		$\boxed{q_4, v_0}$
2	$(q_4, v_0, (q_4, v_0), \epsilon) \vdash \{(q_0, v_0, (q_0, v_0), \epsilon)\}$		$\boxed{q_4, v_0} \xleftarrow[q_5, \epsilon]{} \boxed{q_0, v_0}$
3	$(q_0, v_0, (q_0, v_0), \epsilon) \vdash \{(q_1, v_0, (q_0, v_0), R^{q_0, v_0}_{q_1, v_0})\}$	$[(q_0, v_0), (q_1, v_0)] \leftarrow a$	

	Configuration	Path	Stack
1	$\vdash (q_4, v_0, (q_4, v_0), \epsilon)$		$\boxed{q_4, v_0}$
2	$(q_4, v_0, (q_4, v_0), \epsilon) \vdash \{(q_0, v_0, (q_0, v_0), \epsilon)\}$		$\boxed{q_4, v_0} \xleftarrow{q_5, \epsilon} \boxed{q_0, v_0}$
3	$(q_0, v_0, (q_0, v_0), \epsilon) \vdash \{(q_1, v_0, (q_0, v_0), R^{q_0, v_0}_{q_1, v_0})\}$	$[(q_0, v_0), (q_1, v_0)] \leftarrow a$	
4	$(q_1, v_0, (q_0, v_0), R^{q_0, v_0}_{q_1, v_0}) \vdash \{\cancel{(q_0, v_0, (q_0, v_0), \epsilon)}, (q_3, v_1, (q_0, v_0), R^{q_0, v_0}_{q_3, v_1})\}$	$[(q_1, v_0), (q_3, v_1)] \leftarrow b$ $[(q_0, v_0), (q_3, v_1)] \leftarrow (q_1, v_0)$	

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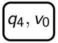
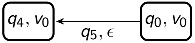
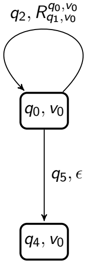
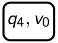
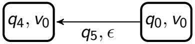
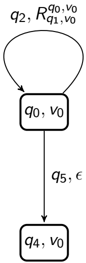
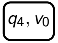
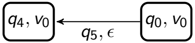
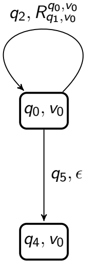
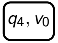
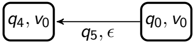
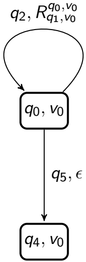
1	$\vdash (q_4, v_0, (q_4, v_0), \epsilon)$		
2	$(q_4, v_0, (q_4, v_0), \epsilon) \vdash \{(q_0, v_0, (q_0, v_0), \epsilon)\}$		
3	$(q_0, v_0, (q_0, v_0), \epsilon) \vdash \{(q_1, v_0, (q_0, v_0), R^{q_0, v_0}_{q_1, v_0})\}$	$[(q_0, v_0), (q_1, v_0)] \leftarrow a$	
4	$(q_1, v_0, (q_0, v_0), R^{q_0, v_0}_{q_1, v_0}) \vdash \{\cancel{(q_0, v_0, (q_0, v_0), \epsilon)}, (q_3, v_1, (q_0, v_0), R^{q_0, v_0}_{q_3, v_1})\}$	$[(q_1, v_0), (q_3, v_1)] \leftarrow b$ $[(q_0, v_0), (q_3, v_1)] \leftarrow (q_1, v_0)$	
5	$(q_3, v_1, (q_0, v_0), R^{q_0, v_0}_{q_3, v_1}) \vdash \{(q_2, v_1, (q_0, v_0), R^{q_0, v_0}_{q_2, v_1}), \boxed{(q_5, v_1, (q_4, v_0), R^{q_4, v_0}_{q_5, v_1})}\}$	$[(q_4, v_0), (q_5, v_1)] \leftarrow S$ $[(q_1, v_0), (q_2, v_1)] \leftarrow S$ $[(q_0, v_0), (q_2, v_1)] \leftarrow (q_1, v_0)$	

