arm_pendulum_modeling_arm_back

October 22, 2021

1 Arm Motion Modeling

1.1 System Description

A double-pendulum system hanging in gravity is shown in the figure above. $q = [\theta_1, \theta_2]$ are the system configuration variables. We assume the z-axis is pointing out from the screen/paper, thus the positive direction of rotation is counter-clockwise. The solution steps are: 1. Computing the Lagrangian of the system. 2. Computing the Euler-Lagrange equations, and solve them for $\ddot{\theta}_1$ and $\ddot{\theta}_2$. 3. Numerically evaluating the solutions for τ_1 and τ_2, and simulating the system for θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$ and $\ddot{\theta}_2$. 4. Animating the simulation.

[11]:
```
from IPython.core.display import HTML
display(HTML("<table><tr><td><img src='./double-pendulum-diagram.png'␣
↪width=450' height='300'></table>"))
```

```
<IPython.core.display.HTML object>
```

1.2 Import Libraries and Define System Constants

Import libraries:

[12]:
```
# Imports required for data processing
import os
import csv
import pandas as pd

# Imports required for dynamics calculations
import sympy
from sympy.abc import t
from sympy import symbols, Eq, Function, solve, sin, cos, Matrix, Subs,␣
↪substitution, Derivative, simplify, symbols, lambdify
import math
from math import pi
import numpy as np
import matplotlib.pyplot as plt

# Imports required for animation
from plotly.offline import init_notebook_mode, iplot
from IPython.display import display, HTML
```

```python
import plotly.graph_objects as go
```

Define the system's constants:

[74]:
```python
# Masses, length and center-of-mass positions (calculated using the lab␣
↪measurements)
# Mass calculations (mass unit is kg)
m_body = 53
m_u = 0.028 * m_body                         # Average upper arm weights relative␣
↪to body weight, from "Biomechanics
                                             # and Motor Control of Human Movement"␣
↪by David Winter (2009), 4th edition
m_l = 0.7395                                 # Average lower prosthetics weights,␣
↪calculated using lab measurements
# m_lower = 0.022 * m_body                   # Average lower arm weights relative␣
↪to body weight, from "Biomechanics
                                             # and Motor Control of Human Movement"␣
↪by David Winter (2009), 4th edition
# Arm length calculations (length unit is m)
H_body = 1.62
L_u = 0.186 * H_body                         # Average upper arm length relative to␣
↪body height
                                             # from "Biomechanics and Motor Control␣
↪of Human Movement" by David
                                             # Winter (2009), 4th edition
# L_l = (0.146 + 0.108) * H_body             # Average lower arm length relative to␣
↪body height
                                             # from "Biomechanics and Motor Control␣
↪of Human Movement" by David
                                             # Winter (2009), 4th edition
L_l = 0.42                                   # Average lower prosthetics length,␣
↪calculated using lab measurements

# Arm center of mass length calculations (length unit is m)
L_u_c = 0.436 * L_u                          # Average upper arm length from␣
↪shoulder to center of mass relative
                                             # to upper arm length, from␣
↪"Biomechanics and Motor Control of Human
                                             # Movement" by David Winter (2009),␣
↪4th edition
L_l_c = 0.2388                               # Average lower prosthetics length␣
↪from elbow to center of mass,
                                             # calculated using lab measurements
# L_l_c = 0.682 * L_l                        # Average lower arm length from␣
↪shoulder to center of mass relative
```

```
                                                    # to upper arm length, from␣
↪"Biomechanics and Motor Control of Human
                                                    # Movement" by David Winter (2009),␣
↪4th edition
```

1.3 Extracting Data

Extracting angles data and computing angular velocities and angular accelerations from the angles:

[94]:
```
def calculate_vel(ang_list, time_list, index):
    return ((ang_list[index + 1] - ang_list[index])
           / (time_list[index + 1] - time_list[index]))


def calculate_acc(vel_list, time_list, index):
    return ((vel_list[index + 1] - vel_list[index])
           / (time_list[index + 1] - time_list[index]))


data_csv_dir = '../../data/hand_back_motion_data/CSV Converted Files'
frame_frequency = 100
print("current directory: ", os.getcwd())

walking_vel_list = []
time_list = []
elbow_ang_list, sholder_ang_list = [], []
elbow_vel_list, sholder_vel_list = [], []
elbow_acc_list, sholder_acc_list = [], []
elbow_acc_data_list, sholder_acc_data_list = [], []
back_ang_list, back_pos_list, back_vel_list = [], [], []

folder_list = os.listdir(data_csv_dir)
folder_list.sort()

for folder in folder_list:

    data_trial_dir = os.path.join(data_csv_dir, folder)
    if os.path.isdir(data_trial_dir):
        file_list = os.listdir(data_trial_dir)

        for file in file_list:
            if "00B429F8" in file:
                if file.endswith(".csv"):
                    file_name = file[:-4]
                    walking_vel = file.split("_")[4][:5]

                    frame = 0
```

```
                    file_time_list = []
                    file_sholder_ang_list, file_sholder_vel_list,␣
↪file_sholder_acc_list, file_sholder_acc_data_list = [], [], [], []

                    # Cutting out weird data behavior on data edges
                    data_path = os.path.join(data_csv_dir, folder, file)
                    data_rows = open(data_path).read().strip().split("\n")[7500:
↪9500]

                    # Extract time [sec], elbow angles [rad], and shoulder␣
↪angles [rad] from data
                    for row in data_rows:
                        splitted_row = row.strip().split("\t")

                        # Check if loop finished all data
                        if not len(splitted_row):
                            break

                        file_time_list.append(frame / frame_frequency)
                        file_sholder_ang_list.append(float(splitted_row[31]) *␣
↪2*pi/360)
                        file_sholder_acc_data_list.
↪append(float(splitted_row[14]))
                        frame += 1

                    # Extract elbow and shoulder velocities [rad/sec] from␣
↪angles
                    for i in range(len(file_time_list) - 1):
                        sholder_vel = calculate_vel(file_sholder_ang_list,␣
↪file_time_list, i)
                        file_sholder_vel_list.append(sholder_vel)

                    # Extract elbow and shoulder Accelerations [rad/sec^2] from␣
↪velocities
                    for i in range(len(file_time_list) - 2):
                        sholder_acc = calculate_acc(file_sholder_vel_list,␣
↪file_time_list, i)
                        file_sholder_acc_list.append(sholder_acc)

                    # Adjust lists length
                    adjusted_file_time_list = file_time_list[:-2]
                    adjusted_file_sholder_ang_list = file_sholder_ang_list[:-2]
                    adjusted_file_sholder_vel_list = file_sholder_vel_list[:-1]
                    adjusted_file_sholder_acc_data_list =␣
↪file_sholder_acc_data_list[:-2]
```

```
                    time_list.append(adjusted_file_time_list)
                    walking_vel_list.append(walking_vel)

                    sholder_ang_list.append(adjusted_file_sholder_ang_list)
                    sholder_vel_list.append(adjusted_file_sholder_vel_list)
                    sholder_acc_list.append(file_sholder_acc_list)
                    sholder_acc_data_list.
↪append(adjusted_file_sholder_acc_data_list)
                    break

        for file in file_list:
            if "00B429E2" in file:
                if file.endswith(".csv"):
                    file_name = file[:-4]
                    walking_vel = file.split("_")[4][:5]

                    frame = 0
                    file_time_list = []
                    file_elbow_ang_list, file_elbow_vel_list,␣
↪file_elbow_acc_list, file_elbow_acc_data_list = [], [], [], []

                    # Cutting out weird data behavior on data edges
                    data_path = os.path.join(data_csv_dir, folder, file)
                    data_rows = open(data_path).read().strip().split("\n")[7500:
↪9500]

                    # Extract time [sec], elbow angles [rad], and shoulder␣
↪angles [rad] from data
                    for i in range(len(data_rows)):
                        splitted_row = data_rows[i].strip().split("\t")

                        # Check if loop finished all data
                        if not len(splitted_row):
                            break

                        file_time_list.append(frame / frame_frequency)
                        file_elbow_ang_list.append((float(splitted_row[31]) -␣
↪file_sholder_ang_list[i]) * 2*pi/360)
                        file_elbow_acc_data_list.append(float(splitted_row[14]))
                        frame += 1

                    # Extract elbow and shoulder velocities [rad/sec] from␣
↪angles
                    for i in range(len(file_time_list) - 1):
                        elbow_vel = calculate_vel(file_elbow_ang_list,␣
↪file_time_list, i)
```

```
                        file_elbow_vel_list.append(elbow_vel)

                    # Extract elbow and shoulder Accelerations [rad/sec^2] from␣
↪velocities
                    for i in range(len(file_time_list) - 2):
                        elbow_acc = calculate_acc(file_elbow_vel_list,␣
↪file_time_list, i)
                        file_elbow_acc_list.append(elbow_acc)

                    # Adjust lists length
                    adjusted_file_elbow_ang_list = file_elbow_ang_list[:-2]
                    adjusted_file_elbow_vel_list = file_elbow_vel_list[:-1]
                    adjusted_file_elbow_acc_data_list =␣
↪file_elbow_acc_data_list[:-2]

                    elbow_ang_list.append(adjusted_file_elbow_ang_list)
                    elbow_vel_list.append(adjusted_file_elbow_vel_list)
                    elbow_acc_list.append(file_elbow_acc_list)
                    elbow_acc_data_list.append(file_elbow_acc_data_list)
                    break

        for file in file_list:
            if "00B43D0C" in file:
                if file.endswith(".csv"):
                    file_name = file[:-4]
                    walking_vel = file.split("_")[4][:5]
                    if walking_vel == "1.4ms":
                        continue

                    frame = 0
                    file_time_list = []
                    file_back_ang_list, file_back_pos_list, file_back_vel_list␣
↪= [], [], []

                    # Cutting out weird data behavior on data edges
                    data_path = os.path.join(data_csv_dir, folder, file)
                    data_rows = open(data_path).read().strip().split("\n")[7500:
↪9500]

                    # Extract time [sec], elbow angles [rad], and shoulder␣
↪angles [rad] from data
                    for i in range(len(data_rows)):
                        splitted_row = data_rows[i].strip().split("\t")

                        # Check if loop finished all data
                        if not len(splitted_row):
```

```
                            break

                        file_time_list.append(frame / frame_frequency)

                        file_back_ang_list.append(float(splitted_row[31]) *␣
↪2*pi/360)
                        file_back_pos_list.append(float(splitted_row[21]))
                        file_back_vel_list.append(float(splitted_row[24]))
                        frame += 1

                    # Adjust lists length
                    adjusted_file_back_ang_list = file_back_ang_list[:-2]
                    adjusted_file_back_pos_list = file_back_pos_list[:-2]
                    adjusted_file_back_vel_list = file_back_vel_list[:-2]

                    back_ang_list.append(adjusted_file_back_ang_list)
                    back_pos_list.append(adjusted_file_back_pos_list)
                    back_vel_list.append(adjusted_file_back_vel_list)
                    break
```

```
current directory:
/home/yael/Documents/MSR_Courses/ME499-Final_Project/Motorized-Prosthetic-
Arm/motor_control/arm_pendulum_modeling
```

