Killara High School

MATHEMATICS DEPARTMENT

Student Number

2024 **Year 12 Examination**

Mathematics Extension 1

HSC Trial Examination

Date: 14 August 2024

Q	Marks
MC	10
11	15
12	15
13	15
14	15
Total	/70

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- Show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 70

Section I - 10 marks

- Allow about 15 minutes for this section

Section II - 60 marks

- Allow about 1 hour and 45 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 30% of the course.

Section I

10 marks
Attempt Questions 1– 10
Allow about 15 minutes for this section.

Use the multiple — choice sheet for Question 1– 10.

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1 It is given that $\sin\left(\frac{11\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$.

Which of the following is the value of $\sin^{-1}\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$?

A. $\frac{\pi}{12}$

B. $\frac{11\pi}{12}$

C. $-\frac{\pi}{12}$

D. $-\frac{11\pi}{12}$

2 Which of the following expressions could be one of the factors of the polynomial shown below?

$$P(x) = x^3 + x^2 - 5x - 2$$

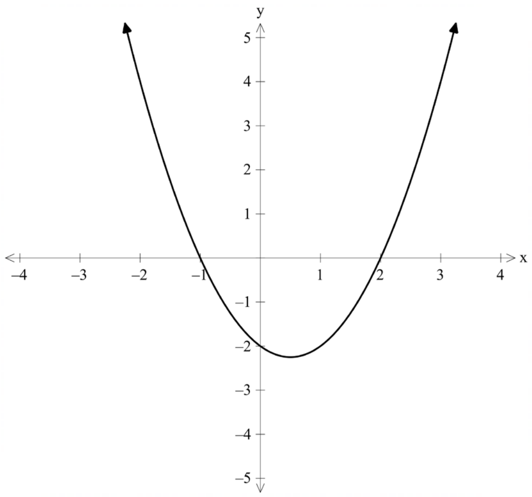
A. $x - 1$

B. $x + 1$

C. $x - 2$

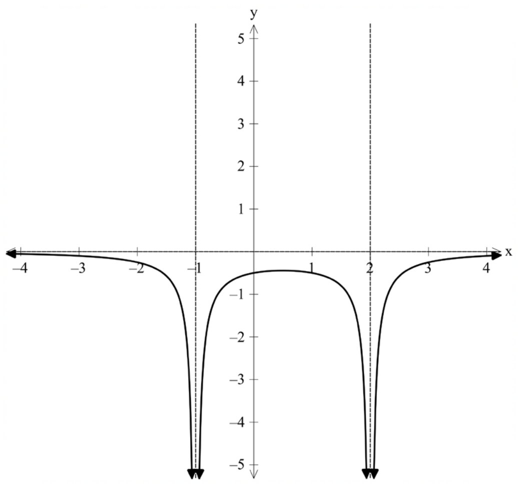
D. $x + 2$

3 The graph of the function $y = f(x)$ is below.

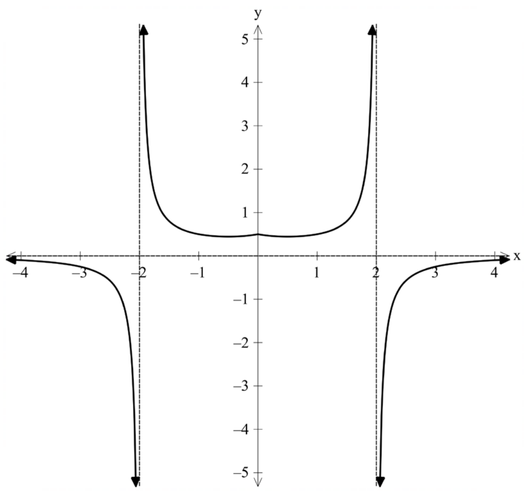


Which of the following is a graph of $y = -\dfrac{1}{f(|x|)}$?

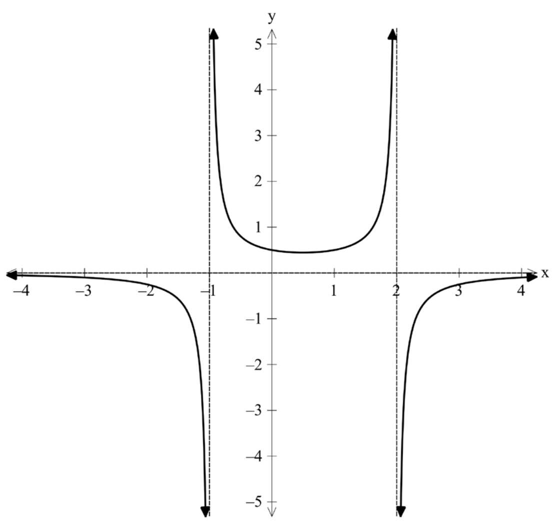
A.