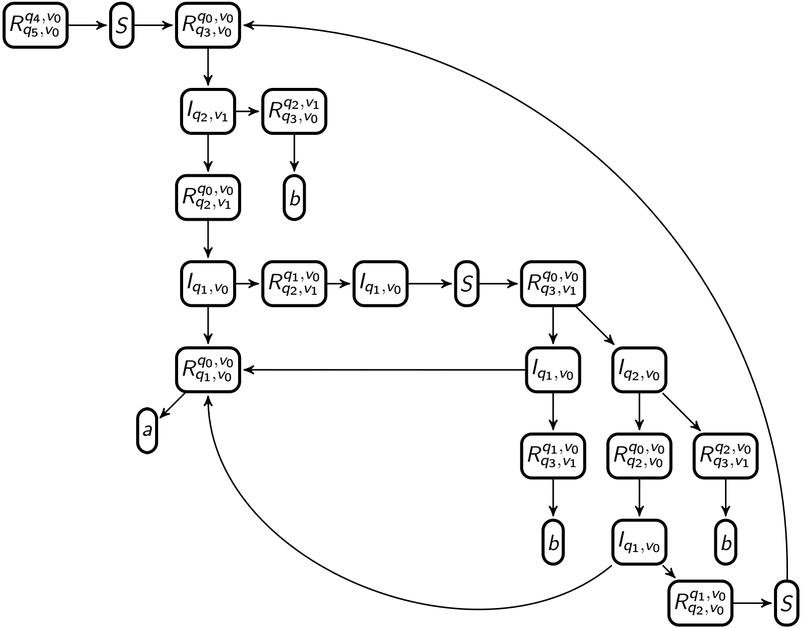
	Configuration	Path	Stack
1	$\vdash (q_4, v_0, (q_4, v_0), \epsilon)$		
2	$(q_4, v_0, (q_4, v_0), \epsilon) \vdash \{(q_0, v_0, (q_0, v_0), \epsilon)\}$		
3	$(q_0, v_0, (q_0, v_0), \epsilon) \vdash \{(q_1, v_0, (q_0, v_0), R^{q_0, v_0}_{q_1, v_0})\}$	$[(q_0, v_0), (q_1, v_0)] \leftarrow a$	
4	$(q_1, v_0, (q_0, v_0), R^{q_0, v_0}_{q_1, v_0}) \vdash \{\cancel{(q_0, v_0, (q_0, v_0), \epsilon)}, (q_3, v_1, (q_0, v_0), R^{q_0, v_0}_{q_3, v_1})\}$	$[(q_1, v_0), (q_3, v_1)] \leftarrow b$ $[(q_0, v_0), (q_3, v_1)] \leftarrow (q_1, v_0)$	
5	$(q_3, v_1, (q_0, v_0), R^{q_0, v_0}_{q_3, v_1}) \vdash \{(q_2, v_1, (q_0, v_0), R^{q_0, v_0}_{q_2, v_1}), \boxed{(q_5, v_1, (q_4, v_0), R^{q_4, v_0}_{q_5, v_1})}\}$	$[(q_4, v_0), (q_5, v_1)] \leftarrow S$ $[(q_1, v_0), (q_2, v_1)] \leftarrow S$ $[(q_0, v_0), (q_2, v_1)] \leftarrow (q_1, v_0)$	
6	$(q_2, v_1, (q_0, v_0), R^{q_0, v_0}_{q_2, v_1}) \vdash \{(q_3, v_0, (q_0, v_0), R^{q_0, v_0}_{q_3, v_0})\}$	$[(q_2, v_1), (q_3, v_0)] \leftarrow b$ $[(q_0, v_0), (q_3, v_0)] \leftarrow (q_2, v_1)$	

