arm_pendulum_modeling

October 19, 2021

1 Arm Motion Modeling

1.1 System Description

A double-pendulum system hanging in gravity is shown in the figure above. $q = [\theta_1, \theta_2]$ are the system configuration variables. We assume the z-axis is pointing out from the screen/paper, thus the positive direction of rotation is counter-clockwise. The solution steps are: 1. Computing the Lagrangian of the system. 2. Computing the Euler-Lagrange equations, and solve them for $\ddot{\theta}_1$ and $\ddot{\theta}_2$. 3. Numerically evaluating the solutions for τ_1 and τ_2, and simulating the system for θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$ and $\ddot{\theta}_2$. 4. Animating the simulation.

[1]:
```
from IPython.core.display import HTML
display(HTML("<table><tr><td><img src='./double-pendulum-diagram.png'␣
↪width=450' height='300'></table>"))
```

```
<IPython.core.display.HTML object>
```

1.2 Import Libraries and Define System Constants

Import libraries:

[9]:
```
# Imports required for data processing
import os
import csv
import pandas as pd

# Imports required for dynamics calculations
import sympy
from sympy.abc import t
from sympy import symbols, Eq, Function, solve, sin, cos, Matrix, Subs,␣
↪substitution, Derivative, simplify, symbols, lambdify
import math
from math import pi
import numpy as np
import matplotlib.pyplot as plt

# Imports required for animation
from plotly.offline import init_notebook_mode, iplot
from IPython.display import display, HTML
```

```python
import plotly.graph_objects as go
```

Define the system's constants:

[10]:
```python
# Masses, length and center-of-mass positions (calculated using the lab␣
↪measurements)
# Mass calculations (mass unit is kg)
# m_body = 90.6                         # Average weights for American adult␣
↪male
#                                       # from "Anthropometric Reference Data␣
↪for Children and Adults:
#                                       # United States, 2015-2018"
m_body_dict = {'ID': 51, 'JD': 79.5, 'JR': 76, 'KS': 59.3, 'KW': 63.8, 'LC': 61.
↪2,
               'LD': 97.3, 'LS': 82.2, 'MK': 93.5, 'MV': 98.5, 'SM': 68.5, 'TD':
↪ 70,
               'TM': 66.2}

# m_upper_arm = 0.028 * m_body          # Average upper arm weights relative␣
↪to body weight, from "Biomechanics
#                                       # and Motor Control of Human Movement"␣
↪by David Winter (2009), 4th edition
m_upper_arm_dict = {'ID': 0.028 * m_body_dict['ID'], 'JD': 0.028 *␣
↪m_body_dict['JD'],
                    'JR': 0.028 * m_body_dict['JR'], 'KS': 0.028 *␣
↪m_body_dict['KS'],
                    'KW': 0.028 * m_body_dict['KW'], 'LC': 0.028 *␣
↪m_body_dict['LC'],
                    'LD': 0.028 * m_body_dict['LD'], 'LS': 0.028 *␣
↪m_body_dict['LS'],
                    'MK': 0.028 * m_body_dict['MK'], 'MV': 0.028 *␣
↪m_body_dict['MV'],
                    'SM': 0.028 * m_body_dict['SM'], 'TD': 0.028 *␣
↪m_body_dict['TD'],
                    'TM': 0.028 * m_body_dict['TM']}

m_lower_arm = 0.7395                    # Average lower prosthetics weights,␣
↪calculated using lab measurements



# Arm length calculations (length unit is m)
# H_body = 1.769                        # Average height for American adult␣
↪male, from "Height and body-mass
#                                       # index trajectories of school-aged␣
↪children and adolescents from
```

```
#                                        # 1985 to 2019 in 200 countries and␣
↪territories: a pooled analysis
#                                        # of 2181 population-based studies␣
↪with 65 million participants"
H_body_dict = {'ID': 1.62, 'JD': 1.76, 'JR': 1.77, 'KS': 1.64, 'KW': 1.62, 'LC':
↪ 1.58,
               'LD': 1.875, 'LS': 1.635, 'MK': 1.78, 'MV': 1.805, 'SM': 1.79,␣
↪'TD': 1.69,
               'TM': 1.735}

# L_upper_arm = 0.186 * H_body           # Average upper arm length relative to␣
↪body height
#                                        # from "Biomechanics and Motor Control␣
↪of Human Movement" by David
#                                        # Winter (2009), 4th edition
L_upper_arm_dict = {'ID': 0.186 * H_body_dict['ID'], 'JD': 0.186 *␣
↪H_body_dict['JD'],
                    'JR': 0.186 * H_body_dict['JR'], 'KS': 0.186 *␣
↪H_body_dict['KS'],
                    'KW': 0.186 * H_body_dict['KW'], 'LC': 0.186 *␣
↪H_body_dict['LC'],
                    'LD': 0.186 * H_body_dict['LD'], 'LS': 0.186 *␣
↪H_body_dict['LS'],
                    'MK': 0.186 * H_body_dict['MK'], 'MV': 0.186 *␣
↪H_body_dict['MV'],
                    'SM': 0.186 * H_body_dict['SM'], 'TD': 0.186 *␣
↪H_body_dict['TD'],
                    'TM': 0.186 * H_body_dict['TM']}

L_lower_arm = 0.42                      # Average lower prosthetics length,␣
↪calculated using lab measurements



# Arm center of mass length calculations (length unit is m)
# L_upper_arm_COM = 0.436 * L_upper_arm  # Average upper arm length from␣
↪shoulder to center of mass relative
#                                        # to upper arm length, from␣
↪"Biomechanics and Motor Control of Human
#                                        # Movement" by David Winter (2009),␣
↪4th edition
L_upper_arm_COM_dict = {'ID': 0.436 * L_upper_arm_dict['ID'], 'JD': 0.436 *␣
↪L_upper_arm_dict['JD'],
                        'JR': 0.436 * L_upper_arm_dict['JR'], 'KS': 0.436 *␣
↪L_upper_arm_dict['KS'],
```

```
                           'KW': 0.436 * L_upper_arm_dict['KW'], 'LC': 0.436 *␣
↪L_upper_arm_dict['LC'],
                           'LD': 0.436 * L_upper_arm_dict['LD'], 'LS': 0.436 *␣
↪L_upper_arm_dict['LS'],
                           'MK': 0.436 * L_upper_arm_dict['MK'], 'MV': 0.436 *␣
↪L_upper_arm_dict['MV'],
                           'SM': 0.436 * L_upper_arm_dict['SM'], 'TD': 0.436 *␣
↪L_upper_arm_dict['TD'],
                           'TM': 0.436 * L_upper_arm_dict['TM']}

L_lower_arm_COM = 0.2388               # Average lower prosthetics length from␣
↪elbow to center of mass,
                                       # calculated using lab measurements
```

1.3 Extracting Data

Extracting angles data and computing angular velocities and angular accelerations from the angles:

[27]:
```
def calculate_Vel(Ang_list, time_list, index):
    return ((Ang_list[index + 1] - Ang_list[index])
          / (time_list[index + 1] - time_list[index]))


def calculate_Acc(Vel_list, time_list, index):
    return ((Vel_list[index + 1] - Vel_list[index])
          / (time_list[index + 1] - time_list[index]))


data_csv_dir = '../../data/control_data/CSV Converted Files'
frame_frequency = 120
print("current directory: ", os.getcwd())

participants_list = []
time_list = []
Elbow_Ang_list = []
sholder_Ang_list = []
Elbow_Vel_list = []
sholder_Vel_list = []
Elbow_Acc_list = []
sholder_Acc_list = []

for file in os.listdir(data_csv_dir):
    file_name = file.split(".")[0]
    participant_name = file.split("_")[0]

    if file.endswith(".csv"):
        frame = 0
```

```
        file_time_list = []
        file_R_Elbow_Ang_list = []
        file_R_sholder_Ang_list = []
        file_L_Elbow_Ang_list = []
        file_L_sholder_Ang_list = []
        file_R_Elbow_Vel_list = []
        file_R_sholder_Vel_list = []
        file_L_Elbow_Vel_list = []
        file_L_sholder_Vel_list = []
        file_R_Elbow_Acc_list = []
        file_R_sholder_Acc_list = []
        file_L_Elbow_Acc_list = []
        file_L_sholder_Acc_list = []

        data_path = os.path.join(data_csv_dir, file)

        # Cutting out weird data behavior on data edges
        if file == 'TD_WN7.csv':
            data_rows = open(data_path).read().strip().split("\n")[40:]
        elif file == 'TD_WN4.csv':
            data_rows = open(data_path).read().strip().split("\n")[24:-12]
        elif file == 'TD_WN11.csv':
            data_rows = open(data_path).read().strip().split("\n")[24:-3]
        else:
            data_rows = open(data_path).read().strip().split("\n")[24:]

        # Extract time [sec], elbow angles [rad], and shoulder angles [rad]␣
↪from data
        for row in data_rows:
            splitted_row = row.strip().split("\t")

            # Check if loop finished all data
            if len(splitted_row) < 80:
                break

            file_time_list.append(frame/frame_frequency)
            file_R_sholder_Ang_list.append(float(splitted_row[11]) * 2*pi/360)
            file_R_Elbow_Ang_list.append(float(splitted_row[9]) * 2*pi/360)
            file_L_sholder_Ang_list.append(float(splitted_row[23]) * 2*pi/360)
            file_L_Elbow_Ang_list.append(float(splitted_row[21]) * 2*pi/360)
            frame += 1

        # Extract elbow and shoulder velocities [rad/sec] from angles
        for i in range(len(file_time_list) - 1):
            R_Elbow_Vel = calculate_Vel(file_R_Elbow_Ang_list, file_time_list,␣
↪i)
```

```
            R_sholder_Vel = calculate_Vel(file_R_sholder_Ang_list,␣
↪file_time_list, i)
            L_Elbow_Vel = calculate_Vel(file_L_Elbow_Ang_list, file_time_list,␣
↪i)
            L_sholder_Vel = calculate_Vel(file_L_sholder_Ang_list,␣
↪file_time_list, i)

            file_R_Elbow_Vel_list.append(R_Elbow_Vel)
            file_R_sholder_Vel_list.append(R_sholder_Vel)
            file_L_Elbow_Vel_list.append(L_Elbow_Vel)
            file_L_sholder_Vel_list.append(L_sholder_Vel)

        # Extract elbow and shoulder Accelerations [rad/sec^2] from velocities
        for i in range(len(file_time_list) - 2):
            R_Elbow_Acc = calculate_Acc(file_R_Elbow_Vel_list, file_time_list,␣
↪i)
            R_sholder_Acc = calculate_Acc(file_R_sholder_Vel_list,␣
↪file_time_list, i)
            L_Elbow_Acc = calculate_Acc(file_L_Elbow_Vel_list, file_time_list,␣
↪i)
            L_sholder_Acc = calculate_Acc(file_L_sholder_Vel_list,␣
↪file_time_list, i)

            file_R_Elbow_Acc_list.append(R_Elbow_Acc)
            file_R_sholder_Acc_list.append(R_sholder_Acc)
            file_L_Elbow_Acc_list.append(L_Elbow_Acc)
            file_L_sholder_Acc_list.append(L_sholder_Acc)

        # Adjust lists length
        file_time_list = file_time_list[:-2]
        file_R_Elbow_Ang_list = file_R_Elbow_Ang_list[:-2]
        file_R_sholder_Ang_list = file_R_sholder_Ang_list[:-2]
        file_L_Elbow_Ang_list = file_L_Elbow_Ang_list[:-2]
        file_L_sholder_Ang_list = file_L_sholder_Ang_list[:-2]

        file_R_Elbow_Vel_list = file_R_Elbow_Vel_list[:-1]
        file_R_sholder_Vel_list = file_R_sholder_Vel_list[:-1]
        file_L_Elbow_Vel_list = file_L_Elbow_Vel_list[:-1]
        file_L_sholder_Vel_list = file_L_sholder_Vel_list[:-1]

        participants_list.append(participant_name)
        participants_list.append(participant_name)

        time_list.append(file_time_list)
        time_list.append(file_time_list)
```

```
        Elbow_Ang_list.append(file_R_Elbow_Ang_list)
        sholder_Ang_list.append(file_R_sholder_Ang_list)
        Elbow_Ang_list.append(file_L_Elbow_Ang_list)
        sholder_Ang_list.append(file_L_sholder_Ang_list)
        Elbow_Vel_list.append(file_R_Elbow_Vel_list)
        sholder_Vel_list.append(file_R_sholder_Vel_list)
        Elbow_Vel_list.append(file_L_Elbow_Vel_list)
        sholder_Vel_list.append(file_L_sholder_Vel_list)
        Elbow_Acc_list.append(file_R_Elbow_Acc_list)
        sholder_Acc_list.append(file_R_sholder_Acc_list)
        Elbow_Acc_list.append(file_L_Elbow_Acc_list)
        sholder_Acc_list.append(file_L_sholder_Acc_list)
```

```
current directory:
/home/yael/Documents/MSR_Courses/ME499-Final_Project/Motorized-Prosthetic-
Arm/motor_control/arm_pendulum_modeling
```

