

ASSIGNMENT 3

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Download all python codes from

<https://github.com/Y.Nagarani/ASSIGNMENT2/tree/main/CODES>

and latex-tikz codes from

<https://github.com/Y.Nagarani/ASSIGNMENT2/tree/main>

1 QUESTION No 2.32

Find the shortest distance between lines

$$\mathbf{L}_1 : \mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad (1.0.1)$$

$$\mathbf{L}_2 : \mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad (1.0.2)$$

2 SOLUTION

$$\text{Let, } \mathbf{A}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{m}_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{A}_2 = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \mathbf{m}_2 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad (2.0.2)$$

The lines will intersect if

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad (2.0.3)$$

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} \quad (2.0.4)$$

The augmented matrix for the above equation is row reduced form

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & -3 \\ 1 & 2 & 2 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & -2 \\ 1 & 2 & 2 \end{pmatrix} \quad (2.0.5)$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.6)$$

\therefore The above matrix has rank=3. Hence the lines do not intersect. Given lines are not parallel but they lie on parallel planes. such lines are known as skew lines.

\therefore the distance between given two lines are

$$\frac{|\mathbf{n}^T (\mathbf{A}_2 - \mathbf{A}_1)|}{\|\mathbf{n}\|} = \frac{|(\mathbf{A}_2 - \mathbf{A}_1)^T (\mathbf{m}_1 \times \mathbf{m}_2)|}{\|\mathbf{m}_1 \times \mathbf{m}_2\|} \quad (2.0.7)$$

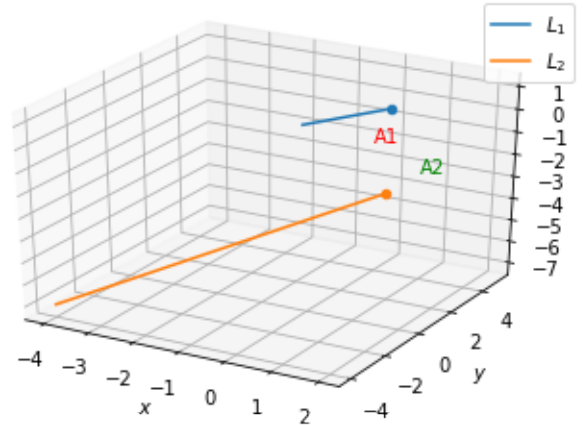


Fig. 0: Skew Lines