KINROSS WOLAROI SCHOOL

2023

Year 12 Mathematics Extension 1
Trial HSC Examination

Teacher Setting Paper: Mrs Vere
Head of Department: Mr Doyle

General Instructions
- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculator may be used
- Write your answers for Section I on the multiple-choice answer sheet provided
- A reference sheet is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations.

Total marks – 70

Section I – Multiple-Choice

10 marks

Attempt Questions 1-10

Allow 15 minutes for this section

Section II – Free Response

60 marks

Attempt questions 11 - 14

Allow 1 hour and 45 minutes for this section

This examination paper does not necessarily reflect the content or
format of the Higher School Certificate Examination in this subject.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 - 10.

QUESTION 1

What is the remainder when $P(x) = -2x^3 + 8x - 10$ is divided by $x+1$?

    (A)   −20

    (B)   −16

    (C)   −4

    (D)   0

QUESTION 2

The graph of $y = g(x)$ is shown to the right.
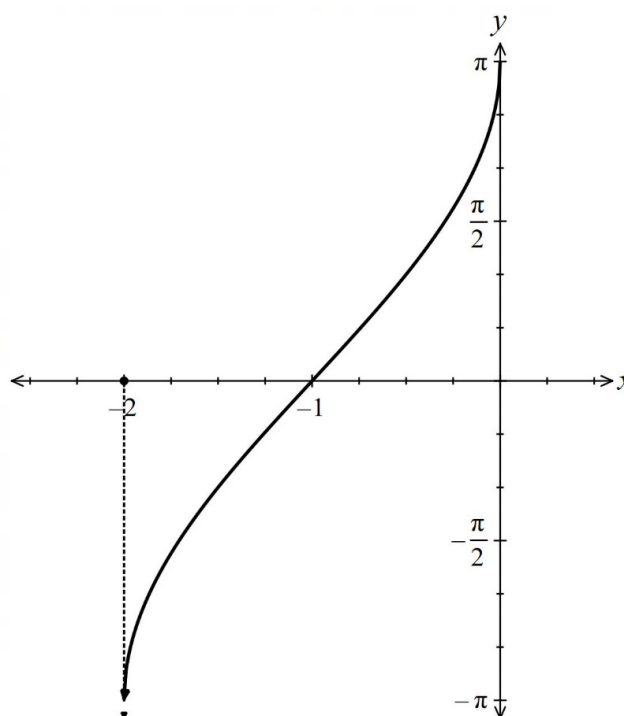
Which equation could describe g(x)?

    (A) $g(x) = \frac{1}{2}\cos^{-1}(x+1)$

    (B) $g(x) = 2\cos^{-1}(x+1)$

    (C) $g(x) = \frac{1}{2}\sin^{-1}(x+1)$

    (D) $g(x) = 2\sin^{-1}(x+1)$

QUESTION 3

Which of the following integrals is an expression for $\int \sin^2 3x\,dx$?

(A) $\quad \int \dfrac{(1+\cos 6x)}{2}\,dx$

(B) $\quad \int \dfrac{(1-\cos 6x)}{2}\,dx$

(C) $\quad \int \dfrac{(1+\sin 6x)}{2}\,dx$

(D) $\quad \int \dfrac{(1-\sin 6x)}{2}\,dx$

QUESTION 4

Which one of the following vectors is parallel to the vector $\overrightarrow{OE} = 3\underset{\sim}{i} - 6\underset{\sim}{j}$?

(A) $\quad \overrightarrow{OA} = 2\underset{\sim}{i} + 4\underset{\sim}{j}$

(B) $\quad \overrightarrow{OB} = 5\underset{\sim}{i} + 10\underset{\sim}{j}$

(C) $\quad \overrightarrow{OC} = -2\underset{\sim}{i} + 4\underset{\sim}{j}$

(D) $\quad \overrightarrow{OD} = -4\underset{\sim}{i} - 8\underset{\sim}{j}$

QUESTION 5

Consider the differential equation $\dfrac{dy}{dx} = \dfrac{2x}{y}$.

