ASSIGNMENT 3

Y.Nagarani

Download all python codes from

https://github.com/Y.Nagarani/ASSIGNMENT2/tree/main/CODES

and latex-tikz codes from

https://github.com/Y.Nagarani/ASSIGNMENT2/tree/main

1 Question No 2.32

Find the shortest distance between lines

$$\mathbf{L_1} : \mathbf{x} = \begin{pmatrix}1\\2\\1\end{pmatrix} + \lambda_1 \begin{pmatrix}1\\-1\\1\end{pmatrix} \tag{1.0.1}$$

$$\mathbf{L_2} : \mathbf{x} = \begin{pmatrix}2\\-1\\-1\end{pmatrix} + \lambda_2 \begin{pmatrix}2\\1\\2\end{pmatrix} \tag{1.0.2}$$

2 SOLUTION

$$Let, \mathbf{A_1} = \begin{pmatrix}1\\2\\1\end{pmatrix}, \mathbf{m_1} = \begin{pmatrix}1\\-1\\1\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{A_2} = \begin{pmatrix}2\\-1\\-1\end{pmatrix}, \mathbf{m_2} = \begin{pmatrix}2\\1\\2\end{pmatrix} \tag{2.0.2}$$

The lines will interest if

$$\begin{pmatrix}1\\2\\1\end{pmatrix} + \lambda_1 \begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}2\\-1\\-1\end{pmatrix} + \lambda_2 \begin{pmatrix}2\\1\\2\end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix}1&2\\-1&1\\1&2\end{pmatrix}\begin{pmatrix}\lambda_1\\\lambda_2\end{pmatrix} = \begin{pmatrix}1\\-3\\-2\end{pmatrix} \tag{2.0.4}$$

The augmented matrix for the above equation is row reduced form

$$\begin{pmatrix}1&2&1\\-1&1&-3\\1&2&2\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2+R_1} \begin{pmatrix}1&2&1\\0&3&-2\\1&2&2\end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_3 \leftarrow R_3-R_1} \begin{pmatrix}1&2&1\\0&3&-2\\0&0&1\end{pmatrix} \tag{2.0.6}$$

$\therefore$ The above matrix has rank=3. Hence the line do not interest. Given lines are not parallel but they lie on parallel planes. such lines are known as skew lines.

$\therefore$ the distance between given two lines are

$$\frac{\left|\mathbf{n}^T(\mathbf{A_2}-\mathbf{A_1})\right|}{\|\mathbf{n}\|} = \frac{\left|(\mathbf{A_2}-\mathbf{A_1})^T(\mathbf{m_1}\times\mathbf{m_2})\right|}{\|\mathbf{m_1}\times\mathbf{m_2}\|} \tag{2.0.7}$$

Distance between given two lines is 4.5

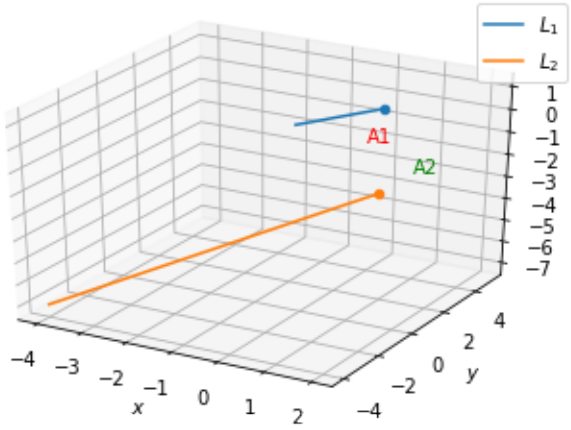

Fig. 0: Skew Lines