ASSIGNMENT 4

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Download all python codes from

https://github.com/Y.Nagarani/Assignment4/tree/main/Assignment4

and latex-tikz codes from

https://github.com/Y.Nagaranj/Assignment4/tree/main/Assignment4

1 Question No 2.19(Quad forms)

Find the zero's of the polynomial $x^2 - 3$ and verify the relationship between the zero's and the coefficients

2 SOLUTION

Given

$$y = x^2 - 3 \tag{2.0.1}$$

$$x^2 - y - 3 = 0 \tag{2.0.2}$$

compare with standard form of equation

$$ax^2 + bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.3}$$

a = 1 , b = 0 , c = 0 , d = 0 , e = $\frac{-1}{2}$, f = -3
Here,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}, f = -3 \tag{2.0.4}$$

$$|V| = \left|\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right| = 0 \tag{2.0.5}$$

Find the eigen values of corresponding **V**

$$|V - \lambda I| = 0 \tag{2.0.6}$$

$$\begin{pmatrix} 1-\lambda & 0 \\ 0 & -\lambda \end{pmatrix} = 0 \tag{2.0.7}$$

$$\lambda = 0, 1 \tag{2.0.8}$$

$Therefore$, the corresponding roots are 0 , 1.
Eigen vectors corresponding to $\lambda = 0, 1$ respectively.

$$p_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.9}$$

$$p_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.10}$$

Using eigenvalue decomposition,

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.11}$$

Now,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p_1}^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p_1} - \mathbf{u} \end{pmatrix} \tag{2.0.12}$$

$\therefore$Vertex $\mathbf{c}$ is given by

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.13}$$

$$\implies \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{2.0.14}$$

$$\implies \mathbf{c} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \tag{2.0.15}$$

Now,

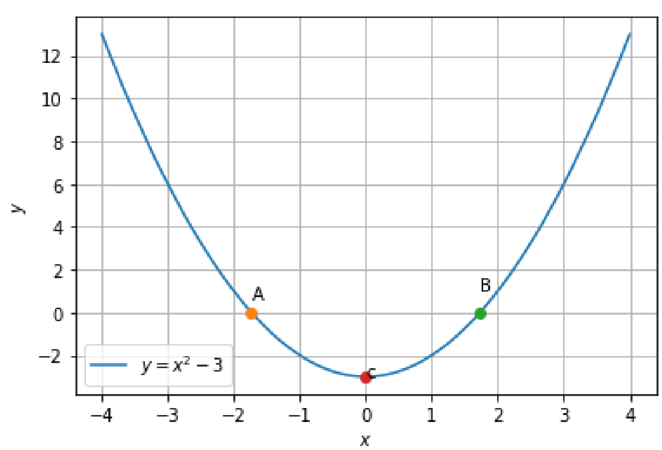

Fig. 2.1: $y = x^2 - 3$

$$\mathbf{p_1}^T\mathbf{c} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -3 \end{pmatrix} \tag{2.0.16}$$
$$= -3 \tag{2.0.17}$$

and,

$$\mathbf{p_2}^T\mathbf{V}\mathbf{p_2} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.18}$$
$$= 1 \tag{2.0.19}$$

$\because$

$$(\mathbf{p_1}^T\mathbf{c})(\mathbf{p_2}^T\mathbf{V}\mathbf{p_2}) = -3 < 0 \tag{2.0.20}$$

Hence,the given equation has real roots.