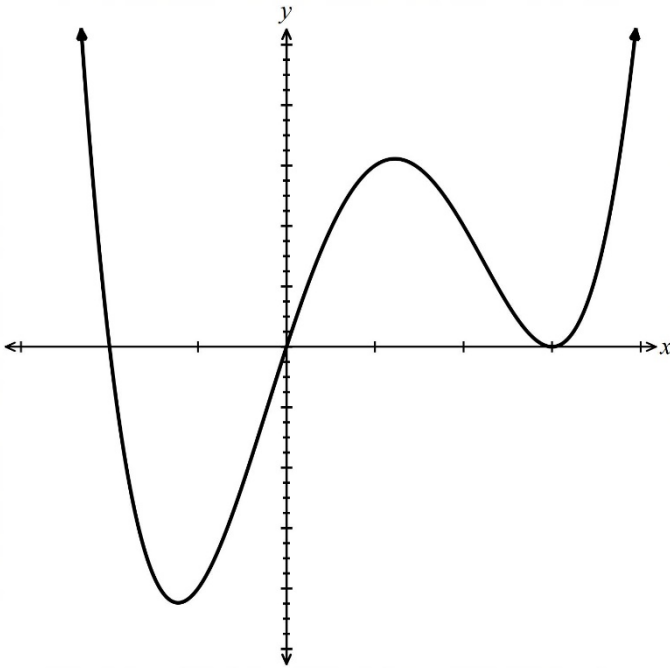Which of the following equations best represents this relationship between x and y?

(A) $\quad y^2 = x^2 + C$

(B) $\quad y^2 = 2x^2 + C$

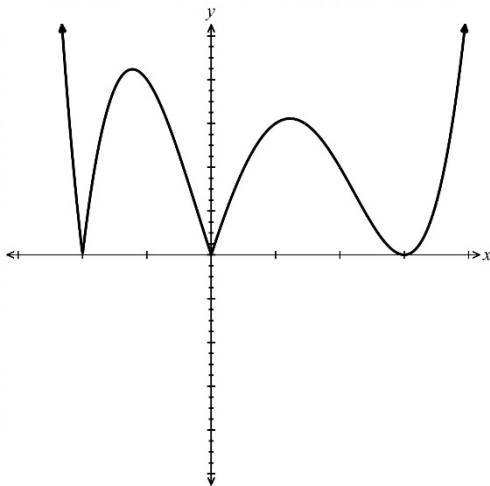(C) $\quad y = x\ln|y| + C$

(D) $\quad y = 2x\ln|y| + C$

QUESTION 6

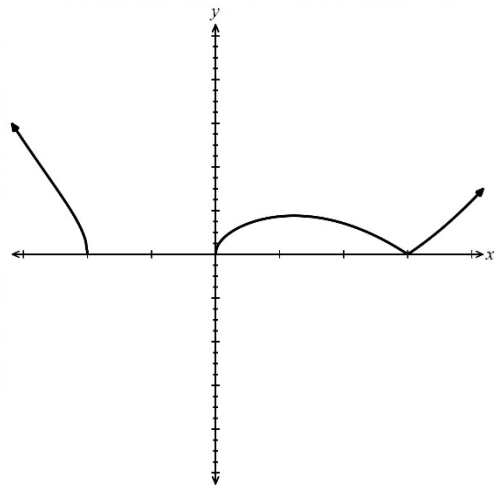The graph of $y = f(x)$ is shown below
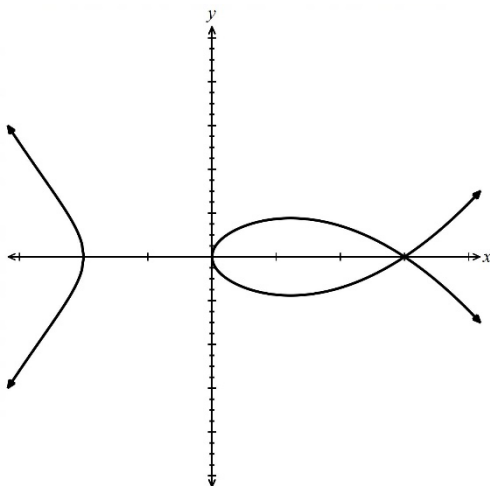

