

# Задача поиска изоморфного подграфа с использованием GPU

Обзор существующих решений

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# Постановка задачи

- Исследовать алгоритмы поиска изоморфного подграфа (subgraph matching), использующие GPU
- Реализовать 2 алгоритма

# Проблема поиска изоморфного подграфа

- $G = (V_G, E_G, L_G, l_G)$  - data graph
- $l : V \rightarrow L$  - labeling function
- $q = (V_q, E_q, L_q, l_q)$  - query graph
- Ищем подграф  $g = (V_g, E_g)$  графа  $G$ : существует биекция  $f : V_q \rightarrow V_g$ , такая что
$$\forall v \in V_q \quad l_q(v) = l_G(f(v))$$
$$\forall e = (u, v) \in E_q, (f(u), f(v)) \in E_g$$

# Актуальность

- Программный анализ
- Поиск химического соединения
- Анализ социальных сетей

# Термины

- $v \in V_G$  называется **кандидатом**  $u \in V_q$ , если  $|u| = |v|$  и  $\deg(u) \leq \deg(v)$

# Методы решения

- Индексация (indexing)
- Обход графа (graph traversal)
- С помощью возвратов (backtracking)
- С помощью комбинирования кандидатов (filtering-and-joining strategy)

# Основные эвристические подходы

- Порядок обхода вершин
- Удаление неперспективных кандидатов как можно раньше

# Efficient Subgraph Matching Using GPUs

1. Использует STwig алгоритм
2. Только операция join на GPU

# STwig алгоритм

## 1. Декомпозиция запроса

Минимизировать число STwigs и найти порядок STwig  
 $\langle O_1, O_2, \dots, O_n \rangle$ , такой что:  $\forall i \geq 2, \exists j < i$  корень  
STwig  $O_i$  является листом  $O_j$

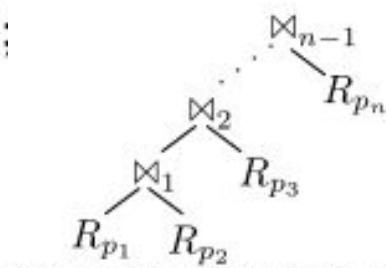
## 2. Сопоставление графа с STwig (matching STwigs)

$$R_N = \{\{n\} \times T_{l_1} \times T_{l_2} \times \dots \times T_{l_{|L_1|}}\};$$

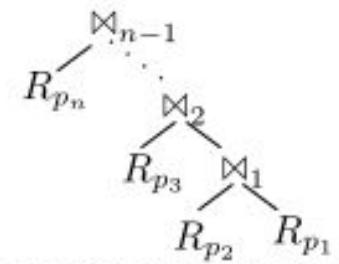
## 3. Комбинирование решений (join)

