ASSIGNMENT 3

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Download all python codes from

https://github.com/Y.Nagarani/ASSIGNMENT2/tree/main/CODES

and latex-tikz codes from

https://github.com/Y.Nagarani/ASSIGNMENT2/tree/main

1 Question No 2.32

Find the shortest distance between lines

$$\mathbf{L_1} : \mathbf{x} = \begin{pmatrix}1\\2\\1\end{pmatrix} + \lambda_1 \begin{pmatrix}1\\-1\\1\end{pmatrix} \tag{1.0.1}$$

$$\mathbf{L_2} : \mathbf{x} = \begin{pmatrix}2\\-1\\-1\end{pmatrix} + \lambda_2 \begin{pmatrix}2\\1\\2\end{pmatrix} \tag{1.0.2}$$

2 SOLUTION

$$Let, \mathbf{A_1} = \begin{pmatrix}1\\2\\1\end{pmatrix}, \mathbf{m_1} = \begin{pmatrix}1\\-1\\1\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{A_2} = \begin{pmatrix}2\\-1\\-1\end{pmatrix}, \mathbf{m_2} = \begin{pmatrix}2\\1\\2\end{pmatrix} \tag{2.0.2}$$

The lines will interest if

$$\begin{pmatrix}1\\2\\1\end{pmatrix} + \lambda_1 \begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}2\\-1\\-1\end{pmatrix} + \lambda_2 \begin{pmatrix}2\\1\\2\end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix}1 & 2\\-1 & 1\\1 & 2\end{pmatrix} \begin{pmatrix}\lambda_1\\\lambda_2\end{pmatrix} = \begin{pmatrix}1\\-3\\-2\end{pmatrix} \tag{2.0.4}$$

The augmented matrix for the above equation is row reduced form

$$\begin{pmatrix}1 & 2 & 1\\-1 & 1 & -3\\1 & 2 & 2\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix}1 & 2 & 1\\0 & 3 & -2\\1 & 2 & 2\end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix}1 & 2 & 1\\0 & 3 & -2\\0 & 0 & 1\end{pmatrix} \tag{2.0.6}$$

$\therefore$ The above matrix has rank=3. Hence the line do not interest. Given lines are not parallel but they lie on parallel planes. such lines are known as skew lines.
$\therefore$ the distance between given two lines are

$$\frac{\left|\mathbf{n}^T (A_2 - A_1)\right|}{\|n\|} = \frac{\left|(A_2 - A_1)^T (m_1 \times m_2)\right|}{\|m_1 \times m_2\|} \tag{2.0.7}$$

By using least square method .
$Therefore,$

$$L_1 : 1 + \lambda_1, 2 - \lambda_1, 1 + \lambda_1 \tag{2.0.8}$$

$$L_2 : 2 + 2\lambda_2, -1 + \lambda_2, -1 + 2\lambda_2 \tag{2.0.9}$$

Square of distance between points in 3d ,

$$d = (-1 + \lambda_1 - 2\lambda_2)^2 + (3 - \lambda_1 - \lambda_2)^2 + (2 + \lambda_1 - 2\lambda_2)^2 \tag{2.0.10}$$

$$d = 14 + 3{\lambda_1}^2 + 9{\lambda_2}^2 - 4\lambda_1 - 6\lambda_1\lambda_2 - 10\lambda_2 \tag{2.0.11}$$

$$\frac{\partial d}{\partial \lambda_1} = 6\lambda_1 - 6\lambda_2 - 4 = 0 \tag{2.0.12}$$

$$\frac{\partial d}{\partial \lambda_2} = 18\lambda_2 - 6\lambda_1 - 10 = 0 \tag{2.0.13}$$

From above two equations we get

$$\lambda_2 = \frac{7}{6} \tag{2.0.14}$$

substitute above value in (2.0.12) we get

$$\lambda_1 = \frac{11}{6} \tag{2.0.15}$$

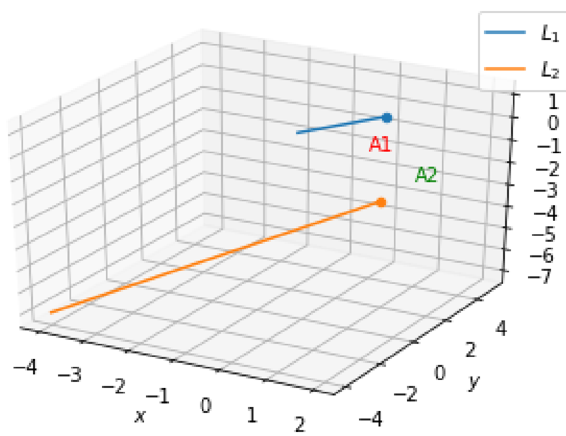

Fig. 0: Skew Lines