Which graph shows $y^2 = f(x)$?
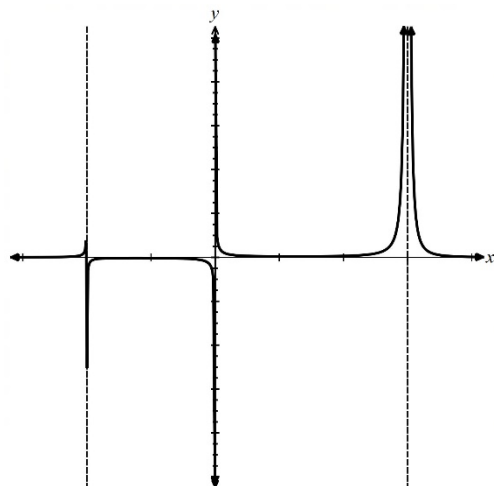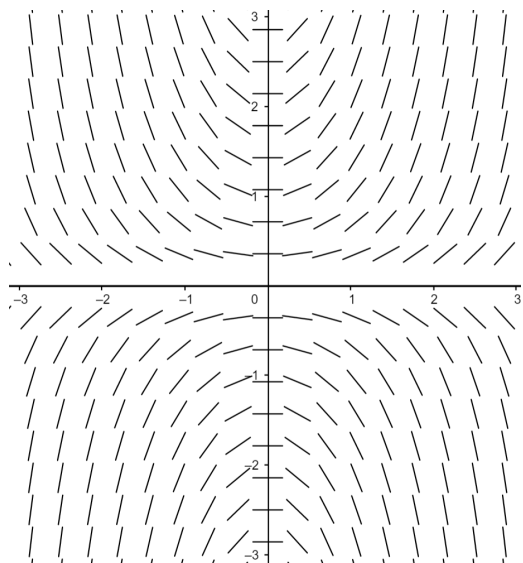
(A)

(B)

(C)

(D)

QUESTION 7

Consider the slope field below.

Which of the following equations could have generated the above slope field?

(A) $\dfrac{dy}{dx} = xy$

(B) $\dfrac{dy}{dx} = x^2 y$

(C) $\dfrac{dy}{dx} = xy^2$

(D) $\dfrac{dy}{dx} = x^2 y^2$

QUESTION 8

If $\dfrac{dN}{dt} = 0.1(N - 200)$ which of the following is a correct expression for N ?

(A) $-200 - 100e^{0.1t}$

(B) $-100 - 200e^{0.1t}$

(C) $200 + 100e^{0.1t}$

(D) $100 + 200e^{0.1t}$

QUESTION 9

Find the term independent of x in the expansion of $\left(x - \dfrac{2}{x^2} \right)^6$.

(A) 12

(B) 30

(C) 60

(D) 160

QUESTION 10

Which of the following options best describes the shape of the graph defined by the pair of parametric

equations: $x(t) = 3\cos(2t) - 2$, $y(t) = 3\sin(2t) + 2$ with $-\dfrac{\pi}{4} \le t \le \dfrac{\pi}{4}$?

(A) The right half of a circle with radius 3 and centre $(2, -2)$

(B) The right half of a circle with radius 3 and centre $(-2, 2)$

(C) The upper half of a circle with radius 3 and centre $(2, -2)$

(D) The upper half of a circle with radius 3 and centre $(-2, 2)$

END OF SECTION I

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hour 45 minutes for this section

Answer each question in the booklets provided. Extra writing booklets are available.

| **QUESTION 11**   (16 marks)   **Start a new writing booklet** | **Marks** |

(a)   Solve $|3x - 5| \le 7$ and show the solution on a number line.   **2**

(b)   A spherical balloon is being inflated at constant rate of $4\,\text{cm}^3$ per second. Find the rate at which the radius of the balloon is increasing when its volume is $250\,\text{cm}^3$, correct to two decimal places.   **2**

(c)   Find the exact value of the magnitude of $3\underset{\sim}{u} - 2\underset{\sim}{v}$ **2**
If $\underset{\sim}{u} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

(d)   Find the angle between the lines generated by the vectors: **2**
$\underset{\sim}{u} = 4\underset{\sim}{i} - 6\underset{\sim}{j}$ and $\underset{\sim}{v} = \underset{\sim}{i} + 5\underset{\sim}{j}$

(e)   Use the principle of mathematical induction to prove that for all positive integers   **3**
$n \ge 1$: $1 + 5 + 25 + \ldots\ldots + 5^{n-1} = \dfrac{1}{4}(5^n - 1)$

(f)   Find the exact value of $2\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) - \cos^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}(-1)$.   **2**

(g)   A committee containing 5 men and 3 women is to be formed from a group of 9 men and 7 women. In how many different ways can the committee be formed?   **1**

(h)   Find $\dfrac{d}{dx}(2x\sin^{-1} x)$.   **2**

QUESTION 12 (16 marks)   **Start a new writing booklet**

Marks

(a)   Use the substitution $u = \tan x$ to evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^2 x \sec^2 x \, dx$.

