

1 $\frac{dy}{dx} = y(x) = \frac{1}{2} \sin(2x) - \frac{1}{3} \cos(3x) + \frac{4}{3}$; $0 \leq x \leq 1$; $y(0) = 1$ can $h = .25$

$$\frac{dy}{dx} = \frac{1}{2} \sin(2x) - \frac{1}{3} \cos(3x) + \frac{4}{3} \Rightarrow y = -\frac{1}{4} \cos(2x) - \frac{1}{9} \sin(3x) + \frac{4}{3}x + C$$

$$1 = -\frac{1}{4} \cos(0) - \frac{1}{9} \sin(0) + \frac{4}{3}(0) + C \Rightarrow 1 = -\frac{1}{4} + C \Rightarrow C = \frac{5}{4}$$

$$y = -\frac{1}{4} \cos(2x) - \frac{1}{9} \sin(3x) + \frac{4}{3}x + \frac{5}{4}$$

	y Euler	y Ver. de 1 ^o ord
0	1	1
	$y_0 = 1$	$(-\frac{1}{4} \cos(2 \cdot 0)) + (-\frac{1}{9} \sin(3 \cdot 0)) + \frac{4}{3}(0) + \frac{5}{4}$
.25	1.25	1.2822
	$1 + .25((\frac{1}{2} \sin(2 \cdot 0)) + (\frac{1}{3} \cos(3 \cdot 0)) + \frac{4}{3})$	$(-\frac{1}{4} \cos(.5)) + (-\frac{1}{9} \sin(.75)) + \frac{4}{3} + \frac{5}{4}$
.5	1.5823	1.6708
	$1.25 + .25((\frac{1}{2} \sin(2 \cdot .25)) + (\frac{1}{3} \cos(3 \cdot .25)) + \frac{4}{3})$	$(-\frac{1}{4} \cos(1)) + (-\frac{1}{9} \sin(1.5)) + .66 + \frac{5}{4}$
.75	2.0149	2.1454
	$1.5823 + .25((\frac{1}{2} \sin(1)) + (\frac{1}{3} \cos(1.5)) + \frac{4}{3})$	$(-\frac{1}{4} \cos(1.5)) + (-\frac{1}{9} \sin(\frac{4.5}{4})) + 1 + \frac{5}{4}$
1	2.5253	2.6717
	$2.0149 + .25((\frac{1}{2} \sin(1.5)) + (\frac{1}{3} \cos(2.25)) + \frac{4}{3})$	$(-\frac{1}{4} \cos(2)) + (-\frac{1}{9} \sin(3)) + \frac{4}{3} + \frac{5}{4}$

2. $\frac{1}{10} \int_0^1 (x+2x^2) dx = \frac{1}{10}$

	Y_{Euler}	$Y_{midpoint}$
0	0	0
	$2 \cdot 0$	$2 \cdot (0 + 2(0)^2) \cdot \frac{1}{10} = 0$
25	1.25	1.25
	$1.25 + 2(1 + 2(0.25)^2) \cdot \frac{1}{10}$	$2 \cdot (1.25 + 2(0.25)^2) \cdot \frac{1}{10} = 1$
5	1.2575	1.2525
	$1.2575 + 2(1.25 + 2(0.25)^2) \cdot \frac{1}{10}$	$2 \cdot (1.25 + 2(0.25)^2) \cdot \frac{1}{10} = 1$
25	2.2575	2.2525
	$2.2575 + 2(2.25 + 2(0.25)^2) \cdot \frac{1}{10}$	$2 \cdot (2.25 + 2(0.25)^2) \cdot \frac{1}{10} = 1$
5	4.2575	4.2525
	$4.2575 + 2(4.25 + 2(0.25)^2) \cdot \frac{1}{10}$	$2 \cdot (4.25 + 2(0.25)^2) \cdot \frac{1}{10} = 1$
25	6.2575	6.2525
	$6.2575 + 2(6.25 + 2(0.25)^2) \cdot \frac{1}{10}$	$2 \cdot (6.25 + 2(0.25)^2) \cdot \frac{1}{10} = 1$

3. $\frac{dy}{dx} = \frac{1}{1+x^2} - 2x^2, 0 \leq x \leq 1, y(0) = 1, y(1) = 0.5$
 $\int \frac{1}{1+x^2} - 2x^2 dx = \tan^{-1}(x) - 2x^3 + C$ $\Rightarrow \tan^{-1}(1) - 2(1)^3 + C = 0.5 \Rightarrow C = 0.5$