1.4 System Modeling - Single Pendulum

Computing the Lagrangian of the system:

[95]:
```
m, g, R, R_c = symbols(r'm, g, R, R_c')

# The system torque variables as function of t
tau = Function(r'tau')(t)

# The system configuration variables as function of t
theta = Function(r'theta')(t)

# The velocity as derivative of position wrt t
theta_dot = theta.diff(t)

# The acceleration as derivative of velocity wrt t
theta_ddot = theta_dot.diff(t)

# Converting the polar coordinates to cartesian coordinates
x = R_c * sin(theta)
y = -R_c * cos(theta)

# Calculating the kinetic and potential energy of the system
KE = 1/2 * m * ((x.diff(t))**2 + (y.diff(t))**2)
```

```
PE = m * g * y

# Computing the Lagrangian
L = simplify(KE - PE)
Lagrange = Function(r'L')(t)
display(Eq(Lagrange, L))
```

$$L(t) = R_c m \left(0.5 R_c \left(\frac{d}{dt}\theta(t)\right)^2 + g\cos(\theta(t))\right)$$

Computing the Euler-Lagrange equations:

[96]:
```
# Define the derivative of L wrt the functions: x, xdot
L_dtheta = L.diff(theta)
L_dtheta_dot = L.diff(theta_dot)

# Define the derivative of L_dxdot wrt to time t
L_dtheta_dot_dt = L_dtheta_dot.diff(t)

# Define the right hand side of the the Euler-Lagrange as a matrix
rhs = simplify(L_dtheta_dot_dt - L_dtheta)

# Define the left hand side of the the Euler-Lagrange as a Matrix
lhs = tau

# Compute the Euler-Lagrange equations as a matrix
EL_eqns = Eq(lhs, rhs)

print('Euler-Lagrange matrix for this systems:')
display(EL_eqns)
```

```
Euler-Lagrange matrix for this systems:
```

$$\tau(t) = R_c m \left(1.0 R_c \frac{d^2}{dt^2}\theta(t) + g\sin(\theta(t))\right)$$

Simulating the system:

[104]:
```
# Substitute the derivative variables with a dummy variables and plug-in the␣
↪constants
solution_subs = rhs

theta_dot_dummy = symbols('thetadot')
theta_ddot_dummy = symbols('thetaddot')

solution_subs = solution_subs.subs([(g, 9.81)])

solution_subs = solution_subs.subs([((theta.diff(t)).diff(t),␣
↪theta_ddot_dummy)])
```

```
solution_subs = solution_subs.subs([(theta.diff(t), theta_dot_dummy)])


# Lambdify the thetas and its derivatives
func = lambdify([theta, theta_dot_dummy, theta_ddot_dummy,
                 m, R, R_c], solution_subs, modules = sympy)

# Initialize the torque and power lists
elbow_tau_list = []
elbow_current_list = []
elbow_power_list = []

motor_kv = 115
torque_const = 8.27 / motor_kv

for i in range(len(time_list)):
    # Initialize the torque and power lists
    tau_list = []
    current_list = []
    power_list = []

    t_list = time_list[i]
    theta_list = elbow_ang_list[i]
    dtheta_list = elbow_vel_list[i]
    ddtheta_list = elbow_acc_list[i]

    # Plug-in the angles, angular velocities and angular accelerations for␣
 ↪every time step to find the torques
    for j in range(len(t_list)):
        tau_list.append(func(theta_list[j], dtheta_list[j], ddtheta_list[j],␣
 ↪m_l, L_l, L_l_c))

        # Calculate the current required to reach the required joints torques␣
 ↪for every time step
        current_list.append(torque_const * tau_list[j])

        # Calculate the power required to reach the required angular velocities␣
 ↪and joints torques for every time step
        power_list.append(dtheta_list[j] * tau_list[j])

    elbow_tau_list.append(tau_list)
    elbow_current_list.append(current_list)
    elbow_power_list.append(power_list)

    print(f"Velocity {walking_vel_list[i]}\t max torque: {format(max(tau_list),␣
 ↪'.3f')}[Nm]\t max angular velocity: {format(max(dtheta_list), '.3f')}[rad/
 ↪sec]\t max power: {format(max(power_list), '.3f')}[W]")
```

```
Velocity 0.5ms   max torque: -0.021[Nm]  max angular velocity: 1.625[rad/sec]
max power: 0.954[W]
Velocity 0.6ms   max torque: -0.057[Nm]  max angular velocity: 1.731[rad/sec]
max power: 1.175[W]
Velocity 0.7ms   max torque: 0.137[Nm]   max angular velocity: 2.556[rad/sec]
max power: 1.862[W]
Velocity 0.8ms   max torque: 0.795[Nm]   max angular velocity: 3.695[rad/sec]
max power: 3.006[W]
Velocity 0.9ms   max torque: 0.202[Nm]   max angular velocity: 3.370[rad/sec]
max power: 2.815[W]
Velocity 1.0ms   max torque: 0.423[Nm]   max angular velocity: 3.588[rad/sec]
max power: 2.420[W]
Velocity 1.1ms   max torque: 0.565[Nm]   max angular velocity: 2.989[rad/sec]
max power: 1.962[W]
Velocity 1.2ms   max torque: 0.495[Nm]   max angular velocity: 3.567[rad/sec]
max power: 2.478[W]
Velocity 1.3ms   max torque: 0.890[Nm]   max angular velocity: 4.525[rad/sec]
max power: 3.748[W]
Velocity 1.4ms   max torque: 1.278[Nm]   max angular velocity: 4.931[rad/sec]
max power: 5.166[W]
Velocity chang   max torque: 0.368[Nm]   max angular velocity: 3.920[rad/sec]
max power: 2.851[W]
```

Calculation summary:

[105]:
```
max_elbow_tau, max_elbow_power, max_elbow_vel = -10, -10, -10
max_elbow_tau_index, max_elbow_power_index, max_elbow_vel_index = -10, -10, -10

for i in range(len(elbow_tau_list)):
    if max_elbow_vel < max(elbow_vel_list[i]):
        max_elbow_vel = max(elbow_vel_list[i])
        max_elbow_vel_index = i

    if max_elbow_tau < max(elbow_tau_list[i]):
        max_elbow_tau = max(elbow_tau_list[i])
        max_elbow_tau_index = i

    if max_elbow_power < max(elbow_power_list[i]):
        max_elbow_power = max(elbow_power_list[i])
        max_elbow_power_index = i

print(f"maximum elbow angular velocity is {format(max_elbow_vel, '.3f')} [rad/
 ↪sec] ({format(max_elbow_vel*60/(2*pi), '.3f')} [rpm]), in velocity␣
 ↪{walking_vel_list[max_elbow_vel_index]} (trial {max_elbow_vel_index})")
print(f"maximum elbow torque is {format(max_elbow_tau, '.3f')} [Nm], in␣
 ↪velocity {walking_vel_list[max_elbow_tau_index]} (trial␣
 ↪{max_elbow_tau_index})")
```

```
print(f"maximum elbow power is {format(max_elbow_power, '.3f')} [W], in␣
↪velocity {walking_vel_list[max_elbow_power_index]} (trial␣
↪{max_elbow_power_index})")


# The torque equations for the maximum power:
solution_subs = solution_subs.subs([(m, m_l), (R, L_l), (R_c, L_l_c), (g, 9.
↪81)])

print("\nThe torque equations for the maximum torque:")
display(Eq(tau, solution_subs))

# display(Elbow_Ang_list[max_Elbow_tau_index])
# display(Elbow_Vel_list[max_Elbow_tau_index])
# display(Elbow_Acc_list[max_Elbow_tau_index])
# display(Elbow_tau_list[max_Elbow_tau_index])
# display(Elbow_Ang_list[2])
# display(Elbow_tau_list[2])
```

```
maximum elbow angular velocity is 4.931 [rad/sec] (47.091 [rpm]), in velocity
1.4ms (trial 9)
maximum elbow torque is 1.278 [Nm], in velocity 1.4ms (trial 9)
maximum elbow power is 5.166 [W], in velocity 1.4ms (trial 9)

The torque equations for the maximum torque:
```

$$\tau(t) = 0.042170312288\ddot{\theta} + 1.732373406\sin(\theta(t))$$

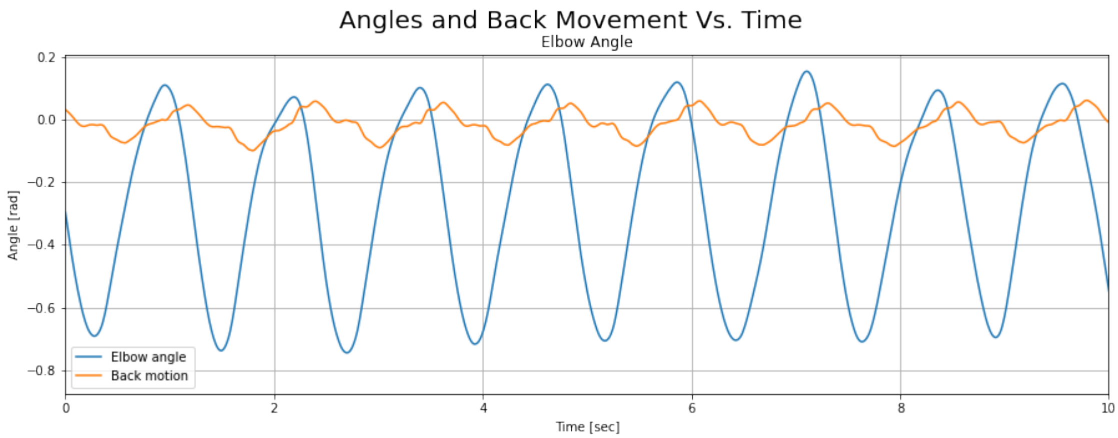
Example for the trial with the largest elbow torque & power:

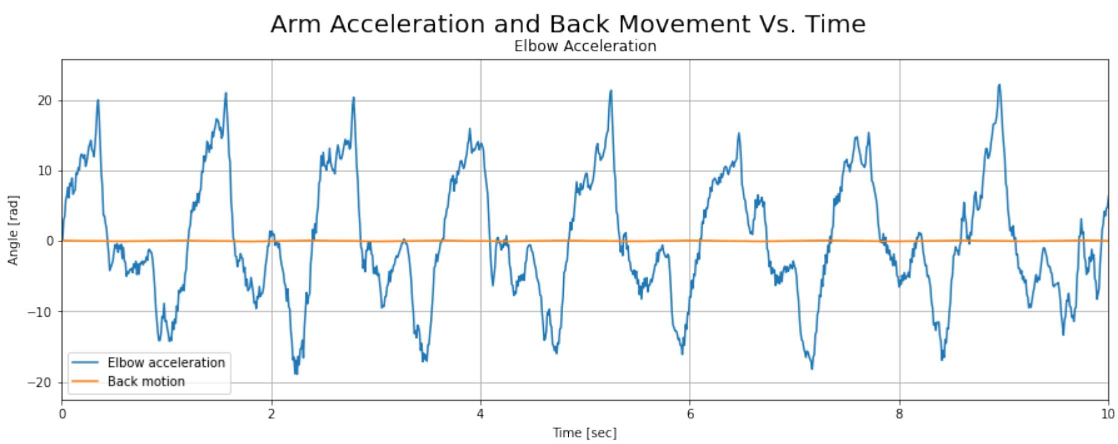
[106]:
```
index = 2
t_list = time_list[index]
theta_list = elbow_ang_list[index]
dtheta_list = elbow_vel_list[index]
ddtheta_list = elbow_acc_list[index]
tau_list = elbow_tau_list[index]
current_list = elbow_current_list[index]
power_list = elbow_power_list[index]

back_rotation_list = back_ang_list[index]
back_position_list = back_pos_list[index]
back_velocity_list = back_vel_list[index]

elbow_acceleration_list = elbow_acc_data_list[index]

# Compute the trajectory of the arm's motion
N = int((max(t_list) - min(t_list))/(1/frame_frequency))
tvec = np.linspace(min(t_list), max(t_list), N)
```

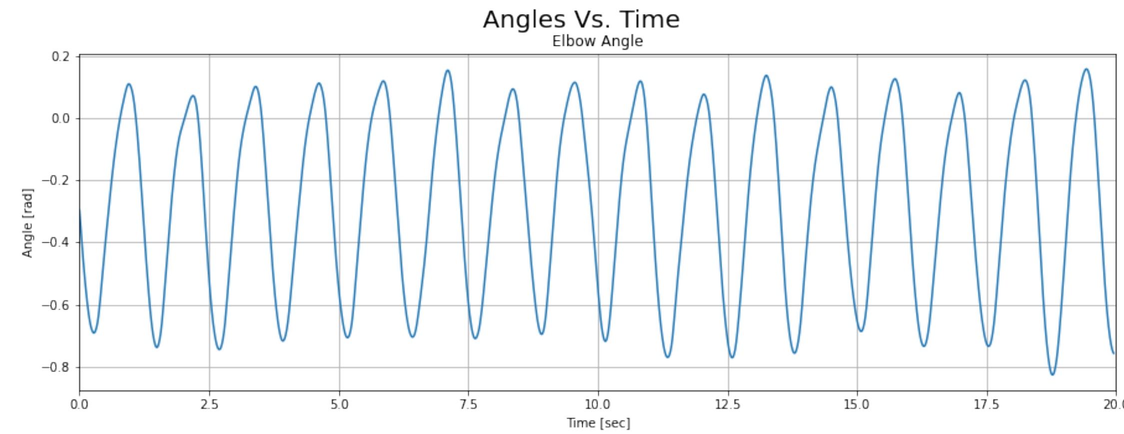
```
traj = np.zeros((3, N))
back_traj = np.zeros((3, N))
acc_traj = np.zeros((2, N))
partial_traj = np.zeros((3, N))

for i in range(N):
    traj[0, i] = theta_list[i]
    traj[1, i] = dtheta_list[i]
    traj[2, i] = ddtheta_list[i]


    back_traj[0, i] = back_rotation_list[i]
    back_traj[1, i] = back_position_list[i]
    back_traj[2, i] = back_velocity_list[i]

    acc_traj[0, i] = elbow_acceleration_list[i]

for i in range(500):
    partial_traj[0, i] = theta_list[i]
    partial_traj[1, i] = dtheta_list[i]
    partial_traj[2, i] = ddtheta_list[i]

# Calculate the length difference between the time list and the trajectory lists
diff = (len(t_list) - len(traj[0]))

# Plot the trajectory lists (angles, velocities, accelerations, torques, and␣
 ↪power)
plt.figure(figsize=(15,5))
plt.suptitle('Angles and Back Movement Vs. Time', fontsize=20)
plt.plot(t_list[:-diff], traj[0], label="Elbow angle")
plt.plot(t_list[:-diff], back_traj[0], label="Back motion")
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec)/2)])
plt.grid()
plt.legend()
plt.title('Elbow Angle')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Arm Acceleration and Back Movement Vs. Time', fontsize=20)
plt.plot(t_list[:-diff], traj[2], label="Elbow acceleration")
plt.plot(t_list[:-diff], back_traj[0], label="Back motion")
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec)/2)])
plt.grid()
```

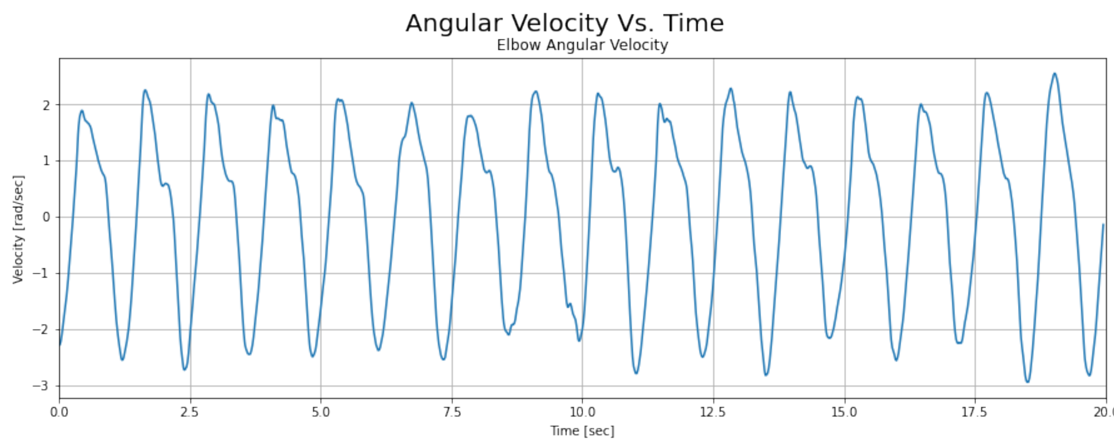
```
plt.legend()
plt.title('Elbow Acceleration')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Angles Vs. Time', fontsize=20)
plt.plot(t_list[:-diff], traj[0])
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Angle')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Angular Velocity Vs. Time', fontsize=20)
plt.plot(t_list[:-diff], traj[1])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Angular Velocity')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Angular Acceleration Vs. Time', fontsize=20)
plt.plot(t_list[:-diff], traj[2])
plt.ylabel('Acceleration [rad/sec^2]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Angular Acceleration')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Torque Vs. Time', fontsize=20)
plt.plot(t_list, tau_list)
plt.ylabel('Torque [Nm]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Torque')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Power Vs. Time', fontsize=20)
plt.plot(t_list, power_list)
```