	Configuration	Path	Stack
1	$\vdash (q_4, v_0, (q_4, v_0), \epsilon)$		
2	$(q_4, v_0, (q_4, v_0), \epsilon) \vdash \{(q_0, v_0, (q_0, v_0), \epsilon)\}$		
3	$(q_0, v_0, (q_0, v_0), \epsilon) \vdash \{(q_1, v_0, (q_0, v_0), R^{q_0, v_0}_{q_1, v_0})\}$	$[(q_0, v_0), (q_1, v_0)] \leftarrow a$	
4	$(q_1, v_0, (q_0, v_0), R^{q_0, v_0}_{q_1, v_0}) \vdash \{\cancel{(q_0, v_0, (q_0, v_0), \epsilon)}, (q_3, v_1, (q_0, v_0), R^{q_0, v_0}_{q_3, v_1})\}$	$[(q_1, v_0), (q_3, v_1)] \leftarrow b$ $[(q_0, v_0), (q_3, v_1)] \leftarrow (q_1, v_0)$	
5	$(q_3, v_1, (q_0, v_0), R^{q_0, v_0}_{q_3, v_1}) \vdash \{(q_2, v_1, (q_0, v_0), R^{q_0, v_0}_{q_2, v_1}), \boxed{(q_5, v_1, (q_4, v_0), R^{q_4, v_0}_{q_5, v_1})}\}$	$[(q_4, v_0), (q_5, v_1)] \leftarrow S$ $[(q_1, v_0), (q_2, v_1)] \leftarrow S$ $[(q_0, v_0), (q_2, v_1)] \leftarrow (q_1, v_0)$	
6	$(q_2, v_1, (q_0, v_0), R^{q_0, v_0}_{q_2, v_1}) \vdash \{(q_3, v_0, (q_0, v_0), R^{q_0, v_0}_{q_3, v_0})\}$	$[(q_2, v_1), (q_3, v_0)] \leftarrow b$ $[(q_0, v_0), (q_3, v_0)] \leftarrow (q_2, v_1)$	
7	$(q_3, v_0, (q_0, v_0), R^{q_0, v_0}_{q_3, v_0}) \vdash \{(q_2, v_0, (q_0, v_0), R^{q_0, v_0}_{q_2, v_0}), \boxed{(q_5, v_0, (q_4, v_0), R^{q_4, v_0}_{q_5, v_0})}\}$	$[(q_4, v_0), (q_5, v_0)] \leftarrow S$ $[(q_1, v_0), (q_2, v_0)] \leftarrow S$ $[(q_0, v_0), (q_2, v_0)] \leftarrow (q_1, v_0)$	

	Configuration	Path	Stack
1	$\vdash (q_4, v_0, (q_4, v_0), \epsilon)$		
2 3	$(q_4, v_0, (q_4, v_0), \epsilon) \vdash \{(q_0, v_0, (q_0, v_0), \epsilon)\}$ $(q_0, v_0, (q_0, v_0), \epsilon) \vdash \{(q_1, v_0, (q_0, v_0), R^{q_0,v_0}_{q_1,v_0})\}$	$[(q_0, v_0), (q_1, v_0)] \leftarrow a$	
4	$(q_1, v_0, (q_0, v_0), R^{q_0,v_0}_{q_1,v_0}) \vdash \{\cancel{(q_0, v_0, (q_0, v_0), \epsilon)}, (q_3, v_1, (q_0, v_0), R^{q_0,v_0}_{q_3,v_1})\}$	$[(q_1, v_0), (q_3, v_1)] \leftarrow b$ $[(q_0, v_0), (q_3, v_1)] \leftarrow (q_1, v_0)$	
5	$(q_3, v_1, (q_0, v_0), R^{q_0,v_0}_{q_3,v_1}) \vdash \{(q_2, v_1, (q_0, v_0), R^{q_0,v_0}_{q_2,v_1}), \boxed{(q_5, v_1, (q_4, v_0), R^{q_4,v_0}_{q_5,v_1})}\}$	$[(q_4, v_0), (q_5, v_1)] \leftarrow S$ $[(q_1, v_0), (q_2, v_1)] \leftarrow S$ $[(q_0, v_0), (q_2, v_1)] \leftarrow (q_1, v_0)$	
6	$(q_2, v_1, (q_0, v_0), R^{q_0,v_0}_{q_2,v_1}) \vdash \{(q_3, v_0, (q_0, v_0), R^{q_0,v_0}_{q_3,v_0})\}$	$[(q_2, v_1), (q_3, v_0)] \leftarrow b$ $[(q_0, v_0), (q_3, v_0)] \leftarrow (q_2, v_1)$	
7	$(q_3, v_0, (q_0, v_0), R^{q_0,v_0}_{q_3,v_0}) \vdash \{(q_2, v_0, (q_0, v_0), R^{q_0,v_0}_{q_2,v_0}), \boxed{(q_5, v_0, (q_4, v_0), R^{q_4,v_0}_{q_5,v_0})}\}$	$[(q_4, v_0), (q_5, v_0)] \leftarrow S$ $[(q_1, v_0), (q_2, v_0)] \leftarrow S$ $[(q_0, v_0), (q_2, v_0)] \leftarrow (q_1, v_0)$	
8	$(q_2, v_0, (q_0, v_0), R^{q_0,v_0}_{q_2,v_0}) \vdash \{\cancel{(q_3, v_1, (q_0, v_0), R^{q_0,v_0}_{q_3,v_1})}\}$	$[(q_0, v_0), (q_3, v_1)] \leftarrow (q_2, v_0)$ $[(q_2, v_0), (q_3, v_1)] \leftarrow b$	

