arm_pendulum_modeling_arm_back

October 22, 2021

1 Arm Motion Modeling

1.1 System Description

A double-pendulum system hanging in gravity is shown in the figure above. $q = [\theta_1, \theta_2]$ are the system configuration variables. We assume the z-axis is pointing out from the screen/paper, thus the positive direction of rotation is counter-clockwise. The solution steps are: 1. Computing the Lagrangian of the system. 2. Computing the Euler-Lagrange equations, and solve them for $\ddot{\theta}_1$ and $\ddot{\theta}_2$. 3. Numerically evaluating the solutions for τ_1 and τ_2, and simulating the system for θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$ and $\ddot{\theta}_2$. 4. Animating the simulation.

[11]:

```
from IPython.core.display import HTML
display(HTML("<table><tr><td><img src='./double-pendulum-diagram.png'␣
↪width=450' height='300'></table>"))
```

```
<IPython.core.display.HTML object>
```

1.2 Import Libraries and Define System Constants

Import libraries:

[12]:

```
# Imports required for data processing
import os
import csv
import pandas as pd

# Imports required for dynamics calculations
import sympy
from sympy.abc import t
from sympy import symbols, Eq, Function, solve, sin, cos, Matrix, Subs,␣
↪substitution, Derivative, simplify, symbols, lambdify
import math
from math import pi
import numpy as np
import matplotlib.pyplot as plt

# Imports required for animation
from plotly.offline import init_notebook_mode, iplot
from IPython.display import display, HTML
```

```
import plotly.graph_objects as go
```

Define the system's constants:

[74]:
```
# Masses, length and center-of-mass positions (calculated using the lab␣
 ↪measurements)
# Mass calculations (mass unit is kg)
m_body = 53
m_u = 0.028 * m_body                        # Average upper arm weights relative␣
 ↪to body weight, from "Biomechanics
                                            # and Motor Control of Human Movement"␣
 ↪by David Winter (2009), 4th edition
m_l = 0.7395                                # Average lower prosthetics weights,␣
 ↪calculated using lab measurements
# m_lower = 0.022 * m_body                  # Average lower arm weights relative␣
 ↪to body weight, from "Biomechanics
                                            # and Motor Control of Human Movement"␣
 ↪by David Winter (2009), 4th edition
# Arm length calculations (length unit is m)
H_body = 1.62
L_u = 0.186 * H_body                        # Average upper arm length relative to␣
 ↪body height
                                            # from "Biomechanics and Motor Control␣
 ↪of Human Movement" by David
                                            # Winter (2009), 4th edition
# L_l = (0.146 + 0.108) * H_body            # Average lower arm length relative to␣
 ↪body height
                                            # from "Biomechanics and Motor Control␣
 ↪of Human Movement" by David
                                            # Winter (2009), 4th edition
L_l = 0.42                                  # Average lower prosthetics length,␣
 ↪calculated using lab measurements

# Arm center of mass length calculations (length unit is m)
L_u_c = 0.436 * L_u                         # Average upper arm length from␣
 ↪shoulder to center of mass relative
                                            # to upper arm length, from␣
 ↪"Biomechanics and Motor Control of Human
                                            # Movement" by David Winter (2009),␣
 ↪4th edition
L_l_c = 0.2388                              # Average lower prosthetics length␣
 ↪from elbow to center of mass,
                                            # calculated using lab measurements
# L_l_c = 0.682 * L_l                       # Average lower arm length from␣
 ↪shoulder to center of mass relative
```

```
                                                    # to upper arm length, from␣
↪"Biomechanics and Motor Control of Human
                                                    # Movement" by David Winter (2009),␣
↪4th edition
```

1.3 Extracting Data

Extracting angles data and computing angular velocities and angular accelerations from the angles:

[75]:
```
def calculate_Vel(Ang_list, time_list, index):
    return ((Ang_list[index + 1] - Ang_list[index])
            / (time_list[index + 1] - time_list[index]))


def calculate_Acc(Vel_list, time_list, index):
    return ((Vel_list[index + 1] - Vel_list[index])
            / (time_list[index + 1] - time_list[index]))


data_csv_dir = '../../data/hand_back_motion_data/CSV Converted Files'
frame_frequency = 100
print("current directory: ", os.getcwd())

walking_vel_list = []
time_list = []
Elbow_Ang_list, Sholder_Ang_list = [], []
Elbow_Vel_list, Sholder_Vel_list = [], []
Elbow_Acc_list, Sholder_Acc_list = [], []
Elbow_Acc_data_list, Sholder_Acc_data_list = [], []
Back_Ang_list, Back_Pos_list, Back_Vel_list = [], [], []

folder_list = os.listdir(data_csv_dir)
folder_list.sort()

for folder in folder_list:

    data_trial_dir = os.path.join(data_csv_dir, folder)
    if os.path.isdir(data_trial_dir):
        file_list = os.listdir(data_trial_dir)

        for file in file_list:
            if "00B429F8" in file:
                if file.endswith(".csv"):
                    file_name = file[:-4]
                    walking_vel = file.split("_")[4][:5]

                    frame = 0
```

```
                    file_time_list = []
                    file_Sholder_Ang_list, file_Sholder_Vel_list,␣
↪file_Sholder_Acc_list, file_Sholder_Acc_data_list = [], [], [], []

                    # Cutting out weird data behavior on data edges
                    data_path = os.path.join(data_csv_dir, folder, file)
                    data_rows = open(data_path).read().strip().split("\n")[7500:
↪9500]

                    # Extract time [sec], elbow angles [rad], and shoulder␣
↪angles [rad] from data
                    for row in data_rows:
                        splitted_row = row.strip().split("\t")

                        # Check if loop finished all data
                        if not len(splitted_row):
                            break

                        file_time_list.append(frame / frame_frequency)
                        file_Sholder_Ang_list.append(float(splitted_row[31]) *␣
↪2*pi/360)
                        file_Sholder_Acc_data_list.
↪append(float(splitted_row[14]))
                        frame += 1

                    # Extract elbow and shoulder velocities [rad/sec] from␣
↪angles
                    for i in range(len(file_time_list) - 1):
                        Sholder_Vel = calculate_Vel(file_Sholder_Ang_list,␣
↪file_time_list, i)
                        file_Sholder_Vel_list.append(Sholder_Vel)

                    # Extract elbow and shoulder Accelerations [rad/sec^2] from␣
↪velocities
                    for i in range(len(file_time_list) - 2):
                        Sholder_Acc = calculate_Acc(file_Sholder_Vel_list,␣
↪file_time_list, i)
                        file_Sholder_Acc_list.append(Sholder_Acc)

                    # Adjust lists length
                    adjusted_file_time_list = file_time_list[:-2]
                    adjusted_file_Sholder_Ang_list = file_Sholder_Ang_list[:-2]
                    adjusted_file_Sholder_Vel_list = file_Sholder_Vel_list[:-1]
                    adjusted_file_Sholder_Acc_data_list =␣
↪file_Sholder_Acc_data_list[:-2]
```

```
                    time_list.append(adjusted_file_time_list)
                    walking_vel_list.append(walking_vel)

                    Sholder_Ang_list.append(adjusted_file_Sholder_Ang_list)
                    Sholder_Vel_list.append(adjusted_file_Sholder_Vel_list)
                    Sholder_Acc_list.append(file_Sholder_Acc_list)
                    Sholder_Acc_data_list.
↪append(adjusted_file_Sholder_Acc_data_list)
                    break

        for file in file_list:
            if "00B429E2" in file:
                if file.endswith(".csv"):
                    file_name = file[:-4]
                    walking_vel = file.split("_")[4][:5]

                    frame = 0
                    file_time_list = []
                    file_Elbow_Ang_list, file_Elbow_Vel_list,␣
↪file_Elbow_Acc_list, file_Elbow_Acc_data_list = [], [], [], []

                    # Cutting out weird data behavior on data edges
                    data_path = os.path.join(data_csv_dir, folder, file)
                    data_rows = open(data_path).read().strip().split("\n")[7500:
↪9500]

                    # Extract time [sec], elbow angles [rad], and shoulder␣
↪angles [rad] from data
                    for i in range(len(data_rows)):
                        splitted_row = data_rows[i].strip().split("\t")

                        # Check if loop finished all data
                        if not len(splitted_row):
                            break

                        file_time_list.append(frame / frame_frequency)
                        file_Elbow_Ang_list.append((float(splitted_row[31]) -␣
↪file_Sholder_Ang_list[i]) * 2*pi/360)
                        file_Elbow_Acc_data_list.append(float(splitted_row[14]))
                        frame += 1

                    # Extract elbow and shoulder velocities [rad/sec] from␣
↪angles
                    for i in range(len(file_time_list) - 1):
                        Elbow_Vel = calculate_Vel(file_Elbow_Ang_list,␣
↪file_time_list, i)
```

```
                        file_Elbow_Vel_list.append(Elbow_Vel)

                    # Extract elbow and shoulder Accelerations [rad/sec^2] from velocities
                    for i in range(len(file_time_list) - 2):
                        Elbow_Acc = calculate_Acc(file_Elbow_Vel_list, file_time_list, i)
                        file_Elbow_Acc_list.append(Elbow_Acc)

                    # Adjust lists length
                    adjusted_file_Elbow_Ang_list = file_Elbow_Ang_list[:-2]
                    adjusted_file_Elbow_Vel_list = file_Elbow_Vel_list[:-1]
                    adjusted_file_Elbow_Acc_data_list = file_Elbow_Acc_data_list[:-2]

                    Elbow_Ang_list.append(adjusted_file_Elbow_Ang_list)
                    Elbow_Vel_list.append(adjusted_file_Elbow_Vel_list)
                    Elbow_Acc_list.append(file_Elbow_Acc_list)
                    Elbow_Acc_data_list.append(file_Elbow_Acc_data_list)
                    break

        for file in file_list:
            if "00B43D0C" in file:
                if file.endswith(".csv"):
                    file_name = file[:-4]
                    walking_vel = file.split("_")[4][:5]
                    if walking_vel == "1.4ms":
                        continue

                    frame = 0
                    file_time_list = []
                    file_Back_Ang_list, file_Back_Pos_list, file_Back_Vel_list = [], [], []

                    # Cutting out weird data behavior on data edges
                    data_path = os.path.join(data_csv_dir, folder, file)
                    data_rows = open(data_path).read().strip().split("\n")[7500:9500]

                    # Extract time [sec], elbow angles [rad], and shoulder angles [rad] from data
                    for i in range(len(data_rows)):
                        splitted_row = data_rows[i].strip().split("\t")

                        # Check if loop finished all data
                        if not len(splitted_row):
```

```
                            break

                        file_time_list.append(frame / frame_frequency)

                        file_Back_Ang_list.append(float(splitted_row[31]) *␣
↪2*pi/360)
                        file_Back_Pos_list.append(float(splitted_row[21]))
                        file_Back_Vel_list.append(float(splitted_row[24]))
                        frame += 1

                    # Adjust lists length
                    adjusted_file_Back_Ang_list = file_Back_Ang_list[:-2]
                    adjusted_file_Back_Pos_list = file_Back_Pos_list[:-2]
                    adjusted_file_Back_Vel_list = file_Back_Vel_list[:-2]

                    Back_Ang_list.append(adjusted_file_Back_Ang_list)
                    Back_Pos_list.append(adjusted_file_Back_Pos_list)
                    Back_Vel_list.append(adjusted_file_Back_Vel_list)
                    break
```

```
current directory:
/home/yael/Documents/MSR_Courses/ME499-Final_Project/Motorized-Prosthetic-
Arm/motor_control/arm_pendulum_modeling
```

1.4 System Modeling

Computing the Lagrangian of the system:

[76]:
```
m1, m2, g, R1, R1_c, R2, R2_c = symbols(r'm1, m2, g, R1, R1_c, R2, R2_c')

# The system torque variables as function of t
tau1 = Function(r'tau1')(t)
tau2 = Function(r'tau2')(t)

# The system configuration variables as function of t
theta1 = Function(r'theta1')(t)
theta2 = Function(r'theta2')(t)

# The velocity as derivative of position wrt t
theta1_dot = theta1.diff(t)
theta2_dot = theta2.diff(t)

# The acceleration as derivative of velocity wrt t
theta1_ddot = theta1_dot.diff(t)
theta2_ddot = theta2_dot.diff(t)

# Converting the polar coordinates to cartesian coordinates
x1 = R1_c * sin(theta1)
```

```
x2 = R1 * sin(theta1) + R2_c * sin(theta1 + theta2)

y1 = -R1_c * cos(theta1)
y2 = -R1 * cos(theta1) - R2_c * cos(theta1 + theta2)

# Calculating the kinetic and potential energy of the system
KE = 1/2 * m1 * ((x1.diff(t))**2 + (y1.diff(t))**2) + 1/2 * m2 * ((x2.
↪diff(t))**2 + (y2.diff(t))**2)
PE = m1 * g * y1 + m2 * g * y2

# Computing the Lagrangian
L = simplify(KE - PE)
Lagrange = Function(r'L')(t)
display(Eq(Lagrange, L))
```

$$L(t) = 0.5R_{1c}^2 m_1 \left(\frac{d}{dt}\theta_1(t)\right)^2 + R_{1c} g m_1 \cos\left(\theta_1(t)\right) + g m_2 \left(R_1 \cos\left(\theta_1(t)\right) + R_{2c} \cos\left(\theta_1(t) + \theta_2(t)\right)\right) +$$
$$0.5m_2 \left(R_1^2 \left(\frac{d}{dt}\theta_1(t)\right)^2 + 2R_1 R_{2c} \cos\left(\theta_2(t)\right) \left(\frac{d}{dt}\theta_1(t)\right)^2 + 2R_1 R_{2c} \cos\left(\theta_2(t)\right) \frac{d}{dt}\theta_1(t) \frac{d}{dt}\theta_2(t) + R_{2c}^2 \left(\frac{d}{dt}\theta_1(t)\right)^2 + 2I$$

Computing the Euler-Lagrange equations:

[77]:
```
# Define the derivative of L wrt the functions: x, xdot
L_dtheta1 = L.diff(theta1)
L_dtheta2 = L.diff(theta2)

L_dtheta1_dot = L.diff(theta1_dot)
L_dtheta2_dot = L.diff(theta2_dot)

# Define the derivative of L_dxdot wrt to time t
L_dtheta1_dot_dt = L_dtheta1_dot.diff(t)
L_dtheta2_dot_dt = L_dtheta2_dot.diff(t)

# Define the left hand side of the the Euler-Lagrange as a matrix
lhs = Matrix([simplify(L_dtheta1_dot_dt - L_dtheta1),
              simplify(L_dtheta2_dot_dt - L_dtheta2)])

# Define the right hand side of the the Euler-Lagrange as a Matrix
rhs = Matrix([tau1, tau2])

# Compute the Euler-Lagrange equations as a matrix
EL_eqns = Eq(lhs, rhs)

print('Euler-Lagrange matrix for this systems:')
display(EL_eqns)
```

```
Euler-Lagrange matrix for this systems:
```

$$\left[\begin{array}{r}1.0R_{1c}^2 m_1 \frac{d^2}{dt^2}\theta_1(t) + R_{1c} g m_1 \sin(\theta_1(t)) + g m_2 \left(R_1 \sin(\theta_1(t)) + R_{2c} \sin(\theta_1(t) + \theta_2(t))\right) + m_2 \left(R_1^2 \frac{d^2}{dt^2}\theta_1(t) - 2R_1 R_{2c}\right. \\ R_{2c} m_2 \left(R_1 \sin(\theta_2(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 + R_1 \cos\right.\end{array}\right.$$

$$\begin{bmatrix}\tau_1(t) \\ \tau_2(t)\end{bmatrix}$$

Solve the equations for τ_1 and τ_2:

[78]:
```python
# Solve the Euler-Lagrange equations for the shoulder and elbow torques
T = Matrix([tau1, tau2])
soln = solve(EL_eqns, T, dict=True)

# Initialize the solutions
solution = [0, 0]
i = 0

for sol in soln:
    for v in T:
        solution[i] = simplify(sol[v])
        display(Eq(T[i], solution[i]))
        i =+ 1
```

$$\tau_1(t) = R_1^2 m_2 \frac{d^2}{dt^2}\theta_1(t) - 2.0R_1 R_{2c} m_2 \sin(\theta_2(t)) \frac{d}{dt}\theta_1(t) \frac{d}{dt}\theta_2(t) - R_1 R_{2c} m_2 \sin(\theta_2(t)) \left(\frac{d}{dt}\theta_2(t)\right)^2 + 2.0R_1 R_{2c} m_2 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_1(t) + R_1 R_{2c} m_2 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_2(t) + R_1 g m_2 \sin(\theta_1(t)) + R_{1c}^2 m_1 \frac{d^2}{dt^2}\theta_1(t) + R_{1c} g m_1 \sin(\theta_1(t)) + R_{2c}^2 m_2 \frac{d^2}{dt^2}\theta_1(t) + R_{2c}^2 m_2 \frac{d^2}{dt^2}\theta_2(t) + R_{2c} g m_2 \sin(\theta_1(t) + \theta_2(t))$$

$$\tau_2(t) = R_{2c} m_2 \left(R_1 \sin(\theta_2(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 + R_1 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_1(t) + R_{2c} \frac{d^2}{dt^2}\theta_1(t) + R_{2c} \frac{d^2}{dt^2}\theta_2(t) + g \sin(\theta_1(t) + \theta_2(t\right.$$

Simulating the system:

[79]:
```python
# Substitute the derivative variables with a dummy variables and plug-in the
↪constants
solution_0_subs = solution[0]
solution_1_subs = solution[1]

theta1_dot_dummy = symbols('thetadot1')
theta2_dot_dummy = symbols('thetadot2')
theta1_ddot_dummy = symbols('thetaddot1')
theta2_ddot_dummy = symbols('thetaddot2')

solution_0_subs = solution_0_subs.subs([(g, 9.81)])
solution_1_subs = solution_1_subs.subs([(g, 9.81)])

solution_0_subs = solution_0_subs.subs([((theta1.diff(t)).diff(t),
↪theta1_ddot_dummy),
```

```
                                                ((theta2.diff(t)).diff(t),␣
↪theta2_ddot_dummy)])
solution_1_subs = solution_1_subs.subs([((theta1.diff(t)).diff(t),␣
↪theta1_ddot_dummy),
                                                ((theta2.diff(t)).diff(t),␣
↪theta2_ddot_dummy)])

solution_0_subs = solution_0_subs.subs([(theta1.diff(t), theta1_dot_dummy),
                                                (theta2.diff(t), theta2_dot_dummy)])
solution_1_subs = solution_1_subs.subs([(theta1.diff(t), theta1_dot_dummy),
                                                (theta2.diff(t), theta2_dot_dummy)])

# Lambdify the thetas and its derivatives
func1 = lambdify([theta1, theta2, theta1_dot_dummy, theta2_dot_dummy,␣
↪theta1_ddot_dummy,
                  theta2_ddot_dummy, m1, m2, R1, R2, R1_c, R2_c],␣
↪solution_0_subs, modules = sympy)
func2 = lambdify([theta1, theta2, theta1_dot_dummy, theta2_dot_dummy,␣
↪theta1_ddot_dummy,
                  theta2_ddot_dummy, m1, m2, R1, R2, R1_c, R2_c],␣
↪solution_1_subs, modules = sympy)

# Initialize the torque and power lists
Sholder_tau_list, Elbow_tau_list = [], []
Sholder_current_list, Elbow_current_list = [], []
Sholder_power_list, Elbow_power_list = [], []

motor_kv = 115
torque_const = 8.27 / motor_kv

for i in range(len(time_list)):
    # Initialize the torque and power lists
    tau1_list, tau2_list = [], []
    current1_list, current2_list = [], []
    power1_list, power2_list = [], []

    t_list = time_list[i]
    theta1_list = Sholder_Ang_list[i]
    theta2_list = Elbow_Ang_list[i]
    dtheta1_list = Sholder_Vel_list[i]
    dtheta2_list = Elbow_Vel_list[i]
    ddtheta1_list = Sholder_Acc_list[i]
    ddtheta2_list = Elbow_Acc_list[i]

    # Plug-in the angles, angular velocities and angular accelerations for␣
↪every time step to find the torques
```

```
    for j in range(len(t_list)):
        tau1_list.append(func1(theta1_list[j], theta2_list[j],
                               dtheta1_list[j], dtheta2_list[j],
                               ddtheta1_list[j], ddtheta2_list[j],
                               m_u, m_l, L_u, L_l, L_u_c, L_l_c))

        tau2_list.append(func2(theta1_list[j], theta2_list[j],
                               dtheta1_list[j], dtheta2_list[j],
                               ddtheta1_list[j], ddtheta2_list[j],
                               m_u, m_l, L_u, L_l, L_u_c, L_l_c))

        # Calculate the current required to reach the required joints torques␣
↪for every time step
        current1_list.append(torque_const * tau1_list[j])
        current2_list.append(torque_const * tau2_list[j])

        # Calculate the power required to reach the required angular velocities␣
↪and joints torques for every time step
        power1_list.append(dtheta1_list[j] * tau1_list[j])
        power2_list.append(dtheta2_list[j] * tau2_list[j])

    Sholder_tau_list.append(tau1_list)
    Elbow_tau_list.append(tau2_list)
    Sholder_current_list.append(current1_list)
    Elbow_current_list.append(current2_list)
    Sholder_power_list.append(power1_list)
    Elbow_power_list.append(power2_list)

    print(f"Velocity {walking_vel_list[i]}\t max torque:␣
↪{format(max(tau2_list), '.3f')}[Nm]\t max angular velocity:␣
↪{format(max(dtheta2_list), '.3f')}[rad/sec]\t max power:␣
↪{format(max(power2_list), '.3f')}[W]")
```

```
Velocity 0.5ms   max torque: 0.228[Nm]   max angular velocity: 1.625[rad/sec]
max power: 1.693[W]
Velocity 0.6ms   max torque: 0.332[Nm]   max angular velocity: 1.731[rad/sec]
max power: 1.798[W]
Velocity 0.7ms   max torque: 0.552[Nm]   max angular velocity: 2.556[rad/sec]
max power: 2.379[W]
Velocity 0.8ms   max torque: 0.900[Nm]   max angular velocity: 3.695[rad/sec]
max power: 4.734[W]
Velocity 0.9ms   max torque: 1.497[Nm]   max angular velocity: 3.370[rad/sec]
max power: 4.650[W]
Velocity 1.0ms   max torque: 3.806[Nm]   max angular velocity: 3.588[rad/sec]
max power: 12.103[W]
Velocity 1.1ms   max torque: 1.901[Nm]   max angular velocity: 2.989[rad/sec]
max power: 3.543[W]
```

```
Velocity 1.2ms   max torque: 2.264[Nm]   max angular velocity: 3.567[rad/sec]
max power: 3.809[W]
Velocity 1.3ms   max torque: 3.269[Nm]   max angular velocity: 4.525[rad/sec]
max power: 5.310[W]
Velocity 1.4ms   max torque: 4.890[Nm]   max angular velocity: 4.931[rad/sec]
max power: 8.735[W]
Velocity chang   max torque: 2.238[Nm]   max angular velocity: 3.920[rad/sec]
max power: 6.652[W]
```

Calculation summary:

[80]:
```
max_Elbow_tau, max_Elbow_power, max_Elbow_Vel = 0, 0, 0
max_Elbow_tau_index, max_Elbow_power_index, max_Elbow_Vel_index = 0, 0, 0

for i in range(len(Elbow_tau_list)):
    if max_Elbow_Vel < max(Elbow_Vel_list[i]):
        max_Elbow_Vel = max(Elbow_Vel_list[i])
        max_Elbow_Vel_index = i

    if max_Elbow_tau < max(Elbow_tau_list[i]):
        max_Elbow_tau = max(Elbow_tau_list[i])
        max_Elbow_tau_index = i

    if max_Elbow_power < max(Elbow_power_list[i]):
        max_Elbow_power = max(Elbow_power_list[i])
        max_Elbow_power_index = i

print(f"maximum elbow angular velocity is {format(max_Elbow_Vel, '.3f')} [rad/
 ↪sec] ({format(max_Elbow_Vel*60/(2*pi), '.3f')} [rpm]), in velocity␣
 ↪{walking_vel_list[max_Elbow_Vel_index]} (trial {max_Elbow_Vel_index})")
print(f"maximum elbow torque is {format(max_Elbow_tau, '.3f')} [Nm], in␣
 ↪velocity {walking_vel_list[max_Elbow_tau_index]} (trial␣
 ↪{max_Elbow_tau_index})")
print(f"maximum elbow power is {format(max_Elbow_power, '.3f')} [W], in␣
 ↪velocity {walking_vel_list[max_Elbow_power_index]} (trial␣
 ↪{max_Elbow_power_index})")


# The torque equations for the maximum power:
solution_0_subs = solution_0_subs.subs([(m1, m_u), (m2, m_l), (R1, L_u), (R2,␣
 ↪L_l), (R1_c, L_u_c), (R2_c, L_l_c), (g, 9.81)])
solution_1_subs = solution_1_subs.subs([(m1, m_u), (m2, m_l), (R1, L_u), (R2,␣
 ↪L_l), (R1_c, L_u_c), (R2_c, L_l_c), (g, 9.81)])

print("\nThe torque equations for the maximum torque:")
display(Eq(T[0], solution_0_subs))
display(Eq(T[1], solution_1_subs))
```

```
# display(Elbow_Ang_list[max_Elbow_tau_index])
# display(Elbow_Vel_list[max_Elbow_tau_index])
# display(Elbow_Acc_list[max_Elbow_tau_index])
# display(Elbow_tau_list[max_Elbow_tau_index])
# display(Elbow_Ang_list[2])
# display(Elbow_tau_list[2])
```

```
maximum elbow angular velocity is 4.931 [rad/sec] (47.091 [rpm]), in velocity
1.4ms (trial 9)
maximum elbow torque is 4.890 [Nm], in velocity 1.4ms (trial 9)
maximum elbow power is 12.103 [W], in velocity 1.0ms (trial 5)

The torque equations for the maximum torque:
```