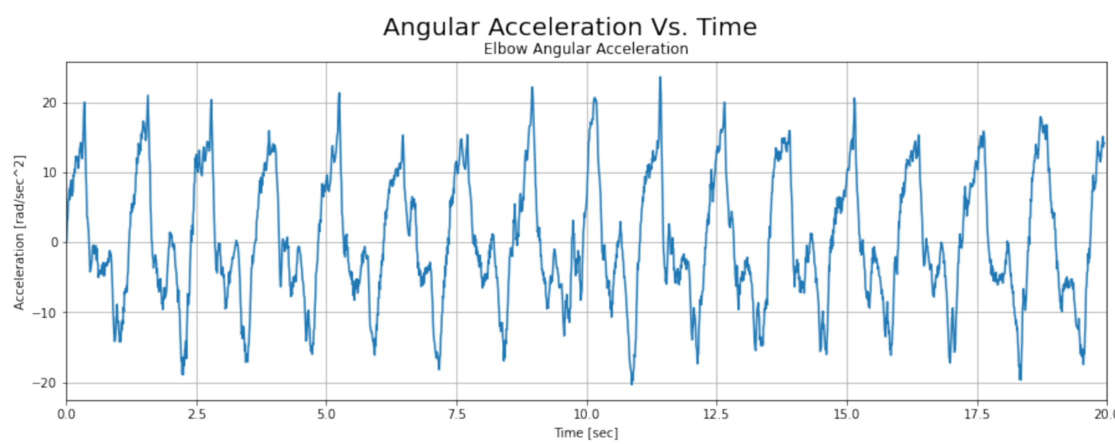
```
plt.ylabel('Power [W]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Power')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Speed Vs. Torque', fontsize=20)
plt.plot(tau_list[:-diff], traj[1])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Torque [Nm]')
plt.grid()
plt.title('Elbow Speed-Torque')
plt.show()
```

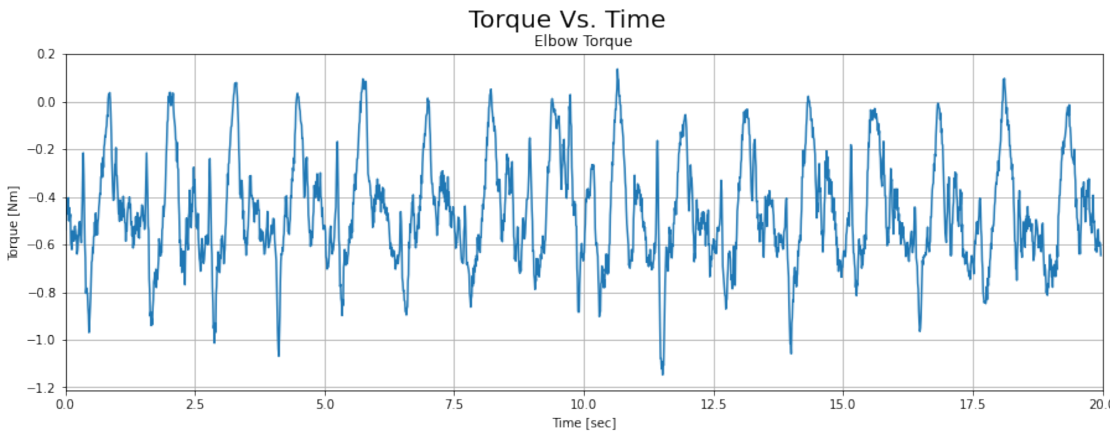


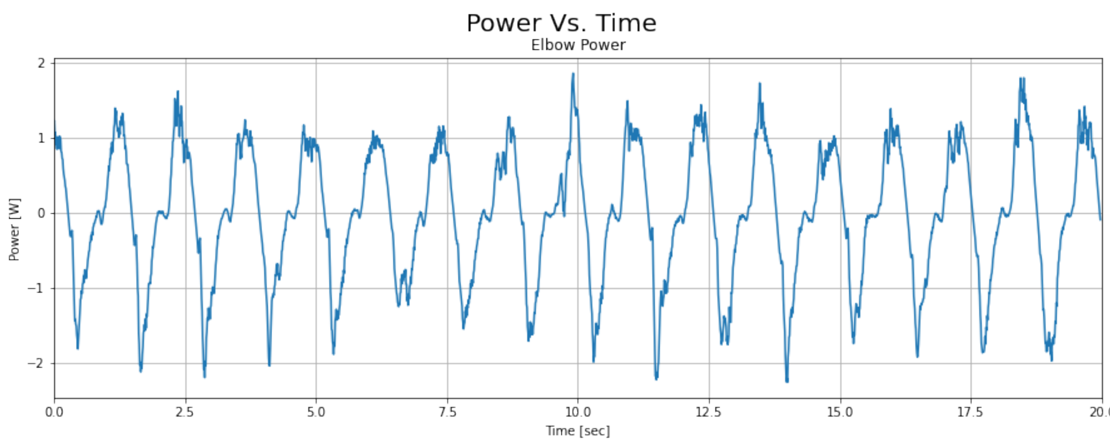


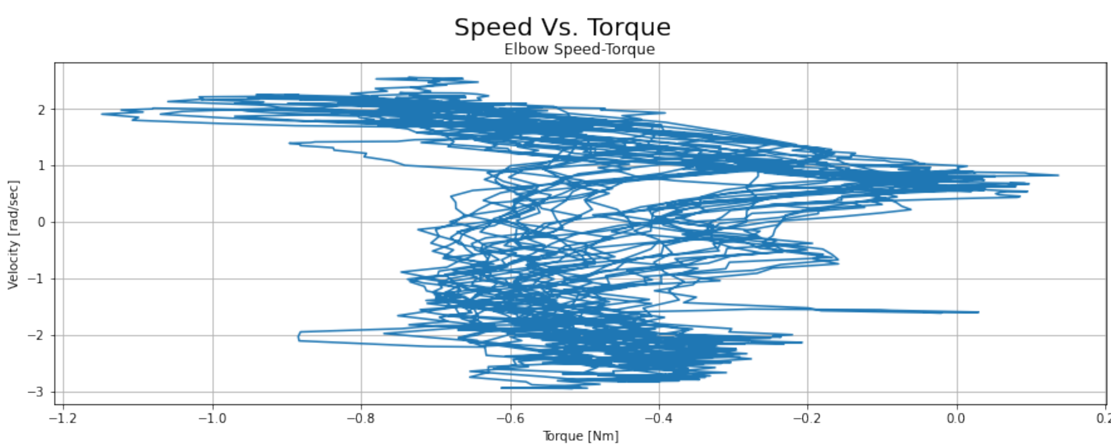

Angles Vs. Time
Elbow Angle
Angle [rad]
Time [sec]
0.2
0.0
−0.2
−0.4
−0.6
−0.8
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


Angular Velocity Vs. Time
Elbow Angular Velocity
Velocity [rad/sec]
Time [sec]
2
1
0
−1
−2
−3
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


Angular Acceleration Vs. Time
Elbow Angular Acceleration
Acceleration [rad/sec^2]
Time [sec]
20
10
0
−10
−20
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0







Animating the simulation:

```
[107]: def animate_double_pend(traj, L, L_c, T):
    """
    Function to generate web-based animation of double-pendulum system

    Parameters:
        traj:         trajectory of theta1 and theta2
        L:            length of the lower arm
        L_c:          length of the center of mass of the lower arm from the␣
 ↪elbow
        T:            length/seconds of animation duration

    Returns: None
    """

    # Browser configuration
    def configure_plotly_browser_state():
        import IPython
        display(IPython.core.display.HTML('''
            <script src="/static/components/requirejs/require.js"></script>
            <script>
              requirejs.config({
                paths: {
                  base: '/static/base',
                  plotly: 'https://cdn.plot.ly/plotly-1.5.1.min.js?noext',
                },
              });
            </script>
            '''))
    configure_plotly_browser_state()
    init_notebook_mode(connected=False)

    # Getting data from pendulum angle trajectories
    xx = L * np.sin(traj[0])
    yy = -L * np.cos(traj[0])
    xx_c = L_c * np.sin(traj[0])
    yy_c = -L_c * np.cos(traj[0])
    N = len(traj[0])

    # Using these to specify axis limits
    xm = np.min(xx)
    xM = np.max(xx)
    ym = np.min(yy) - 0.6
    yM = np.max(yy) + 0.6

    # Defining data dictionary
    data = [dict(x=xx, y=yy,
                 mode='lines', name='Arm',
```

```
                line=dict(width=5, color='blue')
               ),
           dict(x=xx_c, y=yy_c,
                mode='lines', name='Lower Arm Center of Mass',
                line=dict(width=2, color='green')
               ),
           dict(x=xx, y=yy,
                mode='markers', name='Elbow Trajectory',
                marker=dict(color="green", size=2)
               )
         ]

    # Preparing simulation layout
    layout = dict(xaxis=dict(range=[xm, xM], autorange=False,␣
↪zeroline=False,dtick=1),
                 yaxis=dict(range=[ym, yM], autorange=False,␣
↪zeroline=False,scaleanchor = "x",dtick=1),
                 title='Simulation of Arm Modeled as a Double Pendulum',
                 hovermode='closest',
                 updatemenus= [{'type': 'buttons',
                               'buttons': [{'label': 'Play', 'method':␣
↪'animate',
                                            'args': [None, {'frame':␣
↪{'duration': T, 'redraw': False}}]},
                                           {'args': [[None], {'frame':␣
↪{'duration': T, 'redraw': False}, 'mode': 'immediate',
                                            'transition': {'duration':␣
↪0}}],'label': 'Pause', 'method': 'animate'}
                                          ]
                               }]
                )

    # Defining the frames of the simulation
    frames = [dict(data=[dict(x=[0, xx[k]],
                            y=[0, yy[k]],
                            mode='lines',
                            line=dict(color='red', width=4)),
                        go.Scatter(
                            x=[xx_c[k]],
                            y=[yy_c[k]],
                            mode="markers",
                            marker=dict(color="blue", size=12))
                       ]) for k in range(N)]

    # Putting it all together and plotting
    figure = dict(data=data, layout=layout, frames=frames)
```

```
    iplot(figure)

# Animate the system
L = L_l
L_c = L_l_c
T = 5

animate_double_pend(partial_traj, L, L_c, T)
```

```
<IPython.core.display.HTML object>
```

1.5 System Modeling - Double Pendulum

Computing the Lagrangian of the system:

[108]:
```
m1, m2, g, R1, R1_c, R2, R2_c = symbols(r'm1, m2, g, R1, R1_c, R2, R2_c')

# The system torque variables as function of t
tau1 = Function(r'tau1')(t)
tau2 = Function(r'tau2')(t)

# The system configuration variables as function of t
theta1 = Function(r'theta1')(t)
theta2 = Function(r'theta2')(t)

# The velocity as derivative of position wrt t
theta1_dot = theta1.diff(t)
theta2_dot = theta2.diff(t)

# The acceleration as derivative of velocity wrt t
theta1_ddot = theta1_dot.diff(t)
theta2_ddot = theta2_dot.diff(t)

# Converting the polar coordinates to cartesian coordinates
x1 = R1_c * sin(theta1)
x2 = R1 * sin(theta1) + R2_c * sin(theta1 + theta2)

y1 = -R1_c * cos(theta1)
y2 = -R1 * cos(theta1) - R2_c * cos(theta1 + theta2)

# Calculating the kinetic and potential energy of the system
KE = 1/2 * m1 * ((x1.diff(t))**2 + (y1.diff(t))**2) + 1/2 * m2 * ((x2.
 ↪diff(t))**2 + (y2.diff(t))**2)
PE = m1 * g * y1 + m2 * g * y2

# Computing the Lagrangian
L = simplify(KE - PE)
Lagrange = Function(r'L')(t)
```

```
display(Eq(Lagrange, L))
```

$$L(t) = 0.5R_{1c}^2 m_1 \left(\frac{d}{dt}\theta_1(t)\right)^2 + R_{1c} g m_1 \cos\left(\theta_1(t)\right) + g m_2 \left(R_1 \cos\left(\theta_1(t)\right) + R_{2c} \cos\left(\theta_1(t) + \theta_2(t)\right)\right) +$$
$$0.5 m_2 \left(R_1^2 \left(\frac{d}{dt}\theta_1(t)\right)^2 + 2R_1 R_{2c} \cos\left(\theta_2(t)\right) \left(\frac{d}{dt}\theta_1(t)\right)^2 + 2R_1 R_{2c} \cos\left(\theta_2(t)\right) \frac{d}{dt}\theta_1(t) \frac{d}{dt}\theta_2(t) + R_{2c}^2 \left(\frac{d}{dt}\theta_1(t)\right)^2 + 2R\right.$$

Computing the Euler-Lagrange equations:

[109]:
```
# Define the derivative of L wrt the functions: x, xdot
L_dtheta1 = L.diff(theta1)
L_dtheta2 = L.diff(theta2)

L_dtheta1_dot = L.diff(theta1_dot)
L_dtheta2_dot = L.diff(theta2_dot)

# Define the derivative of L_dxdot wrt to time t
L_dtheta1_dot_dt = L_dtheta1_dot.diff(t)
L_dtheta2_dot_dt = L_dtheta2_dot.diff(t)

# Define the left hand side of the the Euler-Lagrange as a matrix
lhs = Matrix([simplify(L_dtheta1_dot_dt - L_dtheta1),
              simplify(L_dtheta2_dot_dt - L_dtheta2)])

# Define the right hand side of the the Euler-Lagrange as a Matrix
rhs = Matrix([tau1, tau2])

# Compute the Euler-Lagrange equations as a matrix
EL_eqns = Eq(lhs, rhs)

print('Euler-Lagrange matrix for this systems:')
display(EL_eqns)
```

```
Euler-Lagrange matrix for this systems:
```

$$\left[\begin{matrix} 1.0R_{1c}^2 m_1 \frac{d^2}{dt^2}\theta_1(t) + R_{1c} g m_1 \sin\left(\theta_1(t)\right) + g m_2 \left(R_1 \sin\left(\theta_1(t)\right) + R_{2c} \sin\left(\theta_1(t) + \theta_2(t)\right)\right) + m_2 \left(R_1^2 \frac{d^2}{dt^2}\theta_1(t) - 2R_1 R_{2c} \right. \\ R_{2c} m_2 \left(R_1 \sin\left(\theta_2(t)\right) \left(\frac{d}{dt}\theta_1(t)\right)^2 + R_1 \cos \right. \end{matrix}\right.$$
$$\left[\begin{matrix}\tau_1(t) \\ \tau_2(t)\end{matrix}\right]$$

Solve the equations for τ_1 and τ_2:

[110]:
```
# Solve the Euler-Lagrange equations for the shoulder and elbow torques
T = Matrix([tau1, tau2])
soln = solve(EL_eqns, T, dict=True)

# Initialize the solutions
solution = [0, 0]
```

```
i = 0

for sol in soln:
    for v in T:
        solution[i] = simplify(sol[v])
        display(Eq(T[i], solution[i]))
        i =+ 1
```

$$\tau_1(t) = R_1^2 m_2 \frac{d^2}{dt^2}\theta_1(t) - 2.0 R_1 R_{2c} m_2 \sin(\theta_2(t)) \frac{d}{dt}\theta_1(t) \frac{d}{dt}\theta_2(t) - R_1 R_{2c} m_2 \sin(\theta_2(t)) \left(\frac{d}{dt}\theta_2(t)\right)^2 + 2.0 R_1 R_{2c} m_2 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_1(t) + R_1 R_{2c} m_2 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_2(t) + R_1 g m_2 \sin(\theta_1(t)) + R_{1c}^2 m_1 \frac{d^2}{dt^2}\theta_1(t) + R_{1c} g m_1 \sin(\theta_1(t)) + R_{2c}^2 m_2 \frac{d^2}{dt^2}\theta_1(t) + R_{2c}^2 m_2 \frac{d^2}{dt^2}\theta_2(t) + R_{2c} g m_2 \sin(\theta_1(t) + \theta_2(t))$$

$$\tau_2(t) = R_{2c} m_2 \left(R_1 \sin(\theta_2(t)) \left(\frac{d}{dt}\theta_1(t)\right)^2 + R_1 \cos(\theta_2(t)) \frac{d^2}{dt^2}\theta_1(t) + R_{2c} \frac{d^2}{dt^2}\theta_1(t) + R_{2c} \frac{d^2}{dt^2}\theta_2(t) + g \sin(\theta_1(t) + \theta_2(t$$

Simulating the system:

[113]:
```
# Substitute the derivative variables with a dummy variables and plug-in the␣
↪constants
solution_0_subs = solution[0]
solution_1_subs = solution[1]

theta1_dot_dummy = symbols('thetadot1')
theta2_dot_dummy = symbols('thetadot2')
theta1_ddot_dummy = symbols('thetaddot1')
theta2_ddot_dummy = symbols('thetaddot2')

solution_0_subs = solution_0_subs.subs([(g, 9.81)])
solution_1_subs = solution_1_subs.subs([(g, 9.81)])

solution_0_subs = solution_0_subs.subs([((theta1.diff(t)).diff(t),␣
↪theta1_ddot_dummy),
                                        ((theta2.diff(t)).diff(t),␣
↪theta2_ddot_dummy)])
solution_1_subs = solution_1_subs.subs([((theta1.diff(t)).diff(t),␣
↪theta1_ddot_dummy),
                                        ((theta2.diff(t)).diff(t),␣
↪theta2_ddot_dummy)])

solution_0_subs = solution_0_subs.subs([(theta1.diff(t), theta1_dot_dummy),
                                        (theta2.diff(t), theta2_dot_dummy)])
solution_1_subs = solution_1_subs.subs([(theta1.diff(t), theta1_dot_dummy),
                                        (theta2.diff(t), theta2_dot_dummy)])

# Lambdify the thetas and its derivatives
```

```
func1 = lambdify([theta1, theta2, theta1_dot_dummy, theta2_dot_dummy,␣
↪theta1_ddot_dummy,
                  theta2_ddot_dummy, m1, m2, R1, R2, R1_c, R2_c],␣
↪solution_0_subs, modules = sympy)
func2 = lambdify([theta1, theta2, theta1_dot_dummy, theta2_dot_dummy,␣
↪theta1_ddot_dummy,
                  theta2_ddot_dummy, m1, m2, R1, R2, R1_c, R2_c],␣
↪solution_1_subs, modules = sympy)

# Initialize the torque and power lists
sholder_tau_list, elbow_tau_list = [], []
sholder_current_list, elbow_current_list = [], []
sholder_power_list, elbow_power_list = [], []

motor_kv = 115
torque_const = 8.27 / motor_kv

for i in range(len(time_list)):
    # Initialize the torque and power lists
    tau1_list, tau2_list = [], []
    current1_list, current2_list = [], []
    power1_list, power2_list = [], []

    t_list = time_list[i]
    theta1_list = sholder_ang_list[i]
    theta2_list = elbow_ang_list[i]
    dtheta1_list = sholder_vel_list[i]
    dtheta2_list = elbow_vel_list[i]
    ddtheta1_list = sholder_acc_list[i]
    ddtheta2_list = elbow_acc_list[i]

    # Plug-in the angles, angular velocities and angular accelerations for␣
↪every time step to find the torques
    for j in range(len(t_list)):
        tau1_list.append(func1(theta1_list[j], theta2_list[j],
                               dtheta1_list[j], dtheta2_list[j],
                               ddtheta1_list[j], ddtheta2_list[j],
                               m_u, m_l, L_u, L_l, L_u_c, L_l_c))

        tau2_list.append(func2(theta1_list[j], theta2_list[j],
                               dtheta1_list[j], dtheta2_list[j],
                               ddtheta1_list[j], ddtheta2_list[j],
                               m_u, m_l, L_u, L_l, L_u_c, L_l_c))

        # Calculate the current required to reach the required joints torques␣
↪for every time step
```

```
        current1_list.append(torque_const * tau1_list[j])
        current2_list.append(torque_const * tau2_list[j])

        # Calculate the power required to reach the required angular velocities␣
↪and joints torques for every time step
        power1_list.append(dtheta1_list[j] * tau1_list[j])
        power2_list.append(dtheta2_list[j] * tau2_list[j])

    sholder_tau_list.append(tau1_list)
    elbow_tau_list.append(tau2_list)
    sholder_current_list.append(current1_list)
    elbow_current_list.append(current2_list)
    sholder_power_list.append(power1_list)
    elbow_power_list.append(power2_list)

    print(f"Velocity {walking_vel_list[i]}\t max torque:␣
↪{format(max(tau2_list), '.3f')}[Nm]\t max angular velocity:␣
↪{format(max(dtheta2_list), '.3f')}[rad/sec]\t max power:␣
↪{format(max(power2_list), '.3f')}[W]")
```

```
Velocity 0.5ms   max torque: 0.228[Nm]   max angular velocity: 1.625[rad/sec]
max power: 1.693[W]
Velocity 0.6ms   max torque: 0.332[Nm]   max angular velocity: 1.731[rad/sec]
max power: 1.798[W]
Velocity 0.7ms   max torque: 0.552[Nm]   max angular velocity: 2.556[rad/sec]
max power: 2.379[W]
Velocity 0.8ms   max torque: 0.900[Nm]   max angular velocity: 3.695[rad/sec]
max power: 4.734[W]
Velocity 0.9ms   max torque: 1.497[Nm]   max angular velocity: 3.370[rad/sec]
max power: 4.650[W]
Velocity 1.0ms   max torque: 3.806[Nm]   max angular velocity: 3.588[rad/sec]
max power: 12.103[W]
Velocity 1.1ms   max torque: 1.901[Nm]   max angular velocity: 2.989[rad/sec]
max power: 3.543[W]
Velocity 1.2ms   max torque: 2.264[Nm]   max angular velocity: 3.567[rad/sec]
max power: 3.809[W]
Velocity 1.3ms   max torque: 3.269[Nm]   max angular velocity: 4.525[rad/sec]
max power: 5.310[W]
Velocity 1.4ms   max torque: 4.890[Nm]   max angular velocity: 4.931[rad/sec]
max power: 8.735[W]
Velocity chang   max torque: 2.238[Nm]   max angular velocity: 3.920[rad/sec]
max power: 6.652[W]
```