	Y_{Euler}	$Y_{midpoint}$	ER%		Y_{Euler}	$Y_{midpoint}$	ER%
				0			
				1	1	0.997	0.003%
0	0	0		2	0.9970	0.997	0.003%
2	1	0.997	0.003%	3	2.994	2.990	0.003%
4	3.988	3.985	0.003%	4	4.988	4.984	0.003%
6	5.982	5.980	0.003%	5	5.982	5.978	0.003%
8	7.976	7.974	0.003%	6	6.976	6.972	0.003%
1	8.970	8.968	0.003%	7	7.970	7.966	0.003%
				8	8.964	8.960	0.003%
				9	9.958	9.954	0.003%
				10	10.952	10.948	0.003%

	y L.L.	y u.L.L.	E %		y L.L.	y u.L.L.	E %
0	0	0		15	984	984	0.00%
0.1	0.1	0.1	0.05%	16	987	987	0.00%
1	0.975	0.975	0.05%	17	989	989	0.00%
11	0.971	0.971	0.05%	18	990	990	0.00%
20	0.972	0.972	0.05%	19	992	992	0.00%
20	0.974	0.974	0.05%	20	995	995	0.00%
20	0.975	0.975	0.05%	21	996	996	0.00%
20	0.975	0.975	0.05%	22	997	997	0.00%
20	0.975	0.975	0.05%	23	998	998	0.00%
20	0.975	0.975	0.05%	24	999	999	0.00%
20	0.975	0.975	0.05%	25	1000	1000	0.00%
20	0.975	0.975	0.05%	26	1000	1000	0.00%
20	0.975	0.975	0.05%	27	1000	1000	0.00%
20	0.975	0.975	0.05%	28	1000	1000	0.00%
20	0.975	0.975	0.05%	29	1000	1000	0.00%
20	0.975	0.975	0.05%	30	1000	1000	0.00%

4 $\frac{dy}{dx} = y \sin y$ w/d $y(0) = y = 0.1$ Heun

x_i	x_{i+1}	y_i	k_1	k_2	y_{i+1}
0	0.01	0.1	0.0099	0.0099	0.1009
1	0.02	0.1009	0.0099	0.0099	0.1018
2	0.03	0.1027	0.0099	0.0099	0.1036
3	0.04	0.1045	0.0099	0.0099	0.1054
4	0.05	0.1063	0.0099	0.0099	0.1072
5	0.06	0.1081	0.0099	0.0099	0.1090
6	0.07	0.1099	0.0099	0.0099	0.1108
7	0.08	0.1117	0.0099	0.0099	0.1126
8	0.09	0.1135	0.0099	0.0099	0.1144
9	0.10	0.1153	0.0099	0.0099	0.1162
10	0.11	0.1171	0.0099	0.0099	0.1180
11	0.12	0.1189	0.0099	0.0099	0.1198
12	0.13	0.1207	0.0099	0.0099	0.1216
13	0.14	0.1225	0.0099	0.0099	0.1234
14	0.15	0.1243	0.0099	0.0099	0.1252

1.4	35.43	20.74	1.412
1.5	32.43	19.4	1.346
1.6	30.73	17.73	1.277
1.8	27.13	15.27	1.121
2	23.12	12.71	1.011