1.4 System Modeling

Computing the Lagrangian of the system:

[28]:
```
m1, m2, g, R1, R1_COM, R2, R2_COM = symbols(r'm1, m2, g, R1, R1_COM, R2,␣
 ↪R2_COM')

# The system torque variables as function of t
tau1 = Function(r'tau1')(t)
tau2 = Function(r'tau2')(t)

# The system configuration variables as function of t
theta1 = Function(r'theta1')(t)
theta2 = Function(r'theta2')(t)

# The velocity as derivative of position wrt t
theta1_dot = theta1.diff(t)
theta2_dot = theta2.diff(t)

# The acceleration as derivative of velocity wrt t
theta1_ddot = theta1_dot.diff(t)
theta2_ddot = theta2_dot.diff(t)

# Converting the polar coordinates to cartesian coordinates
x1 = R1_COM * sin(theta1)
x2 = R1 * sin(theta1) + R2_COM * sin(theta1 + theta2)

y1 = -R1_COM * cos(theta1)
y2 = -R1 * cos(theta1) - R2_COM * cos(theta1 + theta2)

# Calculating the kinetic and potential energy of the system
```

```
KE = 1/2 * m1 * ((x1.diff(t))**2 + (y1.diff(t))**2) + 1/2 * m2 * ((x2.
 ↪diff(t))**2 + (y2.diff(t))**2)
PE = m1 * g * y1 + m2 * g * y2

# Computing the Lagrangian
L = simplify(KE - PE)
print('L: ')
display(L)
```

L:

$$0.5R_{1COM}^2 m_1 \left(\frac{d}{dt}\theta_1(t)\right)^2 + R_{1COM} g m_1 \cos(\theta_1(t)) + g m_2 (R_1 \cos(\theta_1(t)) + R_{2COM} \cos(\theta_1(t) + \theta_2(t))) +$$
$$0.5m_2 \left(R_1^2 \left(\frac{d}{dt}\theta_1(t)\right)^2 + 2R_1 R_{2COM} \cos(\theta_2(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 + 2R_1 R_{2COM} \cos(\theta_2(t)) \frac{d}{dt}\theta_1(t) \frac{d}{dt}\theta_2(t) + R_{2COM}^2 \left(\frac{d}{dt}\theta\right.\right.$$

Computing the Euler-Lagrange equations:

[29]:
```
# Define the derivative of L wrt the functions: x, xdot
L_dtheta1 = L.diff(theta1)
L_dtheta2 = L.diff(theta2)

L_dtheta1_dot = L.diff(theta1_dot)
L_dtheta2_dot = L.diff(theta2_dot)

# Define the derivative of L_dxdot wrt to time t
L_dtheta1_dot_dt = L_dtheta1_dot.diff(t)
L_dtheta2_dot_dt = L_dtheta2_dot.diff(t)

# Define the left hand side of the the Euler-Lagrange as a matrix
lhs = Matrix([simplify(L_dtheta1_dot_dt - L_dtheta1),
              simplify(L_dtheta2_dot_dt - L_dtheta2)])

# Define the right hand side of the the Euler-Lagrange as a Matrix
rhs = Matrix([tau1, tau2])

# Compute the Euler-Lagrange equations as a matrix
EL_eqns = Eq(lhs, rhs)

print('Euler-Lagrange matrix for this systems:')
display(EL_eqns)
```

Euler-Lagrange matrix for this systems:

$$\left[\begin{matrix} 1.0R_{1COM}^2 m_1 \frac{d^2}{dt^2}\theta_1(t) + R_{1COM} g m_1 \sin(\theta_1(t)) + g m_2 (R_1 \sin(\theta_1(t)) + R_{2COM} \sin(\theta_1(t) + \theta_2(t))) + m_2 \left(R_1^2 \frac{d^2}{dt^2}\theta_1(t\right. \\ R_{2COM} m_2 \left(R_1 \sin(\theta_2(t)) \left(\frac{d}{dt}\right.\right. \end{matrix}\right.$$

$$\begin{bmatrix} \tau_1(t) \\ \tau_2(t) \end{bmatrix}$$

Solve the equations for τ_1 and τ_2:

[30]:
```
# Solve the Euler-Lagrange equations for the shoulder and elbow torques
T = Matrix([tau1, tau2])
soln = solve(EL_eqns, T, dict=True)

# Initialize the solutions
solution = [0, 0]
i = 0

for sol in soln:
    for v in T:
        solution[i] = simplify(sol[v])
        display(Eq(T[i], solution[i]))
        i =+ 1
```

$$\tau_1(t) = R_1^2 m_2 \frac{d^2}{dt^2}\theta_1(t) - 2.0 R_1 R_{2COM} m_2 \sin(\theta_2(t)) \frac{d}{dt}\theta_1(t) \frac{d}{dt}\theta_2(t) - R_1 R_{2COM} m_2 \sin(\theta_2(t)) \left(\frac{d}{dt}\theta_2(t)\right)^2 + 2.0 R_1 R_{2COM} m_2 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_1(t) + R_1 R_{2COM} m_2 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_2(t) + R_1 g m_2 \sin(\theta_1(t)) + R_{1COM}^2 m_1 \frac{d^2}{dt^2}\theta_1(t) + R_{1COM} g m_1 \sin(\theta_1(t)) + R_{2COM}^2 m_2 \frac{d^2}{dt^2}\theta_1(t) + R_{2COM}^2 m_2 \frac{d^2}{dt^2}\theta_2(t) + R_{2COM} g m_2 \sin(\theta_1(t) + \theta_2(t))$$

$$\tau_2(t) = R_{2COM} m_2 \left(R_1 \sin(\theta_2(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 + R_1 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_1(t) + R_{2COM} \frac{d^2}{dt^2}\theta_1(t) + R_{2COM} \frac{d^2}{dt^2}\theta_2(t) + g \sin$$

Simulating the system:

[31]:
```
# Substitute the derivative variables with a dummy variables and plug-in the␣
↪constants
solution_0_subs = solution[0]
solution_1_subs = solution[1]

theta1_dot_dummy = symbols('thetadot1')
theta2_dot_dummy = symbols('thetadot2')
theta1_ddot_dummy = symbols('thetaddot1')
theta2_ddot_dummy = symbols('thetaddot2')

solution_0_subs = solution_0_subs.subs([(g, 9.81)])
solution_1_subs = solution_1_subs.subs([(g, 9.81)])

solution_0_subs = solution_0_subs.subs([((theta1.diff(t)).diff(t),␣
↪theta1_ddot_dummy),
                                        ((theta2.diff(t)).diff(t),␣
↪theta2_ddot_dummy)])
solution_1_subs = solution_1_subs.subs([((theta1.diff(t)).diff(t),␣
↪theta1_ddot_dummy),
```

```
                                        ((theta2.diff(t)).diff(t),␣
↪theta2_ddot_dummy)])

solution_0_subs = solution_0_subs.subs([(theta1.diff(t), theta1_dot_dummy),
                                        (theta2.diff(t), theta2_dot_dummy)])
solution_1_subs = solution_1_subs.subs([(theta1.diff(t), theta1_dot_dummy),
                                        (theta2.diff(t), theta2_dot_dummy)])

# Lambdify the thetas and its derivatives
func1 = lambdify([theta1, theta2, theta1_dot_dummy, theta2_dot_dummy,␣
↪theta1_ddot_dummy,
                  theta2_ddot_dummy, m1, m2, R1, R2, R1_COM, R2_COM],␣
↪solution_0_subs, modules = sympy)
func2 = lambdify([theta1, theta2, theta1_dot_dummy, theta2_dot_dummy,␣
↪theta1_ddot_dummy,
                  theta2_ddot_dummy, m1, m2, R1, R2, R1_COM, R2_COM],␣
↪solution_1_subs, modules = sympy)

# Initialize the torque and power lists
sholder_tau_list, Elbow_tau_list = [], []
sholder_current_list, Elbow_current_list = [], []
sholder_power_list, Elbow_power_list = [], []

motor_kv = 115
torque_const = 8.27 / motor_kv

for i in range(len(time_list)):
    # Initialize the torque and power lists
    tau1_list, tau2_list = [], []
    current1_list, current2_list = [], []
    power1_list, power2_list = [], []

    t_list = time_list[i]
    theta1_list = sholder_Ang_list[i]
    theta2_list = Elbow_Ang_list[i]
    dtheta1_list = sholder_Vel_list[i]
    dtheta2_list = Elbow_Vel_list[i]
    ddtheta1_list = sholder_Acc_list[i]
    ddtheta2_list = Elbow_Acc_list[i]

    # Plug-in the angles, angular velocities and angular accelerations for␣
↪every time step to find the torques
    for j in range(len(t_list)):
        tau1_list.append(func1(theta1_list[j], theta2_list[j], dtheta1_list[j],␣
↪dtheta2_list[j],
```

```
                                  ddtheta1_list[j], ddtheta2_list[j],␣
↪m_upper_arm_dict[participants_list[i]],
                                  m_lower_arm,␣
↪L_upper_arm_dict[participants_list[i]], L_lower_arm,
                                  L_upper_arm_COM_dict[participants_list[i]],␣
↪L_lower_arm_COM))

        tau2_list.append(func2(theta1_list[j], theta2_list[j], dtheta1_list[j],␣
↪dtheta2_list[j],
                                  ddtheta1_list[j], ddtheta2_list[j],␣
↪m_upper_arm_dict[participants_list[i]],
                                  m_lower_arm,␣
↪L_upper_arm_dict[participants_list[i]], L_lower_arm,
                                  L_upper_arm_COM_dict[participants_list[i]],␣
↪L_lower_arm_COM))

        # Calculate the current required to reach the required joints torques␣
↪for every time step
        current1_list.append(torque_const * tau1_list[j])
        current2_list.append(torque_const * tau2_list[j])

        # Calculate the power required to reach the required angular velocities␣
↪and joints torques for every time step
        power1_list.append(dtheta1_list[j] * tau1_list[j])
        power2_list.append(dtheta2_list[j] * tau2_list[j])

    sholder_tau_list.append(tau1_list)
    Elbow_tau_list.append(tau2_list)

    sholder_current_list.append(current1_list)
    Elbow_current_list.append(current2_list)

    sholder_power_list.append(power1_list)
    Elbow_power_list.append(power2_list)

    print(f"Trial {i}/{len(time_list) - 1} finished \t maximum torque is␣
↪{format(max(tau2_list), '.3f')} [Nm]\t maximum power is␣
↪{format(max(power2_list), '.3f')} [W]")
```

```
Trial 0/203 finished     maximum torque is 1.724 [Nm]    maximum power is 0.549
[W]
Trial 1/203 finished     maximum torque is 2.262 [Nm]    maximum power is 1.483
[W]
Trial 2/203 finished     maximum torque is 2.195 [Nm]    maximum power is 3.079
[W]
Trial 3/203 finished     maximum torque is 2.162 [Nm]    maximum power is 2.907
[W]
```

```
Trial 4/203 finished     maximum torque is 2.016 [Nm]    maximum power is 3.069
[W]
Trial 5/203 finished     maximum torque is 2.346 [Nm]    maximum power is 3.666
[W]
Trial 6/203 finished     maximum torque is 2.157 [Nm]    maximum power is 1.753
[W]
Trial 7/203 finished     maximum torque is 3.857 [Nm]    maximum power is 1.155
[W]
Trial 8/203 finished     maximum torque is 2.269 [Nm]    maximum power is 3.146
[W]
Trial 9/203 finished     maximum torque is 2.113 [Nm]    maximum power is 2.225
[W]
Trial 10/203 finished    maximum torque is 2.101 [Nm]    maximum power is 2.142
[W]
Trial 11/203 finished    maximum torque is 2.669 [Nm]    maximum power is 4.011
[W]
Trial 12/203 finished    maximum torque is 1.848 [Nm]    maximum power is 2.279
[W]
Trial 13/203 finished    maximum torque is 1.527 [Nm]    maximum power is 1.073
[W]
Trial 14/203 finished    maximum torque is 2.246 [Nm]    maximum power is 2.050
[W]
Trial 15/203 finished    maximum torque is 1.959 [Nm]    maximum power is 2.019
[W]
Trial 16/203 finished    maximum torque is 2.151 [Nm]    maximum power is 1.810
[W]
Trial 17/203 finished    maximum torque is 2.532 [Nm]    maximum power is 3.662
[W]
Trial 18/203 finished    maximum torque is 1.784 [Nm]    maximum power is 1.620
[W]
Trial 19/203 finished    maximum torque is 2.044 [Nm]    maximum power is 3.226
[W]
Trial 20/203 finished    maximum torque is 2.215 [Nm]    maximum power is 2.900
[W]
Trial 21/203 finished    maximum torque is 2.092 [Nm]    maximum power is 2.660
[W]
Trial 22/203 finished    maximum torque is 2.098 [Nm]    maximum power is 1.675
[W]
Trial 23/203 finished    maximum torque is 1.903 [Nm]    maximum power is 1.349
[W]
Trial 24/203 finished    maximum torque is 1.877 [Nm]    maximum power is 0.604
[W]
Trial 25/203 finished    maximum torque is 2.079 [Nm]    maximum power is 1.186
[W]
Trial 26/203 finished    maximum torque is 1.862 [Nm]    maximum power is 2.632
[W]
Trial 27/203 finished    maximum torque is 1.639 [Nm]    maximum power is 1.742
[W]
```

```
Trial 28/203 finished    maximum torque is 2.330 [Nm]    maximum power is 3.189
[W]
Trial 29/203 finished    maximum torque is 2.033 [Nm]    maximum power is 1.699
[W]
Trial 30/203 finished    maximum torque is 1.781 [Nm]    maximum power is 1.580
[W]
Trial 31/203 finished    maximum torque is 1.444 [Nm]    maximum power is 1.096
[W]
Trial 32/203 finished    maximum torque is 1.926 [Nm]    maximum power is 3.662
[W]
Trial 33/203 finished    maximum torque is 2.176 [Nm]    maximum power is 4.196
[W]
Trial 34/203 finished    maximum torque is 1.834 [Nm]    maximum power is 2.813
[W]
Trial 35/203 finished    maximum torque is 2.126 [Nm]    maximum power is 3.386
[W]
Trial 36/203 finished    maximum torque is 1.714 [Nm]    maximum power is 1.628
[W]
Trial 37/203 finished    maximum torque is 1.604 [Nm]    maximum power is 1.134
[W]
Trial 38/203 finished    maximum torque is 1.760 [Nm]    maximum power is 1.442
[W]
Trial 39/203 finished    maximum torque is 1.981 [Nm]    maximum power is 2.659
[W]
Trial 40/203 finished    maximum torque is 1.888 [Nm]    maximum power is 0.964
[W]
Trial 41/203 finished    maximum torque is 2.331 [Nm]    maximum power is 1.417
[W]
Trial 42/203 finished    maximum torque is 1.755 [Nm]    maximum power is 1.278
[W]
Trial 43/203 finished    maximum torque is 1.558 [Nm]    maximum power is 0.905
[W]
Trial 44/203 finished    maximum torque is 2.182 [Nm]    maximum power is 2.536
[W]
Trial 45/203 finished    maximum torque is 2.260 [Nm]    maximum power is 4.148
[W]
Trial 46/203 finished    maximum torque is 1.812 [Nm]    maximum power is 0.658
[W]
Trial 47/203 finished    maximum torque is 2.467 [Nm]    maximum power is 1.763
[W]
Trial 48/203 finished    maximum torque is 1.725 [Nm]    maximum power is 2.373
[W]
Trial 49/203 finished    maximum torque is 1.601 [Nm]    maximum power is 1.229
[W]
Trial 50/203 finished    maximum torque is 2.490 [Nm]    maximum power is 6.810
[W]
Trial 51/203 finished    maximum torque is 2.403 [Nm]    maximum power is 2.430
[W]
```

```
Trial 52/203 finished    maximum torque is 2.869 [Nm]    maximum power is 1.375
[W]
Trial 53/203 finished    maximum torque is 1.446 [Nm]    maximum power is 1.393
[W]
Trial 54/203 finished    maximum torque is 1.987 [Nm]    maximum power is 1.902
[W]
Trial 55/203 finished    maximum torque is 1.536 [Nm]    maximum power is 1.430
[W]
Trial 56/203 finished    maximum torque is 2.087 [Nm]    maximum power is 2.283
[W]
Trial 57/203 finished    maximum torque is 2.191 [Nm]    maximum power is 2.616
[W]
Trial 58/203 finished    maximum torque is 2.107 [Nm]    maximum power is 1.425
[W]
Trial 59/203 finished    maximum torque is 2.375 [Nm]    maximum power is 1.704
[W]
Trial 60/203 finished    maximum torque is 1.938 [Nm]    maximum power is 0.834
[W]
Trial 61/203 finished    maximum torque is 1.921 [Nm]    maximum power is 1.301
[W]
Trial 62/203 finished    maximum torque is 2.376 [Nm]    maximum power is 5.876
[W]
Trial 63/203 finished    maximum torque is 2.244 [Nm]    maximum power is 3.340
[W]
Trial 64/203 finished    maximum torque is 2.061 [Nm]    maximum power is 2.558
[W]
Trial 65/203 finished    maximum torque is 2.100 [Nm]    maximum power is 2.625
[W]
Trial 66/203 finished    maximum torque is 1.663 [Nm]    maximum power is 1.312
[W]
Trial 67/203 finished    maximum torque is 2.116 [Nm]    maximum power is 3.063
[W]
Trial 68/203 finished    maximum torque is 1.895 [Nm]    maximum power is 2.606
[W]
Trial 69/203 finished    maximum torque is 3.397 [Nm]    maximum power is 5.689
[W]
Trial 70/203 finished    maximum torque is 1.729 [Nm]    maximum power is 1.053
[W]
Trial 71/203 finished    maximum torque is 1.908 [Nm]    maximum power is 2.222
[W]
Trial 72/203 finished    maximum torque is 2.131 [Nm]    maximum power is 2.015
[W]
Trial 73/203 finished    maximum torque is 2.156 [Nm]    maximum power is 1.733
[W]
Trial 74/203 finished    maximum torque is 1.723 [Nm]    maximum power is 1.493
[W]
Trial 75/203 finished    maximum torque is 1.992 [Nm]    maximum power is 2.875
[W]
```

```
Trial 76/203 finished    maximum torque is 1.952 [Nm]    maximum power is 0.733
[W]
Trial 77/203 finished    maximum torque is 1.864 [Nm]    maximum power is 0.840
[W]
Trial 78/203 finished    maximum torque is 1.709 [Nm]    maximum power is 1.120
[W]
Trial 79/203 finished    maximum torque is 1.709 [Nm]    maximum power is 2.075
[W]
Trial 80/203 finished    maximum torque is 2.027 [Nm]    maximum power is 2.190
[W]
Trial 81/203 finished    maximum torque is 2.463 [Nm]    maximum power is 3.874
[W]
Trial 82/203 finished    maximum torque is 1.872 [Nm]    maximum power is 2.846
[W]
Trial 83/203 finished    maximum torque is 1.907 [Nm]    maximum power is 2.994
[W]
Trial 84/203 finished    maximum torque is 1.912 [Nm]    maximum power is 0.776
[W]
Trial 85/203 finished    maximum torque is 2.063 [Nm]    maximum power is 0.752
[W]
Trial 86/203 finished    maximum torque is 1.982 [Nm]    maximum power is 2.788
[W]
Trial 87/203 finished    maximum torque is 1.975 [Nm]    maximum power is 2.702
[W]
Trial 88/203 finished    maximum torque is 2.201 [Nm]    maximum power is 2.370
[W]
Trial 89/203 finished    maximum torque is 1.958 [Nm]    maximum power is 1.672
[W]
Trial 90/203 finished    maximum torque is 2.029 [Nm]    maximum power is 1.471
[W]
Trial 91/203 finished    maximum torque is 2.095 [Nm]    maximum power is 1.522
[W]
Trial 92/203 finished    maximum torque is 2.051 [Nm]    maximum power is 2.388
[W]
Trial 93/203 finished    maximum torque is 1.967 [Nm]    maximum power is 1.547
[W]
Trial 94/203 finished    maximum torque is 1.895 [Nm]    maximum power is 1.728
[W]
Trial 95/203 finished    maximum torque is 2.213 [Nm]    maximum power is 3.315
[W]
Trial 96/203 finished    maximum torque is 1.765 [Nm]    maximum power is 1.787
[W]
Trial 97/203 finished    maximum torque is 1.572 [Nm]    maximum power is 1.189
[W]
Trial 98/203 finished    maximum torque is 2.047 [Nm]    maximum power is 2.742
[W]
Trial 99/203 finished    maximum torque is 1.863 [Nm]    maximum power is 2.993
[W]
```

```
Trial 100/203 finished   maximum torque is 2.041 [Nm]    maximum power is 2.836
[W]
Trial 101/203 finished   maximum torque is 2.331 [Nm]    maximum power is 3.646
[W]
Trial 102/203 finished   maximum torque is 1.822 [Nm]    maximum power is 1.137
[W]
Trial 103/203 finished   maximum torque is 2.022 [Nm]    maximum power is 2.148
[W]
Trial 104/203 finished   maximum torque is 1.972 [Nm]    maximum power is 1.944
[W]
Trial 105/203 finished   maximum torque is 2.942 [Nm]    maximum power is 3.463
[W]
Trial 106/203 finished   maximum torque is 2.223 [Nm]    maximum power is 1.528
[W]
Trial 107/203 finished   maximum torque is 2.191 [Nm]    maximum power is 2.091
[W]
Trial 108/203 finished   maximum torque is 2.169 [Nm]    maximum power is 2.189
[W]
Trial 109/203 finished   maximum torque is 1.932 [Nm]    maximum power is 1.626
[W]
Trial 110/203 finished   maximum torque is 1.763 [Nm]    maximum power is 1.116
[W]
Trial 111/203 finished   maximum torque is 1.837 [Nm]    maximum power is 0.803
[W]
Trial 112/203 finished   maximum torque is 1.744 [Nm]    maximum power is 1.151
[W]
Trial 113/203 finished   maximum torque is 1.861 [Nm]    maximum power is 2.469
[W]
Trial 114/203 finished   maximum torque is 2.658 [Nm]    maximum power is 3.810
[W]
Trial 115/203 finished   maximum torque is 1.858 [Nm]    maximum power is 1.957
[W]
Trial 116/203 finished   maximum torque is 2.187 [Nm]    maximum power is 2.410
[W]
Trial 117/203 finished   maximum torque is 1.917 [Nm]    maximum power is 1.363
[W]
Trial 118/203 finished   maximum torque is 1.834 [Nm]    maximum power is 1.439
[W]
Trial 119/203 finished   maximum torque is 1.637 [Nm]    maximum power is 0.672
[W]
Trial 120/203 finished   maximum torque is 2.274 [Nm]    maximum power is 3.163
[W]
Trial 121/203 finished   maximum torque is 2.209 [Nm]    maximum power is 1.725
[W]
Trial 122/203 finished   maximum torque is 2.421 [Nm]    maximum power is 3.402
[W]
Trial 123/203 finished   maximum torque is 1.784 [Nm]    maximum power is 3.929
[W]
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Trial 124/203 finished   maximum torque is 2.082 [Nm]    maximum power is 1.888
[W]
Trial 125/203 finished   maximum torque is 3.691 [Nm]    maximum power is 10.920
[W]
Trial 126/203 finished   maximum torque is 1.976 [Nm]    maximum power is 2.872
[W]
Trial 127/203 finished   maximum torque is 2.707 [Nm]    maximum power is 4.604
[W]
Trial 128/203 finished   maximum torque is 2.103 [Nm]    maximum power is 1.594
[W]
Trial 129/203 finished   maximum torque is 2.238 [Nm]    maximum power is 1.743
[W]
Trial 130/203 finished   maximum torque is 2.155 [Nm]    maximum power is 0.910
[W]
Trial 131/203 finished   maximum torque is 1.895 [Nm]    maximum power is 1.619
[W]
Trial 132/203 finished   maximum torque is 1.766 [Nm]    maximum power is 1.680
[W]
Trial 133/203 finished   maximum torque is 1.435 [Nm]    maximum power is 0.837
[W]
Trial 134/203 finished   maximum torque is 1.886 [Nm]    maximum power is 1.764
[W]
Trial 135/203 finished   maximum torque is 1.689 [Nm]    maximum power is 1.243
[W]
Trial 136/203 finished   maximum torque is 2.039 [Nm]    maximum power is 2.892
[W]
Trial 137/203 finished   maximum torque is 1.969 [Nm]    maximum power is 2.807
[W]
Trial 138/203 finished   maximum torque is 2.024 [Nm]    maximum power is 2.436
[W]
Trial 139/203 finished   maximum torque is 2.062 [Nm]    maximum power is 3.563
[W]
Trial 140/203 finished   maximum torque is 1.750 [Nm]    maximum power is 1.527
[W]
Trial 141/203 finished   maximum torque is 1.509 [Nm]    maximum power is 0.916
[W]
Trial 142/203 finished   maximum torque is 2.147 [Nm]    maximum power is 3.096
[W]
Trial 143/203 finished   maximum torque is 2.105 [Nm]    maximum power is 3.047
[W]
Trial 144/203 finished   maximum torque is 1.832 [Nm]    maximum power is 0.497
[W]
Trial 145/203 finished   maximum torque is 1.894 [Nm]    maximum power is 0.975
[W]
Trial 146/203 finished   maximum torque is 1.854 [Nm]    maximum power is 0.655
[W]
Trial 147/203 finished   maximum torque is 2.585 [Nm]    maximum power is 2.828
[W]
```

```
Trial 148/203 finished   maximum torque is 1.915 [Nm]    maximum power is 1.598
[W]
Trial 149/203 finished   maximum torque is 2.064 [Nm]    maximum power is 1.749
[W]
Trial 150/203 finished   maximum torque is 1.519 [Nm]    maximum power is 0.714
[W]
Trial 151/203 finished   maximum torque is 1.837 [Nm]    maximum power is 1.194
[W]
Trial 152/203 finished   maximum torque is 1.748 [Nm]    maximum power is 2.013
[W]
Trial 153/203 finished   maximum torque is 1.572 [Nm]    maximum power is 1.280
[W]
Trial 154/203 finished   maximum torque is 1.954 [Nm]    maximum power is 1.517
[W]
Trial 155/203 finished   maximum torque is 1.508 [Nm]    maximum power is 1.341
[W]
Trial 156/203 finished   maximum torque is 2.255 [Nm]    maximum power is 2.891
[W]
Trial 157/203 finished   maximum torque is 2.211 [Nm]    maximum power is 2.954
[W]
Trial 158/203 finished   maximum torque is 1.795 [Nm]    maximum power is 0.623
[W]
Trial 159/203 finished   maximum torque is 2.029 [Nm]    maximum power is 0.894
[W]
Trial 160/203 finished   maximum torque is 1.904 [Nm]    maximum power is 1.085
[W]
Trial 161/203 finished   maximum torque is 1.929 [Nm]    maximum power is 1.754
[W]
Trial 162/203 finished   maximum torque is 2.149 [Nm]    maximum power is 2.040
[W]
Trial 163/203 finished   maximum torque is 2.391 [Nm]    maximum power is 3.745
[W]
Trial 164/203 finished   maximum torque is 1.838 [Nm]    maximum power is 2.772
[W]
Trial 165/203 finished   maximum torque is 1.939 [Nm]    maximum power is 2.873
[W]
Trial 166/203 finished   maximum torque is 1.782 [Nm]    maximum power is 1.068
[W]
Trial 167/203 finished   maximum torque is 1.762 [Nm]    maximum power is 1.961
[W]
Trial 168/203 finished   maximum torque is 1.934 [Nm]    maximum power is 1.954
[W]
Trial 169/203 finished   maximum torque is 1.821 [Nm]    maximum power is 2.794
[W]
Trial 170/203 finished   maximum torque is 2.213 [Nm]    maximum power is 3.121
[W]
Trial 171/203 finished   maximum torque is 2.065 [Nm]    maximum power is 2.912
[W]
```