Calculation summary:

[116]:
```
max_elbow_tau, max_elbow_power, max_elbow_vel = 0, 0, 0
max_elbow_tau_index, max_elbow_power_index, max_elbow_vel_index = 0, 0, 0
```

```
for i in range(len(elbow_tau_list)):
    if max_elbow_vel < max(elbow_vel_list[i]):
        max_elbow_vel = max(elbow_vel_list[i])
        max_elbow_vel_index = i

    if max_elbow_tau < max(elbow_tau_list[i]):
        max_elbow_tau = max(elbow_tau_list[i])
        max_elbow_tau_index = i

    if max_elbow_power < max(elbow_power_list[i]):
        max_elbow_power = max(elbow_power_list[i])
        max_elbow_power_index = i

print(f"maximum elbow angular velocity is {format(max_elbow_vel, '.3f')} [rad/
 ↪sec] ({format(max_elbow_vel*60/(2*pi), '.3f')} [rpm]), in velocity␣
 ↪{walking_vel_list[max_elbow_vel_index]} (trial {max_elbow_vel_index})")
print(f"maximum elbow torque is {format(max_elbow_tau, '.3f')} [Nm], in␣
 ↪velocity {walking_vel_list[max_elbow_tau_index]} (trial␣
 ↪{max_elbow_tau_index})")
print(f"maximum elbow power is {format(max_elbow_power, '.3f')} [W], in␣
 ↪velocity {walking_vel_list[max_elbow_power_index]} (trial␣
 ↪{max_elbow_power_index})")


# The torque equations for the maximum power:
solution_0_subs = solution_0_subs.subs([(m1, m_u), (m2, m_l), (R1, L_u), (R2,␣
 ↪L_l), (R1_c, L_u_c), (R2_c, L_l_c), (g, 9.81)])
solution_1_subs = solution_1_subs.subs([(m1, m_u), (m2, m_l), (R1, L_u), (R2,␣
 ↪L_l), (R1_c, L_u_c), (R2_c, L_l_c), (g, 9.81)])

print("\nThe torque equations for the maximum torque:")
display(Eq(T[0], solution_0_subs))
display(Eq(T[1], solution_1_subs))

# display(Elbow_Ang_list[max_Elbow_tau_index])
# display(Elbow_Vel_list[max_Elbow_tau_index])
# display(Elbow_Acc_list[max_Elbow_tau_index])
# display(Elbow_tau_list[max_Elbow_tau_index])
# display(Elbow_Ang_list[2])
# display(Elbow_tau_list[2])
```

```
maximum elbow angular velocity is 4.931 [rad/sec] (47.091 [rpm]), in velocity
1.4ms (trial 9)
maximum elbow torque is 4.890 [Nm], in velocity 1.4ms (trial 9)
maximum elbow power is 12.103 [W], in velocity 1.0ms (trial 5)

The torque equations for the maximum torque:
```

$\tau_1(t) = 0.1064217644464\ddot{\theta}_1 \cos(\theta_2(t)) + 0.1349254238316621\ddot{\theta}_1 + 0.0532108822232\ddot{\theta}_2 \cos(\theta_2(t)) + 0.042170312288\ddot{\theta}_2 - 0.1064217644464\dot{\theta}_1\dot{\theta}_2 \sin(\theta_2(t)) - 0.0532108822232\dot{\theta}_2^2 \sin(\theta_2(t)) + 1.732373406 \sin(\theta_1(t) + \theta_2(t)) + 4.0984945085808 \sin(\theta_1(t))$

$\tau_2(t) = 0.0532108822232\ddot{\theta}_1 \cos(\theta_2(t)) + 0.042170312288\ddot{\theta}_1 + 0.042170312288\ddot{\theta}_2 + 0.0532108822232\dot{\theta}_1^2 \sin(\theta_2(t)) + 1.732373406 \sin(\theta_1(t) + \theta_2(t))$

Example for the trial with the largest elbow torque & power:

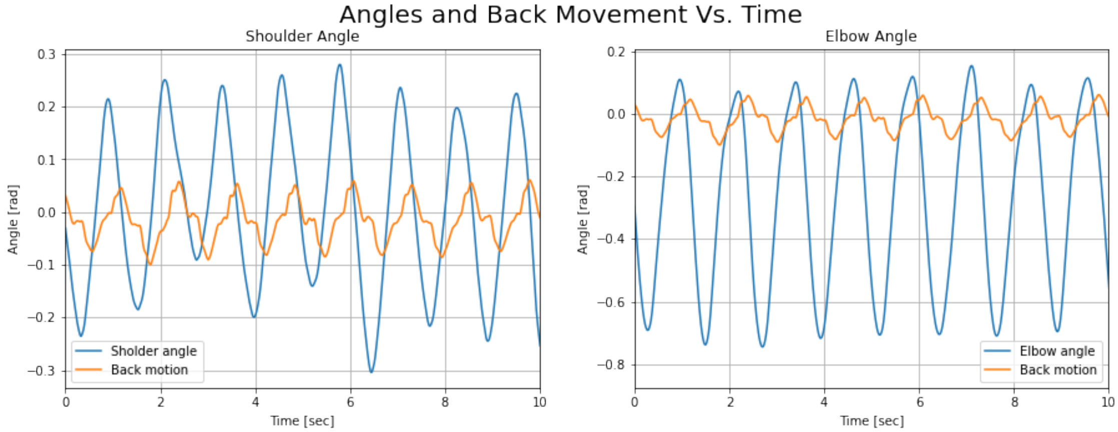
[118]:
```
index = 2
t_list = time_list[index]
theta1_list = sholder_ang_list[index]
theta2_list = elbow_ang_list[index]
dtheta1_list = sholder_vel_list[index]
dtheta2_list = elbow_vel_list[index]
ddtheta1_list = sholder_acc_list[index]
ddtheta2_list = elbow_acc_list[index]
tau1_list = sholder_tau_list[index]
tau2_list = elbow_tau_list[index]
current1_list = sholder_current_list[index]
current2_list = elbow_current_list[index]
power1_list = sholder_power_list[index]
power2_list = elbow_power_list[index]

back_rotation_list = back_ang_list[index]
back_position_list = back_pos_list[index]
back_velocity_list = back_vel_list[index]

elbow_acceleration_list = elbow_acc_data_list[index]
sholder_acceleration_list = sholder_acc_data_list[index]

# Compute the trajectory of the arm's motion
N = int((max(t_list) - min(t_list))/(1/frame_frequency))
tvec = np.linspace(min(t_list), max(t_list), N)
traj = np.zeros((6, N))
back_traj = np.zeros((3, N))
acc_traj = np.zeros((2, N))
partial_traj = np.zeros((6, N))

for i in range(N):
    traj[0, i] = theta1_list[i]
    traj[1, i] = theta2_list[i]
    traj[2, i] = dtheta1_list[i]
    traj[3, i] = dtheta2_list[i]
    traj[4, i] = ddtheta1_list[i]
    traj[5, i] = ddtheta2_list[i]
```