Pauli Matrix

0	.0000	1	1.6	1.9991	2.0009	2.0002
0.001	.0003	1.0001	1.7	2.119	2.1009	2.0009
0.002	.0013	1.0004	1.8	2.2899	2.2199	2.1499
0.003	.0027	1.0009	1.9	2.5099	2.3199	2.2099
0.004	.0040	1.0017	2	2.7699	2.4699	2.3799
0.005	.0053	1.0028	2.1	3.0699	2.6699	2.5799
0.010	.0107	1.0057	2.2	3.4099	2.8699	2.7799
0.015	.0160	1.0087	2.3	3.7899	3.0699	2.9799
0.020	.0213	1.0119	2.4	4.2099	3.2699	3.1799
0.030	.0320	1.0178	2.5	4.6699	3.4699	3.3799
0.040	.0427	1.0239	2.6	5.1699	3.6699	3.5799
0.050	.0533	1.0302	2.7	5.7099	3.8699	3.7799
0.060	.0640	1.0367	2.8	6.2899	4.0699	3.9799
0.070	.0747	1.0434	2.9	6.9099	4.2699	4.1799
0.080	.0853	1.0503	3	7.5699	4.4699	4.3799
0.090	.0960	1.0574	3.1	8.2699	4.6699	4.5799
0.100	.1067	1.0647	3.2	9.0099	4.8699	4.7799
0.110	.1173	1.0722	3.3	9.7899	5.0699	4.9799
0.120	.1280	1.0800	3.4	10.6099	5.2699	5.1799
0.130	.1387	1.0880	3.5	11.4699	5.4699	5.3799
0.140	.1493	1.0962	3.6	12.3699	5.6699	5.5799
0.150	.1600	1.1047	3.7	13.3099	5.8699	5.7799
0.160	.1707	1.1134	3.8	14.2899	6.0699	5.9799
0.170	.1813	1.1223	3.9	15.3099	6.2699	6.1799
0.180	.1920	1.1314	4	16.3699	6.4699	6.3799
0.190	.2027	1.1407	4.1	17.4699	6.6699	6.5799
0.200	.2133	1.1502	4.2	18.6099	6.8699	6.7799
0.210	.2240	1.1600	4.3	19.7899	7.0699	6.9799
0.220	.2347	1.1700	4.4	21.0099	7.2699	7.1799
0.230	.2453	1.1802	4.5	22.2699	7.4699	7.3799
0.240	.2560	1.1906	4.6	23.5699	7.6699	7.5799
0.250	.2667	1.2012	4.7	24.9099	7.8699	7.7799
0.260	.2773	1.2120	4.8	26.2899	8.0699	7.9799
0.270	.2880	1.2230	4.9	27.7099	8.2699	8.1799
0.280	.2987	1.2342	5	29.1699	8.4699	8.3799
0.290	.3093	1.2456	5.1	30.6699	8.6699	8.5799
0.300	.3200	1.2572	5.2	32.2099	8.8699	8.7799
0.310	.3307	1.2690	5.3	33.7899	9.0699	8.9799
0.320	.3413	1.2810	5.4	35.4099	9.2699	9.1799
0.330	.3520	1.2932	5.5	37.0699	9.4699	9.3799
0.340	.3627	1.3056	5.6	38.7699	9.6699	9.5799
0.350	.3733	1.3182	5.7	40.5099	9.8699	9.7799
0.360	.3840	1.3310	5.8	42.2899	10.0699	9.9799
0.370	.3947	1.3440	5.9	44.1099	10.2699	10.1799
0.380	.4053	1.3572	6	45.9699	10.4699	10.3799
0.390	.4160	1.3706	6.1	47.8699	10.6699	10.5799
0.400	.4267	1.3842	6.2	49.8099	10.8699	10.7799
0.410	.4373	1.3980	6.3	51.7899	11.0699	10.9799
0.420	.4480	1.4120	6.4	53.8099	11.2699	11.1799
0.430	.4587	1.4262	6.5	55.8699	11.4699	11.3799
0.440	.4693	1.4406	6.6	57.9699	11.6699	11.5799
0.450	.4800	1.4552	6.7	60.1099	11.8699	11.7799
0.460	.4907	1.4700	6.8	62.2899	12.0699	11.9799
0.470	.5013	1.4850	6.9	64.5099	12.2699	12.1799
0.480	.5120	1.5002	7	66.7699	12.4699	12.3799
0.490	.5227	1.5156	7.1	69.0699	12.6699	12.5799
0.500	.5333	1.5312	7.2	71.4099	12.8699	12.7799
0.510	.5440	1.5470	7.3	73.7899	13.0699	12.9799
0.520	.5547	1.5630	7.4	76.2099	13.2699	13.1799
0.530	.5653	1.5792	7.5	78.6699	13.4699	13.3799
0.540	.5760	1.5956	7.6	81.1699	13.6699	13.5799
0.550	.5867	1.6122	7.7	83.7099	13.8699	13.7799
0.560	.5973	1.6290	7.8	86.2899	14.0699	13.9799
0.570	.6080	1.6460	7.9	88.9099	14.2699	14.1799
0.580	.6187	1.6632	8	91.5699	14.4699	14.3799
0.590	.6293	1.6806	8.1	94.2699	14.6699	14.5799
0.600	.6400	1.6982	8.2	97.0099	14.8699	14.7799
0.610	.6507	1.7160	8.3	99.7899	15.0699	14.9799
0.620	.6613	1.7340	8.4	102.6099	15.2699	15.1799
0.630	.6720	1.7522	8.5	105.4699	15.4699	15.3799
0.640	.6827	1.7706	8.6	108.3699	15.6699	15.5799
0.650	.6933	1.7892	8.7	111.3099	15.8699	15.7799
0.660	.7040	1.8080	8.8	114.2899	16.0699	15.9799
0.670	.7147	1.8270	8.9	117.3099	16.2699	16.1799
0.680	.7253	1.8462	9	120.3699	16.4699	16.3799
0.690	.7360	1.8656	9.1	123.4699	16.6699	16.5799
0.700	.7467	1.8852	9.2	126.6099	16.8699	16.7799
0.710	.7573	1.9050	9.3	129.7899	17.0699	16.9799
0.720	.7680	1.9250	9.4	133.0099	17.2699	17.1799
0.730	.7787	1.9452	9.5	136.2699	17.4699	17.3799
0.740	.7893	1.9656	9.6	139.5699	17.6699	17.5799
0.750	.8000	1.9862	9.7	142.9099	17.8699	17.7799
0.760	.8107	2.0070	9.8	146.2899	18.0699	17.9799
0.770	.8213	2.0280	9.9	149.7099	18.2699	18.1799
0.780	.8320	2.0492	10	153.1699	18.4699	18.3799
0.790	.8427	2.0706	10.1	156.6699	18.6699	18.5799
0.800	.8533	2.0922	10.2	160.2099	18.8699	18.7799
0.810	.8640	2.1140	10.3	163.7899	19.0699	18.9799
0.820	.8747	2.1360	10.4	167.4099	19.2699	19.1799
0.830	.8853	2.1582	10.5	171.0699	19.4699	19.3799
0.840	.8960	2.1806	10.6	174.7699	19.6699	19.5799
0.850	.9067	2.2032	10.7	178.5099	19.8699	19.7799
0.860	.9173	2.2260	10.8	182.2899	20.0699	19.9799
0.870	.9280	2.2490	10.9	186.1099	20.2699	20.1799
0.880	.9387	2.2722	11	190.0099	20.4699	20.3799
0.890	.9493	2.2956	11.1	193.9499	20.6699	20.5799
0.900	.9600	2.3192	11.2	197.9299	20.8699	20.7799
0.910	.9707	2.3430	11.3	201.9499	21.0699	20.9799
0.920	.9813	2.3670	11.4	206.0099	21.2699	21.1799
0.930	.9920	2.3912	11.5	210.1099	21.4699	21.3799
0.940	1.0027	2.4156	11.6	214.2499	21.6699	21.5799
0.950	1.0133	2.4402	11.7	218.4299	21.8699	21.7799
0.960	1.0240	2.4650	11.8	222.6499	22.0699	21.9799
0.970	1.0347	2.4900	11.9	226.9099	22.2699	22.1799
0.980	1.0453	2.5152	12	231.2099	22.4699	22.3799
0.990	1.0560	2.5406	12.1	235.5499	22.6699	22.5799
1.000	1.0667	2.5662	12.2	239.9299	22.8699	22.7799