# Особенности реализации join

$RN = \{\{n\} \times T_{l_1} \times T_{l_2} \times \dots \times T_{l_{|L_1|}}\}$



(a) Left-deep join tree

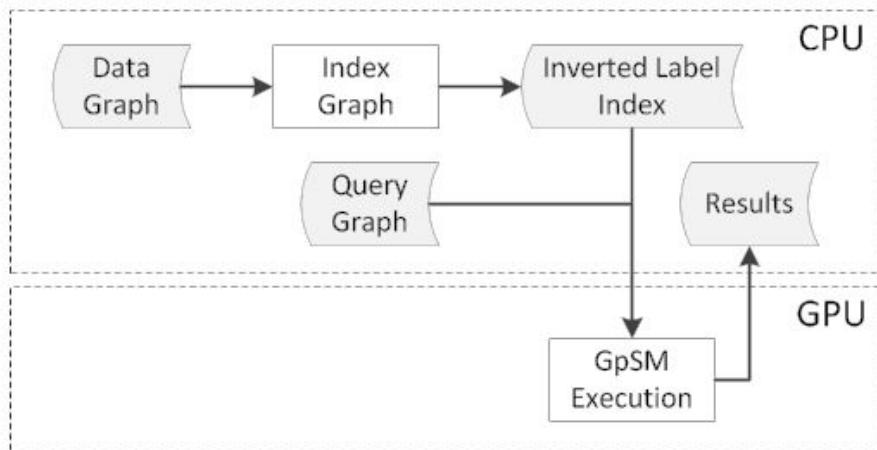


(b) Right-deep join tree

# Fast Subgraph Matching on Large Graphs using Graphics Processors (GpSM)

1. Инициализация порядка обхода запроса
2. Фаза фильтрации (filtering phase)
  - a. Инициализация кандидатов (initializing candidate vertices)
  - b. Улучшение кандидатов (refining candidate vertices)
3. Фаза присоединения (joining phase)
  - a. Поиск всех подходящих ребер кандидатов
  - b. Комбинирование решений

# GpSM для очень больших графов



# Towards GPU-Based Common-Sense Reasoning: Using Fast Subgraph Matching (GpSense)

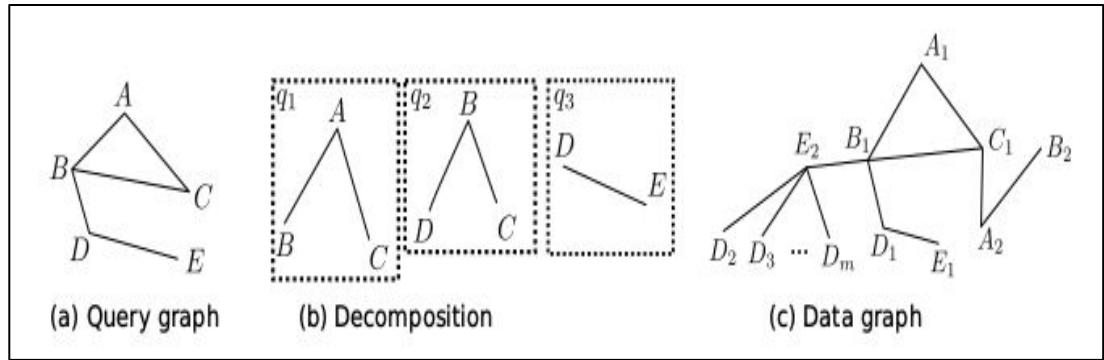
1. Стратегия фильтрации и комбинирования кандидатов  
(filtering-and-joining strategy)
2. Common-Sense Knowledge графы
  - a. « концепция-отношение-концепция »
  - b. Направленный граф и имеет метки на ребрах
  - c. Граф-запрос помимо вершин с метками имеет вершины с неизвестными метками
3. Многоуровневое сжатие графа

# GPU acceleration of subgraph isomorphism search in large scale graph (GPUSI)

1. Инициализация (запрос-дерево)
2. Исследование региона (region exploration)
3. Определение порядка обхода региона (determine matching order)
4. Поиск изоморфного подграфа (subgraph matching)

# Дополнительные материалы

# 1) Efficient Subgraph Matching Using GPUs



## Algorithm 1. Match the First STwig

```
input : Data graph  $G$ , STwig  $o_1 = (r_1, L_1)$  where  $r_1$  is the root node and  $L_1$  is  
the leave label set  
output: Result set  $R$   
Fill the set  $S_1$  with all nodes with  $r_1$ 's label;  
for each  $n$  in  $S_1$  do  
  for each  $l_i$  in  $L_1$  do  
     $T_{l_i} \leftarrow \{m | m \in n.\text{neighbors} \text{ and } m.\text{label} = l_i\}$ ;  
     $R = R \cup \{n\} \times T_{l_1} \times T_{l_2} \times \dots \times T_{l_{|L_1|}}$ ;
```