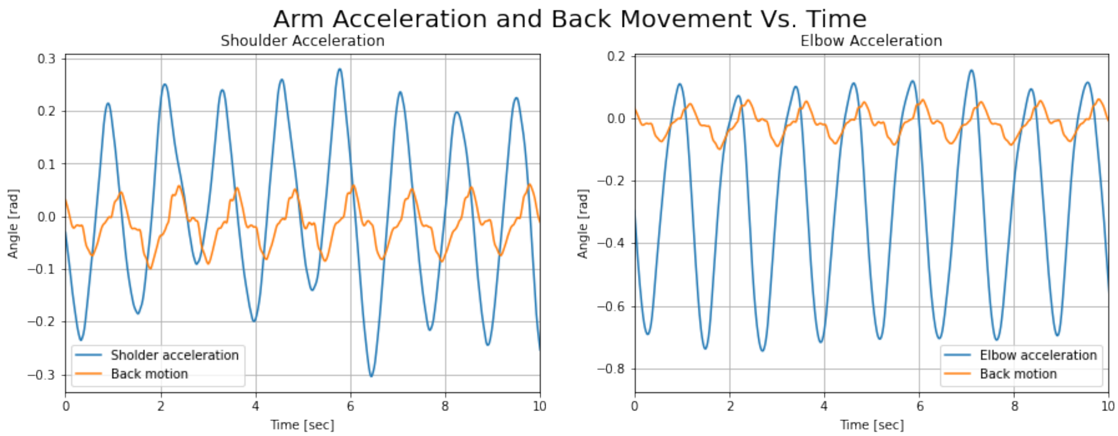
```
    back_traj[0, i] = back_rotation_list[i]
    back_traj[1, i] = back_position_list[i]
    back_traj[2, i] = back_velocity_list[i]

    acc_traj[0, i] = elbow_acceleration_list[i]
    acc_traj[1, i] = sholder_acceleration_list[i]

for i in range(100):
    partial_traj[0, i] = theta1_list[i]
    partial_traj[1, i] = theta2_list[i]
    partial_traj[2, i] = dtheta1_list[i]
    partial_traj[3, i] = dtheta2_list[i]
    partial_traj[4, i] = ddtheta1_list[i]
    partial_traj[5, i] = ddtheta2_list[i]

# Calculate the length difference between the time list and the trajectory lists
diff = (len(t_list) - len(traj[0]))

# Plot the trajectory lists (angles, velocities, accelerations, torques, and␣
 ↪power)
plt.figure(figsize=(15,5))
plt.suptitle('Angles and Back Movement Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[0], label="Sholder angle")
plt.plot(t_list[:-diff], back_traj[0], label="Back motion")
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec)/2)])
plt.grid()
plt.legend()
plt.title('Shoulder Angle')

plt.subplot(122)
plt.plot(t_list[:-diff], traj[1], label="Elbow angle")
plt.plot(t_list[:-diff], back_traj[0], label="Back motion")
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec)/2)])
plt.grid()
plt.legend()
plt.title('Elbow Angle')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Arm Acceleration and Back Movement Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[0], label="Sholder acceleration")
```

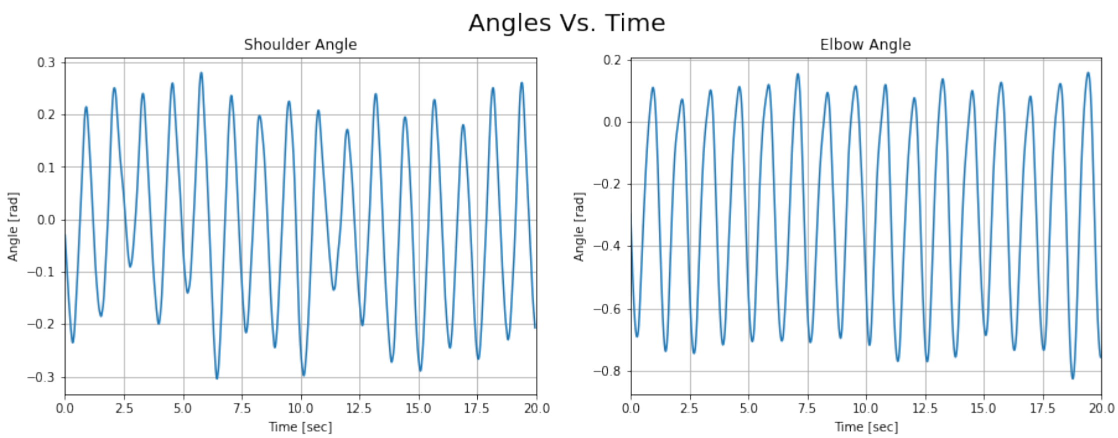
```
plt.plot(t_list[:-diff], back_traj[0], label="Back motion")
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec)/2)])
plt.grid()
plt.legend()
plt.title('Shoulder Acceleration')

plt.subplot(122)
plt.plot(t_list[:-diff], traj[1], label="Elbow acceleration")
plt.plot(t_list[:-diff], back_traj[0], label="Back motion")
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec)/2)])
plt.grid()
plt.legend()
plt.title('Elbow Acceleration')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Angles Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[0])
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Shoulder Angle')

plt.subplot(122)
plt.plot(t_list[:-diff], traj[1])
plt.ylabel('Angle [rad]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Angle')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Angular Velocity Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[2])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Shoulder Angular Velocity')
```

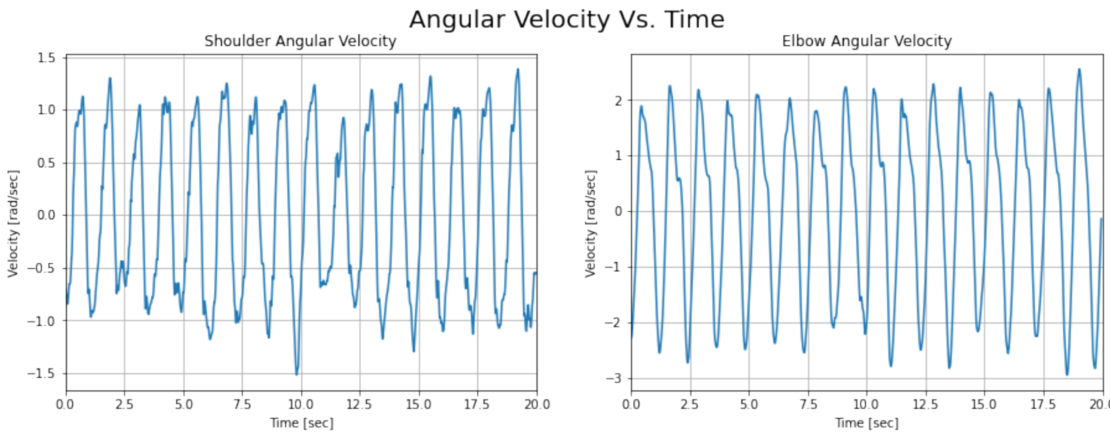
```
plt.subplot(122)
plt.plot(t_list[:-diff], traj[3])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Angular Velocity')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Angular Acceleration Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list[:-diff], traj[4])
plt.ylabel('Acceleration [rad/sec^2]')
plt.xlabel('Time [sec]')
plt.grid()
plt.title('Shoulder Angular Acceleration')

plt.subplot(122)
plt.plot(t_list[:-diff], traj[5])
plt.ylabel('Acceleration [rad/sec^2]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Angular Acceleration')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Torque Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list, tau1_list)
plt.ylabel('Torque [Nm]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Shoulder Torque')

plt.subplot(122)
plt.plot(t_list, tau2_list)
plt.ylabel('Torque [Nm]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Torque')
plt.show()
```

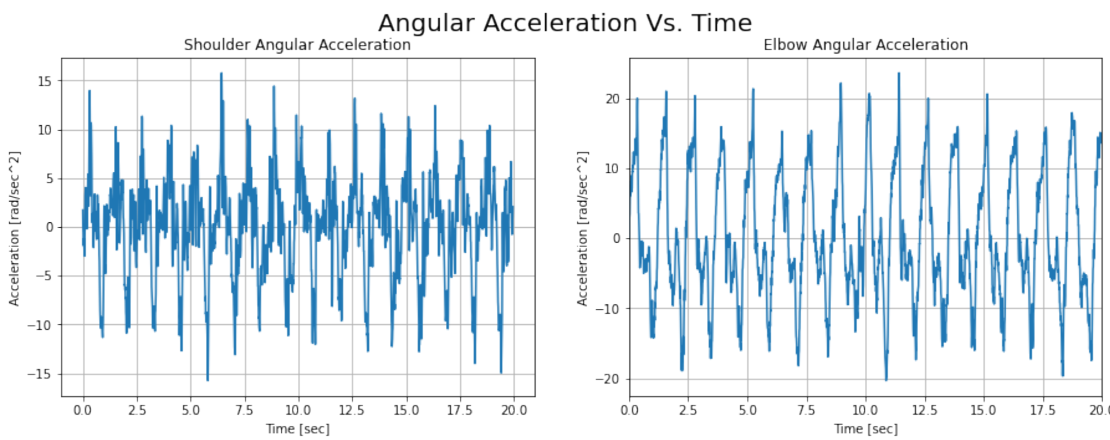
```
plt.figure(figsize=(15,5))
plt.suptitle('Power Vs. Time', fontsize=20)
plt.subplot(121)
plt.plot(t_list, power1_list)
plt.ylabel('Power [W]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Shoulder Power')

plt.subplot(122)
plt.plot(t_list, power2_list)
plt.ylabel('Power [W]')
plt.xlabel('Time [sec]')
plt.xlim([0, math.ceil(max(tvec))])
plt.grid()
plt.title('Elbow Power')
plt.show()

plt.figure(figsize=(15,5))
plt.suptitle('Speed Vs. Torque', fontsize=20)
plt.subplot(121)
plt.plot(tau1_list[:-diff], traj[2])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Torque [Nm]')
plt.grid()
plt.title('Shoulder Speed-Torque')

plt.subplot(122)
plt.plot(tau2_list[:-diff], traj[3])
plt.ylabel('Velocity [rad/sec]')
plt.xlabel('Torque [Nm]')
plt.grid()
plt.title('Elbow Speed-Torque')
plt.show()
```