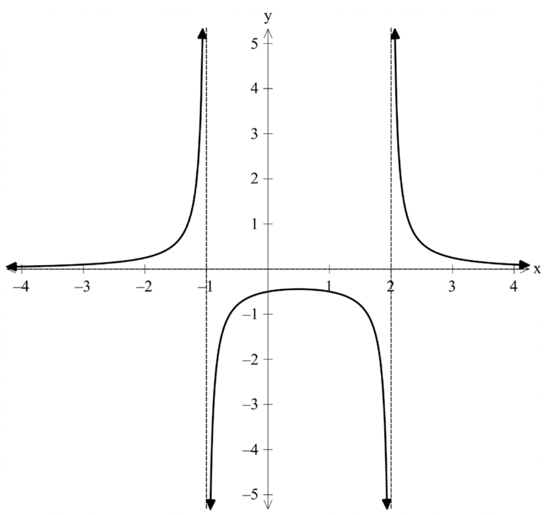
B.



C.



D.



4 Which of the following is the coefficient of the x^3 term in the expansion of $(2-x)^8$?

A. 448

B. -448

C. 1792

D. -1792

5 Which inequality below has the same solution as $|x+5|+|x-1|=6$?

A. $\dfrac{1}{x^2-4x-5} \leq 0$

B. $x^2-4x-5<0$

C. $|x+2| \leq 3$

D. $|x+2| \geq 3$

6 A team of r student representatives are chosen from class of n students where $r<n$.
This team should include a captain and a vice captain.
How many ways can this team be selected?

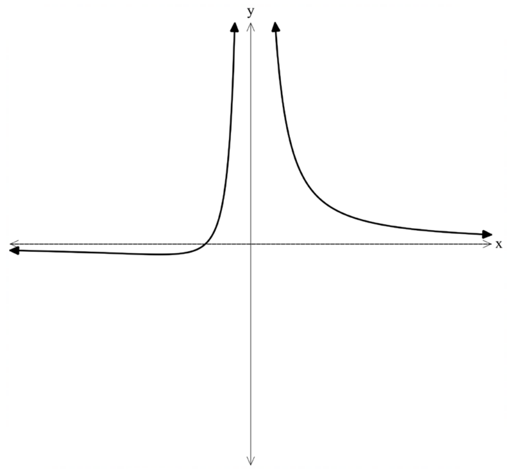
A. $n(n-1)\binom{n}{r}$

B. $2! \times \binom{n}{r}$

C. $r(r-1)\binom{n-2}{r}$

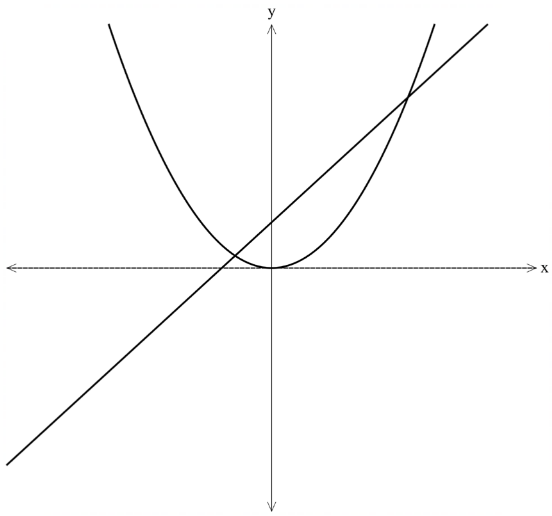
D. $n(n-1)\binom{n-2}{r-2}$

7 The diagram shows the graph of the function $h(x) = \dfrac{g(x)}{f(x)}$.

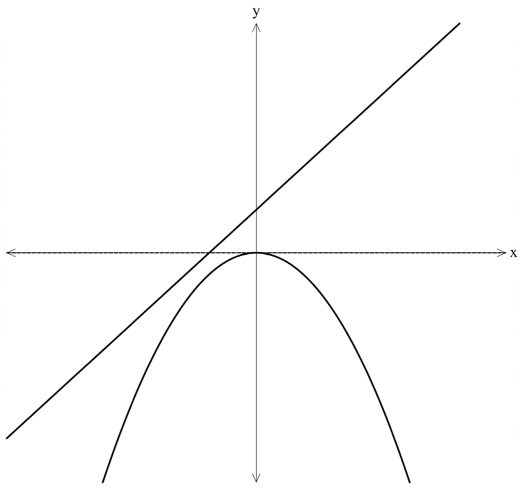


Which of the following best represent the graphs of both $f(x)$ and $g(x)$?

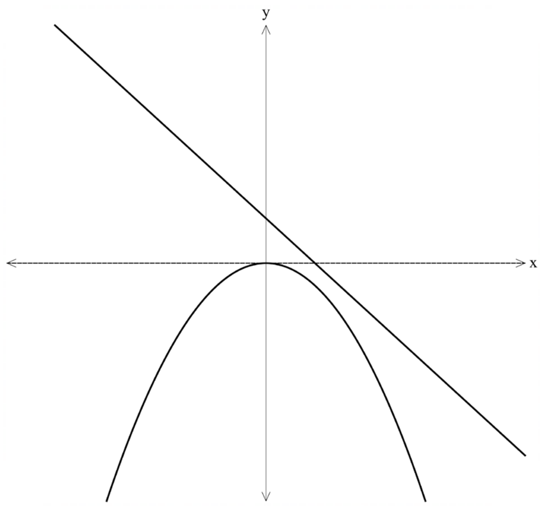
A.