# 1) Efficient Subgraph Matching Using GPUs

**Algorithm 2.** Join algorithm for each thread

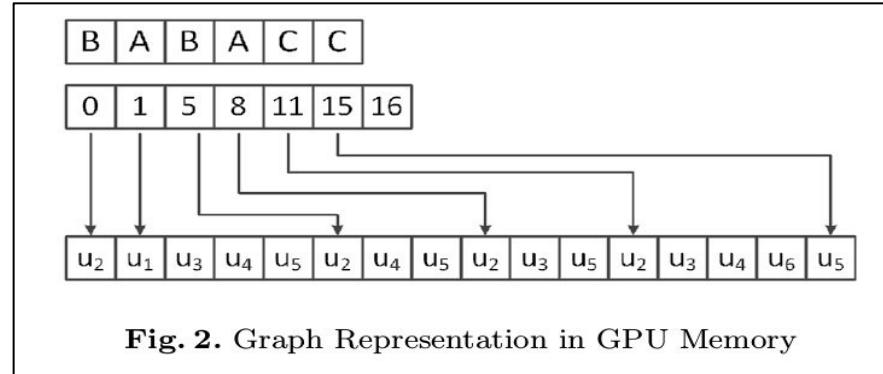
```
Input: Probe relation  $R_{p_1}$  and hash tables  $H_2, H_3, \dots, H_n$ 
1 begin
    // pos indicates this thread is processing subjoin  $\bowtie_{pos}$ 
2    pos  $\leftarrow 1$ ;
3    itArray[1]  $\leftarrow \text{none}$ ;
4    while true do
5        while itArray[pos] = none and pos  $\neq 1$  do
6            pos  $\leftarrow pos - 1$ ;
7        while itArray[1] = none do
8            i  $\leftarrow \text{GetNewRecordsOrQuit}()$ ;
9            set imResult based on  $R_{p_1}[i]$ ;
10           itArray[1]  $\leftarrow H_2.\text{match}(imResult)$ ;
11           set imResult based on pos and itArray[pos].record();
12           itArray[pos]  $\leftarrow itArray[pos].\text{next}()$ ;
13           if pos = n - 1 then
14               output imResult as a final result;
15           else
16               itArray[pos + 1]  $\leftarrow H_{pos+2}.\text{match}(imResult)$ ;
17               pos  $\leftarrow pos + 1$ ;
```

## 2) Fast Subgraph Matching on Large Graphs using Graphics Processors (GpSM)

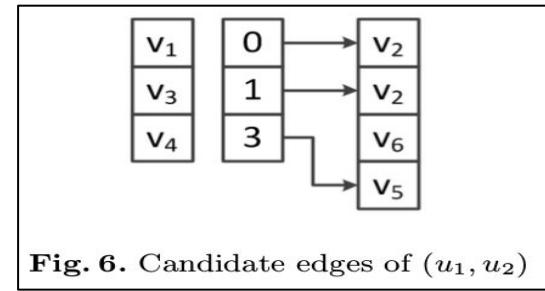
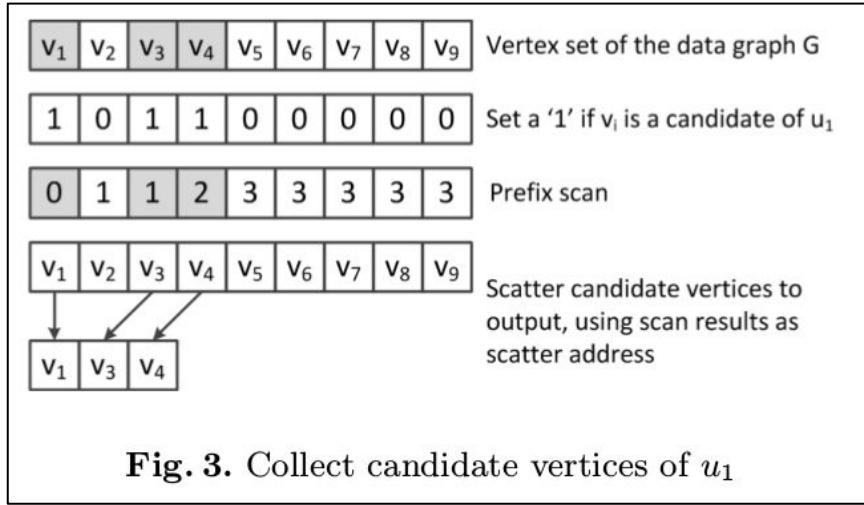
### Algorithm 1. GpSM( $q, g$ )