Trial 172/203 finished   maximum torque is 1.623 [Nm]   maximum power is 1.938 [W]
Trial 173/203 finished   maximum torque is 1.539 [Nm]   maximum power is 1.021 [W]
Trial 174/203 finished   maximum torque is 1.684 [Nm]   maximum power is 0.736 [W]
Trial 175/203 finished   maximum torque is 1.990 [Nm]   maximum power is 1.582 [W]
Trial 176/203 finished   maximum torque is 2.001 [Nm]   maximum power is 3.070 [W]
Trial 177/203 finished   maximum torque is 1.943 [Nm]   maximum power is 1.621 [W]
Trial 178/203 finished   maximum torque is 1.602 [Nm]   maximum power is 0.497 [W]
Trial 179/203 finished   maximum torque is 1.916 [Nm]   maximum power is 0.848 [W]
Trial 180/203 finished   maximum torque is 2.081 [Nm]   maximum power is 1.403 [W]
Trial 181/203 finished   maximum torque is 2.197 [Nm]   maximum power is 2.198 [W]
Trial 182/203 finished   maximum torque is 2.069 [Nm]   maximum power is 2.887 [W]
Trial 183/203 finished   maximum torque is 1.959 [Nm]   maximum power is 3.850 [W]
Trial 184/203 finished   maximum torque is 2.478 [Nm]   maximum power is 2.407 [W]
Trial 185/203 finished   maximum torque is 2.325 [Nm]   maximum power is 2.783 [W]
Trial 186/203 finished   maximum torque is 1.818 [Nm]   maximum power is 1.410 [W]
Trial 187/203 finished   maximum torque is 1.918 [Nm]   maximum power is 2.348 [W]
Trial 188/203 finished   maximum torque is 1.702 [Nm]   maximum power is 1.669 [W]
Trial 189/203 finished   maximum torque is 1.904 [Nm]   maximum power is 2.223 [W]
Trial 190/203 finished   maximum torque is 2.137 [Nm]   maximum power is 1.441 [W]
Trial 191/203 finished   maximum torque is 1.854 [Nm]   maximum power is 2.063 [W]
Trial 192/203 finished   maximum torque is 2.015 [Nm]   maximum power is 3.539 [W]
Trial 193/203 finished   maximum torque is 2.479 [Nm]   maximum power is 1.987 [W]
Trial 194/203 finished   maximum torque is 2.281 [Nm]   maximum power is 1.948 [W]
Trial 195/203 finished   maximum torque is 2.028 [Nm]   maximum power is 1.481 [W]

```
Trial 196/203 finished   maximum torque is 1.795 [Nm]    maximum power is 2.051
[W]
Trial 197/203 finished   maximum torque is 1.500 [Nm]    maximum power is 0.869
[W]
Trial 198/203 finished   maximum torque is 1.994 [Nm]    maximum power is 2.683
[W]
Trial 199/203 finished   maximum torque is 1.902 [Nm]    maximum power is 3.710
[W]
Trial 200/203 finished   maximum torque is 1.741 [Nm]    maximum power is 0.904
[W]
Trial 201/203 finished   maximum torque is 1.956 [Nm]    maximum power is 2.327
[W]
Trial 202/203 finished   maximum torque is 2.137 [Nm]    maximum power is 1.852
[W]
Trial 203/203 finished   maximum torque is 1.377 [Nm]    maximum power is 0.993
[W]
```

