2024	YEAR 12 TRIAL HSC **EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Student ID below, the top of page 7 and all working pages of your booklets

Total marks: 70

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7–13)

- Attempt Questions 11–14
- Use a separate writing booklet for each question
- Allow about 1 hour and 45 minutes for this section

STUDENT ID ______________________________________

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the provided answer sheet for Questions 1-10.

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1 Consider the polynomial $x^3 - 3x^2 - 2x + 1 = 0$ with roots α, β and γ.

Which of the following is a polynomial with roots $3\alpha, 3\beta$ and 3γ?

A. $x^3 - 9x^2 - 6x + 3 = 0$

B. $x^3 - 9x^2 - 6x + 27 = 0$

C. $x^3 + 9x^2 - 18x - 27 = 0$

D. $x^3 - 9x^2 - 18x + 27 = 0$

2 Four people are running for president of a school's Student Representative Council committee (SRC).

What is the minimum number of votes needed to win the election outright if 302 students cast their vote?

A. 75

B. 76

C. 77

D. 78

3 The growth of a population of insects can be modelled using the differential equation,

$$\frac{dP}{dt} = 0.025P\left(1 - \frac{P}{500}\right)$$

What is the population, P, of insects, when the rate of growth is maximum?

A. 25

B. 50

C. 250

D. 500

4 A circular oil spill is expanding such that its radius increases at a constant rate of two metres per minute.

At what rate is the area of the oil spill increasing when the radius is 5 metres?

A. 10π square metres per minute

B. 20π square metres per minute

C. 40π square metres per minute

D. 50π square metres per minute

5 If $\tan x = -\frac{1}{2}$ and $\frac{\pi}{2} < x < \pi$, what is the value of $\sin 2x$?

A. $-\dfrac{4}{\sqrt{5}}$

B. $-\dfrac{4}{5}$

C. $\dfrac{4}{\sqrt{5}}$

D. $\dfrac{4}{5}$

6 The sides of a parallelogram are represented by the vectors $4\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{i} + \underset{\sim}{j}$.

What is the acute angle between the diagonals of the parallelogram?

A. $\cos^{-1} \dfrac{1}{5\sqrt{2}}$

B. $\cos^{-1} \dfrac{23}{5\sqrt{29}}$

C. $\cos^{-1} \dfrac{1}{\sqrt{14}}$

D. $\cos^{-1} \dfrac{23}{\sqrt{147}}$

7 Which of the following vectors is perpendicular to the line $y = 3x - 1$?

A. $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

B. $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

C. $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

D. $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

8 Which of the following is the constant term in the expansion of $\left(2x^3 + \frac{1}{x}\right)^8$?

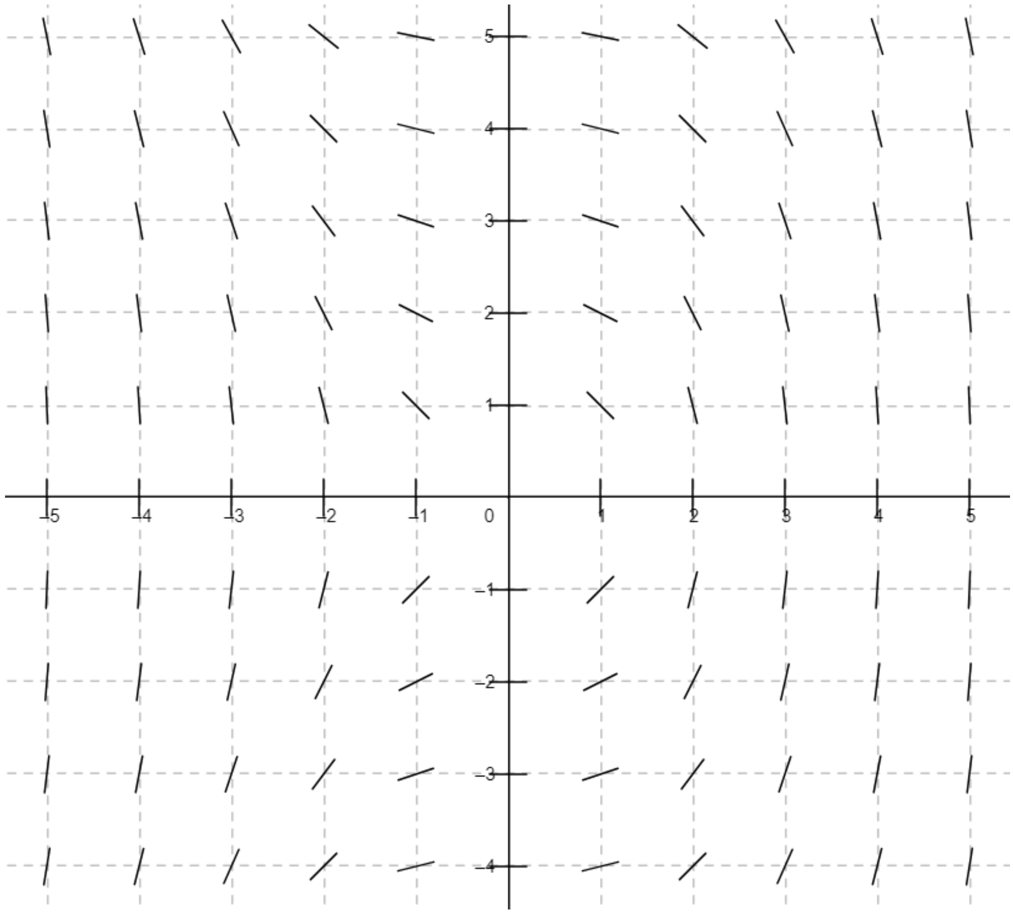
A. 92

B. 98

C. 108

D. 112

9 A differential equation has a direction field shown.



Which differential equation best matches the direction field?