**Input:** query graph  $q$ , data graph  $g$   
**Output:** all matches of  $q$  in  $g$

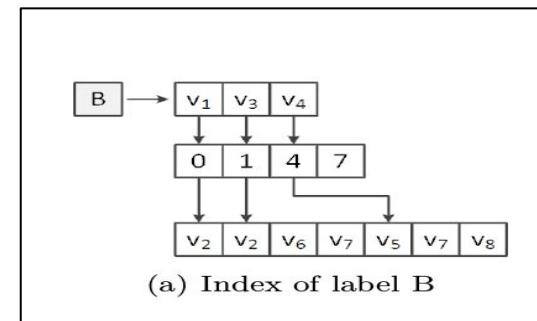
- 1  $C := \text{InitializeCandidateVertices}(q, g);$
- 2  $C := \text{RefineCandidateVertices}(q, g, C);$
- 3  $E := \text{FindCandidateEdges}(q, g, C);$
- 4  $M := \text{JoinCandidateEdges}(q, g, E);$
- 5 **return**  $M$



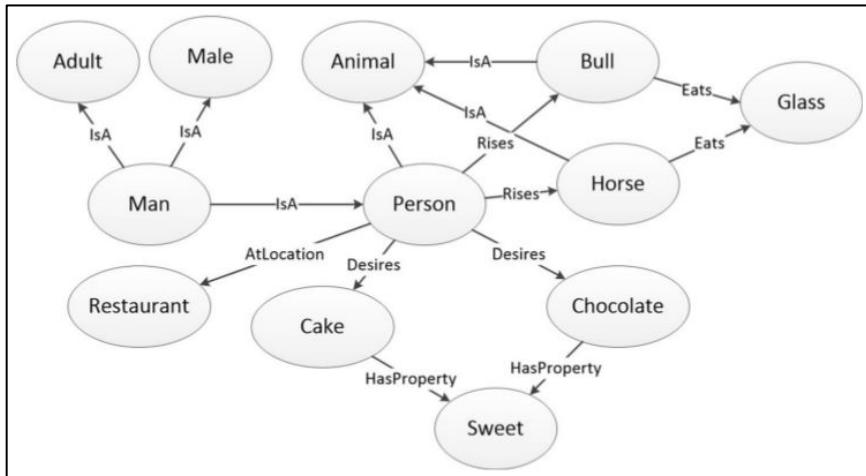
## 2) Fast Subgraph Matching on Large Graphs using Graphics Processors (GpSM)



**Fig. 6.** Candidate edges of  $(u_1, u_2)$



# 3) Towards GPU-Based Common-Sense Reasoning: Using Fast Subgraph Matching



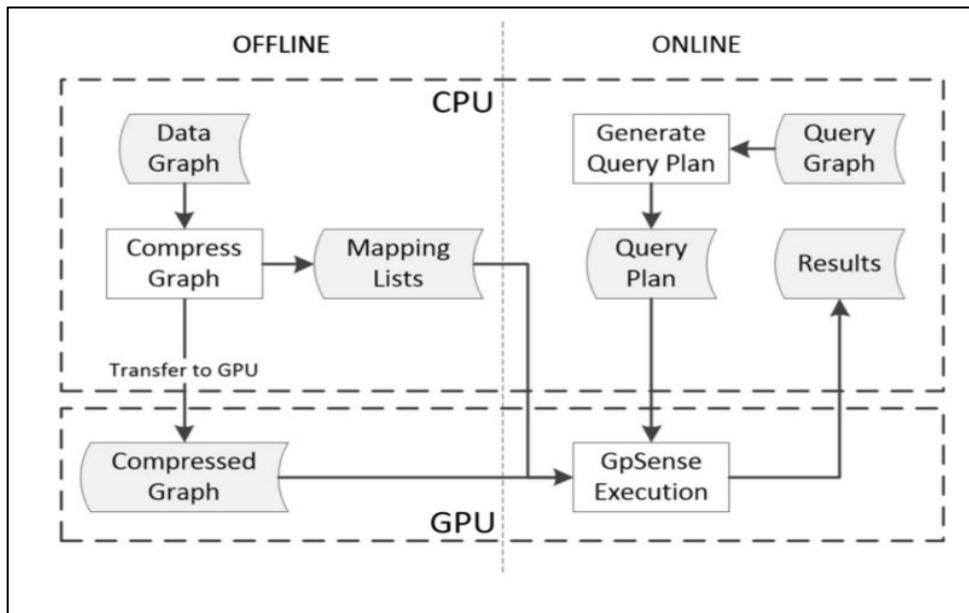
**Algorithm 1:** GPUSubgraphMatching (  $q(V, E, L)$ ,  $g(V', E', L')$  )