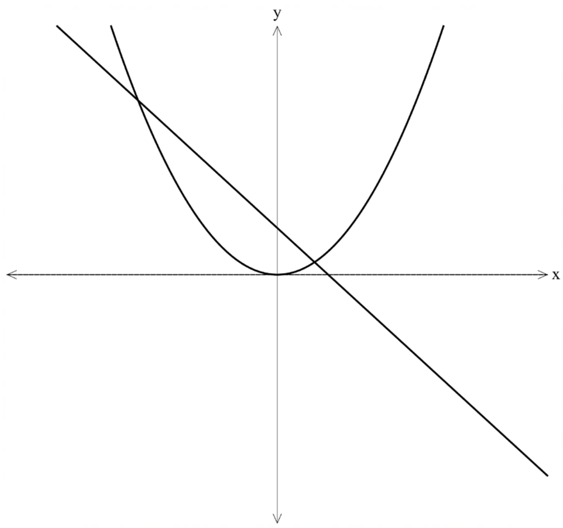
B.



C.



D.



8 Which one of the following is $\sin^{-1}(\cos\theta)$, given that $\frac{3\pi}{2} < \theta < 2\pi$?

A. $\frac{\pi}{2} - \theta$

B. $\frac{3\pi}{2} - \theta$

C. $-\frac{3\pi}{2} + \theta$

D. $2\pi - \theta$

9 The chance of being sunny on three consecutive days is 0.512. The probability that it is Sunny on exactly two days during a period of n days is 0.0512. What is the value of n? Assume the probability of being Sunny is the same every day.

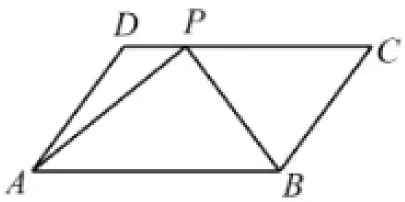
A. 4

B. 5

C. 6

D. 7

10 Inside the parallelogram ABCD below. $|\overrightarrow{AB}| = 8, |\overrightarrow{AD}| = 5, \overrightarrow{CP} = 3\overrightarrow{PD}$ and $\overrightarrow{AP} \cdot \overrightarrow{BP} = 2$. Which of the following is the value of $\overrightarrow{AB} \cdot \overrightarrow{AD}$?



A. 17

B. 19

C. 22

D. 24

End of Section I

Section II

60 marks
Attempt Questions 11–14.
Allow about 1 hour 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Answer each question in a NEW booklet.

(a) Solve: **3**

$$\frac{2x}{x+3} \leq 1$$

(b) Find the cartesian equation of the curve whose parametric equations are: **2**

$$x = 2\cos t,\ y = 2\sin t\,, 0 \leq t \leq \pi$$

(c) Three men and three women are to sit at a round table. How many seating arrangements are possible:

(i) If there are no restrictions? **1**

(ii) If two particular women must sit together? **2**

(d) (i) Express $\cos x - \sqrt{3}\sin x$ in the form $rcos(x+\alpha)$. **1**

(ii) Hence, solve the equation $\cos x - \sqrt{3}\sin x = -\sqrt{3},\ 0 \leq x \leq 2\pi$. **2**

(e) Vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are given by

$$\underset{\sim}{u} = \binom{2}{3} \text{ and } \underset{\sim}{v} = \binom{-2}{4}.$$

Find the magnitude and direction of $\underset{\sim}{u} - \underset{\sim}{v}$. **2**

(f) Evaluate

$$\int_0^{\frac{1}{3}} \frac{dx}{1 + 9x^2}.$$ **2**

End of Question 11

Question 12 (15 marks) Answer each question in a NEW booklet.

(a) Differentiate **2**

$$y = 3x\sin^{-1}(2x).$$

(b) Find the following definite integral using the substitution $x = \cos\theta$. . **3**
Express your answer in exact value.

$$\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}}\,dx$$

(c) A direction field is to be drawn for the differential equation **1**

$$\frac{dy}{dx} = \frac{x+2y}{2y}.$$

On the diagram on the extra page, clearly draw the correct slopes of the direction field at the points Q and R.

(d) It is given that **3**

$$\frac{{}^nP_{r-1}}{a} = \frac{{}^nP_r}{b} = \frac{{}^nP_{r+1}}{c}$$

Prove that $b^2 = a(b+c)$.