$$\tau_1(t) = 0.1064217644464\ddot{\theta}_1 \cos(\theta_2(t)) + 0.1349254238316621\ddot{\theta}_1 + 0.0532108822232\ddot{\theta}_2 \cos(\theta_2(t)) + 0.042170312288\ddot{\theta}_2 - 0.1064217644464\dot{\theta}_1\dot{\theta}_2 \sin(\theta_2(t)) - 0.0532108822232\dot{\theta}_2^2 \sin(\theta_2(t)) + 1.732373406 \sin(\theta_1(t) + \theta_2(t)) + 4.0984945085808 \sin(\theta_1(t))$$

$$\tau_2(t) = 0.0532108822232\ddot{\theta}_1 \cos(\theta_2(t)) + 0.042170312288\ddot{\theta}_1 + 0.042170312288\ddot{\theta}_2 + 0.0532108822232\dot{\theta}_1^2 \sin(\theta_2(t)) + 1.732373406 \sin(\theta_1(t) + \theta_2(t))$$

Example for the trial with the largest elbow torque & power:

[81]:
```
index = 2
t_list = time_list[index]
theta1_list = Sholder_Ang_list[index]
theta2_list = Elbow_Ang_list[index]
dtheta1_list = Sholder_Vel_list[index]
dtheta2_list = Elbow_Vel_list[index]
ddtheta1_list = Sholder_Acc_list[index]
ddtheta2_list = Elbow_Acc_list[index]
tau1_list = Sholder_tau_list[index]
tau2_list = Elbow_tau_list[index]
current1_list = Sholder_current_list[index]
current2_list = Elbow_current_list[index]
power1_list = Sholder_power_list[index]
power2_list = Elbow_power_list[index]

back_rotation_list = Back_Ang_list[index]
back_position_list = Back_Pos_list[index]
back_velocity_list = Back_Vel_list[index]

Elbow_Acceleration_list = Elbow_Acc_data_list[index]
Sholder_Acceleration_list = Sholder_Acc_data_list[index]

# Compute the trajectory of the arm's motion
N = int((max(t_list) - min(t_list))/(1/frame_frequency))
tvec = np.linspace(min(t_list), max(t_list), N)
traj = np.zeros((6, N))
```

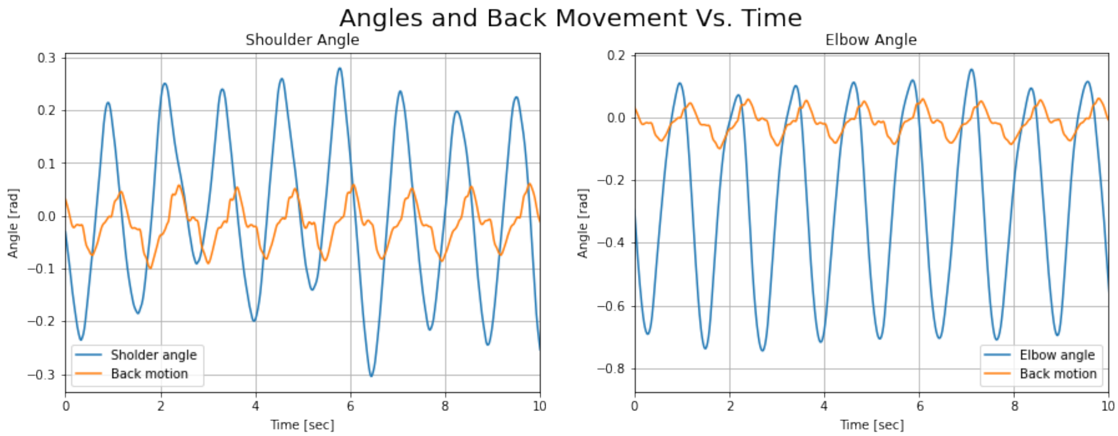
```
back_traj = np.zeros((3, N))
acc_traj = np.zeros((2, N))
partial_traj = np.zeros((6, N))

for i in range(N):
    traj[0, i] = theta1_list[i]
    traj[1, i] = theta2_list[i]
    traj[2, i] = dtheta1_list[i]
    traj[3, i] = dtheta2_list[i]
    traj[4, i] = ddtheta1_list[i]
    traj[5, i] = ddtheta2_list[i]


    back_traj[0, i] = back_rotation_list[i]
    back_traj[1, i] = back_position_list[i]
    back_traj[2, i] = back_velocity_list[i]

    acc_traj[0, i] = Elbow_Acceleration_list[i]
    acc_traj[1, i] = Sholder_Acceleration_list[i]

for i in range(100):
    partial_traj[0, i] = theta1_list[i]
    partial_traj[1, i] = theta2_list[i]
    partial_traj[2, i] = dtheta1_list[i]
    partial_traj[3, i] = dtheta2_list[i]
    partial_traj[4, i] = ddtheta1_list[i]
    partial_traj[5, i] = ddtheta2_list[i]

# Calculate the length difference between the time list and the trajectory lists
diff = (len(t_list) - len(traj[0]))

# Plot the trajectory lists (angles, velocities, accelerations, torques, and␣
 ↪power)
plt.figure(figsize=(15,5))
plt.suptitle('Angles and Back Movement Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[0], label="Sholder angle")
plt.plot(t_list[:-diff], back_traj[0], label="Back motion")
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec)/2)])
plt.grid()
plt.legend()
plt.title('Shoulder Angle')

plt.subplot(122)
plt.plot(t_list[:-diff], traj[1], label="Elbow angle")
```

```
plt.plot(t_list[:-diff], back_traj[0], label="Back motion")
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec)/2)])
plt.grid()
plt.legend()
plt.title('Elbow Angle')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Arm Acceleration and Back Movement Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[0], label="Sholder acceleration")
plt.plot(t_list[:-diff], back_traj[0], label="Back motion")
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec)/2)])
plt.grid()
plt.legend()
plt.title('Shoulder Acceleration')

plt.subplot(122)
plt.plot(t_list[:-diff], traj[1], label="Elbow acceleration")
plt.plot(t_list[:-diff], back_traj[0], label="Back motion")
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec)/2)])
plt.grid()
plt.legend()
plt.title('Elbow Acceleration')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Angles Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[0])
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Shoulder Angle')

plt.subplot(122)
plt.plot(t_list[:-diff], traj[1])
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
```