**Input:** query graph  $q$ , data graph  $g$

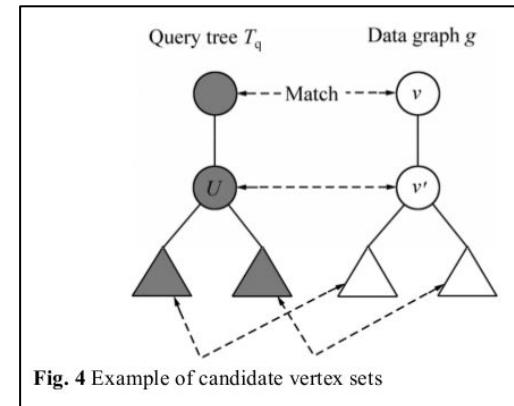
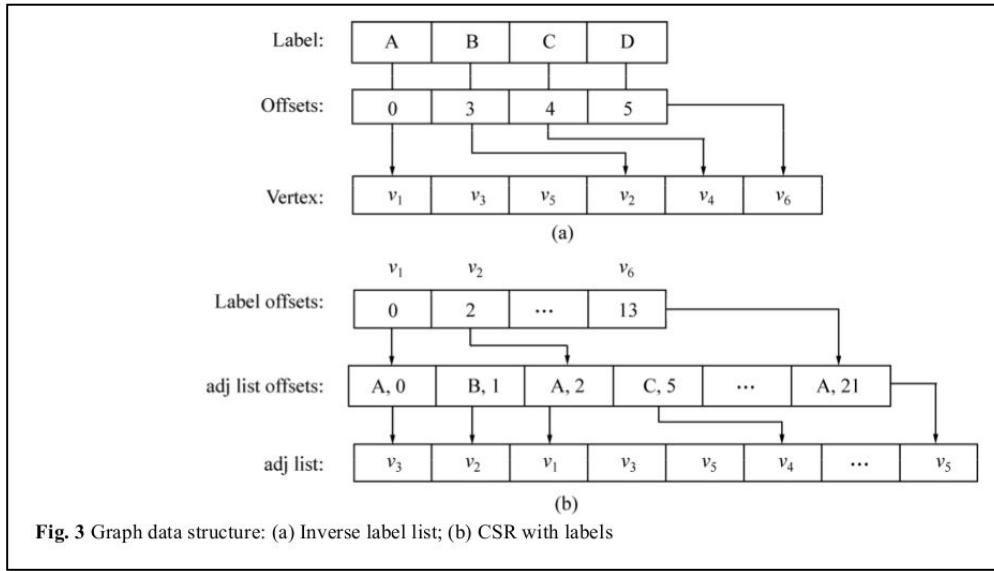
**Output:** all matches of  $q$  in  $g$

```
1 P := generate_query_plan( $q, g$ );
2 forall the node  $u \in P$  do
3   if  $u$  is not filtered then
4     c_set( $u$ ) := identify_node_candidates( $u, g$ );
5   c_array( $u$ ) := collect_edge_candidates( $c\_set(u)$ );
6   c_set := filter_neighbor_candidates( $c\_array(u), q, g$ );
7 refine_node_candidates( $c\_set, q, g$ );
8 forall the edge  $e (u,v) \in E$  do
9   EC( $e$ ) := collect_edge_candidates( $e, c\_set, q, g$ );
10 M := combine_edge_candidates( $EC, q, g$ );
11 return  $M$ 
```