3

(b)   Using the *t*-formula solve the equation $\dfrac{5\sin x}{2} + \cos x = 2$ for $0° \le x \le 180°$.

Answer correct to the nearest minute.

3

(c)   Factorise the polynomial $P(x) = 2x^3 + 3x^2 - 29x - 60$ completely.

3

(d)   Given that $f(x) = \tan^{-1}(e^{2x-1})$, find the value of $f'\left(\dfrac{1}{2}\right)$.

2

(e)   At a film festival there are nine feature films. On the opening night there are three sessions, at 6 pm, 8 pm and 10 pm. Every feature film is shown at each of the three sessions.

There are 200 film critics at the festival and each one attends three different films on the opening night and writes a review of each.

Use the pigeonhole principle to show that at least one combination of three films will be reviewed by at least three different critics.
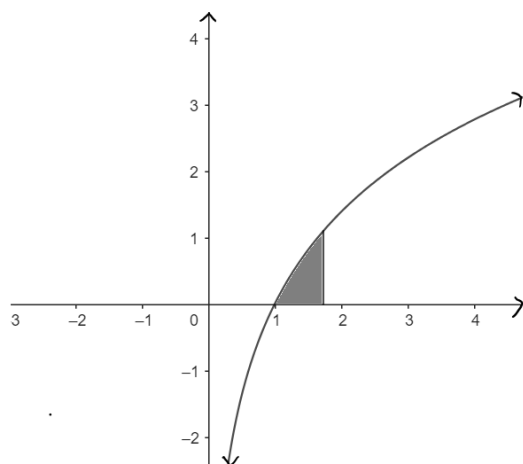
2

(f)   i) Show that $x^2 + 2x + 2$ can be expressed as $(x+1)^2 + 1$.

1

ii) Hence find $\int \dfrac{1}{x^2 + 2x + 2} \, dx$.

2

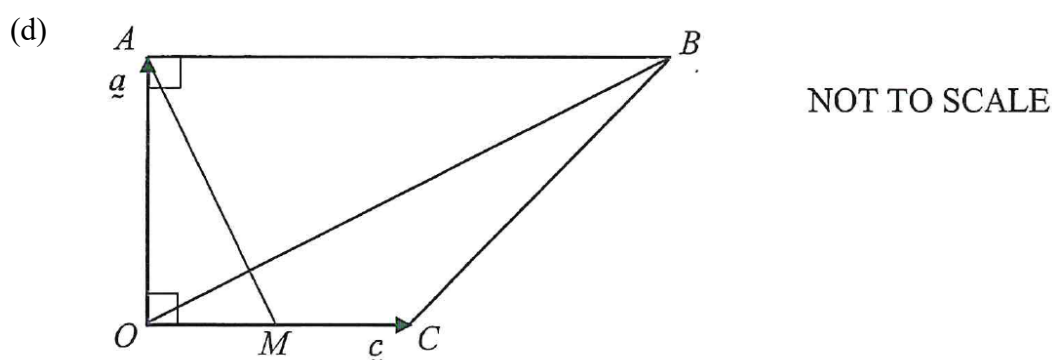**QUESTION 13** (14 marks)   **Start a new writing booklet**

(a)   Find, in the form $y = f(x)$, the solution of the differential equation $\dfrac{dy}{dx} = 2\dfrac{\sqrt{y}}{1+x^2}$,   **3**

given that $y = 0$ when $x = 1$.

(b)   Find the exact volume of the solid formed by rotating the region between

$y = 2\ln x$, the x axis and the line $x = \sqrt{3}$, about the **y-axis.**

4

(c)   Use the result $\cos 3x = 4\cos^3 x - 3\cos x$ to solve the equation   **3**

$\cos 3x = 2(\cos 2x + 1)$ for $0 \le x \le 2\pi$.