A. $\dfrac{dy}{dx} = \dfrac{1}{x^2}$

B. $\dfrac{dy}{dx} = \dfrac{-1}{x^2}$

C. $\dfrac{dy}{dx} = \dfrac{y}{x}$

D. $\dfrac{dy}{dx} = \dfrac{-x^2}{y}$

10 Which one of the following pairs of parametric equations does NOT fully represent the Cartesian equation below?

$$y = x^2 - 2x + 1$$

A. $x = \sqrt{t}$

$y = t - 2\sqrt{t} + 1$

B. $x = 1 - t$

$y = t^2$

C. $x = \ln t$

$y = (\ln t)^2 - \ln t^2 + 1$

D. $x = \tan t$

$y = \sec^2 t - 2\tan t$

Student ID: ____________________

2024 **HSC TRIAL EXAMINATION**

Mathematics Extension 1

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hours and 45 minutes for this section

Instructions

- Answer each question in a SEPARATE writing booklet.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Write your Student ID above

Please turn over

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Solve **2**

$$\frac{4}{x-1} \geq 5$$

(b) For the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 2\underset{\sim}{j}$, find the following.

(i) $\underset{\sim}{a} - 2\underset{\sim}{b}$ **1**

(ii) $\underset{\sim}{a} \cdot \underset{\sim}{b}$ **1**

(c) Consider the polynomial $P(x) = ax^3 + 3x^2 - 12x - 20$.

(i) If $P(x)$ is divisible by $x + 2$, show that $a = 2$. **1**

(ii) Hence, find all solutions to $P(x) = 0$. **3**

(d) The vectors $\underset{\sim}{a} = \begin{pmatrix} k \\ k+1 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 2 \\ 2k+5 \end{pmatrix}$ are parallel. **3**

What are the possible values of k?

(e) On a 36°C day, James takes an ice cream out of the freezer. The freezer is set at an optimal temperature of −18°C.

After one minute, the temperature of the ice cream is −10°C.

The differential equation $\frac{dT}{dt} = -k(T - 36)$ models this situation where T is the temperature of the ice cream and t is the minutes out of the freezer.

(i) Show that the equation $T = 36 + Ae^{-kt}$, where A is a constant, satisfies the differential equation provided. **1**

(ii) It is known that ice cream begins to melt at 0°C. After how many minutes will the ice cream begin to melt? Give your answer to one decimal place. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Find: **2**

$$\int \frac{2\,dx}{9+3x^2}$$

(b) Express $\sqrt{2}\cos(x) - \sqrt{6}\sin(x)$ in the form $R\cos(x+\alpha)$. **2**

(c) If $f(x) = 5x^3 + x + 6$ passes through the point (1 , 12), find the value of the gradient of the tangent to $f^{-1}(x)$ at $x = 12$. **2**

(d) Nathan is the goal kicker for the local rugby league team. His goal kicking success rate is 83% for sideline conversions.

If Nathan attempts 9 goals from the sideline during the team's next match, find, to three significant figures:

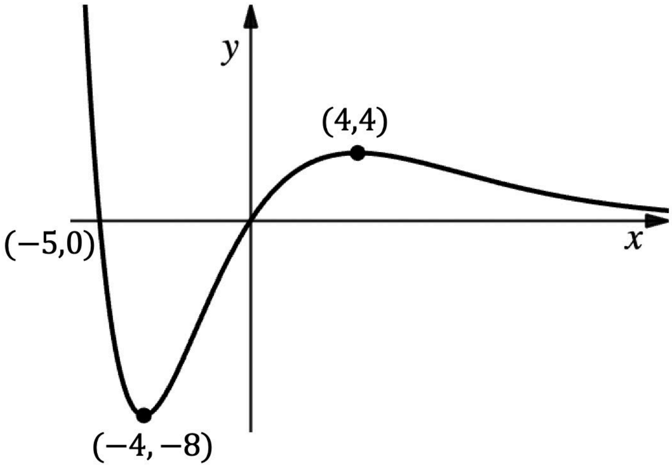
(i) the probability he successfully kicks exactly 6 goals. **1**

(ii) the probability he successfully kicks at least 8 goals. **2**

Question 12 continues on page 10

Question 12 (continued)

(e) The diagram shows the graph of the function $y = f(x)$. **3**



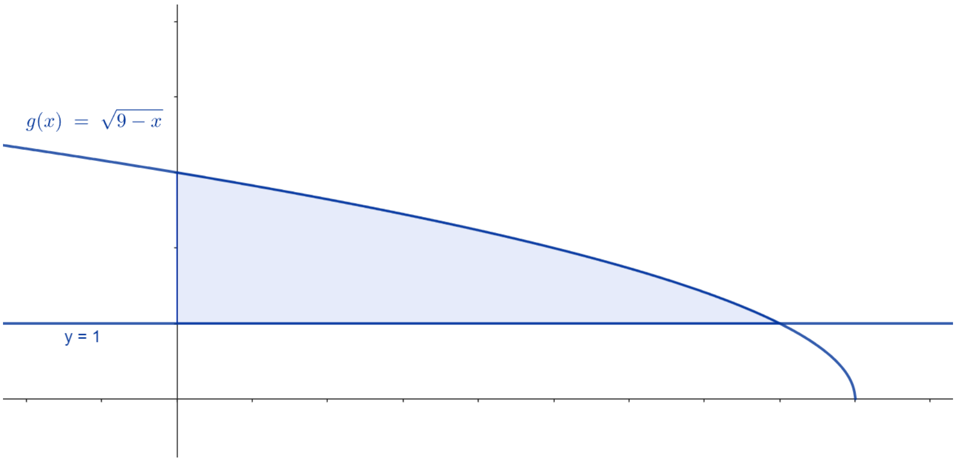
Sketch the graph of the curve $y^2 = f(x)$, showing all important features and the transformation of the given points.

(f) Use mathematical induction to prove that $9^n - 3^n$ is divisible by 6 for all integers $n \geq 1$. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram below shows the region bounded by the curve $g(x) = \sqrt{9 - x}$, the line $y = 1$ and the y-axis. The diagram is not to scale. **3**



If the region is rotated about the x-axis, find the volume of the solid of revolution.