Calculation summary:

[34]:
```
# print(f"Shoulder max angular velocity:\t{format(max(dtheta1_list), '.3f')}␣
↪[rad/sec]\t\t Shoulder average angular velocity:\t{format(sum(dtheta1_list) /
↪ float(len(dtheta1_list)), '.3f')} [rad/sec]")
# print(f"Shoulder max angular velocity:\t{format(max(dtheta1_list)*60/(2*pi),␣
↪'.3f')} [rpm]\t\t Shoulder average angular velocity:
↪\t{format((sum(dtheta1_list) / float(len(dtheta1_list)))*60/(2*pi), '.3f')}␣
↪[rpm]")
# print(f"Elbow max angular velocity:\t{format(max(dtheta2_list), '.3f')} [rad/
↪sec]\t\t Elbow average angular velocity:\t{format(sum(dtheta2_list) /␣
↪float(len(dtheta2_list)), '.3f')} [rad/sec]")
# print(f"Elbow max angular velocity:\t{format(max(dtheta2_list)*60/(2*pi), '.
↪3f')} [rpm]\t\t Elbow average angular velocity:\t{format((sum(dtheta2_list) /
↪ float(len(dtheta2_list)))*60/(2*pi), '.3f')} [rpm]\n")
# print(f"Shoulder max torque:\t\t{format(max(tau1_list), '.3f')} [Nm]\t\t␣
↪Shoulder average torque:\t\t{format(sum(tau1_list) / float(len(tau1_list)),␣
↪'.3f')} [Nm]")
# print(f"Elbow max torque:\t\t{format(max(tau2_list), '.3f')} [Nm]\t\t Elbow␣
↪average torque:\t\t\t{format(sum(tau2_list) / float(len(tau2_list)), '.3f')}␣
↪[Nm]\n")
# print(f"Shoulder max power:\t\t{format(max(power1_list), '.3f')} [W]\t\t␣
↪Shoulder average power:\t\t{format(sum(power1_list) /␣
↪float(len(power1_list)), '.3f')} [W]")
# print(f"Elbow max power:\t\t{format(max(power2_list), '.3f')} [W]\t\t Elbow␣
↪average power:\t\t\t{format(sum(power2_list) / float(len(power2_list)), '.
↪3f')} [W]\n")


max_Elbow_tau, max_Elbow_power, max_Elbow_Vel = 0, 0, 0
max_Elbow_tau_index, max_Elbow_power_index, max_Elbow_Vel_index = 0, 0, 0
```

```python
for i in range(len(Elbow_tau_list)):
    if max_Elbow_Vel < max(Elbow_Vel_list[i]):
        max_Elbow_Vel = max(Elbow_Vel_list[i])
        max_Elbow_Vel_index = i

    if max_Elbow_tau < max(Elbow_tau_list[i]):
        max_Elbow_tau = max(Elbow_tau_list[i])
        max_Elbow_tau_index = i

    if max_Elbow_power < max(Elbow_power_list[i]):
        max_Elbow_power = max(Elbow_power_list[i])
        max_Elbow_power_index = i

print(f"maximum elbow angular velocity is {format(max_Elbow_Vel, '.3f')} [rad/sec] ({format(max_Elbow_Vel*60/(2*pi), '.3f')} [rpm]), in trial {max_Elbow_Vel_index}")
print(f"maximum elbow torque is {format(max_Elbow_tau, '.3f')} [Nm], in trial {max_Elbow_tau_index}")
print(f"maximum elbow power is {format(max_Elbow_power, '.3f')} [W], in trial {max_Elbow_power_index}")


# The torque equations for the maximum power:
solution_0_subs = solution_0_subs.subs([(m1, m_upper_arm_dict[participants_list[max_Elbow_tau_index]]), (m2, m_lower_arm), (R1, L_upper_arm_dict[participants_list[max_Elbow_tau_index]]), (R2, L_lower_arm), (R1_COM, L_upper_arm_COM_dict[participants_list[max_Elbow_tau_index]]), (R2_COM, L_lower_arm_COM), (g, 9.81)])
solution_1_subs = solution_1_subs.subs([(m1, m_upper_arm_dict[participants_list[max_Elbow_tau_index]]), (m2, m_lower_arm), (R1, L_upper_arm_dict[participants_list[max_Elbow_tau_index]]), (R2, L_lower_arm), (R1_COM, L_upper_arm_COM_dict[participants_list[max_Elbow_tau_index]]), (R2_COM, L_lower_arm_COM), (g, 9.81)])

print("\nThe torque equations for the maximum torque:")
display(Eq(T[0], solution_0_subs))
display(Eq(T[1], solution_1_subs))

display(Elbow_Ang_list[max_Elbow_tau_index])
display(Elbow_Vel_list[max_Elbow_tau_index])
display(Elbow_Acc_list[max_Elbow_tau_index])
display(Elbow_tau_list[max_Elbow_tau_index])
```

```
maximum elbow angular velocity is 4.199 [rad/sec] (40.100 [rpm]), in trial 125
maximum elbow torque is 3.857 [Nm], in trial 7
maximum elbow power is 10.920 [W], in trial 125

The torque equations for the maximum torque:
```

$$\tau_1(t) = 0.1110202357768\ddot{\theta}_1 \cos(\theta_2(t)) + 0.1520553387246 74\ddot{\theta}_1 + 0.0555101178884\ddot{\theta}_2 \cos(\theta_2(t)) + 0.042170312288\ddot{\theta}_2 - 0.1110202357768\dot{\theta}_1\dot{\theta}_2 \sin(\theta_2(t)) - 0.0555101178884\dot{\theta}_2^2 \sin(\theta_2(t)) + 1.732373406 \sin(\theta_1(t) + \theta_2(t)) + 4.915563608124 \sin(\theta_1(t))$$

$$\tau_2(t) = 0.0555101178884\ddot{\theta}_1 \cos(\theta_2(t)) + 0.042170312288\ddot{\theta}_1 + 0.042170312288\ddot{\theta}_2 + 0.0555101178884\dot{\theta}_1^2 \sin(\theta_2(t)) + 1.732373406 \sin(\theta_1(t) + \theta_2(t))$$

```
[0.6678676915681501,
 0.6644293929417213,
 0.6608689212676528,
 0.6571862765459449,
 0.6533290988990375,
 0.6493497482044903,
 0.6452482244623037,
 0.6410768875500372,
 0.6369404572228107,
 0.6330832795759032,
 0.6297497007045939,
 0.6271666134116423,
 0.6254736440372078,
 0.6246882458738104,
 0.6247580590438901,
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 0.6220004388257392,
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-0.172236078311690,
-0.218053861609067,
-0.254125221966570,
-0.332573835278804,
-0.359359753764963,
-0.396445298401289,
-0.413686105876258,
-0.379876475363388,
-0.358301428671233,
-0.257237524308390,
-0.0993482943393087,
0.0542037399476356,
0.201007937894225,
0.333093002092513,
0.469949078196805,
0.604593772843395,
0.707286268824358,
0.799633575126767,
0.884289996080361,
0.958125814916900,
1.01033858336348,
1.05239762119660,
1.08076066405343,
1.09797352590896,
1.08165930053362,
1.10655900360224,
1.09885657303963,
1.12345575025865,
1.16977961526083,
1.11929043187398,
1.13462651767264,
1.12692233682151,
1.13015674807508,

```
1.15119814499059,
1.17587974750144,
1.19721721445396,
1.24599023752414,
1.20973378331380,
1.21956556162985,
1.13924853463546,
1.00267253933263,
0.857684073170001,
0.716626637620560,
0.609486062366760,
0.492424185092715,
0.505181527671631,
3.85669298166448,
-0.218339563032717]
```

Example for the trial with the largest elbow torque & power:

[35]:
```
index = 125
# index = 195
t_list = time_list[index]
theta1_list = sholder_Ang_list[index]
theta2_list = Elbow_Ang_list[index]
dtheta1_list = sholder_Vel_list[index]
dtheta2_list = Elbow_Vel_list[index]
ddtheta1_list = sholder_Acc_list[index]
ddtheta2_list = Elbow_Acc_list[index]
tau1_list = sholder_tau_list[index]
tau2_list = Elbow_tau_list[index]
current1_list = sholder_current_list[index]
current2_list = Elbow_current_list[index]
power1_list = sholder_power_list[index]
power2_list = Elbow_power_list[index]

# Compute the trajectory of the arm's motion
N = int((max(t_list) - min(t_list))/(1/frame_frequency))
tvec = np.linspace(min(t_list), max(t_list), N)
traj = np.zeros((6, N))
for i in range(N):
    traj[0, i] = theta1_list[i]
    traj[1, i] = theta2_list[i]
    traj[2, i] = dtheta1_list[i]
    traj[3, i] = dtheta2_list[i]
    traj[4, i] = ddtheta1_list[i]
    traj[5, i] = ddtheta2_list[i]

# Calculate the length difference between the time list and the trajectory lists
diff = (len(t_list) - len(traj[0]))
```

```
# Plot the trajectory lists (angles, velocities, accelerations, torques, and␣
↪power)
plt.figure(figsize=(15,5))
plt.suptitle('Angles Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[0])
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Shoulder Angle')

plt.subplot(122)
plt.plot(t_list[:-diff], traj[1])
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Elbow Angle')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Angular Velocity Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[2])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Shoulder Angular Velocity')

plt.subplot(122)
plt.plot(t_list[:-diff], traj[3])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Elbow Angular Velocity')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Angular Acceleration Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[4])
plt.ylabel('Acceleration [rad/sec^2]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Shoulder Angular Acceleration')

plt.subplot(122)
```

```
plt.plot(t_list[:-diff], traj[5])
plt.ylabel('Acceleration [rad/sec^2]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Elbow Angular Acceleration')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Torque Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list, tau1_list)
plt.ylabel('Torque [Nm]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Shoulder Torque')

plt.subplot(122)
plt.plot(t_list, tau2_list)
plt.ylabel('Torque [Nm]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Elbow Torque')
plt.show()

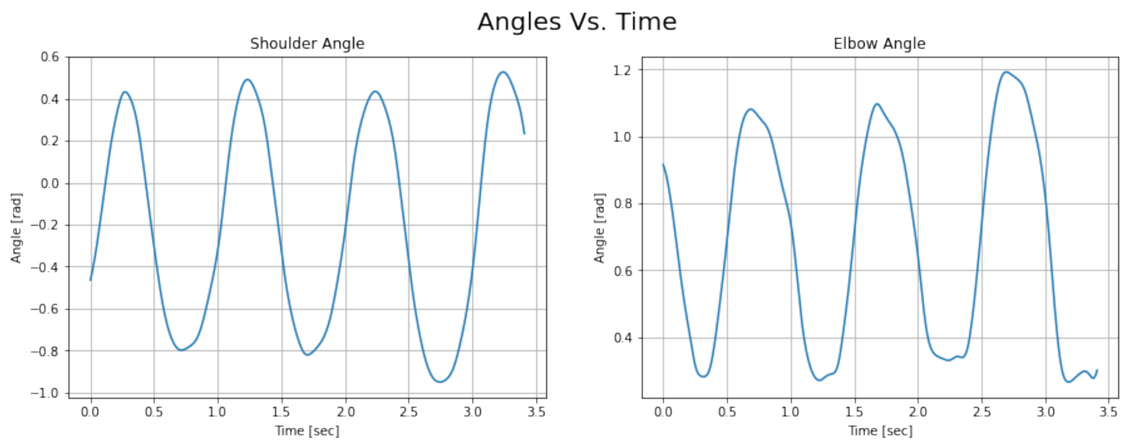
plt.figure(figsize=(15,5))
plt.suptitle('Power Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list, power1_list)
plt.ylabel('Power [W]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Shoulder Power')