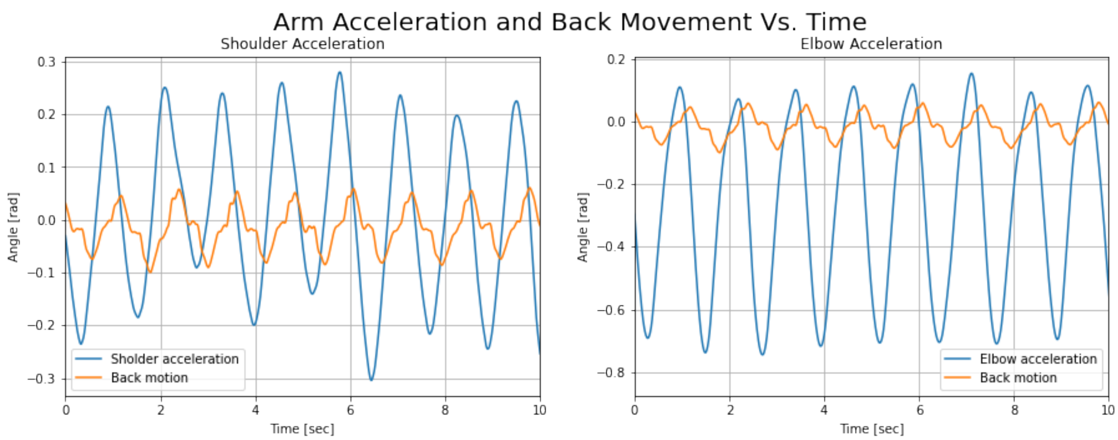
```
plt.grid()
plt.title('Elbow Angle')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Angular Velocity Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[2])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Shoulder Angular Velocity')

plt.subplot(122)
plt.plot(t_list[:-diff], traj[3])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Angular Velocity')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Angular Acceleration Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[4])
plt.ylabel('Acceleration [rad/sec^2]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Shoulder Angular Acceleration')

plt.subplot(122)
plt.plot(t_list[:-diff], traj[5])
plt.ylabel('Acceleration [rad/sec^2]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Angular Acceleration')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Torque Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list, tau1_list)
plt.ylabel('Torque [Nm]')
plt.xlabel('Time [sec]')
```

```
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Shoulder Torque')

plt.subplot(122)
plt.plot(t_list, tau2_list)
plt.ylabel('Torque [Nm]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Torque')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Power Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list, power1_list)
plt.ylabel('Power [W]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Shoulder Power')

plt.subplot(122)
plt.plot(t_list, power2_list)
plt.ylabel('Power [W]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Power')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Speed Vs. Torque', fontsize=20)
plt.subplot(121)
plt.plot(tau1_list[:-diff], traj[2])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Torque [Nm]')
plt.grid()
plt.title('Shoulder Speed-Torque')

plt.subplot(122)
plt.plot(tau2_list[:-diff], traj[3])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Torque [Nm]')
plt.grid()
plt.title('Elbow Speed-Torque')
```