(b) According to a study conducted in 2023, 85% of people who receive the flu shot experience no side effects. **3**

A random sample of 120 people who took the flu shot was considered.

By considering a normal approximation to the sample proportion, find the probability that more than 100 patients will show no side effects.

You may use the extract shown from a table giving values of $P(Z < z)$, where Z has a standard normal distribution.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830

Question 13 continues on page 12

Question 13 (continued)

(c) Find the particular solution to the differential equation $\frac{dy}{dx} = \sqrt{16 - 3y^2}$ that passes through the point $(0, 2)$. **3**

(d) A 1000 litre tank initially contains 20 kg of sugar.

A tap is turned on and releases pure water into the tank at a rate of 5 litres per minute. The liquid in the tank is constantly stirred.

During this time, liquid is pumped out of the tank at a rate of 5 litres per minute. After t minutes, there is m kg of sugar dissolved in the tank.

(i) Show that **1**

$$\frac{dm}{dt} = -\frac{m}{200}$$

(ii) Hence, or otherwise, show that **3**

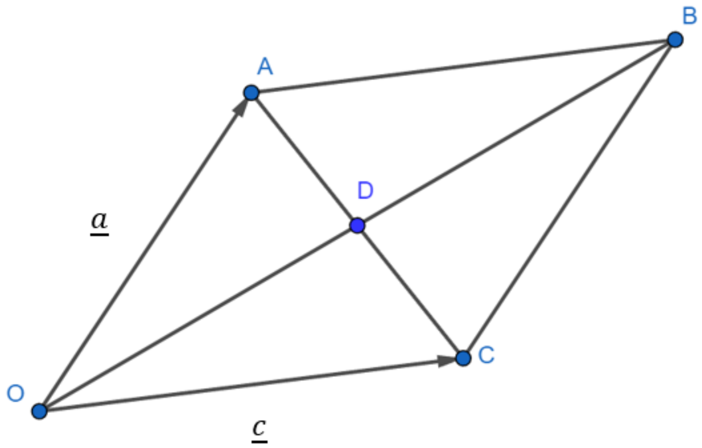
$$m = 20e^{-\frac{t}{200}}$$

(iii) When will the concentration of sugar in the tank be 0.01 kg per litre? Give your answer to the nearest minute. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) In the parallogram shown below, $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.



(i) Find the vector $\overrightarrow{AC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$. **1**

(ii) Using vector methods, prove that the diagonals of the paralleogram bisect each other. **3**

(b) (i) Show that: **2**

$$\sin^2 6x = 4\sin^2 3x - 4\sin^4 3x$$

(ii) Evaluate **3**

$$\int sin^2 6x \cos 3x \ dx$$

using the substitution $u = \sin 3x$.

(c) (i) Show that : **2**

$$\cos 3A = 4\cos^3 A - 3\cos A$$

(ii) By letting $x = r\cos\theta$, or otherwise, solve the equation **4**

$$x^3 - 3x + 1 = 0$$

End of paper

Student ID: ______________________________

2024 **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

Mathematics Extension 1
Year 12 Trial HSC
Section I Answer Sheet

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response circle completely.

1	A ○	B ○	C ○	D ○
2	A ○	B ○	C ○	D ○
3	A ○	B ○	C ○	D ○
4	A ○	B ○	C ○	D ○
5	A ○	B ○	C ○	D ○
6	A ○	B ○	C ○	D ○
7	A ○	B ○	C ○	D ○
8	A ○	B ○	C ○	D ○
9	A ○	B ○	C ○	D ○
10	A ○	B ○	C ○	D ○

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$
$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$$

$$S = \frac{a}{1-r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

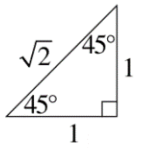
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$

$A = \frac{1}{2}ab\sin C$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

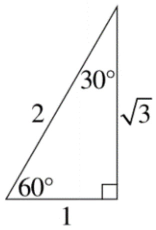


$c^2 = a^2 + b^2 - 2ab\,cos\,C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \frac{1}{2}r^2\theta$



Trigonometric identities

$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$

$\text{cosec}\, A = \frac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$

$cos^2 x + sin^2 x = 1$

Compound angles

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$\cos A = \frac{1-t^2}{1+t^2}$

$\tan A = \frac{2t}{1-t^2}$

$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$

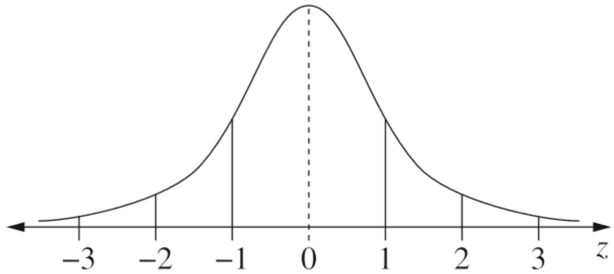
$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \frac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between -1 and 1
- approximately 95% of scores have z-scores between -2 and 2
- approximately 99.7% of scores have z-scores between -3 and 3

$E(X) = \mu$

$\text{Var}(X) = E\left[(X-\mu)^2\right] = E(X^2) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq x) = \int_a^x f(x)dx$

$P(a < X < b) = \int_a^b f(x)dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$