(e) Use mathematical induction to prove **3**

$$\cos 2x \cos 2^2x \ldots \cos 2^n x = \frac{\sin 2^{n+1}x}{2^n \sin 2x},$$

where $\sin 2x \neq 0$ for $n = 1,2,3, \ldots.$

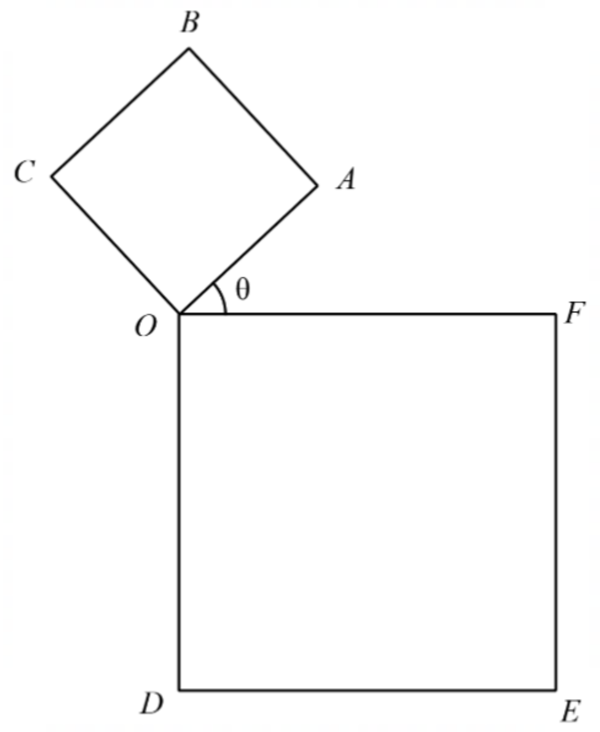
(f) Find the particular solution to the differential equation $\dfrac{dy}{dx} = \dfrac{xy}{x+2}$ that passes through the point $(0,-1)$. **3**

End of Question 12

Question 13 (15 marks) Answer each question in a NEW booklet.

(a) The region between the graph of $f(x) = \sin x \cos x$ and the x − axis over the interval $x \in \left[0, \frac{\pi}{2}\right]$ is revolved about the x − axis. Find the volume of the resulting solid. You may sketch a diagram first. **3**

(b) The diagram below shows two squares $OABC$ and $ODEF$ with $OA = 1,\ OD = 2$ and $\angle AOF = \theta$, where $0° < \theta < 90°$. Let $\overrightarrow{OF} = 2\underset{\sim}{i}, \overrightarrow{OD} = -2\underset{\sim}{j}$ and $\overrightarrow{OA} = \cos\theta\,\underset{\sim}{i} + \sin\theta\,\underset{\sim}{j}$.



(i) Explain why $\overrightarrow{OC} = -\sin\theta\ \underset{\sim}{i} + \cos\theta\ \underset{\sim}{j}$. **1**

(ii) Use vector methods to show that if the points B, A and E are collinear, **3**

$$\cos\theta - \sin\theta = \frac{1}{2}.$$

Question 13 continued on page 12

Question 13 continued

(c) A bottle of soda with an initial temperature of 25°C is placed in a refrigerator with a constant temperature of $6°C$.

The temperature, $T°C$ of the soda after t minutes is given by

$$\frac{dT}{dt} = k(T-6),$$

where k is a constant.

The temperature of the soda drops to $16°C$ after 30 minutes.

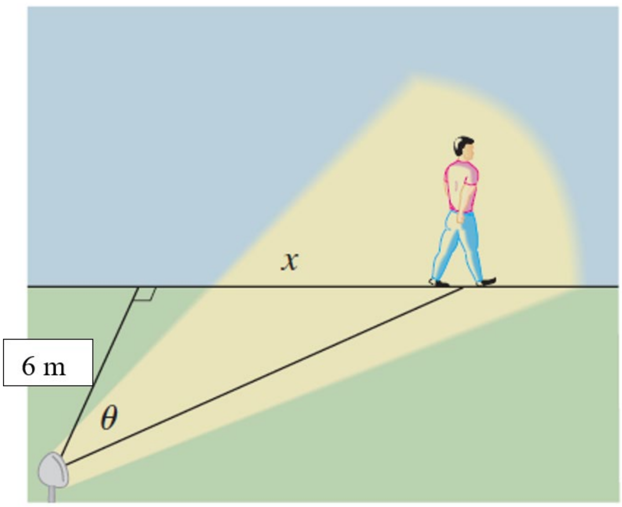
Prove that **2**

$$k = \frac{1}{30}\ln\left(\frac{10}{19}\right)$$

(d) n students have been enrolled into the Higher Mathematics 1A course at university. It is seen that in every maths lecture, 5% of the students will be absent. The probability that no more than 20 students miss the lecture is 0.90658.

Calculate the number of the students n, enrolled in the course. **3**
(A Standard Normal Distribution table is attached with this paper.

(e) A man walks along a straight path at a speed of 1.2 m/s. A searchlight is located on the ground 6 m from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 8 m from the point on the path closest to the searchlight? **3**



End of Question 13

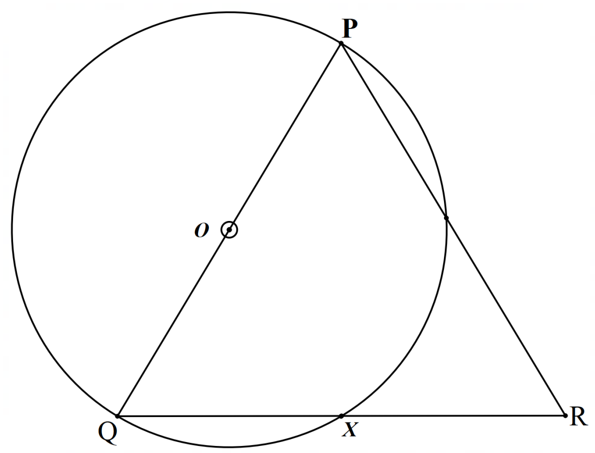
Question 14 (15 marks) Answer each question in a NEW booklet.