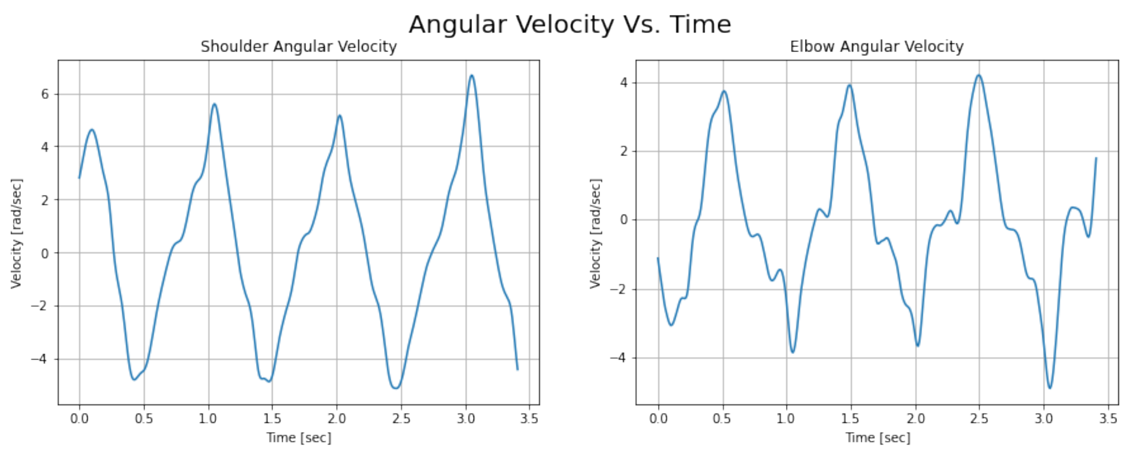
plt.subplot(122)
plt.plot(t_list, power2_list)
plt.ylabel('Power [W]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Elbow Power')
plt.show()

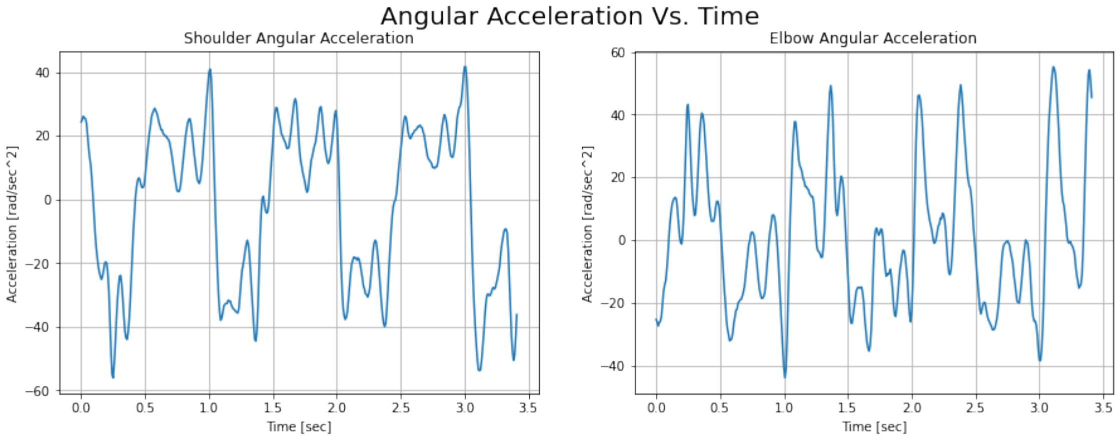
plt.figure(figsize=(15,5))
plt.suptitle('Speed Vs. Torque', fontsize=20)
plt.subplot(121)
plt.plot(tau1_list[:-diff], traj[2])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Torque [Nm]')
```

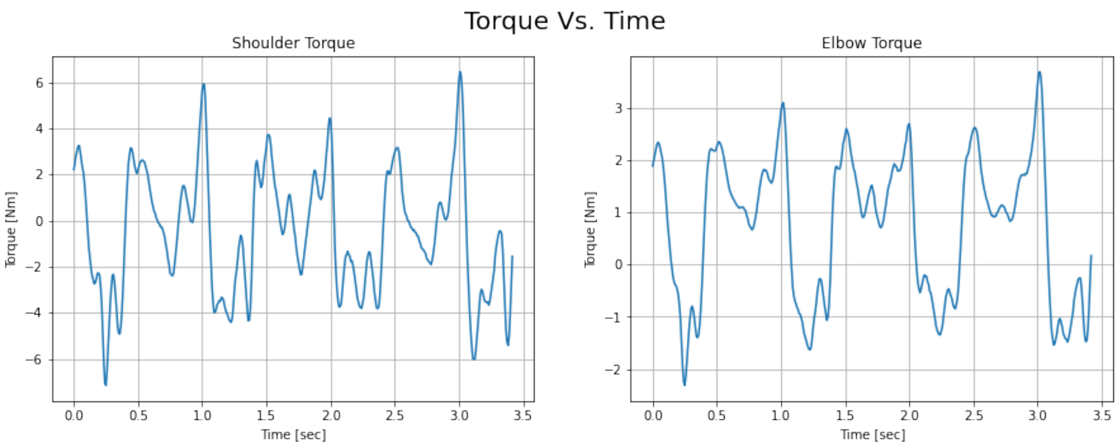
```
plt.grid()
plt.title('Shoulder Speed-Torque')

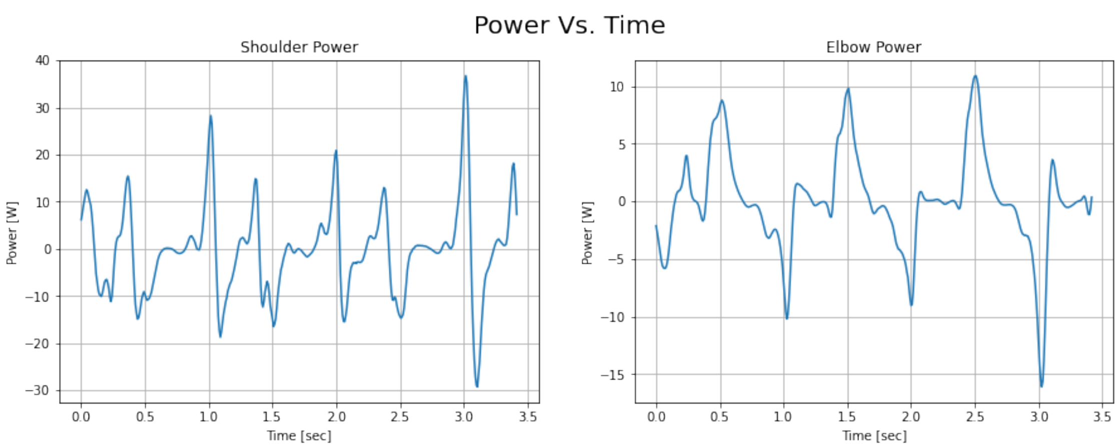
plt.subplot(122)
plt.plot(tau2_list[:-diff], traj[3])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Torque [Nm]')
plt.grid()
plt.title('Elbow Speed-Torque')
plt.show()
```

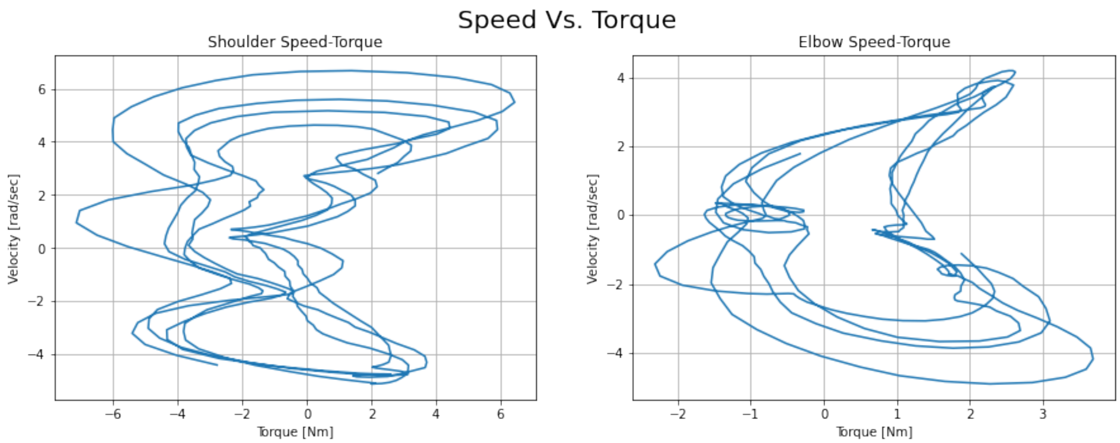





Angular Acceleration Vs. Time
Shoulder Angular Acceleration
Elbow Angular Acceleration
Acceleration [rad/sec^2]
Time [sec]


Torque Vs. Time
Shoulder Torque
Elbow Torque
Torque [Nm]
Time [sec]


Power Vs. Time
Shoulder Power
Elbow Power
Power [W]
Time [sec]



Animating the simulation:

[36]:
```
def animate_double_pend(traj, L1, L2, L1_COM, L2_COM, T):
    """
    Function to generate web-based animation of double-pendulum system

    Parameters:
        traj:         trajectory of theta1 and theta2
        L1:           length of the upper arm
        L2:           length of the lower arm
        L1_COM:       length of the center of mass of the upper arm from the␣
↪shoulder
        L2_COM:       length of the center of mass of the lower arm from the␣
↪elbow
        T:            length/seconds of animation duration

    Returns: None
    """

    # Browser configuration
    def configure_plotly_browser_state():
        import IPython
        display(IPython.core.display.HTML('''
            <script src="/static/components/requirejs/require.js"></script>
            <script>
              requirejs.config({
                paths: {
                  base: '/static/base',
                  plotly: 'https://cdn.plot.ly/plotly-1.5.1.min.js?noext',
                },
              });
            </script>
```

```
          '''))
configure_plotly_browser_state()
init_notebook_mode(connected=False)

# Getting data from pendulum angle trajectories
xx1 = L1 * np.sin(traj[0])
yy1 = -L1 * np.cos(traj[0])
xx1_COM = L1_COM * np.sin(traj[0])
yy1_COM = -L1_COM * np.cos(traj[0])
xx2 = xx1 + L2 * np.sin(traj[0] + traj[1])
yy2 = yy1 - L2 * np.cos(traj[0] + traj[1])
xx2_COM = xx1 + L2_COM * np.sin(traj[0] + traj[1])
yy2_COM = yy1 - L2_COM * np.cos(traj[0] + traj[1])
N = len(traj[0])