$X \sim Bin(n, p)$

$\Rightarrow P(X = x)$

$= \binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \ldots, n$

$E(X) = np$

$\text{Var}(X) = np(1-p)$

Differential Calculus

Function	**Derivative**
$y=f(x)^n$	$\dfrac{dy}{dx}=nf'(x)\left[f(x)\right]^{n-1}$
$y=uv$	$\dfrac{dy}{dx}=u\dfrac{dv}{dx}+v\dfrac{du}{dx}$
$\boldsymbol{y=g(u), u=f(x)}$	$\dfrac{dy}{dx}=\dfrac{dy}{du}\times\dfrac{du}{dx}$
$y=\dfrac{u}{v}$	$\dfrac{dy}{dx}=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$
$y=\sin f(x)$	$\dfrac{dy}{dx}=f'(x)\cos f(x)$
$y=\cos f(x)$	$\dfrac{dy}{dx}=-f'(x)\sin f(x)$
$y=\tan f(x)$	$\dfrac{dy}{dx}=f'(x)\sec^2 f(x)$
$y=e^{f(x)}$	$\dfrac{dy}{dx}=f'(x)e^{f(x)}$
$y=\ln f(x)$	$\dfrac{dy}{dx}=\dfrac{f'(x)}{f(x)}$
$y=a^{f(x)}$	
$\dfrac{dy}{dx}=(\ln a)f'(x)a^{f(x)}$	
$y=\log_a f(x)$	$\dfrac{dy}{dx}=\dfrac{f'(x)}{(\ln a)f(x)}$
$y=\sin^{-1} f(x)$	$\dfrac{dy}{dx}=\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y=\cos^{-1} f(x)$	$\dfrac{dy}{dx}=-\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y=\tan^{-1} f(x)$	$\dfrac{dy}{dx}=\dfrac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx=\frac{1}{n+1}\left[f(x)\right]^{n+1}+c$$

where $n\neq -1$

$$\int f'(x)\sin f(x)dx=-\cos f(x)+c$$

$$\int f'(x)\cos f(x)dx=\sin f(x)+c$$

$$\int f'(x)\sec^2 f(x)dx=\tan f(x)+c$$

$$\int f'(x)e^{f(x)}dx=e^{f(x)}+c$$

$$\int \frac{f'(x)}{f(x)}dx=\ln\left|f(x)\right|+c$$

$$\int f'(x)a^{f(x)}dx=\frac{a^{f(x)}}{\ln a}+c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}dx=\sin^{-1}\frac{f(x)}{a}+c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}dx=\frac{1}{a}\tan^{-1}\frac{f(x)}{a}+c$$

$$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx\approx\frac{b-a}{2n}\left\{f(a)+f(b)+2\left[f(x_1)+\ldots+f(x_{n-1})\right]\right\}$$

where $a=x_0$ and $b=x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^r + \ldots + a^n$$

Vectors

$$|\underset{\sim}{u}| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{\boldsymbol{r}} = \underset{\sim}{\boldsymbol{a}} + \boldsymbol{\lambda}\underset{\sim}{\boldsymbol{b}}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2024 Mathematics Extension 1 Year 12 Trial HSC Marking Guidelines

Section 1

Multiple-choice Answer Key (explanations over page)

Question	Answer
1	D
2	C
3	C
4	B
5	B
6	B
7	A
8	D
9	D
10	A

Explanation of Multiple-choice Answers

Question	Answer	Explanation
1	D	Method 1 – For the given polynomial, $x^3 - 3x^2 - 2x + 1 = 0$, we have: $\alpha + \beta + \gamma = 3$ $\alpha\beta + \beta\gamma + \alpha\gamma = -2$ $\alpha\beta\gamma = -1$ $\therefore$ If the roots are now $3\alpha, 3\beta$ and 3γ, then we have: Sum of the roots = $3\alpha + 3\beta + 3\gamma = 3 \times 3 = 9$ Sum of the roots two at a time = $9\alpha\beta + 9\beta\gamma + 9\alpha\gamma = -2 \times 9 = -18$ Product of the roots = $27\alpha\beta\gamma = -1 \times 27 = -27$ This gives the polynomial equation $x^3 - 9x^2 - 18x + 27 = 0$. Method 2 – Since α, β and γ satisfy the given equation, $x^3 - 3x^2 - 2x + 1 = 0$, then $3\alpha, 3\beta$ and 3γ satisfy the equation $\left(\frac{x}{3}\right)^3 - 3\left(\frac{x}{3}\right)^2 - 2\left(\frac{x}{3}\right) + 1 = 0$ Simplifying the equation gives $x^3 - 9x^2 - 18x + 27 = 0$.
2	C	Using the pigeonhole principle, a candiate would need a minimum of 77 votes to guarantee victory.
3	C	The equation of $\frac{dP}{dt}$ is a quadratic (a concave down parabola). So maximum rate of growth occurs at the vertex, which is halfway between $P = 0$ and $P = 500$. $\therefore P = 250$
4	B	For the area of the circular oil spill, $A = \pi r^2$. $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 2 = 4\pi r$ $\therefore When\ r = 5,\ \frac{dA}{dt} = 4\pi \times 5 = 20\pi$ square meters per minute.
5	B	Since x is in the 2nd quadrant, $\sin x = \frac{1}{\sqrt{5}}$ and $\cos x = \frac{-2}{\sqrt{5}}$. $\sin 2x = 2 \sin x \cos x = 2 \times \frac{1}{\sqrt{5}} \times \frac{-2}{\sqrt{5}}$ $\therefore \sin 2x = \frac{-4}{5}$