(a) The big toy this coming Christmas season is the Tickle Me Jack Stein Doll. When you gently squeeze his tummy, Jack will giggle and tell you stories of his work as a detective. Children love this. **4**
Research shows that the rate of sales is proportional to the product of the number of toys sold and the number in the target audience who have not purchased it yet. Assume that the target audience number is 4 million. 950000 toys have already been sold by 12 am on October 1, and 3.5 million have been sold by 12 am on December 1.

Determine on what day did the company sell to exactly half of its target audience.

You may use the result that $\dfrac{1}{x(a-x)} = \dfrac{1}{ax} + \dfrac{1}{a(a-x)}$.

(b) Triangle PQR is isosceles with $PQ = PR$. A circle with centre O and diameter PQ is drawn, and it cuts QR at X. Let $\overrightarrow{OP} = \underset{\sim}{a}$. **3**



Prove using vector method only that X is the midpoint of QR.

(c) A cereal company is interested to know whether people drink the cereal milk. They asked a random sample of 1012 Australian adults what they do with the milk in the bowl after they have eaten the cereal. Of the respondents, 67% said that they drank it. Suppose that 70% of Australian adults actually drink the cereal milk.

Let $\hat{p}$ be the proportion of people in the sample who drink the cereal milk.

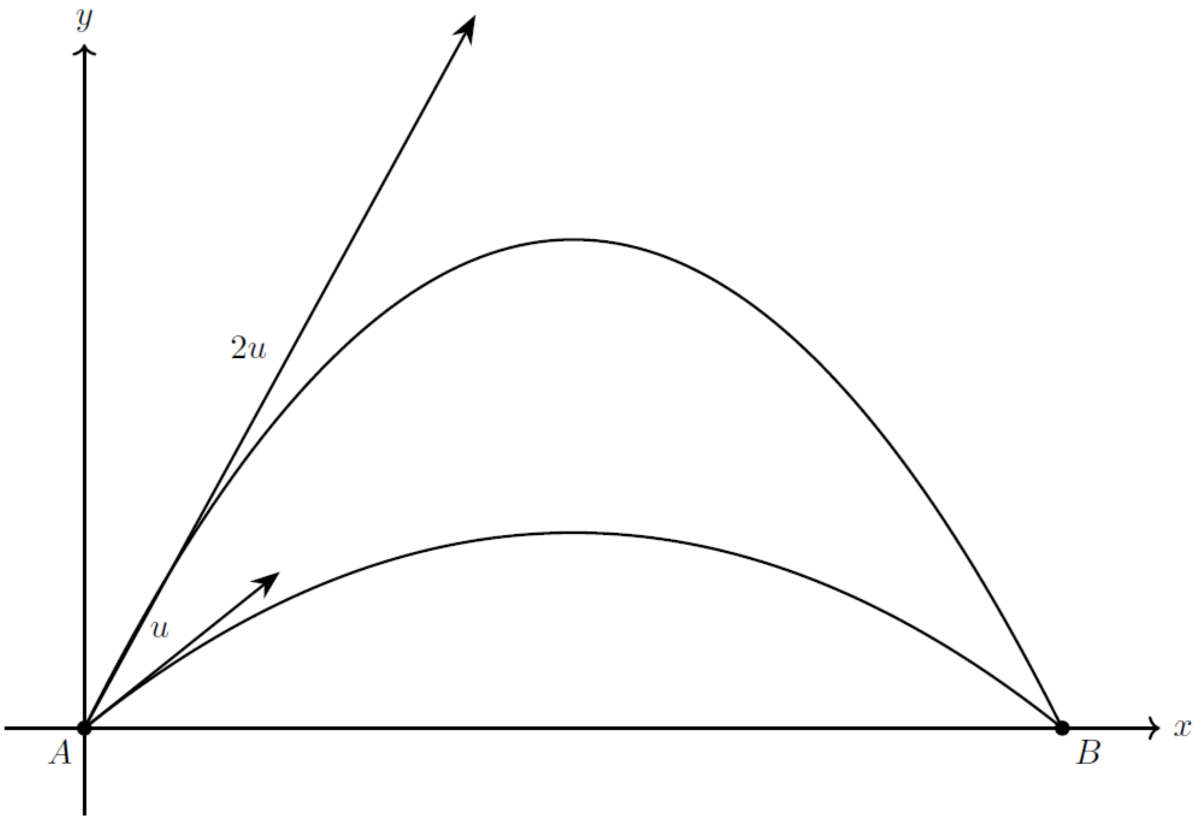
(i) What is the mean and standard deviation of the sampling distribution of $\hat{p}$? **2**

(ii) By calculating the appropriate probability, do you have any doubts about the result of this poll? Justify your solution. **3**

Question 14 continued on page 14

Question 14 continued

(d) Two particles P and Q are projected from the same fixed point A on the horizontal ground. P is projected at an acute angle of θ° to the horizontal with speed $u\ ms^{-1}$, and Q is projected at an acute angle of $2\theta°$ to the horizontal with speed $2u\ ms^{-1}$. **3**



Both P and Q land back on the horizontal ground at the same point B.

If P takes 0.6 seconds to travel from A to B, find the time taken by Q to travel from A to B.