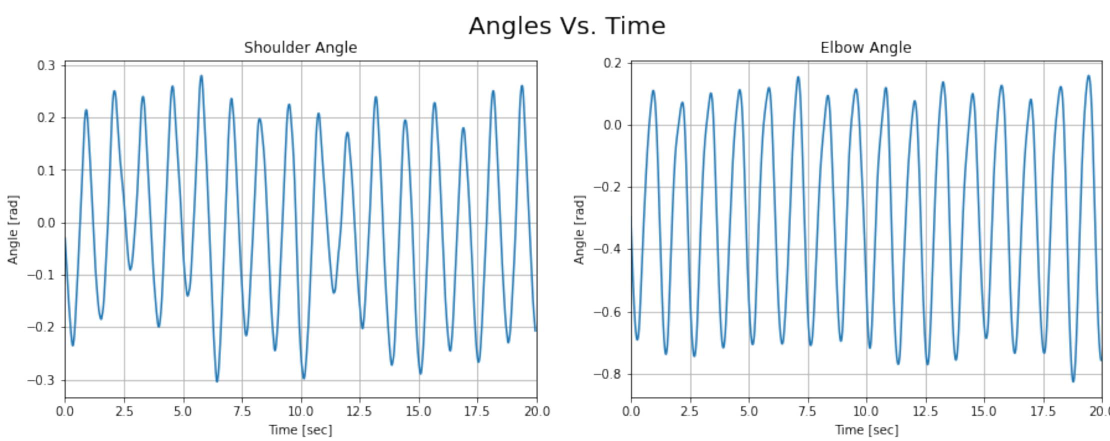
```
plt.show()
```



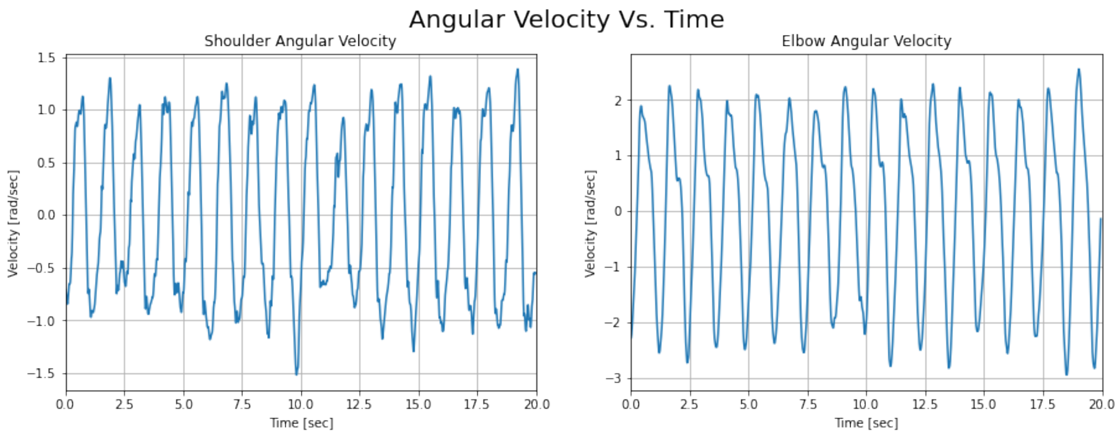
Angles and Back Movement Vs. Time
Shoulder Angle
Elbow Angle
Angle [rad]
Time [sec]
Sholder angle
Back motion
Elbow angle
Back motion

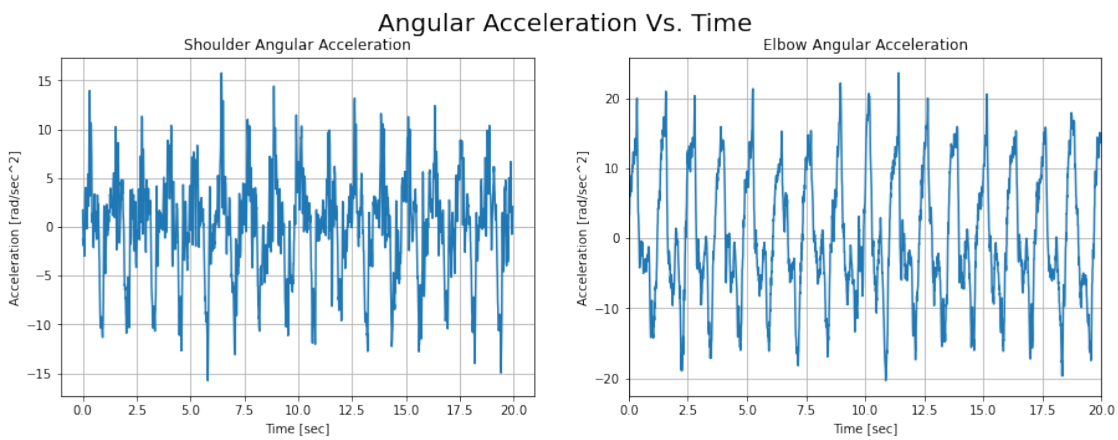


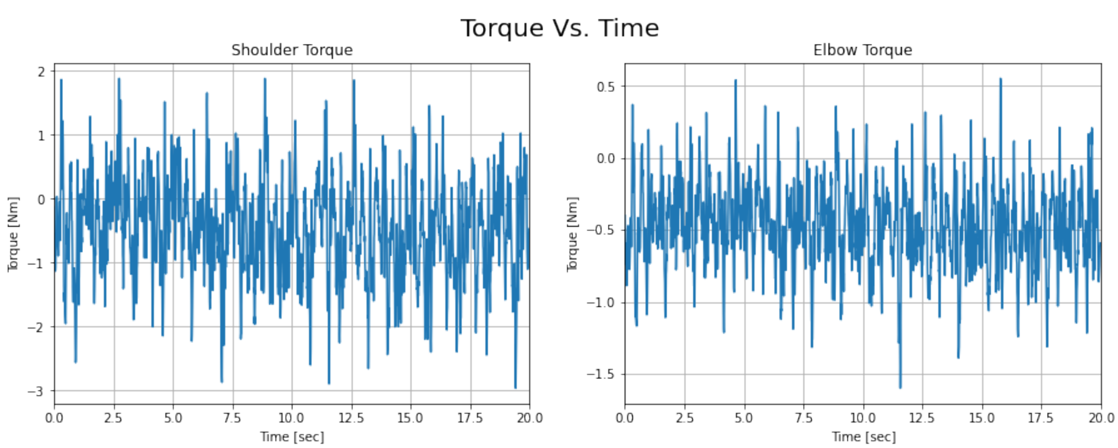
Arm Acceleration and Back Movement Vs. Time
Shoulder Acceleration
Elbow Acceleration
Angle [rad]
Time [sec]
Sholder acceleration
Back motion
Elbow acceleration
Back motion