6	B	The diagonals of the parallelogram are $\left(4\underline{i}-3\underline{j}\right)+\left(\underline{i}+\underline{j}\right)$ and $\left(4\underline{i}-3\underline{j}\right)-\left(\underline{i}+\underline{j}\right)$. That is, $5\underline{i}-2\underline{j}$ and $3\underline{i}-4\underline{j}$ $\cos\theta=\dfrac{\left(5\underline{i}-2\underline{j}\right)\cdot\left(3\underline{i}-4\underline{j}\right)}{\left\|5\underline{i}-2\underline{j}\right\|\left\|3\underline{i}-4\right\|}=\dfrac{23}{\sqrt{5^2+2^2}\times\sqrt{3^2+4^2}}=\dfrac{23}{5\sqrt{29}}$ Hence, $\theta=\cos^{-1}\dfrac{23}{5\sqrt{29}}$
7	A	The given line passes through the points $A(0,-1)$ and $B\left(\frac{1}{3},0\right)$. $\therefore\ \overrightarrow{AB}=\begin{pmatrix}1\\3\end{pmatrix}$ $\therefore$ a vector which is perpendicular to $\overrightarrow{AB}$ is $\begin{pmatrix}-3\\1\end{pmatrix}$ since their dot product gives zero.
8	D	A term in this expansion takes the form: ${}^{8}C_r(2x^3)^r(x^{-1})^{8-r}={}^{8}C_r(2)^r(x)^{r-8+3r}={}^{8}C_r(2)^r(x)^{4r-8}$ For the constant term, we need to find r such that $4r-8=0$. $\therefore r=2$ $\therefore\ {}^{8}C_2(2)^2=112$
9	D	To determine the differential equation which best matches the given slope field, consider the slopes of the various line segments. Since the line segments along the x-axis ($y=0$) are vertical, the differential equation is undefined for $x=0$. Also, consider the horizontal line segments along the y-axis. The slopes of the line segments in the first and second quadrants are negative while the line segments in the third and fourth quadrants are positive. $\therefore$ The required differential equation is $\dfrac{dy}{dx}=\dfrac{-x^2}{y}$
10	A	The Cartesian equation for each option is the same. However, with Option A, $x\geq 0$ only which does not fully represent $y=x^2-2x+1$ which requires $x\in\mathbb{R}$.

Section II

Note: An incorrect answer in a previous part will not necessarily preclude students from achieving full marks in a later part. Answers here are based on correct prior part answers. Marking will need to adapt to pursue correct method with the use of incorrect prior parts.

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Show working towards correct solution	1

Sample answer:

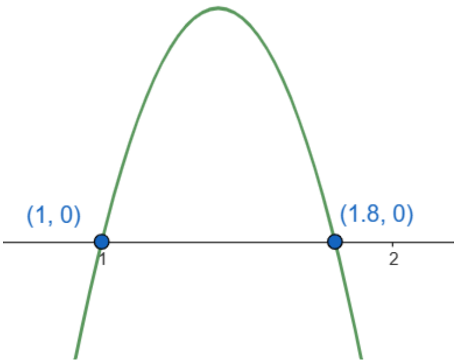
$\frac{4}{x-1} \geq 5, \qquad x \neq 1$

$\times\ (x-1)^2$ to get:

$4(x-1) \geq 5(x-1)^2$

$4(x-1) - 5(x-1)^2 \geq 0$

$(x-1)(9-5x) \geq 0$



$\therefore$ the solution is: $1 < x \leq \frac{9}{5}$

Question 11 (b)(i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$\underset{\sim}{a} - 2\underset{\sim}{b} = 3\underset{\sim}{i} + \underset{\sim}{j} - \left(2\underset{\sim}{i} - 4\underset{\sim}{j}\right) = \underset{\sim}{i} + 5\underset{\sim}{j}$

Question 11 (b)(ii)

Criteria	Marks
Provides correct solution	1

Sample answer:

$\underset{\sim}{a} . \underset{\sim}{b} = \left(3\underset{\sim}{i} + \underset{\sim}{j}\right) . \left(\underset{\sim}{i} - 2\underset{\sim}{j}\right)$
$= 3 - 2$
$= 1$

Question 11 (c)(i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Since $P(x)$ is divisible by $x + 2$, then $P(-2) = 0$.

$P(-2) = a(-2)^3 + 3(-2)^2 - 12(-2) - 20 = 0$
$-8a + 16 = 0$
$\therefore a = 2$

Question 11 (c)(ii)

Criteria	Marks
• Provides correct solution	3
• Correctly finds one of the roots, or equivalent merit	2
• Makes progress using the relationship between roots and coefficients, or equivalent merit	1

Sample answer:

Using part (i), we have $P(x) = 2x^3 + 3x^2 - 12x - 20$.
One of the roots is $x = -2$ since the polynomial is divisible by $x + 2$. Let the remaining two roots be α and β.
By considering sum of the roots and the product of the roots, we have:

$$\alpha + \beta - 2 = -\frac{3}{2}$$
$$\alpha + \beta = \frac{1}{2}$$

and

$$-2\alpha\beta = 10$$
$$\alpha\beta = -5$$

Solving the equations simultaneously gives the remaining two roots, $\alpha = \frac{5}{2}$ and $\beta = -2$ (which is repeated).

Question 11 (d)

Criteria	Marks
• Provides complete solution	3
• Makes significant progress in finding the values of k	2
• Recognises and attempts to use properties of parallel vectors	1

Sample answer:

If the vectors are parallel, then there exists some scalar λ such that $\underline{a} = \lambda \underline{b}$.

That is, $\begin{pmatrix} k \\ k+1 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ 2k+5 \end{pmatrix}$

$\therefore k = 2\lambda$ and $k + 1 = \lambda(2k + 5)$

Rearranging the first equation gives $\lambda = \frac{k}{2}$. Substituting this into the second equation gives,

$k + 1 = (2k + 5) \times \frac{k}{2}$

This gives,

$2k^2 + 3k - 2 = 0$

$(2k - 1)(k + 2) = 0$

$\therefore k = \frac{1}{2}, -2$

Question 11 (e)(i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Consider the given equation: $T = 36 + Ae^{-kt}$.

$\frac{dT}{dt} = -kAe^{-kt}$

Since $T - 36 = Ae^{-kt}$,

$\frac{dT}{dt} = -k(T - 36)$, as required.