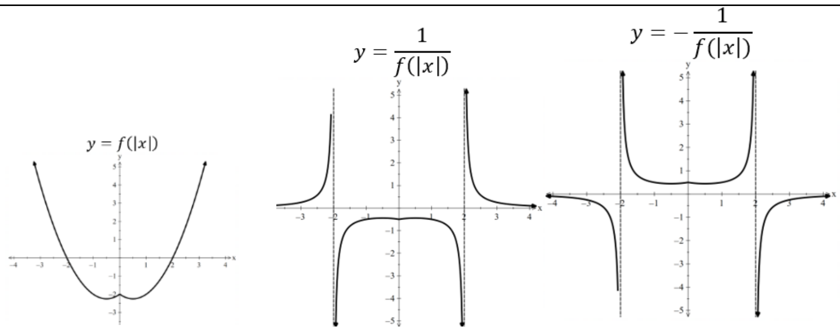
The equations of motion for the particle A is given below. DO NOT PROVE THESE.

$$\underset{\sim}{a} = \begin{pmatrix} 0 \\ -g \end{pmatrix}, \quad \underset{\sim}{v} = \begin{pmatrix} ucos\theta \\ -gt + usin\theta \end{pmatrix}, \quad \underset{\sim}{r} = \begin{pmatrix} ucos\theta\ t \\ -\frac{gt^2}{2} + usin\theta\ t \end{pmatrix}$$

End of paper

2024 Extension 1 Trial marking scheme and feedback KHS

Multiple Choice

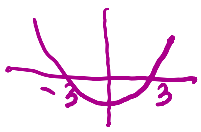
1.	A	Range of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\theta = \sin(\pi - \theta)$ $\therefore \sin^{-1}\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = \pi - \frac{11\pi}{12} = \frac{\pi}{12}$
2.	C	Option A: $P(1) = 1 + 1 - 5 - 2 = -5 \neq 0$ Option B: $P(2) = 8 + 4 - 10 - 2 = 0$ Option C: $P(-2) = -8 + 4 + 10 - 2 = -2 \neq 0$ Option D: $P(-1) = -1 + 1 + 5 - 2 = 3 \neq 0$ $x - 2$ is the factor Answer: C
3.	B	
4.	D	Coefficient of the term $x^3 = \binom{8}{3}(2)^5(-1)^3 = -1792$ Answer: D

5.	C	$x = -5\ and\ x = 1$ have divided the number line into 3 regions $x \leq -5$, $-5 < x < 1$ and $x \geq 1$ $\lvert x+5\rvert + \lvert x-1\rvert = 6$ is true when $-5 \leq x \leq 1$. Note $x = -5\ and\ x = 1$ also satisfy the this equation. The solution to $\frac{1}{x^2-4x-5} \leq 0$ is $-1 < x < 5$ which is not the same. The solution to $x^2 - 4x - 5 < 0$ is $-1 < x < 5$ which is not the same. The solution to $\lvert x+2\rvert \geq 3$ is $x \geq 1\ or x \leq -5$. Still it is not the same. The solution to $\lvert x+2\rvert \leq 3$ is $-3 \leq x+2 \leq 3$. It is basically the same as $-5 \leq x \leq 1$.
6.	D	$n(n-1)^nC_r$ does not make any sense. ${}^nC_r$ computes the number of ways to select r students from n students. In a group of r students selected, we have chosen a captain and a vice captain. Therefore it should be $r(r-1)^nC_r$ However $r(r-1)^nC_r$ has not appeared in either a, b, c or d. $2!^n\ C_r$ is a distractor. $n(n-1)^{n-2}C_{r-2}$ looks weird, but it is correct. First of all we select a captain and then a vice captain from the class, which can be done in $n(n-1)$ ways. Now two people have been selected, so we only need to choose $r-2$ students from the remaining class. $r(r-1)^{n-1}C_{r-1}$ means we need to select a captain and a vice captain from r students chosen, but ${}^{n-1}C_{r-1}$ is wrong. In addition $r(r-1)^nC_r = n(n-1)^{n-2}C_{r-2}$ You can prove this by using the combination formula and the concept of factorials.
7.	A	

8.	C	It is known that $-\frac{\pi}{2} \leq \sin^{-1}(\cos\theta) \leq \frac{\pi}{2}$ and $\frac{3\pi}{2} < \theta < 2\pi$ $\cos\theta = \sin(\frac{\pi}{2} - \theta)$, $$\sin^{-1}(\cos\theta) = \sin^{-1}(\sin(\frac{\pi}{2} - \theta)) \neq \frac{\pi}{2} - \theta$$ $$-2\pi < -\theta < -\frac{3\pi}{2}$$ $-\frac{3\pi}{2} < \frac{\pi}{2} - \theta < -\pi$ This is not $-\frac{\pi}{2} \leq \sin^{-1}(\cos\theta) \leq \frac{\pi}{2}$ Another perspective is that $$\sin^{-1}(\cos\theta) = \sin^{-1}(\sin(\frac{\pi}{2} + \theta)) \neq \frac{\pi}{2} + \theta$$ Because $2\pi < \frac{\pi}{2} + \theta < \frac{5\pi}{2}$; this again is not inside $-\frac{\pi}{2} \leq \sin^{-1}(\cos\theta) \leq \frac{\pi}{2}$ Therefore we have to make it smaller by taking 2π $$0 < \frac{\pi}{2} + \theta - 2\pi < \frac{\pi}{2}$$ $$0 < \theta - \frac{3\pi}{2} < \frac{\pi}{2}$$ This is inside $-\frac{\pi}{2} \leq \sin^{-1}(\cos\theta) \leq \frac{\pi}{2}$. Hence $\sin^{-1}(\cos\theta) = \theta - \frac{3\pi}{2}$ for $\frac{3\pi}{2} < \theta < 2\pi$. Other options will not give the desired range. In the exam, you can choose an angle to test it when necessary.
9.	B	The chance of being sunning on a single day is $\sqrt[3]{0.512} = 0.8$. The probability that it is sunny on exactly two days during a period of n days is $\binom{n}{2} \times 0.8^2 \times 0.2^{n-2} = 0.0512$. This is a binomial distribution $Bin\sim(n, 0.8)$. We can test the 4 different values from a), b), c) and d). The only value that work works is $n = 5$. It would challenging if it was not a multiple choice question.

