CHOICE OF SHIFTS

(1) RAYLEIGH QUOTIENT OF THE LAST COLUMN OF $\underline{Q}^{(k)}$.

$$\mu^{(k)} = (q_m^{(k)})^T A q_m^{(k)}$$

THIS WILL GIVE A CUBIC CONVERGENCE TO THE E-VEC AND E-VAL.

THIS COMES FOR FREE SINCE IT IS THE m, m ENTRY OF $A^{(k)}$.

$$A^{(k)}_{mm} = e_m^T A^{(k)} e_m$$

$$= e_m^T \underline{Q}^{(k)T} A \underline{Q}^{(k)} e_m$$

$$= q_m^{(k)T} A q_m^{(k)}$$

SETTING $\mu^{(k)} = A^{(k)}_{mm}$ IS KNOWN AS THE <u>RAYLEIGH</u> <u>QUOTIENT</u> <u>SHIFT</u>.

(2) ~~WILKO~~ WILKINSON SHIFT

SET B (2x2) AS THE LOWER RIGHT MOST SUBMATRIX OF $A^{(k)}$

③

$$B = \begin{bmatrix} a_{m-1} & b_{m-1} \\ b_{m-1} & a_m \end{bmatrix}$$

SET $\mu^{(k)}$ AS ~~THE~~ EIGENVALUE OF B CLOSEST TO a_m.

$$\mu^{(k)} = a_m - \frac{\text{sign}(\delta)\, b_{m-1}^2}{\left(|\delta| + \sqrt{\delta^2 + b_{m-1}^2}\right)}$$

WHERE

$$\delta = \frac{a_{m-1} - a_m}{2}$$

④

STABILITY

FOR $A \in \mathbb{R}^{m \times m}$ REAL, SYMMETRIC, AND TRIDIAGONAL.

QR ALGORITHM ON A COMPUTER SATISFYING FPA I & II GIVES

$$\tilde{Q}\tilde{\Lambda}\tilde{Q}^T = A + \delta A$$

$$\frac{\|\delta A\|}{\|A\|} = O(\epsilon)$$

FOR SOME $\delta A \in \mathbb{R}^{m \times m}$

QUESTION

(i) $(A^T)^{-1} = (A^{-1})^T$

THIS IS TRUE

$$(A^{-1}A)^T = I$$

$$A^T (A^{-1})^T = I$$

THUS, $(A^{-1})^T = (A^T)^{-1}$

(ii) $(A^{-1})^k = (A^k)^{-1}$

$$\underbrace{(A^{-1} \cdots A^{-1})(A \cdots A)}_{k \text{ TIMES}} = I$$

$$(A^{-1})^k A^k = I$$

THUS,

$$(A^{-1})^k = (A^k)^{-1}.$$

UNSYMMETRIC E-VAL PROBLEM

MANY OF THE METHODS USED IN THE SYMMETRIC ~~THE~~ E-VAL PROBLEM CARRY OVER TO THE UNSYMMETRIC ONE.

- POWER METHOD. (ITERATION)
- INVERSE ~~IN~~ ITERATION.
- "PURE" QR
- "PRACTICAL" QR.

BUT, MANY OF THE IMPORTANT DETAILS (CHOOSING SHIFTS, ETC) BECOME MORE COMPLICATED.

ITERATIVE METHODS

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- WITH DIRECT METHODS OPERATION COUNT TO SOLVE A LINEAR ~~SYM~~ SYSTEM IS $O(m^3)$

- INSTEAD GENERATE A SEQUENCE THAT CONVERGES TO THE SOLUTION (HOPEFULLY) VERY QUICKLY.

- EACH ITERATION COSTS MATRIX-VECTOR MULTIPLICATION

TOTAL: $O(m^2) \times N_{ITER}$

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- ALLOWS USERS TO SOLVE LINEAR SYSTEMS ~~THA~~ USING IMPLICIT MATRIX VECTOR MULTIPLICATION.

- KRYLOV SUBSPACE METHODS.