Question 11 (e)(ii)

Criteria	Marks
• Provides correct solution	3
• Finds the value of A and k, or equivalent merit	2
• Finds the value of A, or equivalent merit	1

Sample answer:

Since the initial (when $t = 0$) temperature of the ice cream is -18°C, we have:
$-18 = 36 + Ae^0$
$\therefore A = -54$

We are also given that $T = -10$°C when $t = 1$.
$-10 = 36 - 54e^{-k}$
$\therefore k = -\ln\left(\frac{23}{27}\right)$

For the ice cream to begin melting, we need to set temperature $T = 0$°C.
$0 = 36 - 54e^{t\times\ln\left(\frac{23}{27}\right)}$
$54e^{t\times\ln\left(\frac{23}{27}\right)} = 36$
$e^{t\times\ln\left(\frac{23}{27}\right)} = \frac{36}{54}$
$t = \frac{\ln\left(\frac{36}{54}\right)}{\ln\left(\frac{23}{27}\right)} = 2.5287\ldots$
$\therefore t = 2.5$ minutes (to one decimal place)

Question 12 (a)

Criteria	Marks
• Provides the correct solution	2
• Determines the required integral gives $\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$ or $\tan^{-1}\left(\frac{\sqrt{3}x}{3}\right)$, or equivalent merit	1

Sample answer:

$$\int \frac{2\,dx}{9+3x^2} = \frac{2}{3}\int \frac{dx}{3+x^2}$$
$$= \frac{2\sqrt{3}}{9}\tan^{-1}\left(\frac{\sqrt{3}x}{3}\right) + C$$

Question 12 (b)

Criteria	Marks
• Provides correct solution	2
• Correctly provides either the value of R or the value of α	1

Sample answer:

$$R = \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{6}\right)^2} = \sqrt{8} = 2\sqrt{2}$$
$$\alpha = \tan^{-1}\left(\frac{\sqrt{6}}{\sqrt{2}}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$
$$\therefore \sqrt{2}\cos(x) - \sqrt{6}\sin(x)$$

$$= 2\sqrt{2}\cos\left(x + \frac{\pi}{3}\right)$$

Question 12 (c)

Criteria	Marks
• Provides correct solution	2
• Makes significant progress using the expression for the derivative of $f^{-1}(x)$, or equivalent merit	1

Sample answer:

Using the fact that:

$$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'\left(f^{-1}(x)\right)}$$

For $f(x) = 5x^3 + x + 6$, $f'(x) = 15x^2 + 1$

$$\therefore \frac{d}{dx}\left(f^{-1}(12)\right) = \frac{1}{f'\left(f^{-1}(12)\right)} = \frac{1}{f'(1)} = \frac{1}{16}$$

Question 12 (d)(i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$${}^{9}C_{6} \times (0.83)^{6} \times (0.17)^{3} = 0.135$$

Question 12 (d)(ii)

Criteria	Marks
• Provides correct solution	2
• Determines the probability of kicking 9 goals, or equivalent merit	1

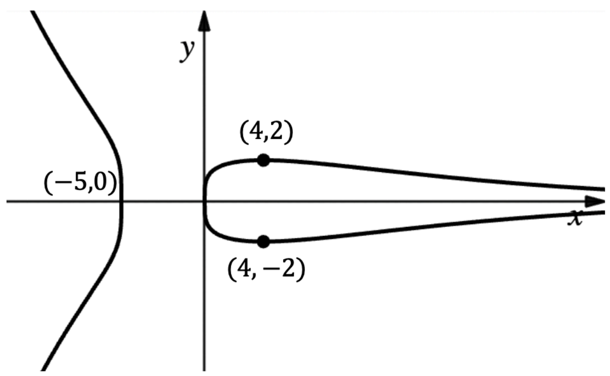
Sample answer:

$${}^{9}C_{8} \times (0.83)^{8} \times (0.17)^{1} + {}^{9}C_{9} \times (0.83)^{9} = 0.532$$

Question 12 (e)

Criteria	Marks
• Provides correct solution	3
• Correctly sketches $y^2 = f(x)$, with some points transformed points incorrect, or equivalent merit	2
• Correctly sketches the graph of $y = \sqrt{f(x)}$, or equivalent merit	1

Sample answer:



Question 12 (f)

Criteria	Marks
• Provides correct solution	3
• Proves true for $n=1$ and incorporates the assumption $P(k)$ into $P(k+1)$	2
• Proves true for $n=1$	1

Sample answer:

Required to prove: $9^n - 3^n$ is divisible by 6.
Consider the base case, that is, for $n = 1$.
$9^1 - 3^1 = 6 = 1 \times 6$
$\therefore$ true for $n = 1$

Assume the statement is true for $n = k$, where $k \in \mathbb{Z}^+$
i.e. $9^k - 3^k = 6M$, where M is some integer.
Now need to prove the statement is true for $n = k + 1$,
i.e. $9^{k+1} - 3^{k+1}$ is divisible by 6.

Rewrite $9^{k+1} - 3^{k+1}$ as $9 \times 9^k - 3^{k+1}$.
Now using assumption, so have:
$9 \times 9^k - 3^{k+1}$
$= 9(6M + 3^k) - 3 \times 3^k$
$= 9 \times 6M + 9 \times 3^k - 3 \times 3^k$
$= 6(9M + 3^k)$, where $9M + 3^k$ is an integer.
$\therefore$ Divisible by 6.

Hence, by mathematical induction, the statement is true for all positive integers $n \geq 1$.

Question 13 (a)

Criteria	Marks
• Provides the correct solution	3
• Integrates correctly and attempts to find the volume required	2
• Writes the correct expression for volume, or equivalent merit	1

Sample answer:

$$V = \pi \int_0^8 \left(\left(\sqrt{9-x} \right)^2 - 1 \right) dx = \pi \int_0^8 (8-x) dy$$

$$V = \pi \left[8x - \frac{x^2}{2} \right]_0^8 = \pi \left[\left(64 - \frac{64}{2} \right) - (0) \right]$$

$\therefore V = 32\pi$ cubic units

Question 13 (b)

Criteria	Marks
• Provides the correct solution	3
• Find the required z score, or equivalent merit	2
• States the value of the standard deviation of $\hat{P}$	1

Sample answer:

Let $\hat{P}$ be the random variable representing the sample proportion of patients showing no side effects.