10.	C	It looks challenging, but we need to be patient. We have to compute $\overrightarrow{AB}\cdot\overrightarrow{AD}$. Although we know their lengths, we do not know the size of $\angle DAB$; otherwise, it would be straightforward. It is given that $DP = 2, PC = 6, AD = 5\ and\ AB = 8$. We need to build a connection between the lengths, $\overrightarrow{AB}\cdot\overrightarrow{AD}$ and $\overrightarrow{AP}\cdot\overrightarrow{BP} = 2$ What we can do is express $\overrightarrow{AP}\ and\ \overrightarrow{BP}$ in terms of $\overrightarrow{AB}$, $\overrightarrow{AD}, \overrightarrow{DP}$ and so on. $$\overrightarrow{AP} = \overrightarrow{AD} + \overrightarrow{DP} = \overrightarrow{AD} + \frac{1}{4}\overrightarrow{AB}$$ $$\overrightarrow{BP} = \overrightarrow{BC} + \overrightarrow{CP} = \overrightarrow{AD} + \frac{3}{4}\overrightarrow{CD} = \overrightarrow{AD} - \frac{3}{4}\overrightarrow{AB}$$ $\overrightarrow{AP}\cdot\overrightarrow{BP} = \left(\overrightarrow{AD} + \frac{1}{4}\overrightarrow{AB}\right)\cdot(\overrightarrow{AD} - \frac{3}{4}\overrightarrow{AB})$ $= \overrightarrow{AD}\cdot\overrightarrow{AD} - \frac{3}{4}\overrightarrow{AB}\cdot\overrightarrow{AD} + \frac{1}{4}\overrightarrow{AB}\cdot\overrightarrow{AD} - \frac{3}{16}\overrightarrow{AB}\cdot\overrightarrow{AB}$ $= 25 - \frac{1}{2}\overrightarrow{AB}\cdot\overrightarrow{AD} - \frac{3}{16}\times 64$ $13 - \frac{1}{2}\overrightarrow{AB}\cdot\overrightarrow{AD} = 2$ $\therefore$ $\frac{1}{2}\overrightarrow{AB}\cdot\overrightarrow{AD} = 11 \quad \overrightarrow{AB}\cdot\overrightarrow{AD} = 22$

Question 11

(a)	$\frac{2x}{x+3} \le 1$ $\frac{2x(x+3)^2}{x+3} \le 1(x+3)^2$ $2x(x+3) \le (x+3)^2$ $(x+3)(2x-x-3) \le 0$ $(x+3)(x-3) \le 0$ $\therefore\ -3 \le x \le 3$ But $x \ne -3,\ then\ -3 < x \le 3$ 	1 mark: multiplies with $(x+3)^2$ on both sides and attempts to simplify 1 mark: correctly solves the inequality and gives $-3 \le x \le 3$ 1 mark: Applies the restriction and gives the correct solution $-3 < x \le 3$	Feedback: A lot of students could solve the inequality, but forgot to consider the domain of $\frac{2x}{x+3}$.
(b)	$x = 2\cos t,\ y = 2\sin t, 0 \le t \le \pi.$ $\cos t = \frac{x}{2}$ $\sin t = \frac{y}{2}$ $\sin^2 t + \cos^2 t = \left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ $\therefore\ x^2 + y^2 = 4$	1 marks: correctly eliminates the parameter t and gives $x^2 + y^2 = 4$ 1 mark: correctly restricts the domain to $0 \le t \le \pi$ and gives the solution $y = \sqrt{4 - x^2}$	Most students could figure out this is a circle $x^2 + y^2 = 4$, but did not consider the range. $0 \le y \le 2$. S Students should make y the subject explicitly.