(d)

NOT TO SCALE

In the diagram, $OABC$ is a trapezium with $\overrightarrow{OA} = \underset{\sim}{a}, \overrightarrow{OC} = \underset{\sim}{c}$,

$\angle COA = \angle BAO = 90°$ and $OA = OC = \dfrac{1}{2}AB$. M is the midpoint of OC.   **4**

Use vector methods to show that $OB \perp AM$.

QUESTION 14 (14 marks) **Start a new writing booklet**

<div align="right">**Marks**</div>

(a) The velocity of a particle is described by the differential equation $\dfrac{dx}{dt} = e^{2x}\cos t$,

where x centimetres is its displacement relative to the origin at time t seconds.

Find an expression for x, in terms of t, if the particle passes through the origin at time

$t = \dfrac{5\pi}{6}$.

<div align="right">**3**</div>

(b) A drone, which is hovering at a height of 5 km, releases a target object which falls under gravity. At the same time, a jet, which is at a height of 3.6 km and is 5km west of the drone, fires a projectile at a speed of 400 m/s at an angle of $\theta°$ to the horizontal toward the target.

5 km
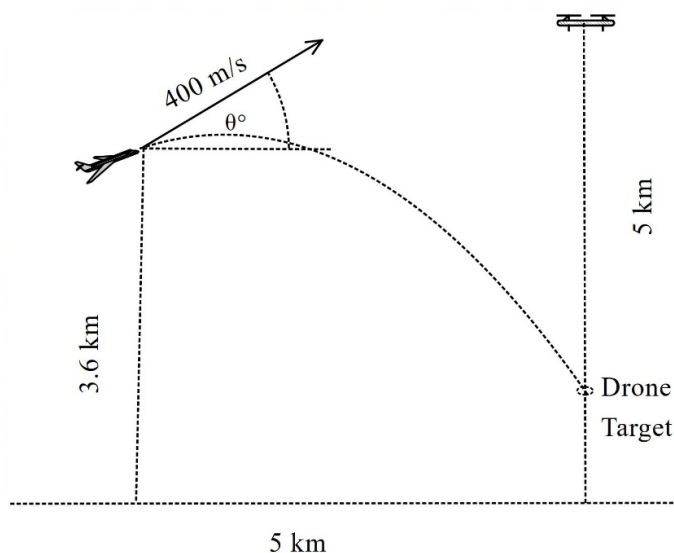
Using a point on the ground directly below the jet as the origin, the positions of the projectile and target at time t seconds after the projectile is launched are as follows:

Projectile   Drone Target

$$\underset{\sim}{p}(t) = \begin{pmatrix} 400t\cos\theta \\ 3600 + 400t\sin\theta - 5t^2 \end{pmatrix} \qquad \underset{\sim}{d}(t) = \begin{pmatrix} 5000 \\ 5000 - 5t^2 \end{pmatrix}$$

    i)   Calculate the size of angle θ, if the projectile is to hit the target.   **3**

            Answer to the nearest minute.

    ii)   Determine how many seconds after the projectile is fired that it hits the target,   **2**

            and the height of the target at that time.

Question 14 continues on the next page.

(c)   Use Mathematical Induction to prove that $13^n + 6^{n-1}$ is divisible by 7 for all positive   **3**
integers $n \geq 1$.

(d)   Using the expansion of $(1+x)^3(1+x)^6$ show that: **3**

$$\binom{3}{0}\binom{6}{6} + \binom{3}{1}\binom{6}{5} + \binom{3}{2}\binom{6}{4} + \binom{3}{3}\binom{6}{3} = \binom{9}{6}$$

END OF PAPER.

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MULTIPLE-CHOICE ANSWER SHEET

For multiple choice questions, choose the best answer A, B, C or D and fill in the correct circle.

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Ⓐ Ⓑ Ⓒ Ⓓ

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Ⓐ Ⓑ Ⓒ Ⓓ

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Ⓐ Ⓑ Ⓒ Ⓓ

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Ⓐ Ⓑ Ⓒ Ⓓ

9. Ⓐ Ⓑ Ⓒ Ⓓ

10. Ⓐ Ⓑ Ⓒ Ⓓ