ASSIGNMENT 9

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Download all python codes from

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https://github.com/Y.Nagarani/ASSIGNMENT9/
    tree/main/CODES
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and latex-tikz codes from

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https://github.com/Y.Nagarani/ASSIGNMENT9/
    tree/main
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1 Question No 2.15

A manufacture produce nuts and bolts.It takes 1 hour of work on a machine A and 3 hours on machine B to produce a package of nuts.It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs17.50 per package on nuts and Rs7.00 per package on bolts.How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day

2 SOLUTION

item	Machine A	Machine N	Earn profit
Nuts	1	3	17.5
Bolts	3	1	7
Hour's a day	12	12	

TABLE 2.1: Manufacturer produces nuts and bolts

Let the number of nuts be x and the number of bolts be y such that

$$x \geq 0 \tag{2.0.1}$$

$$y \geq 0 \tag{2.0.2}$$

According to the question,

$$x + 3y \leq 12 \tag{2.0.3}$$

$$3x + y \leq 12 \tag{2.0.4}$$

$\therefore$ Our problem is

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 17.5 & 7 \end{pmatrix} \mathbf{x} \tag{2.0.5}$$

$$s.t. \quad \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 12 \\ 12 \end{pmatrix} \tag{2.0.6}$$

Lagrangian function is given by

$$\begin{aligned} &L(\mathbf{x}, \boldsymbol{\lambda}) \\ &= \begin{pmatrix} 17.5 & 7 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} - 12 \right] \right. \\ &+ \left[\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} - 12 \right] \\ &+ \left. \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \boldsymbol{\lambda} \end{aligned} \tag{2.0.7}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} \tag{2.0.8}$$

Now,

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = \begin{pmatrix} 17.5 + \begin{pmatrix} 1 & 3 & -1 & 0 \end{pmatrix} \boldsymbol{\lambda} \\ 7 + \begin{pmatrix} 3 & 1 & 0 & -1 \end{pmatrix} \boldsymbol{\lambda} \\ \begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} - 12 \\ \begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} - 12 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{2.0.9}$$

$\therefore$ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 3 & 1 & 0 & -1 \\ 1 & 3 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -17.5 \\ -7 \\ 12 \\ 12 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.10}$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 1 & 3 \\ 0 & 0 & 3 & 1 \\ 1 & 3 & 0 & 0 \\ 3 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -17.5 \\ -7 \\ 12 \\ 12 \end{pmatrix} \tag{2.0.11}$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 3 \\ 0 & 0 & 3 & 1 \\ 1 & 3 & 0 & 0 \\ 3 & 1 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -17.5 \\ -7 \\ 12 \\ 12 \end{pmatrix} \tag{2.0.12}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-1}{8} & \frac{3}{8} \\ 0 & 0 & \frac{3}{8} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{3}{8} & 0 & 0 \\ \frac{3}{8} & \frac{-1}{8} & 0 & 0 \end{pmatrix} \begin{pmatrix} -17.5 \\ -7 \\ 12 \\ 12 \end{pmatrix} \tag{2.0.13}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -0.5 \\ -5.7 \end{pmatrix} \tag{2.0.14}$$

$\because \boldsymbol{\lambda} = \begin{pmatrix} -0.5 \\ -5.7 \end{pmatrix} \succ \mathbf{0}$

$\therefore$ Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \tag{2.0.15}$$

$$Z = \begin{pmatrix} 17.5 & 7 \end{pmatrix} \mathbf{x} \tag{2.0.16}$$

$$= \begin{pmatrix} 17.5 & 7 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \end{pmatrix} \tag{2.0.17}$$

$$= 73.5 \tag{2.0.18}$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \tag{2.0.19}$$

$$Z = 73.49999997 \tag{2.0.20}$$

Hence , $\boxed{x = 3}$ Nuts and $\boxed{y = 3}$ Bolts should be used to maximum time Available profit $\boxed{Z = 73.5}$.

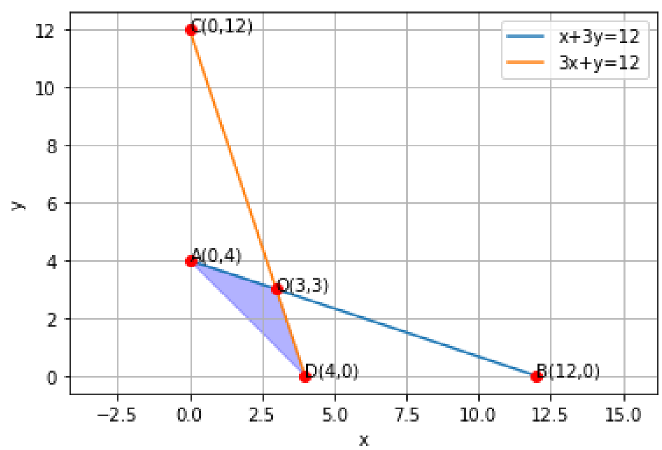

Fig. 2.1: Graphical Solution