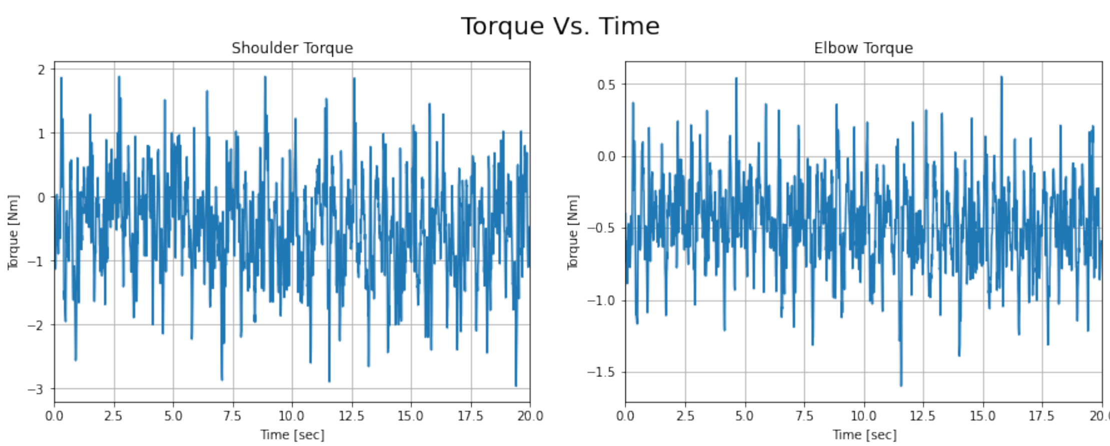

Angles and Back Movement Vs. Time
Shoulder Angle
Elbow Angle
Angle [rad]
Time [sec]
Sholder angle
Back motion
Elbow angle
Back motion

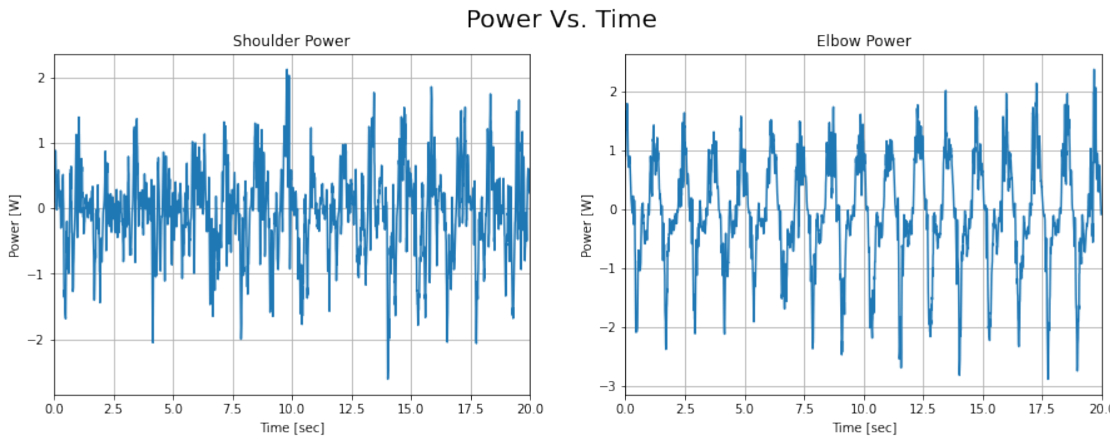

Arm Acceleration and Back Movement Vs. Time
Shoulder Acceleration
Elbow Acceleration
Angle [rad]
Time [sec]
Sholder acceleration
Back motion
Elbow acceleration
Back motion

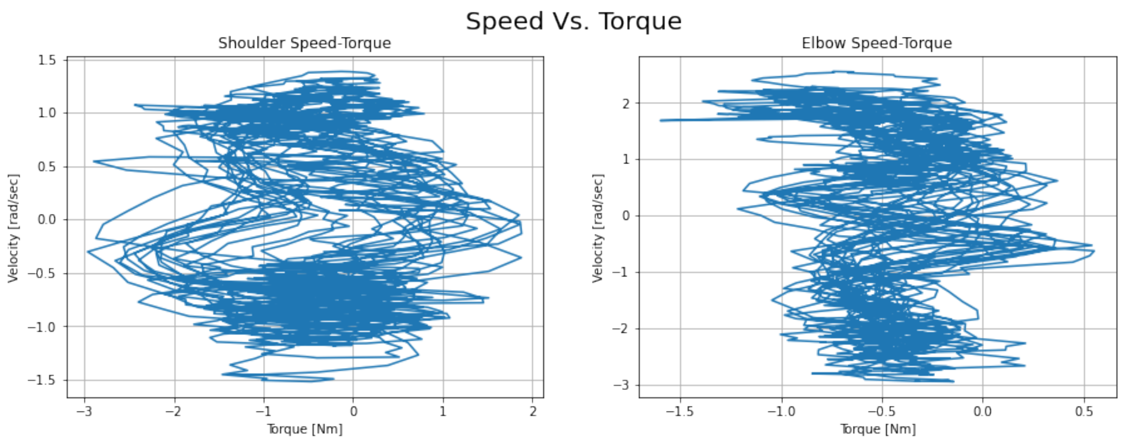

Angles Vs. Time
Shoulder Angle
Elbow Angle
Angle [rad]
Time [sec]


Angular Velocity Vs. Time
Shoulder Angular Velocity
Velocity [rad/sec]
Time [sec]
Elbow Angular Velocity
Velocity [rad/sec]
Time [sec]


Angular Acceleration Vs. Time
Shoulder Angular Acceleration
Acceleration [rad/sec^2]
Time [sec]
Elbow Angular Acceleration
Acceleration [rad/sec^2]
Time [sec]


Torque Vs. Time
Shoulder Torque
Torque [Nm]
Time [sec]
Elbow Torque
Torque [Nm]
Time [sec]





Animating the simulation:

[119]:
```
def animate_double_pend(traj, L1, L2, L1_c, L2_c, T):
    """
    Function to generate web-based animation of double-pendulum system

    Parameters:
        traj:         trajectory of theta1 and theta2
        L1:           length of the upper arm
        L2:           length of the lower arm
        L1_c:         length of the center of mass of the upper arm from the␣
↪shoulder
        L2_c:         length of the center of mass of the lower arm from the␣
↪elbow
        T:            length/seconds of animation duration

    Returns: None
```

```python
"""

# Browser configuration
def configure_plotly_browser_state():
    import IPython
    display(IPython.core.display.HTML('''
        <script src="/static/components/requirejs/require.js"></script>
        <script>
          requirejs.config({
            paths: {
              base: '/static/base',
              plotly: 'https://cdn.plot.ly/plotly-1.5.1.min.js?noext',
            },
          });
        </script>
        '''))
configure_plotly_browser_state()
init_notebook_mode(connected=False)

# Getting data from pendulum angle trajectories
xx1 = L1 * np.sin(traj[0])
yy1 = -L1 * np.cos(traj[0])
xx1_c = L1_c * np.sin(traj[0])
yy1_c = -L1_c * np.cos(traj[0])
xx2 = xx1 + L2 * np.sin(traj[0] + traj[1])
yy2 = yy1 - L2 * np.cos(traj[0] + traj[1])
xx2_c = xx1 + L2_c * np.sin(traj[0] + traj[1])
yy2_c = yy1 - L2_c * np.cos(traj[0] + traj[1])
N = len(traj[0])

# Using these to specify axis limits
xm = np.min(xx1)
xM = np.max(xx1)
ym = np.min(yy1) - 0.6
yM = np.max(yy1) + 0.6

# Defining data dictionary
data = [dict(x=xx1, y=yy1,
             mode='lines', name='Arm',
             line=dict(width=5, color='blue')
            ),
        dict(x=xx1_c, y=yy1_c,
             mode='lines', name='Upper Arm Center of Mass',
             line=dict(width=2, color='green')
            ),
        dict(x=xx2_c, y=yy2_c,
             mode='lines', name='Lower Arm Center of Mass',
```

```
                line=dict(width=2, color='orange')
               ),
           dict(x=xx1, y=yy1,
                mode='markers', name='Elbow Trajectory',
                marker=dict(color="green", size=2)
               ),
           dict(x=xx2, y=yy2,
                mode='markers', name='Hand Trajectory',
                marker=dict(color="orange", size=2)
               )
         ]

   # Preparing simulation layout
   layout = dict(xaxis=dict(range=[xm, xM], autorange=False,␣
↪zeroline=False,dtick=1),
                 yaxis=dict(range=[ym, yM], autorange=False,␣
↪zeroline=False,scaleanchor = "x",dtick=1),
                 title='Simulation of Arm Modeled as a Double Pendulum',
                 hovermode='closest',
                 updatemenus= [{'type': 'buttons',
                               'buttons': [{'label': 'Play', 'method':␣
↪'animate',
                                            'args': [None, {'frame':␣
↪{'duration': T, 'redraw': False}}]},
                                           {'args': [[None], {'frame':␣
↪{'duration': T, 'redraw': False}, 'mode': 'immediate',
                                            'transition': {'duration':␣
↪0}}],'label': 'Pause', 'method': 'animate'}
                                          ]
                               }]
                )

   # Defining the frames of the simulation
   frames = [dict(data=[dict(x=[0,xx1[k],xx2[k]],
                             y=[0,yy1[k],yy2[k]],
                             mode='lines',
                             line=dict(color='red', width=4)),
                        go.Scatter(
                             x=[xx1_c[k]],
                             y=[yy1_c[k]],
                             mode="markers",
                             marker=dict(color="blue", size=12)),
                        go.Scatter(
                             x=[xx2_c[k]],
                             y=[yy2_c[k]],
                             mode="markers",
```

```
                              marker=dict(color="purple", size=12)),
                        ]) for k in range(N)]

    # Putting it all together and plotting
    figure = dict(data=data, layout=layout, frames=frames)
    iplot(figure)

# Animate the system
L1 = L_u
L2 = L_l
L1_c = L_u_c
L2_c = L_l_c
T = 5

animate_double_pend(partial_traj, L1, L2, L1_c, L2_c, T)
```

```
<IPython.core.display.HTML object>
```

[ ]:

```

```