Path Index

$$
\begin{array}{ccc}
 & \begin{matrix} q_0, v_0 & q_1, v_0 & q_2, v_0 & q_3, v_0 & q_4, v_0 & q_5, v_0 & q_6, v_0 & q_0, v_1 & q_1, v_1 & q_2, v_1 & q_3, v_1 & q_4, v_1 & q_5, v_1 & q_6, v_1 \end{matrix} & \\
\begin{matrix} q_0, v_0 \\ q_1, v_0 \\ q_2, v_0 \\ q_3, v_0 \\ q_4, v_0 \\ q_5, v_0 \\ q_6, v_0 \\ q_0, v_1 \\ q_1, v_1 \\ q_2, v_1 \\ q_3, v_1 \\ q_4, v_1 \\ q_5, v_1 \\ q_6, v_1 \end{matrix} &
\begin{pmatrix}
 & \{a\} & \{I_{q_1,v_0}\} & \{I_{q_2,v_1}\} & & & & & & \{I_{q_1,v_0}\} & \{I_{q_1,v_0}, I_{q_2,v_0}\} & & & \\
 & & \{S\} & & & & & & & \{S\} & \{b\} & & & \\
 & & & & & & & & & & \{b\} & & & \\
 & & & & & & & & & & & & & \\
 & & & & & \{S\} & & & & & & & \{S\} & \\
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
 & & & \{b\} & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & &
\end{pmatrix} &
\begin{matrix} q_0, v_0 \\ q_1, v_0 \\ q_2, v_0 \\ q_3, v_0 \\ q_4, v_0 \\ q_5, v_0 \\ q_6, v_0 \\ q_0, v_1 \\ q_1, v_1 \\ q_2, v_1 \\ q_3, v_1 \\ q_4, v_1 \\ q_5, v_1 \\ q_6, v_1 \end{matrix} \\
 & \begin{matrix} q_0, v_0 & q_1, v_0 & q_2, v_0 & q_3, v_0 & q_4, v_0 & q_5, v_0 & q_6, v_0 & q_0, v_1 & q_1, v_1 & q_2, v_1 & q_3, v_1 & q_4, v_1 & q_5, v_1 & q_6, v_1 \end{matrix} &
\end{array}
$$



$R^{q_4,v_0}_{q_5,v_0}$
S
$R^{q_0,v_0}_{q_3,v_0}$
I_{q_2,v_1}
$R^{q_2,v_1}_{q_3,v_0}$
$R^{q_0,v_0}_{q_2,v_1}$
b
I_{q_1,v_0}
$R^{q_1,v_0}_{q_2,v_1}$
I_{q_1,v_0}
S
$R^{q_0,v_0}_{q_3,v_1}$
$R^{q_0,v_0}_{q_1,v_0}$
I_{q_1,v_0}
I_{q_2,v_0}
a
$R^{q_1,v_0}_{q_3,v_1}$
$R^{q_0,v_0}_{q_2,v_0}$
$R^{q_2,v_0}_{q_3,v_1}$
b
I_{q_1,v_0}
b
$R^{q_1,v_0}_{q_2,v_0}$
S