### 3) Towards GPU-Based Common-Sense Reasoning: Using Fast Subgraph Matching



# 4) GPU acceleration of subgraph isomorphism search in large scale graph



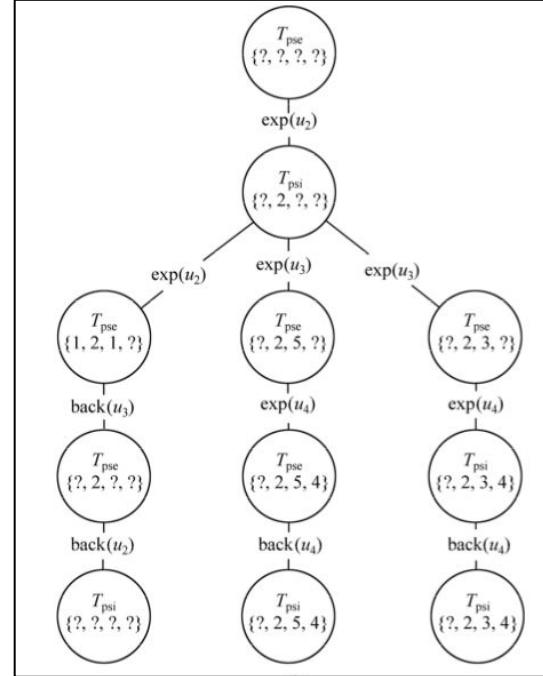
# Термины

- Partial subgraph embedding (**Gpse**) - это структура данных, состоящая из  $|V_q|$  пар  $\langle v_q, v_g \rangle$  ( $v_g = \text{map}(v_q)$ ), где вершине  $v_q$  сопоставляется  $v_g$
- Набор вершин-кандидатов (candidate vertex set) или CVS( $u, v$ ) - набор вершин, содержащий вершины графа  $G$ , которые соответствуют и являются дочерними вершинами  $v$  в дереве построенном DFS обходом и каждое поддерево и соответствует соответствующему поддереву  $v$  в DFS дереве

# 4) GPU acceleration of subgraph isomorphism search in large scale graph

**Algorithm 2:** CPU region exploration

```
1 Explore Region ( $u, V_m, R$ )  
2 for each unvisited data vertex  $v$  in  $V_{adj}$   
3     if  $\deg(u) > \deg(v)$  or  $NLF(u) = FAIL$   
4         then  
5             continue;  
6         mark  $v$  as visited;  
7         append  $v$  into  $CR(u, v_p)$ , where  $v_p$  is the  
parent of  $v$  in DFS tree;  
8         sort child  $u_c$  in ascending order of  
| $dj(v, L(u_c))$ |  
9         for each child vertex  $u_c$  of  $u$  then  
10            if  $explore(u_c, adj(v, L(u_c)), R) = FAIL$   
11            then  
12                for each visited child query vertex  $u_{cc}$   
of  $u$  except  $u_c$  do  
13                    ClearRegion( $u_{cc}, v, R$ );  
14                    remove  $v$  from  $R(u, v_p)$ ;  
15                    return FAIL;  
16                end  
17            end  
18            mark  $v$  as unvisited;  
19        if  $|R(u, v_p)| = 0$  then  
20            return FAIL;  
21        return DONE;  
22    }
```



