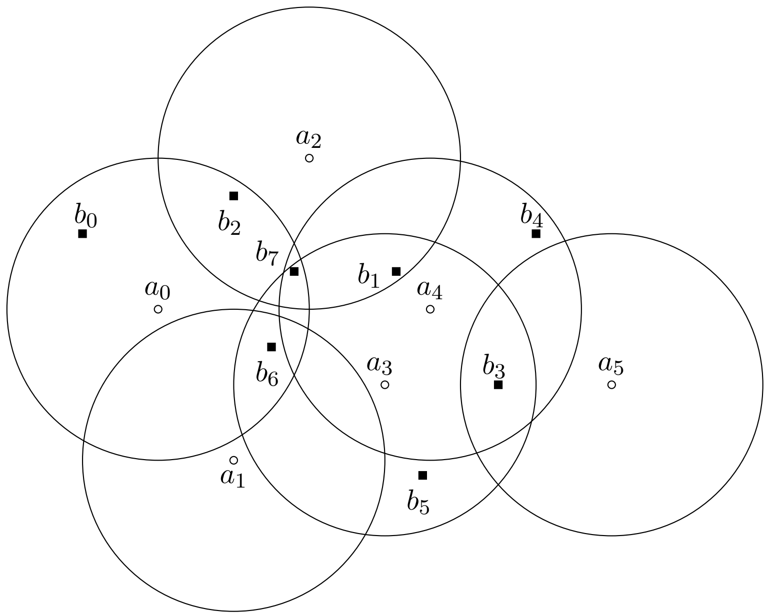
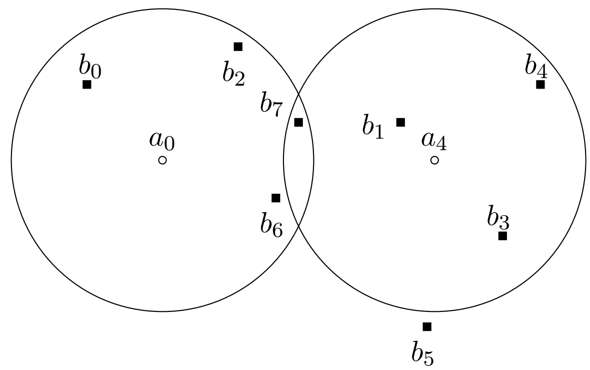
Term Project
EECS 477
Fall 2013

The Problem



You are given a set $A = \{a_0, a_1, \ldots, a_{m-1}\}$ of possible locations for radio antennas and a set $B = \{b_0, b_1, \ldots, b_{n-1}\}$ of locations of base receiver stations that receive signals from these antennas. Unfortunately, if a base receiver station b_i is in range of two or more antennas, interference will prevent b_i from receiving a signal from any of them. You would like to pick a subset of A where you will build antennas so that the maximum number of base receiver stations to receive signals. In the diagram above, $m = 6$ and $n = 8$. The antennas $a_0, a_1, \ldots, a_5$ are the small white circles. The range of each antenna a_i is a large circle centered on a_i. The base stations $b_0, b_1, \ldots, b_7$ are small black squares. It may seem that you should build an antenna at a_3, because you could then communicate with five base receiver stations. However, further antenna building does not increase the number base receiver stations that could receive a signal. Building another antenna at a_0, for example, would allow you to send a signal to b_0 and b_2, but would cause interference for b_6 and b_7. It turns out to be better to build antennas at a_0 and a_4. The result looks like this



In this case, six base receiver stations can receive a signal. This is the maximum number that can be covered in this example.