$E(\hat{P}) = p = 0.85$

Standard deviation $(\hat{P}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.85(1-0.85)}{120}} = \sqrt{\frac{0.85 \times 0.15}{120}}$

We seek $P\left(\hat{P} > \frac{100}{120}\right)$

Now, $z = \frac{\frac{100}{120} - 0.85}{\sqrt{\frac{0.85 \times 0.15}{120}}} = -0.5113 \ldots$

Using z-scores table,

$\therefore P\left(\hat{p} > \frac{100}{120}\right) = P(Z > -0.51) = 0.695$

Question 13 (c)

Criteria	Marks
• Provides the correct solution	3
• Integrates correctly, or equivalent merit	2
• Correctly separates variables, or equivalent merit	1

Sample answer:

Given $\frac{dy}{dx} = \sqrt{16-3y^2}$,

Separate variables to give,
$\frac{dy}{\sqrt{16-3y^2}} = dx$

Integrating both sides gives,
$$\int \frac{dy}{\sqrt{16-3y^2}} = x + C$$
$$\frac{1}{\sqrt{3}}\int \frac{dy}{\sqrt{\frac{16}{3} - y^2}} = x + C$$
$$\frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{\sqrt{3}y}{4}\right) = x + C$$

At the given point $(0,2)$, we have
$C = \frac{\sqrt{3}\pi}{9}$
$\therefore \sin^{-1}\left(\frac{\sqrt{3}y}{4}\right) = \sqrt{3}x + \frac{\pi}{3}$
$\frac{\sqrt{3}y}{4} = \sin\left(\sqrt{3}x + \frac{\pi}{3}\right)$
$y = \frac{4\sqrt{3}}{3}\sin\left(\sqrt{3}x + \frac{\pi}{3}\right)$

Question 13 (d)(i)

Criteria	Marks
• Provides the correct solution	1

Sample answer:

$\frac{dm}{dt}$ = mass rate of sugar pumped into the tank - mass rate of sugar pumped out of the tank
= concentration pumped in (in kg/L) × volume rate pumped in (in L/min) - concentration pumped out (in kg/L) × volume rate pumped out (in L/min)

Pure water is released into the tank, hence no sugar flows in.

$\frac{dm}{dt} = 0 - \frac{m}{Volume\ of\ tank} \times 5 \text{ litres/min}$

$\frac{dm}{dt} = -\frac{m}{200}$

Question 13 (d)(ii)

Criteria	Marks
• Provides the correct solution	3
• Makes significant progress towards finding the correct expression for *m*	2
• Correctly separates variables and attempts to integrate both sides or equivalent merit	1

Sample answer:

Using $\frac{dm}{dt} = -\frac{m}{200}$ from part (i),

Separate variables and integrate both sides to give;

$\int \frac{1}{m}\, dm = -\int \frac{1}{200}\, dt \quad (m \neq 0)$

$\ln|m| = -\frac{t}{200} + C$

$m = Ae^{-\frac{t}{200}}$

where $A = \pm e^C$

When $t = 0, m = 20$,

$\therefore A = 20$

$m = 20e^{-\frac{t}{200}}$

Question 13 (d)(iii)

Criteria	Marks
• Provides the correct solution	2
• Makes significant progress towards the required solution	1

Sample answer:

When the concentration of sugar is 0.01 kg per litre, $m = 10$.

$10 = 20e^{-\frac{t}{200}}$

$\frac{1}{2} = e^{-\frac{t}{200}}$

$t = -200\ln\left(\frac{1}{2}\right) = 138.629\ldots$

$\therefore t = 139$ minutes

Question 14 (a)(i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

Question 14 (a)(ii)

Criteria	Marks
• Provides correct solution	3
• Makes significant progress towards the required result using expressions for $\overrightarrow{OD}$.	2
• Writes an expression for $\overrightarrow{OD}$ in terms of $\overrightarrow{OB}$, or equivalent merit.	1

Sample answer:

$\overrightarrow{OB} = \underset{\sim}{c} + \underset{\sim}{a}$
Since D lies on the vector $\overrightarrow{OB}$, we have:

$\overrightarrow{OD} = \lambda(\overrightarrow{OB}) = \lambda(\underset{\sim}{c} + \underset{\sim}{a})$, for some scalar λ.

We also know that D lies on vector $\overrightarrow{AC}$.
From part (i), we have that
$\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$
$\therefore \overrightarrow{AD} = \mu(\underset{\sim}{c} - \underset{\sim}{a})$
$\therefore \overrightarrow{OD} = \mu(\underset{\sim}{c} - \underset{\sim}{a}) + \underset{\sim}{a}$, for some scalar μ.

Equate the two expressions for $\overrightarrow{OD}$ gives,

$\lambda(\underset{\sim}{c} + \underset{\sim}{a}) = \mu(\underset{\sim}{c} - \underset{\sim}{a}) + \underset{\sim}{a}$
$\lambda\underset{\sim}{c} + \lambda\underset{\sim}{a} = (1 - \mu)\underset{\sim}{a} + \mu\underset{\sim}{c}$

Equating coefficients of $\underset{\sim}{a}$ and $\underset{\sim}{c}$ gives,
$\lambda = \mu$ and $\lambda = 1 - \mu$
Solving these equations simultaneously gives,
$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$
$\therefore \lambda = \mu = \frac{1}{2}$
This shows that D is the midpoint of diagonal OB and of diagonal AC.

Therefore, the diagonals of the paralleogram bisect each other.