# 4) GPU acceleration of subgraph isomorphism search in large scale graph

**Algorithm 3:** Task dispatch of parallel region exploration

```
1 DispatchExploration( $u$ , CVS){  
2 for each category  $\text{CVS}(u, c_{\text{at}}, -)$  of  $\text{CVS}(u, -)$  in parallel  
3     if  $\text{CVS}(u, c_{\text{at}}, -)$  is empty then  
4         return FAIL;  
5     for each child  $u_c$  of  $u$  do  
6         Expand( $u_c$ ,  $\text{CVS}(u, c_{\text{at}}, -)$ , CVS);  
7         DispatchExploration( $u_c$ , CVS)  
8         Backward( $u_c$ ,  $\text{CVS}(u, c_{\text{at}}, -)$ , CVS);  
9         if  $\text{CVSp}(u, -)$  is empty then  
10            break;  
11         if num of children of  $u \geq 1$  then  
12             ClearRegion( $\text{CVS}(u, c_{\text{at}}, -)$ );  
13     return DONE;  
14 }
```

```
15  
16 Expand( $u_c$ ,  $\text{CVS}(u, c_{\text{at}}, -)$ ,  $R$ ){  
17     atomic_enqueue(gpu_emb_queue,  $u_c$ ,  
18     CVS( $u, c_{\text{at}}, -$ ),  $R$ , EXPAND);  
19     barrier();  
20     if cpu_thread_id=0 then  
21         GPUProcessKernel(gpu_emb_queue);  
22     barrier();  
23  
24 Backward( $u_c$ ,  $\text{CVS}(u, c_{\text{at}}, -)$ ,  $R$ ){  
25     atomic_enqueue(gpu_emb_queue,  $u_c$ ,  
26     CVS( $u, c_{\text{at}}, -$ ),  $R$ , BACKWARD);  
27     barrier();  
28     if thread_id=0 then  
29         GPUProcessKernel(gpu_emb_queue);  
30     barrier();  
30 }
```

# 4) GPU acceleration of subgraph isomorphism search in large scale graph

**Algorithm 4:** GPU region exploration

```

1 GPUProcessKernel(gpu_emb_queue){
2 if laneId of a warp=0 then // distribute task to warps
3     { $u_c$ , CVS( $u$ ,  $c_{at}$ ,  $-$ ),  $R$ , offset, operation}=
4         atomic_warp_dequeue (gpu_emb_queue);
5 if operation = EXPAND then
6     WarpForward(offset);
7 else if operation=BACKWARD then
8     WarpBackward(offset, gpu_emb_queue);
9 else then
10    WarpClear(offset, gpu_emb_queue);
11
12 WarpExpand(offset, gpu_emb_queue){
13 WarpLoad(v[tid], CVS( $u$ ,  $c_{at}$ ,  $-$ ), offset);
14 if (v[tid]=visited){
15     mark the flag of v as 0;
16 }
```

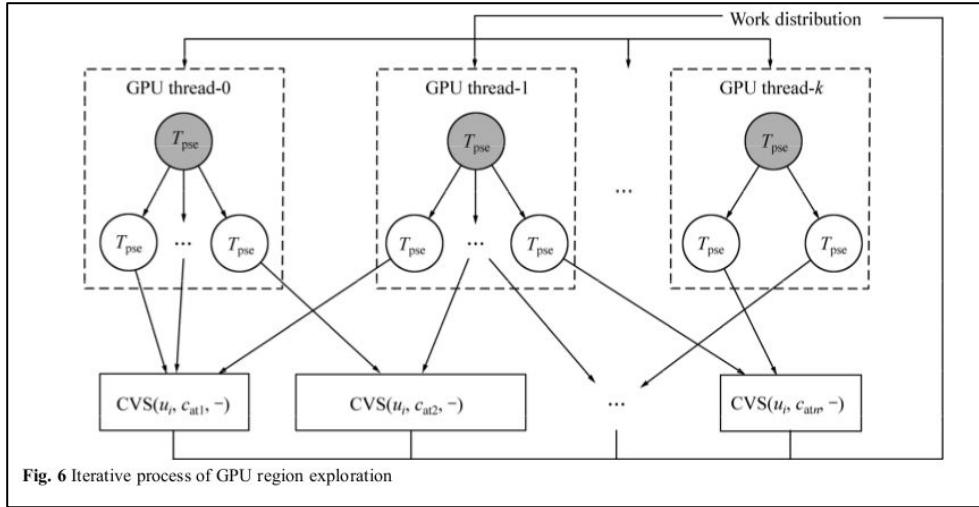
```

17 if (deg( $u$ )>deg( $v$ [tid]) or NLF( $u$ ,  $v$ [tid])=FAIL){
18     mark the flag of v as 0;
19 }
20 while WarpAny( $v$ [tid])=1 do // do if there is a  $v$ [tid]
21     that is not processed)
22     // vie for control of warp
23     if  $v$ [tid] that is not processed then
24         winner=tid;
25     if winner=tid then
26          $v_m=v$ [tid];
27          $v$ [tid]=0;
28     // warp traverse the winner's  $R(u_c, v$ [tid])
29     WarpLoad( $v_c$ [tid], adj( $u_c$ ,  $v_m$ ));
30     sort children  $u_{cs}$  of  $u_c$  in ascending order of
31     |adj( $v$ ,  $L(u_c)$ )|
32      $c_{at}$ =the order;
33     WarpStore(CVS( $u_c$ ,  $c_{at}$ ,  $v_m$ ),  $v_c$ [tid]);
34 }
```

```

35 WarpLoad( $v$ [tid], CVS( $u$ ,  $c_{at}$ ,  $-$ ), offset);
36 while WarpAny( $v$ [tid])=1 do // do if there is a  $v$ [tid]
37     that is not processed)
38     if  $v$ [tid] that is not processed then
39         winner=tid;
40     if winner=tid then
41          $v_{w1}=v$ [tid];
42          $v$ [tid]=0;
43     // warp traverse the winner's  $R(u_c, v$ [tid])
44     WarpLoad( $v_c$ [tid], CVS( $u_c$ ,  $v_1$ ));
45     If flag of  $v_c$ [tid]=matched;
46     WarpStore(CVS( $u_c$ ,  $c_{at}$ ,  $v$ ),  $v_c$ [tid]);
47     if WarpAll(flag of  $v_c$ [tid])=unmatched then
48         if tid=own then
49             mark the flag of  $v$ [tid] as
50             unmatched;
```