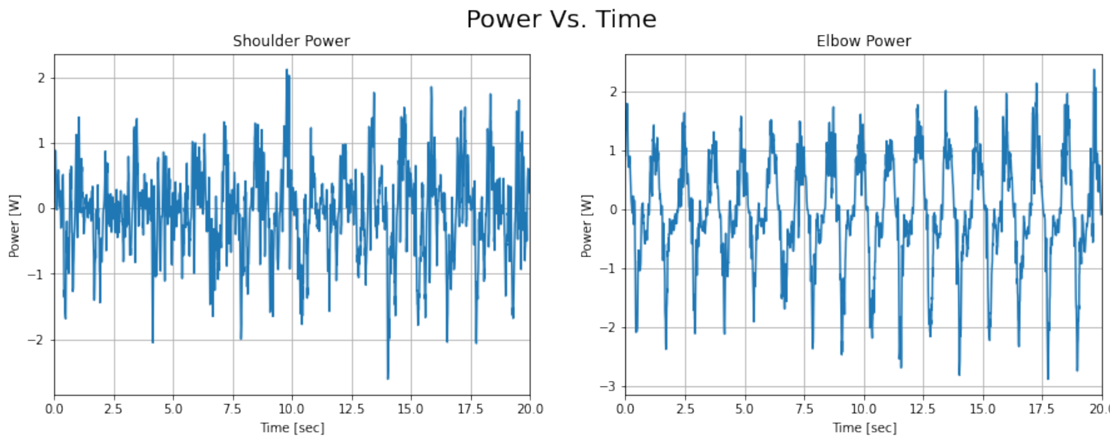


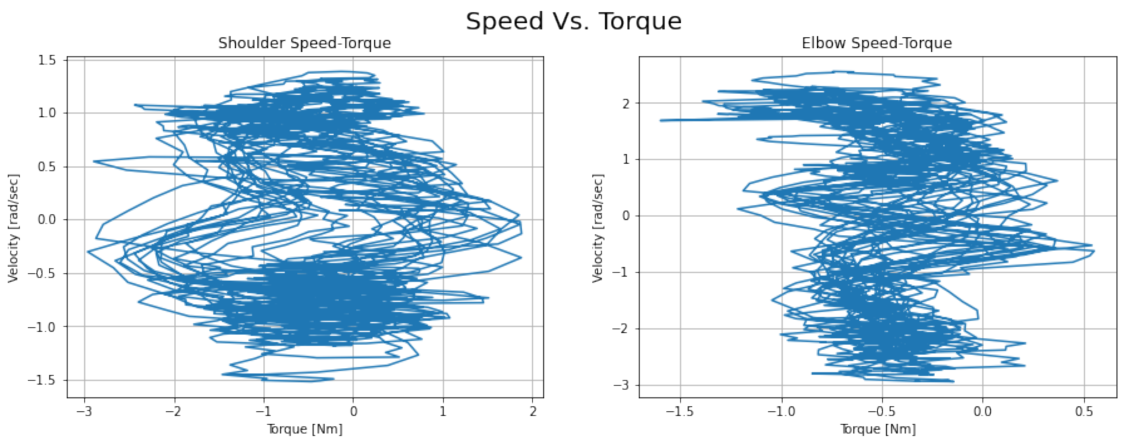
Angles Vs. Time
Shoulder Angle
Elbow Angle
Angle [rad]
Time [sec]


Angular Velocity Vs. Time
Shoulder Angular Velocity
Elbow Angular Velocity
Velocity [rad/sec]
Time [sec]


Angular Acceleration Vs. Time
Shoulder Angular Acceleration
Elbow Angular Acceleration
Acceleration [rad/sec^2]
Time [sec]


Torque Vs. Time
Shoulder Torque
Elbow Torque
Torque [Nm]
Time [sec]





Animating the simulation:

[82]:
```
def animate_double_pend(traj, L1, L2, L1_c, L2_c, T):
    """
    Function to generate web-based animation of double-pendulum system

    Parameters:
        traj:         trajectory of theta1 and theta2
        L1:           length of the upper arm
        L2:           length of the lower arm
        L1_c:         length of the center of mass of the upper arm from the␣
↪shoulder
        L2_c:         length of the center of mass of the lower arm from the␣
↪elbow
        T:            length/seconds of animation duration

    Returns: None
```

```
"""

# Browser configuration
def configure_plotly_browser_state():
    import IPython
    display(IPython.core.display.HTML('''
        <script src="/static/components/requirejs/require.js"></script>
        <script>
          requirejs.config({
            paths: {
              base: '/static/base',
              plotly: 'https://cdn.plot.ly/plotly-1.5.1.min.js?noext',
            },
          });
        </script>
        '''))
configure_plotly_browser_state()
init_notebook_mode(connected=False)

# Getting data from pendulum angle trajectories
xx1 = L1 * np.sin(traj[0])
yy1 = -L1 * np.cos(traj[0])
xx1_c = L1_c * np.sin(traj[0])
yy1_c = -L1_c * np.cos(traj[0])
xx2 = xx1 + L2 * np.sin(traj[0] + traj[1])
yy2 = yy1 - L2 * np.cos(traj[0] + traj[1])
xx2_c = xx1 + L2_c * np.sin(traj[0] + traj[1])
yy2_c = yy1 - L2_c * np.cos(traj[0] + traj[1])
N = len(traj[0])

# Using these to specify axis limits
xm = np.min(xx1)
xM = np.max(xx1)
ym = np.min(yy1) - 0.6
yM = np.max(yy1) + 0.6

# Defining data dictionary
data = [dict(x=xx1, y=yy1,
             mode='lines', name='Arm',
             line=dict(width=5, color='blue')
            ),
        dict(x=xx1_c, y=yy1_c,
             mode='lines', name='Upper Arm Center of Mass',
             line=dict(width=2, color='green')
            ),
        dict(x=xx2_c, y=yy2_c,
             mode='lines', name='Lower Arm Center of Mass',
```

```
                 line=dict(width=2, color='orange')
                ),
            dict(x=xx1, y=yy1,
                 mode='markers', name='Elbow Trajectory',
                 marker=dict(color="green", size=2)
                ),
            dict(x=xx2, y=yy2,
                 mode='markers', name='Hand Trajectory',
                 marker=dict(color="orange", size=2)
                )
          ]

    # Preparing simulation layout
    layout = dict(xaxis=dict(range=[xm, xM], autorange=False,␣
↪zeroline=False,dtick=1),
                  yaxis=dict(range=[ym, yM], autorange=False,␣
↪zeroline=False,scaleanchor = "x",dtick=1),
                  title='Simulation of Arm Modeled as a Double Pendulum',
                  hovermode='closest',
                  updatemenus= [{'type': 'buttons',
                                 'buttons': [{'label': 'Play', 'method':␣
↪'animate',
                                              'args': [None, {'frame':␣
↪{'duration': T, 'redraw': False}}]},
                                             {'args': [[None], {'frame':␣
↪{'duration': T, 'redraw': False}, 'mode': 'immediate',
                                              'transition': {'duration':␣
↪0}}],'label': 'Pause', 'method': 'animate'}
                                            ]
                                }]
                 )

    # Defining the frames of the simulation
    frames = [dict(data=[dict(x=[0,xx1[k],xx2[k]],
                              y=[0,yy1[k],yy2[k]],
                              mode='lines',
                              line=dict(color='red', width=4)),
                         go.Scatter(
                              x=[xx1_c[k]],
                              y=[yy1_c[k]],
                              mode="markers",
                              marker=dict(color="blue", size=12)),
                         go.Scatter(
                              x=[xx2_c[k]],
                              y=[yy2_c[k]],
                              mode="markers",
```

```
                              marker=dict(color="purple", size=12)),
                        ]) for k in range(N)]

    # Putting it all together and plotting
    figure = dict(data=data, layout=layout, frames=frames)
    iplot(figure)

# Animate the system
L1 = L_u
L2 = L_l
L1_c = L_u_c
L2_c = L_l_c
T = 5

animate_double_pend(partial_traj, L1, L2, L1_c, L2_c, T)
```

<IPython.core.display.HTML object>

[ ]:

```

```