Question 14 (b)(i)

Criteria	Marks
• Provides correct solution	2
• Uses the double angle formula or otherwise to make significant progress towards the required solution	1

Sample answer:

Take the LHS,
$sin^2 6x = (\sin 6x)^2 = (2\sin 3x \cos 3x)^2$
$= 4\sin^2 3x \cos^2 3x$
$= 4\sin^2 3x\,(1 - \sin^2 3x)$
$= 4\sin^2 3x - 4\sin^4 3x$
$= RHS$

Question 14 (b)(ii)

Criteria	Marks
• Provides correct solution	3
• Integrates correctly in terms of u, or equilvant merit	2
• Correctly rewrites integral in terms of u, or equivlant merit	1

Sample answer:

Using part (i), rearrange integral to give:
$\int \sin^2 6x \, cos\, 3x \; dx = \int (4\sin^2 3x - 4\sin^4 3x) \cos 3x \; dx$

Using the substitution,

$u = \sin 3x$

$\frac{du}{dx} = 3\cos 3x$

$dx = \frac{du}{3\cos 3x}$

$$\therefore \int (4u^2 - 4u^4)\frac{du}{3} = \frac{4}{3}\int (u^2 - u^4)\, du$$

$$= \frac{4}{3}\left[\frac{u^3}{3} - \frac{u^5}{5}\right] + C$$

$$= \frac{4}{3}\left[\frac{sin^3 3x}{3} - \frac{sin^5 3x}{5}\right] + C$$

$$= \frac{4sin^3 3x}{9} - \frac{4sin^5 3x}{15} + C$$

Question 14 (c)(i)

Criteria	Marks
• Provides correct solution	2
• Makes progress towards finding the required solution	1

Sample answer:

Take the LHS to give:

$\cos 3A = \cos(2A + A)$
$= \cos 2A \cos A - \sin 2A \sin A$
$= (2\cos^2 A - 1) \cos A - (2 \sin A \cos A) \sin A$
$= 2\cos^3 A - \cos A - 2\sin^2 A \cos A$
$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A) \cos A$
$= 4 \cos^3 A - 3 \cos A$
$= RHS$

Question 14 (c)(ii)

Criteria	Marks
• Provides correct solution	4
• Finds solutions to θ, or equivalent merit.	3
• Correctly uses part (i) to obtain $\cos 3\theta = -\frac{1}{2}$, or equivalent merit.	2
• States that $r = 2$, or equivalent merit	1

Sample answer:

Letting $x = r\cos\theta$ gives,
$r^3 cos^3\theta - 3rcos\theta + 1 = 0$
$\therefore r^3 cos^3\theta - 3rcos\theta = -1$

Using part (i), we have $\cos 3A = 4\cos^3 A - 3\cos A$.
$\frac{r^3}{3r} = \frac{4}{3} \implies r^2 = 4$
Taking $r = 2$ gives,
$8cos^3\theta - 6cos\theta = 2(4\cos^3\theta - 3\cos\theta) = -1$

Hence we need to solve
$2\cos 3\theta = -1$
$\cos 3\theta = -\frac{1}{2}$
From this, we have
$3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \ldots$
$\theta = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \ldots$

So, $2\cos\frac{2\pi}{9}$, $2\cos\frac{4\pi}{9}$, $2\cos\frac{8\pi}{9}$, $2\cos\frac{10\pi}{9}, \ldots$ etc are roots of the cubic equation, but they are not all distinct. We require 3 distinct solutions for the cubic (degree 3).

Since $\cos\theta$ is one-to-one for $0 \le \theta \le \pi$, $2cos\frac{2\pi}{9}$, $2cos\frac{4\pi}{9}$, and $2cos\frac{8\pi}{9}$ are the three distinct roots.

2024 Mathematics HSC Trial Extension 1
Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	ME-F2 Polynomials	ME11-1
2	1	ME-A1 Working with Combinatorics	ME11-5
3	1	ME-C3 Applications of Calculus	ME12-4
4	1	ME-C1 Rates of Change	ME11-4
5	1	ME-T2 Further Trigonometric Identities	ME11-3
6	1	ME-V1 Introduction to Vectors	ME12-2
7	1	ME-V1 Introduction to Vectors	ME12-2
8	1	ME-A1 Working with Combinatorics	ME11-5
9	1	ME-C3 Applications of Calculus	ME12-4
10	1	ME-F1 Further Work with Functions	ME11-1

Section II

Question	Marks	Content	Syllabus outcomes
11a	2	ME-F1 Further Work with Functions	ME11-1
11b(i)	1	ME-V1 Introduction to Vectors	ME12-2
11b(ii)	1	ME-V1 Introduction to Vectors	ME12-2
11c(i)	1	ME-F2 Polynomials	ME11-1
11c(ii)	3	ME-F2 Polynomials	ME11-1
11d	3	ME-V1 Introduction to Vectors	ME12-2
11e(i)	1	ME-C1 Rates of Change	ME11-4
11e(ii)	3	ME-C1 Rates of Change	ME11-4
12a	2	ME-C2 Further Calculus Skills	ME12-1
12b	2	ME-T3 Trigonometric Equations	ME12-3
12c	2	ME-C2 Further Calculus Skills	ME12-1
12d(i)	1	ME-S1 The Binomial Distribution	ME12-5
12d(ii)	2	ME-S1 The Binomial Distribution	ME12-5
12e	3	ME-F2 Polynomials	ME11-1
12f	3	ME-P1 Proof by Mathematical Induction	ME12-1
13a	3	ME-C3 Applications of Calculus	ME12-4
13b	3	ME-S1 The Binomial Distribution	ME12-5
13c	3	ME-C3 Applications of Calculus	ME12-4
13d(i)	1	ME-C3 Applications of Calculus	ME12-4
13d(ii)	3	ME-C3 Applications of Calculus	ME12-4
13d(iii)	2	ME-C3 Applications of Calculus	ME12-4
14a(i)	1	ME-V1 Introduction to Vectors	ME12-2
14a(ii)	3	ME-V1 Introduction to Vectors	ME12-2
14b(i)	2	ME-T2 Further Trigonometric Identities	ME11-3
14b(ii)	3	ME-C3 Applications of Calculus	ME12-4
14c(i)	2	ME-T2 Further Trigonometric Identities	ME11-3
14c(ii)	4	ME-T3 Trigonometric Equations	ME12-3