# 4) GPU acceleration of subgraph isomorphism search in large scale graph



# 4) GPU acceleration of subgraph isomorphism search in large scale graph

**Algorithm 5:** CPU graph matching algorithm

```
1 SubgraphMatching(){
2   for each  $u$  in query graph in the matching order
      obtained do
3     expandEmbedding( $u, q, R, \text{emList}$ );
4     if outputEmlist is empty then
5       return FAIL;
6   return emList;
7 }

8

9 expandEmbedding( $u, q, R, \text{emList}$ ){
10  for each partial subgraph embedding  $G_{\text{psi}}$  dequeued
      from InputEmbList
11    matched=true;
12    for each vertex  $v$  in  $R(u, v_p)$  do; // $v_p$  is the
      parent of  $v$ 
13      for each non-tree between  $u$  and the
      matched query vertex  $u_m$ 
14        if IsJoin( $v_m, v$ )=false then; //the vertex
           $v_m$  of  $G_{\text{psi}}$  matching with  $u_m$ 
15          matched=false;
16    if matched=true do
17      new  $G_{\text{psi}}=\text{generateEmbedding}(G_{\text{psi}}, v)$ ;
18      enqueue new  $G_{\text{psi}}$  into OutputEmList;
20  return outputEmlist;
21 }
```

**Algorithm 6:** GPU subgraph matching

```
1 WarpExpandEmbedding(){
2   if laneId of a warp=0 then // distribute task to warp
3     offset=atomic_warp_dequeue(InputEmbList);
4
5   WarpLoad(embedding[tid], offset, InputEmbList);
6   WarpLoad( $v[tid]$ , CVS( $u, v_p$  in embedding)); //  $v_p$  is
      the parent of  $v$ 
7
8   for each non-tree between  $u$  and the matched query
      vertex  $u_m$  do
9     WarpLoad(neighbor of  $v_m$  with the label
       $L(u)$ );
10    matched=false;
11    for each neighbor  $n_{vm}$  of  $v_m$  do
12      if  $n_{vm}=v[tid]$  then
13        matched=true;
14      if matched=false then
15        break;
16    if matched=true then
17      new  $G_{\text{psi}}=\text{generateEmbedding}(G_{\text{psi}}, v)$ ;
18      enqueue new  $G_{\text{psi}}$  into OutputEmbList;
19 }
```