#### 1

# **ASSIGNMENT 3**

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### Download all python codes from

https://github.com/Y.Nagarani/ASSIGNMENT2/tree/main/CODES

and latex-tikz codes from

https://github.com/Y.Nagarani/ASSIGNMENT2/tree/main

#### 1 Question No 2.32

Find the shortest distance between lines

$$\mathbf{L_1} : \mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \tag{1.0.1}$$

$$\mathbf{L_2}: \mathbf{x} = \begin{pmatrix} 2\\-1\\-1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2\\1\\2 \end{pmatrix} \tag{1.0.2}$$

2 SOLUTION

$$Let, \mathbf{A_1} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{m_1} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$
 (2.0.1)

$$\mathbf{A}_2 = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \mathbf{m}_2 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \tag{2.0.2}$$

The lines will interest if

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$
 (2.0.3)

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$$
 (2.0.4)

The augmented matrix for the above equation is row reduced form

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & -3 \\ 1 & 2 & 2 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & -2 \\ 1 & 2 & 2 \end{pmatrix}$$
 (2.0.5)

$$\stackrel{R_3 \leftarrow R_3 - R_1}{\longleftrightarrow} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.6}$$

.. The above matrix has rank=3. Hence the line do not interest. Given lines are not parallel but they lie on parallel planes. such lines are known as skew lines.

: the distance between given two lines are

$$\frac{\left|\mathbf{n}^{T}(\mathbf{A}_{2}-\mathbf{A}_{1})\right|}{\|\mathbf{n}\|} = \frac{\left|(\mathbf{A}_{2}-\mathbf{A}_{1})^{T}(\mathbf{m}_{1}\times\mathbf{m}_{2})\right|}{\|\mathbf{m}_{1}\times\mathbf{m}_{2}\|} \quad (2.0.7)$$

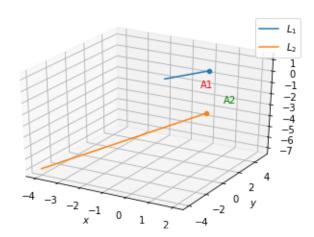


Fig. 0: Skew Lines