ASSIGNMENT 5

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Download all python codes from

https://github.com/Y.Nagarani/Assignment6/tree/main/Assignment6

and latex-tikz codes from

https://github.com/Y.Nagarani/Assignment6/tree/main/Assignment6

1 Question No 2.72(d)

In each of the following find the equation for the ellipse that satisfies the given conditions:

a. Ends of major axis $\begin{pmatrix} \pm 3 \\ 0 \end{pmatrix}$, ends of minor axis $\begin{pmatrix} 0 \\ \pm 2 \end{pmatrix}$

2 Solution

Lemma 2.1. *The standard equation of an ellipse is given by:*

$$\frac{\mathbf{y}^\top D\mathbf{y}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{2.0.1}$$

$$\text{where, } D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{2.0.2}$$

Also,the length of semi major axis, a is

$$a = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} \tag{2.0.3}$$

and the length of semi minor axis, b is

$$b = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} \tag{2.0.4}$$

For major axis a = 3 substitute in (2.0.4)

$$\lambda_1 = \frac{\mathbf{u}^T\mathbf{v}^{-1}\mathbf{u} - f}{9} \tag{2.0.5}$$

For minor axis b= 2 substitute in (2.0.5)

$$\lambda_2 = \frac{\mathbf{u}^T\mathbf{v}^{-1}\mathbf{u} - f}{4} \tag{2.0.6}$$

Using lemma (2.1),the standard equation of ellipse is given by :

$$\frac{\mathbf{y}^\top D\mathbf{y}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{2.0.7}$$

Putting (2.0.6) and (2.0.7) in above equation we get:

$$\implies \frac{\mathbf{y}^\top \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\mathbf{y}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - \mathbf{f}} = 1 \tag{2.0.8}$$

$$\implies \mathbf{y}^\top \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{4} \end{pmatrix}\mathbf{y} = 1 \tag{2.0.9}$$

The Plot of ellipse is:

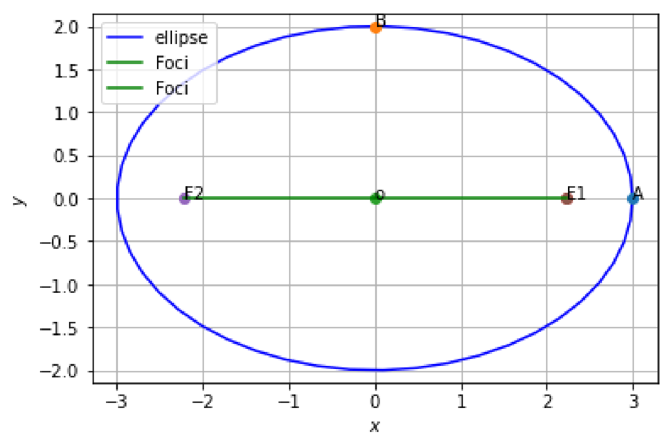

Fig. 2.1: Ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$