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ASSIGNMENT 3

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Download all python codes from

https://github.com/Y.Nagarani/ASSIGNMENT2/tree/main/CODES

and latex-tikz codes from

https://github.com/Y.Nagarani/ASSIGNMENT2/tree/main

1 Question No 2.32

Find the shortest distance between lines

$$\mathbf{L_1}: \mathbf{X} = \begin{pmatrix} 1\\2\\1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1\\-1\\1 \end{pmatrix} \tag{1.0.1}$$

$$\mathbf{L_2}: \mathbf{X} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \tag{1.0.2}$$

2 SOLUTION

$$Let, \mathbf{A_1} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{m_1} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$
 (2.0.1)

$$\mathbf{A_2} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \mathbf{m_2} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \tag{2.0.2}$$

The lines will interest if

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$
 (2.0.3)

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$$
 (2.0.4)

The augmented matrix for the above equation is row reduced form

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & -3 \\ 1 & 2 & 2 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & -2 \\ 1 & 2 & 2 \end{pmatrix}$$
 (2.0.5)

$$\stackrel{R_3 \leftarrow R_3 - R_1}{\longleftrightarrow} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.6}$$

.. The above matrix has rank=3. Hence the line do not interest. Given lines are not parallel but they lie on parallel planes. such lines are known as skew lines

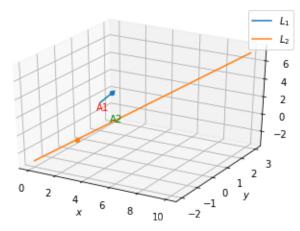


Fig. 0: Skew Lines