Kullback-Leibler Divergence						
x	$p(x)$	$q(x)$	$\frac{p(x)}{q(x)}$	$\ln \frac{p(x)}{q(x)}$	$p(x) \ln \frac{p(x)}{q(x)}$	$D_{KL}(p q)$
1	0.1	0.1	1	0	0	0
2	0.2	0.2	1	0	0	0
3	0.3	0.3	1	0	0	0
4	0.4	0.4	1	0	0	0
5	0.5	0.5	1	0	0	0
6	0.6	0.6	1	0	0	0
7	0.7	0.7	1	0	0	0
8	0.8	0.8	1	0	0	0
9	0.9	0.9	1	0	0	0
10	0.1	0.1	1	0	0	0
11	0.2	0.2	1	0	0	0
12	0.3	0.3	1	0	0	0
13	0.4	0.4	1	0	0	0
14	0.5	0.5	1	0	0	0
15	0.6	0.6	1	0	0	0
16	0.7	0.7	1	0	0	0
17	0.8	0.8	1	0	0	0
18	0.9	0.9	1	0	0	0
19	0.1	0.1	1	0	0	0
20	0.2	0.2	1	0	0	0
21	0.3	0.3	1	0	0	0
22	0.4	0.4	1	0	0	0
23	0.5	0.5	1	0	0	0
24	0.6	0.6	1	0	0	0
25	0.7	0.7	1	0	0	0
26	0.8	0.8	1	0	0	0
27	0.9	0.9	1	0	0	0
28	0.1	0.1	1	0	0	0
29	0.2	0.2	1	0	0	0
30	0.3	0.3	1	0	0	0
31	0.4	0.4	1	0	0	0
32	0.5	0.5	1	0	0	0
33	0.6	0.6	1	0	0	0
34	0.7	0.7	1	0	0	0
35	0.8	0.8	1	0	0	0
36	0.9	0.9	1	0	0	0
37	0.1	0.1	1	0	0	0
38	0.2	0.2	1	0	0	0
39	0.3	0.3	1	0	0	0
40	0.4	0.4	1	0	0	0
41	0.5	0.5	1	0	0	0
42	0.6	0.6	1	0	0	0
43	0.7	0.7	1	0	0	0
44	0.8	0.8	1	0	0	0
45	0.9	0.9	1	0	0	0
46	0.1	0.1	1	0	0	0
47	0.2	0.2	1	0	0	0
48	0.3	0.3	1	0	0	0
49	0.4	0.4	1	0	0	0
50	0.5	0.5	1	0	0	0