# Using these to specify axis limits
xm = np.min(xx1)
xM = np.max(xx1)
ym = np.min(yy1) - 0.6
yM = np.max(yy1) + 0.6

# Defining data dictionary
data = [dict(x=xx1, y=yy1,
             mode='lines', name='Arm',
             line=dict(width=5, color='blue')
            ),
        dict(x=xx1_COM, y=yy1_COM,
             mode='lines', name='Upper Arm Center of Mass',
             line=dict(width=2, color='green')
            ),
        dict(x=xx2_COM, y=yy2_COM,
             mode='lines', name='Lower Arm Center of Mass',
             line=dict(width=2, color='orange')
            ),
        dict(x=xx1, y=yy1,
             mode='markers', name='Elbow Trajectory',
             marker=dict(color="green", size=2)
            ),
        dict(x=xx2, y=yy2,
             mode='markers', name='Hand Trajectory',
             marker=dict(color="orange", size=2)
            )
      ]

# Preparing simulation layout
layout = dict(xaxis=dict(range=[xm, xM], autorange=False,␣
↪zeroline=False,dtick=1),
```

```
                  yaxis=dict(range=[ym, yM], autorange=False,␣
↪zeroline=False,scaleanchor = "x",dtick=1),
                  title='Simulation of Arm Modeled as a Double Pendulum',
                  hovermode='closest',
                  updatemenus= [{'type': 'buttons',
                                 'buttons': [{'label': 'Play', 'method':␣
↪'animate',
                                              'args': [None, {'frame':␣
↪{'duration': T, 'redraw': False}}]},
                                             {'args': [[None], {'frame':␣
↪{'duration': T, 'redraw': False}, 'mode': 'immediate',
                                              'transition': {'duration':␣
↪0}}],'label': 'Pause', 'method': 'animate'}
                                            ]
                                }]
                 )

    # Defining the frames of the simulation
    frames = [dict(data=[dict(x=[0,xx1[k],xx2[k]],
                              y=[0,yy1[k],yy2[k]],
                              mode='lines',
                              line=dict(color='red', width=4)),
                         go.Scatter(
                              x=[xx1_COM[k]],
                              y=[yy1_COM[k]],
                              mode="markers",
                              marker=dict(color="blue", size=12)),
                         go.Scatter(
                              x=[xx2_COM[k]],
                              y=[yy2_COM[k]],
                              mode="markers",
                              marker=dict(color="purple", size=12)),
                        ]) for k in range(N)]

    # Putting it all together and plotting
    figure = dict(data=data, layout=layout, frames=frames)
    iplot(figure)

# Animate the system
L1 = L_upper_arm_dict[participants_list[index]]
L2 = L_lower_arm
L1_COM = L_upper_arm_COM_dict[participants_list[index]]
L2_COM = L_lower_arm_COM
T = 5

animate_double_pend(traj, L1, L2, L1_COM, L2_COM, T)
```

<IPython.core.display.HTML object>

1.5 Motor Selection

Plotting the torque-speed curve of all trials and the torque-speed curve of some motors:

[22]:
```
# Compute the torque and speed vectors of the arm's motion for all trials
tot_dtheta1_list = []
tot_dtheta2_list = []
tot_tau1_list = []
tot_tau2_list = []

for lst in range(len(time_list)):
    for i in range(len(time_list[lst])):
        tot_dtheta1_list.append(sholder_Vel_list[lst][i])
        tot_dtheta2_list.append(Elbow_Vel_list[lst][i])
        tot_tau1_list.append(sholder_tau_list[lst][i])
        tot_tau2_list.append(Elbow_tau_list[lst][i])

# Compute the torque and speed vectors of some motors
######################################## T-Motor␣
↪########################################
# T-Motor, GL80 (KV30):
tau_stall_GL80_30 = 1.75
no_load_speed_GL80_30 = 720*2*pi/60
motor_speed_GL80_30 = [2*no_load_speed_GL80_30, no_load_speed_GL80_30, 0, -.
↪5*no_load_speed_GL80_30]
motor_torque_GL80_30 = [-tau_stall_GL80_30, 0, tau_stall_GL80_30, 1.
↪5*tau_stall_GL80_30]

# T-Motor, GL80 (KV60):
tau_stall_GL80_60 = 2.9
no_load_speed_GL80_60 = 1440*2*pi/60
motor_speed_GL80_60 = [2*no_load_speed_GL80_60, no_load_speed_GL80_60, 0, -.
↪5*no_load_speed_GL80_60]
motor_torque_GL80_60 = [-tau_stall_GL80_60, 0, tau_stall_GL80_60, 1.
↪5*tau_stall_GL80_60]

# T-Motor, G80 (KV30):
tau_stall_G80_30 = 2.9
no_load_speed_G80_30 = 700*2*pi/60
motor_speed_G80_30 = [2*no_load_speed_G80_30, no_load_speed_G80_30, 0, -.
↪5*no_load_speed_G80_30]
motor_torque_G80_30 = [-tau_stall_G80_30, 0, tau_stall_G80_30, 1.
↪5*tau_stall_G80_30]

# T-Motor, G80 (KV60):
```

```
tau_stall_G80_60 = 2.9
no_load_speed_G80_60 = 1400*2*pi/60
motor_speed_G80_60 = [2*no_load_speed_G80_60, no_load_speed_G80_60, 0, -.5*no_load_speed_G80_60]
motor_torque_G80_60 = [-tau_stall_G80_60, 0, tau_stall_G80_60, 1.5*tau_stall_G80_60]

# T-Motor, GL60 (KV25):
tau_stall_GL60_25 = 1.75
no_load_speed_GL60_25 = 600*2*pi/60
motor_speed_GL60_25 = [2*no_load_speed_GL60_25, no_load_speed_GL60_25, 0, -.5*no_load_speed_GL60_25]
motor_torque_GL60_25 = [-tau_stall_GL60_25, 0, tau_stall_GL60_25, 1.5*tau_stall_GL60_25]

# T-Motor, GL60 (KV55):
tau_stall_GL60_55 = 1.75
no_load_speed_GL60_55 = 1200*2*pi/60
motor_speed_GL60_55 = [2*no_load_speed_GL60_55, no_load_speed_GL60_55, 0, -.5*no_load_speed_GL60_55]
motor_torque_GL60_55 = [-tau_stall_GL60_55, 0, tau_stall_GL60_55, 1.5*tau_stall_GL60_55]

# T-Motor, GL100 (KV10):
tau_stall_GL100 = 7.7
no_load_speed_GL100 = 250*2*pi/60
motor_speed_GL100 = [2*no_load_speed_GL100, no_load_speed_GL100, 0, -.5*no_load_speed_GL100]
motor_torque_GL100 = [-tau_stall_GL100, 0, tau_stall_GL100, 1.5*tau_stall_GL100]

# T-Motor, G100 (KV10):
tau_stall_G100 = 7.7
no_load_speed_G100 = 250*2*pi/60
motor_speed_G100 = [2*no_load_speed_G100, no_load_speed_G100, 0, -.5*no_load_speed_G100]
motor_torque_G100 = [-tau_stall_G100, 0, tau_stall_G100, 1.5*tau_stall_G100]

# T-Motor, R60 (KV115):
tau_stall_R60 = 16.96
no_load_speed_R60 = 5520*2*pi/60
motor_speed_R60 = [2*no_load_speed_R60, no_load_speed_R60, 0, -.5*no_load_speed_R60]
motor_torque_R60 = [-tau_stall_R60, 0, tau_stall_R60, 1.5*tau_stall_R60]

# T-Motor, R80 (KV110):
tau_stall_R80 = 17.73
```

```
no_load_speed_R80 = 5280*2*pi/60
motor_speed_R80 = [2*no_load_speed_R80, no_load_speed_R80, 0, -.
↪5*no_load_speed_R80]
motor_torque_R80 = [-tau_stall_R80, 0, tau_stall_R80, 1.5*tau_stall_R80]

# T-Motor, AK60-6:
tau_stall_AK60_6 = 37.49
no_load_speed_AK60_6 = 560*2*pi/60
motor_speed_AK60_6 = [2*no_load_speed_AK60_6, no_load_speed_AK60_6, 0, -.
↪5*no_load_speed_AK60_6]
motor_torque_AK60_6 = [-tau_stall_AK60_6, 0, tau_stall_AK60_6, 1.
↪5*tau_stall_AK60_6]

# T-Motor, AK80-6:
tau_stall_AK80_6 = 80.888
no_load_speed_AK80_6 = 460*2*pi/60
motor_speed_AK80_6 = [2*no_load_speed_AK80_6, no_load_speed_AK80_6, 0, -.
↪5*no_load_speed_AK80_6]
motor_torque_AK80_6 = [-tau_stall_AK80_6, 0, tau_stall_AK80_6, 1.
↪5*tau_stall_AK80_6]

######################################## Maxon␣
↪########################################
# Maxon, 614949:
tau_stall_614949 = 4.3
no_load_speed_614949 = 4300*2*pi/60
motor_speed_614949 = [2*no_load_speed_614949, no_load_speed_614949, 0, -.
↪5*no_load_speed_614949]
motor_torque_614949 = [-tau_stall_614949, 0, tau_stall_614949, 1.
↪5*tau_stall_614949]

######################################## E-S Motor␣
↪########################################
# E-S Motor, 28PG-385SP-19-EN:
tau_stall_28PG = 3.73
no_load_speed_28PG = 310*2*pi/60
motor_speed_28PG = [2*no_load_speed_28PG, no_load_speed_28PG, 0, -.
↪5*no_load_speed_28PG]
motor_torque_28PG = [-tau_stall_28PG, 0, tau_stall_28PG, 1.5*tau_stall_28PG]

# E-S Motor, 36GP-555PM-51-EN 24V:
tau_stall_36GP_51 = 4.90
no_load_speed_36GP_51 = 230*2*pi/60
motor_speed_36GP_51 = [2*no_load_speed_36GP_51, no_load_speed_36GP_51, 0, -.
↪5*no_load_speed_36GP_51]
```

```
motor_torque_36GP_51 = [-tau_stall_36GP_51, 0, tau_stall_36GP_51, 1.
↪5*tau_stall_36GP_51]

# E-S Motor, 36GP-555PM-100-EN 24V:
tau_stall_36GP_100 = 4.90
no_load_speed_36GP_100 = 120*2*pi/60
motor_speed_36GP_100 = [2*no_load_speed_36GP_100, no_load_speed_36GP_100, 0, -.
↪5*no_load_speed_36GP_100]
motor_torque_36GP_100 = [-tau_stall_36GP_100, 0, tau_stall_36GP_100, 1.
↪5*tau_stall_36GP_100]

# E-S Motor, 36GP-555PM-139-EN 24V:
tau_stall_36GP_139 = 4.90
no_load_speed_36GP_139 = 85*2*pi/60
motor_speed_36GP_139 = [2*no_load_speed_36GP_139, no_load_speed_36GP_139, 0, -.
↪5*no_load_speed_36GP_139]
motor_torque_36GP_139 = [-tau_stall_36GP_139, 0, tau_stall_36GP_139, 1.
↪5*tau_stall_36GP_139]

