Assignment 1

Y.Nagarani

Download all python codes from

https://github.com/Y.Nagarani/Matrix−Theory/tree/main/Assignment1/Codes

and latex-tikz codes from

https://github.com/Y.Nagarani/Matrix−Theory/tree/main/Assignment1

1 Question No. 2.1

Construct $\triangle ABC$ of sides $a = 4$, $b = 5$ and c=6

2 Explanation

Let us assume that:

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} p \\ 0 \end{pmatrix} \tag{2.0.1}$$

Then

$$AB = \|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A}\|^2 = c^2 \quad \because \mathbf{B} = \mathbf{0} \tag{2.0.2}$$

$$BC = \|\mathbf{C} - \mathbf{B}\|^2 = \|\mathbf{C}\|^2 = a^2 \tag{2.0.3}$$

$$AC = \|\mathbf{A} - \mathbf{C}\|^2 = b^2 \tag{2.0.4}$$

From (2.0.4),

$$b^2 = \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{C}\|^T \|\mathbf{A} - \mathbf{C}\| \tag{2.0.5}$$

$$= \mathbf{A}^T\mathbf{A} + \mathbf{C}^T\mathbf{C} - \mathbf{A}^T\mathbf{C} - \mathbf{C}^T\mathbf{A} \tag{2.0.6}$$

$$= \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \quad \left(\because \mathbf{A}^T\mathbf{C} = \mathbf{C}^T\mathbf{A}\right) \tag{2.0.7}$$

$$= a^2 + c^2 - 2ap \tag{2.0.8}$$

yielding

$$p = \frac{a^2 + c^2 - b^2}{2 \times a} \tag{2.0.9}$$

$$p = \frac{4^2 + 6^2 - 5^2}{2 \times 4} \tag{2.0.10}$$

$$p = \frac{16 + 36 - 25}{8} \tag{2.0.11}$$

$$p = 3.375 \tag{2.0.12}$$

$$\tag{2.0.13}$$

From (2.0.2),

$$\|\mathbf{A}\|^2 = c^2 = p^2 + q^2 \tag{2.0.14}$$

$$\implies q = \pm\sqrt{c^2 - p^2} \tag{2.0.15}$$

$$q = \pm\sqrt{6^2 - 3.375^2} \tag{2.0.16}$$

$$q = \pm\sqrt{24.609375} \tag{2.0.17}$$

$$q = 4.960783708 \tag{2.0.18}$$

The vertex **B** can be expressed in *polar coordinate form* as

$$\mathbf{B} = c\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2.0.19}$$

From $\triangle ABC$,we use the law of cosines:

$\mathrm{b}^2 = a^2 + c^2 - 2ac\cos B$
$cosB = \frac{a^2+c^2-b^2}{2ab}$
$cosB = \frac{27}{40}$
$cosB = 0.675$
we know that , $\sin^2 B + cos^2 B = 1$
$sinB = \sqrt{1 - cos^2 B}$
$sinB = \sqrt{1 - (0.675)^2}$
$sinB = 0.737817728168$
where
$c = \sqrt{a^2 + b^2} = \sqrt{41}$
$c = 6.403124237432$
B can be expressed as B = c$\begin{pmatrix} cosB \\ sinB \end{pmatrix}$

$$\mathrm{B} = \begin{pmatrix} 4.322108860267 \\ 4.724338578040 \end{pmatrix}$$

So,the vertices of $\triangle ABC$ in fig.

$$\mathbf{A} = \begin{pmatrix} 3.375 \\ 4.960 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4.322 \\ 4.724 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{2.0.20}$$

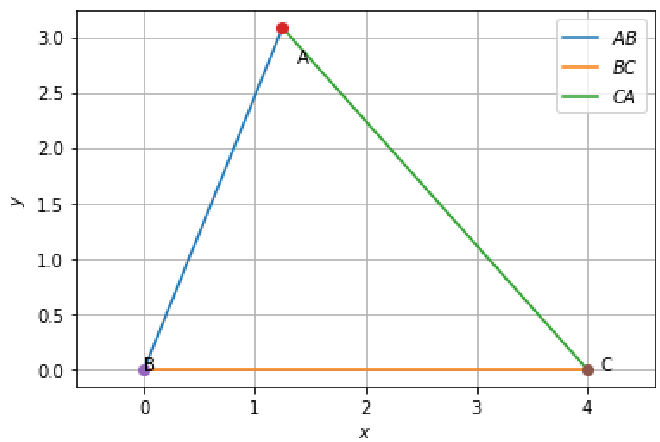

Fig. 0: $\triangle ABC$