In general, because of physical obstructions such as buildings and mountains, the situation may be more complicated and there may not be such a straightforward geometric representation. For that reason, we use the following file format to represent a problem instance. On the first line are integers m, the number of antennas, and n, the number of receiver base stations. This is followed by m lines, one for each of the antennas $a_0, a_1, \ldots, a_{m-1}$. The line for antenna a_i will begin with an integer j_i indicating the number of base stations in the range of antenna a_i. (You may assume there will always be a positive number of base stations in the range of each a_i.) This will be followed by a list of j_i distinct integers (each in the range 0 to $n-1$) listing the base stations in the range of a_i. Note that rather than writing base station b_k, we simply write the index k. For example, the input file corresponding to the diagram on page 1 looks like this:

```
6 8
4 0 2 6 7
1 6
3 1 2 7
5 1 3 5 6 7
4 1 3 4 7
1 3
```

You will write a program in C, C++, Java, or Python that reads this input file and output a set of antenna locations. Your output file will consist of two lines. The first line begins with an integer j in the range $1 \leq j < m$ indicating the number of antenna locations. The this will be followed by j integers (each in the range 0 to $m-1$) listing the antenna locations. The second line will be the number k of base receiver stations that can receive signals. For example, the output file corresponding to the diagram on page 2 looks like this:

```
2 0 4
6
```

When you submit your program, we will run it on a suite of test data sets. For each one we will check that the second line in the output of your program is consistent with first line. If it is we will record this value. Your grade will be determined by how well your program does against programs submitted by other students.

We will post a subset of the test data sets used to grade your program and give you instructions on how to submit your programs soon.

1 How to Approach the Problem

The following decision problem is NP-complete.

> **Exact Cover**
>
> **Given:** Set B of size n and subsets $A_0, A_1, \ldots, A_{m-1}$ of B.
>
> **Question:** Is there a set $I \subseteq \{0, 1, \ldots, m-1\}$ such that $\{A_i \mid i \in I\}$ is a partition of B?

Recall that $\{A_i \mid i \in I\}$ is a partition of B is a partition of B if the sets A_i, $i \in I$, are disjoint and $B = \bigcup_{i \in I}$. A problem instance here is just a slight rephrasing of the problem instance in the last section: we just let A_i be the set of base receivers in B that are in range of antenna a_i. Thus, the answer for an Exact Cover problem instance is yes just in case there is a solution to the optimization problem of the last section where all base receiver stations can receive a signal. Thus, if we could find the optimal solution to the optimization problem in polynomial time, it would follow that P=NP. Since it is unlikely that you will be able to show this, the best you can hope to do is find a good approximation.

Here is a simple reformulation of the Exact Cover problem.

> **Bipartite Graph Reformulation of Exact Cover**
>
> **Given:** Bipartite graph $G = (V, E)$ where $V = A \cup B$, $A \cap B = \emptyset$, each edge in E has one end-point in A and one end-point in B.
>
> **Question:** Is there a set $A' \subset A$ such that every vertex in B is connected to precisely on vertex in A?

We are simply putting an edge between $a_i \in A$ and $b_j \in B$ just in case b_j is an element of A_i in the previous problem. Another way of saying this is that in the bipartite graph G we think of vertices in A as sets, vertices in B as elements, and edges indicate inclusion. However, this is an arbitrary designation. We could just as easily think of the vertices in A as elements and the vertices in B as sets. In that case, we have the following equivalent problem.

> **Exact Hitting Set**
>
> **Given:** Set A of size m and subsets $B_0, B_1, \ldots, B_{n-1}$ of A.
>
> **Question:** Is there a set $A' \subset A$ such that $|A' \cap B_i| = 1$ for each $i < n$?

Exact Cover and Exact Hitting Set are said to be *duals* of each other. The same idea applies to the optimization problem of the previous section. We can formulate it in this way.

Exact Cover Optimization

Given: Set B of size n and subsets $A_0, A_1, \ldots, A_{m-1}$ of B.

Find: $I \subseteq \{0, 1, \ldots, m-1\}$ that maximizes the size of

$$B(I) = \{b \in B \mid b \text{ is covered by precisely one } A_i \text{ where } i \in I\ \}.$$

The equivalent dual problem is the following.

Exact Hitting Set Optimization

Given: Set A of size m and subsets $B_0, B_1, \ldots, B_{n-1}$ of A.

Find: $A' \subset A$ that maximizes $|\{i \mid |A' \cap B_i| = 1\}|$ (the number of subsets B_i that have precisely one element in common with A'.

Begin by looking over some of the techniques in Chapter 11 of the textbook. This may give you some idea about how to begin. You can also use ideas from other sources, but cite them in the comments of your program.

Due Date

The project is due midnight December 6.