######################################## Pololu␣
↪########################################
# Pololu, 150:1 Metal Gearmotor 37Dx73L mm 12V with 64 CPR Encoder (Helical␣
↪Pinion):
tau_stall_37D__12V_150 = 4.805
no_load_speed_37D__12V_150 = 67*2*pi/60
motor_speed_37D__12V_150 = [2*no_load_speed_37D__12V_150,␣
↪no_load_speed_37D__12V_150, 0, -.5*no_load_speed_37D__12V_150]
motor_torque_37D__12V_150 = [-tau_stall_37D__12V_150, 0,␣
↪tau_stall_37D__12V_150, 1.5*tau_stall_37D__12V_150]

# Pololu, 131:1 Metal Gearmotor 37Dx73L mm 12V with 64 CPR Encoder (Helical␣
↪Pinion):
tau_stall_37D__12V_131 = 4.41
no_load_speed_37D__12V_131 = 76*2*pi/60
motor_speed_37D__12V_131 = [2*no_load_speed_37D__12V_131,␣
↪no_load_speed_37D__12V_131, 0, -.5*no_load_speed_37D__12V_131]
motor_torque_37D__12V_131 = [-tau_stall_37D__12V_131, 0,␣
↪tau_stall_37D__12V_131, 1.5*tau_stall_37D__12V_131]

# Pololu, 150:1 Metal Gearmotor 37Dx73L mm 24V with 64 CPR Encoder (Helical␣
↪Pinion):
tau_stall_37D__24V_150 = 5.49
no_load_speed_37D__24V_150 = 68*2*pi/60
motor_speed_37D__24V_150 = [2*no_load_speed_37D__24V_150,␣
↪no_load_speed_37D__24V_150, 0, -.5*no_load_speed_37D__24V_150]
```

```
motor_torque_37D__24V_150 = [-tau_stall_37D__24V_150, 0, tau_stall_37D__24V_150, 1.5*tau_stall_37D__24V_150]

# Pololu, 131:1 Metal Gearmotor 37Dx73L mm 24V with 64 CPR Encoder (Helical Pinion):
tau_stall_37D__24V_131 = 4.61
no_load_speed_37D__24V_131 = 79*2*pi/60
motor_speed_37D__24V_131 = [2*no_load_speed_37D__24V_131, no_load_speed_37D__24V_131, 0, -.5*no_load_speed_37D__24V_131]
motor_torque_37D__24V_131 = [-tau_stall_37D__24V_131, 0, tau_stall_37D__24V_131, 1.5*tau_stall_37D__24V_131]

# ODrive, DUAL SHAFT MOTOR - D5065 270KV:
tau_stall_D5065 = 1.99
no_load_speed_D5065 = 8640*2*pi/60
motor_speed_D5065 = [2*no_load_speed_D5065, no_load_speed_D5065, 0, -.5*no_load_speed_D5065]
motor_torque_D5065 = [-tau_stall_D5065, 0, tau_stall_D5065, 1.5*tau_stall_D5065]

# ODrive, DUAL SHAFT MOTOR - D6374 150KV:
tau_stall_D6374 = 3.86
no_load_speed_D6374 = 5760*2*pi/60
motor_speed_D6374 = [2*no_load_speed_D6374, no_load_speed_D6374, 0, -.5*no_load_speed_D6374]
motor_torque_D6374 = [-tau_stall_D6374, 0, tau_stall_D6374, 1.5*tau_stall_D6374]

# Turnigy, 9235-100KV Brushless Multi-Rotor Motor:
tau_stall_9235 = 4.71
no_load_speed_9235 = 3840*2*pi/60
motor_speed_9235 = [2*no_load_speed_9235, no_load_speed_9235, 0, -.5*no_load_speed_9235]
motor_torque_9235 = [-tau_stall_9235, 0, tau_stall_9235, 1.5*tau_stall_9235]

# Turnigy, SK8 6374-192KV Sensored Brushless Motor (14P):
tau_stall_6374_192KV = 4.31
no_load_speed_6374_192KV = 7373*2*pi/60
motor_speed_6374_192KV = [2*no_load_speed_6374_192KV, no_load_speed_6374_192KV, 0, -.5*no_load_speed_6374_192KV]
motor_torque_6374_192KV = [-tau_stall_6374_192KV, 0, tau_stall_6374_192KV, 1.5*tau_stall_6374_192KV]

# Turnigy, SK8 6374-149KV Sensored Brushless Motor (14P):
tau_stall_6374_149KV = 4.31
no_load_speed_6374_149KV = 7373*2*pi/60
motor_speed_6374_149KV = [2*no_load_speed_6374_149KV, no_load_speed_6374_149KV, 0, -.5*no_load_speed_6374_149KV]
```

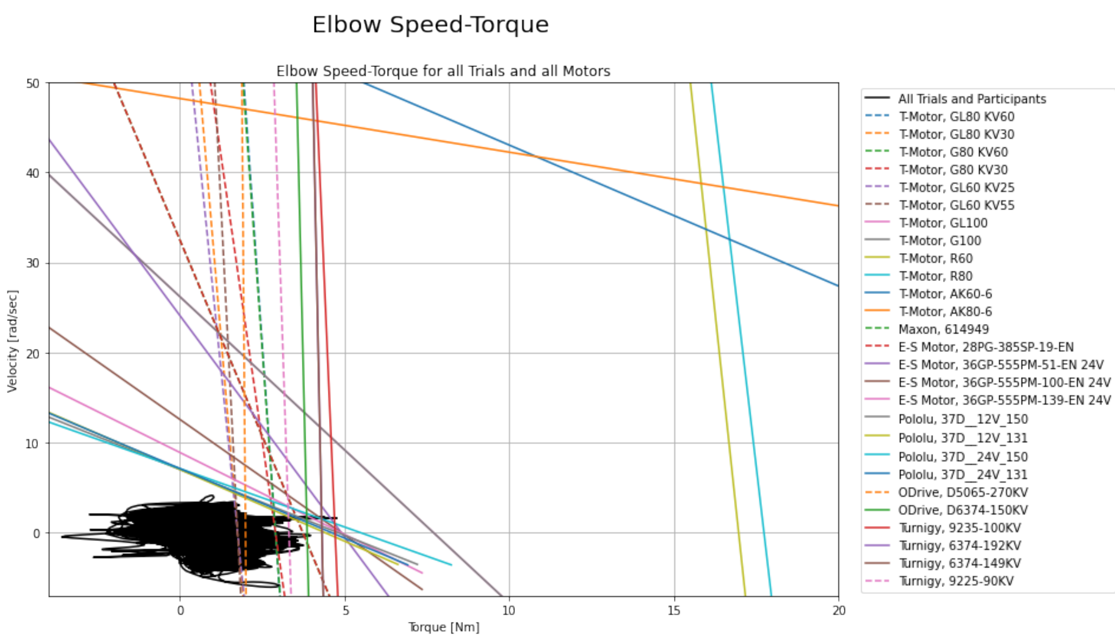
```python
motor_torque_6374_149KV = [-tau_stall_6374_149KV, 0, tau_stall_6374_149KV, 1.5*tau_stall_6374_149KV]

# Turnigy, 9225-90KV Turnigy Multistar Brushless Multi-Rotor Motor:
tau_stall_9225 = 3.31
no_load_speed_9225 = 3456*2*pi/60
motor_speed_9225 = [2*no_load_speed_9225, no_load_speed_9225, 0, -.5*no_load_speed_9225]
motor_torque_9225 = [-tau_stall_9225, 0, tau_stall_9225, 1.5*tau_stall_9225]

# Plotting the torque-speed curves of the arm's motion and the motors
plt.figure(figsize=(12,8))
plt.plot(tot_tau2_list, tot_dtheta2_list, color='black', label='All Trials and Participants')
plt.plot(motor_torque_GL80_60, motor_speed_GL80_60, '--', label='T-Motor, GL80 KV60')
plt.plot(motor_torque_GL80_30, motor_speed_GL80_30, '--', label='T-Motor, GL80 KV30')
plt.plot(motor_torque_G80_60, motor_speed_G80_60, '--', label='T-Motor, G80 KV60')
plt.plot(motor_torque_G80_30, motor_speed_G80_30, '--', label='T-Motor, G80 KV30')
plt.plot(motor_torque_GL60_25, motor_speed_GL60_25, '--', label='T-Motor, GL60 KV25')
plt.plot(motor_torque_GL60_55, motor_speed_GL60_55, '--', label='T-Motor, GL60 KV55')
plt.plot(motor_torque_GL100, motor_speed_GL100, label='T-Motor, GL100')
plt.plot(motor_torque_G100, motor_speed_G100, label='T-Motor, G100')
plt.plot(motor_torque_R60, motor_speed_R60, label='T-Motor, R60')
plt.plot(motor_torque_R80, motor_speed_R80, label='T-Motor, R80')
plt.plot(motor_torque_AK60_6, motor_speed_AK60_6, label='T-Motor, AK60-6')
plt.plot(motor_torque_AK80_6, motor_speed_AK80_6, label='T-Motor, AK80-6')
plt.plot(motor_torque_28PG, motor_speed_28PG, '--', label='Maxon, 614949')
plt.plot(motor_torque_28PG, motor_speed_28PG, '--', label='E-S Motor, 28PG-385SP-19-EN')
plt.plot(motor_torque_36GP_51, motor_speed_36GP_51, label='E-S Motor, 36GP-555PM-51-EN 24V')
plt.plot(motor_torque_36GP_100, motor_speed_36GP_100, label='E-S Motor, 36GP-555PM-100-EN 24V')
plt.plot(motor_torque_36GP_139, motor_speed_36GP_139, label='E-S Motor, 36GP-555PM-139-EN 24V')
plt.plot(motor_torque_37D__12V_150, motor_speed_37D__12V_150, label='Pololu, 37D__12V_150')
plt.plot(motor_torque_37D__12V_131, motor_speed_37D__12V_150, label='Pololu, 37D__12V_131')
```

```python
plt.plot(motor_torque_37D__24V_150, motor_speed_37D__24V_150, label='Pololu, 37D__24V_150')
plt.plot(motor_torque_37D__24V_131, motor_speed_37D__24V_150, label='Pololu, 37D__24V_131')
plt.plot(motor_torque_D5065, motor_speed_D5065, '--', label='ODrive, D5065-270KV')
plt.plot(motor_torque_D6374, motor_speed_D6374, label='ODrive, D6374-150KV')
plt.plot(motor_torque_9235, motor_speed_9235, label='Turnigy, 9235-100KV')
plt.plot(motor_torque_6374_192KV, motor_speed_6374_192KV, label='Turnigy, 6374-192KV')
plt.plot(motor_torque_6374_149KV, motor_speed_6374_149KV, label='Turnigy, 6374-149KV')
plt.plot(motor_torque_9225, motor_speed_9225, linestyle='--', label='Turnigy, 9225-90KV')

plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Torque [Nm]')
plt.xlim([-4, 20])
plt.ylim([-7, 50])
plt.grid()
plt.legend(loc='upper left', bbox_to_anchor=(1.02,1))
plt.suptitle('Elbow Speed-Torque', fontsize=20)
plt.title('Elbow Speed-Torque for all Trials and all Motors')
plt.show()
```



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