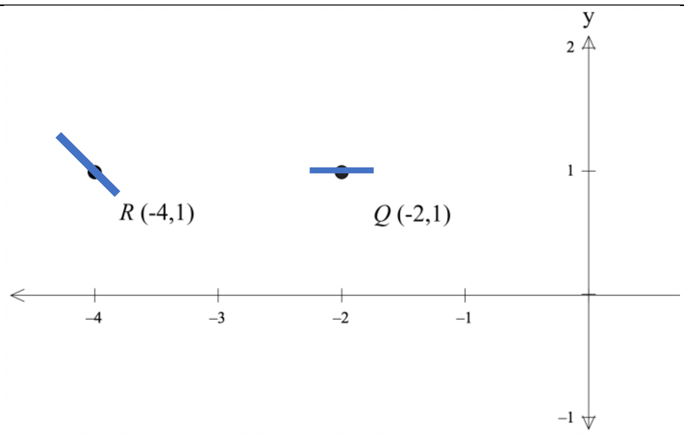
	$0 \le t \le t$, hence, $y = \sqrt{4 - x^2}$		
(c)	(i) 6 people There are $5! = 120\ ways$	 1 mark: correct answer	This was well done.
(c)	(ii) Take the two particular women as one group. Now we have six entities, and can be arranged in 5! Ways = 120 The two women can be arranged in 2! Ways. So, the total is $4! \times 2! = 48\ ways$	1 mark: arranges the 6 entities correctly 1 mark: multiplies by 2!	This part was well done.
(d)(i)	$\cos x - \sqrt{3}\sin x \cong\ rcos(x + \alpha)$ $= rcos\,x\cos\alpha - r\sin x \sin\alpha$ $r\cos\alpha = 1\ ,\qquad rsin\,\alpha = \sqrt{3}$ $r = 2,\qquad \tan\alpha = \sqrt{3}$ $\alpha = \dfrac{\pi}{3}$ $\cos x - \sqrt{3}\sin x \cong\ 2cos\left(x + \dfrac{\pi}{3}\right)$		Mostly done well.
(d) (ii)	$2cos\left(x + \dfrac{\pi}{3}\right) = -\sqrt{3}$ $cos\left(x + \dfrac{\pi}{3}\right) = -\dfrac{\sqrt{3}}{2}$ $x + \dfrac{\pi}{3} = \dfrac{5\pi}{6}, \dfrac{7\pi}{6}$ $x = \dfrac{\pi}{2}, \dfrac{5\pi}{6}$	2 marks: correct solutions from correct working 1 mark: minor error, and gives one correct solution	Mostly done well.
(e)(i)	$\underset{\sim}{u} = \begin{pmatrix}2\\3\end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix}-2\\4\end{pmatrix}$ $\underset{\sim}{u} - \underset{\sim}{v} = \begin{pmatrix}2 - -2\\3 - 4\end{pmatrix} = \begin{pmatrix}4\\-1\end{pmatrix}$ $= \left\lvert\left(\underset{\sim}{u} - \underset{\sim}{v}\right)\right\rvert = \sqrt{4^2 + 1^2} = \sqrt{17}$ Direction = $\tan^{-1}\left(-\frac{1}{4}\right)$ $= \ -14.036 \ldots = -14°2'\ or$ $-0.245\ radians\ or\ 104°N$	1 mark: correct $\underset{\sim}{u} - \underset{\sim}{v}$ and correct magnitude 1 mark: correct bearing from their $\underset{\sim}{u} - \underset{\sim}{v}$	Mostly done well. Some students missed out the direction or left their direction as $\tan^{-1}\left(-\frac{1}{4}\right)$. Most show $-14°\ or$ $104°N$

(f)	$\int_0^{\frac{1}{3}} \frac{dx}{1+9x^2}$ $u = 3x$ $du = 3\,dx$ $u = 0, \quad x = 0$ $u = \frac{1}{3}, \quad x = 1$ $= \frac{1}{3}\int_0^1 \frac{du}{1+u^2}$ $= \frac{1}{3}\,[\tan^{-1} u]_0^1$ $= \frac{1}{3}\left(\frac{\pi}{4} - 0\right)$ $= \frac{\pi}{12}$ OR $\int_0^{\frac{1}{3}} \frac{dx}{9\left(\frac{1}{9} + x^2\right)}$ $= \frac{1}{9} \times 3 \times \left[\tan^{-1} \frac{x}{\frac{1}{3}}\right]_0^{\frac{1}{3}}$ $= \frac{1}{3} \times \left(\frac{\pi}{4} - 0\right)$ $= \frac{\pi}{12}$	2 marks: correct substitution, correct integration and then correct evaluation 1 mark: correct substitution, and attempts to integrate 2 marks: Correct answer from correct working 1 mark: correctly manipulates and minor error in evaluation	Generally done well. Some didn't integrate inverse tan correctly. Common mistake $\int_0^{\frac{1}{3}} \frac{dx}{1+9x^2}$ $= [\tan^{-1} 3x]_0^1$ $= \frac{\pi}{4}$

Question 12

(a)	$y = 3x\sin^{-1}(2x)$ $\frac{dy}{dx} = 3\sin^{-1}(2x) + 3x \times \frac{2}{\sqrt{1-4x^2}}$ $\frac{dy}{dx} = \frac{6x}{\sqrt{1-4x^2}} + 3\sin^{-1}(2x)$	1 mark: correctly applied the product rule or differentiate inverse sine function correctly 2 marks: correct answers	Very well done. Very few students forgot to apply the chain rule when differentiating $\sin^{-1}(2x)$.

(b)	$x = \cos\theta$ $\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}}dx$ $\frac{dx}{d\theta} = -\sin\theta$ $dx = -\sin\theta \cdot dx$ $x_{lower\ limit} = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$ $x_{upper\ limit} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ 1 mark $\int_{-\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{x}{\sqrt{1-x^2}}dx$ $= -\int_{\frac{3\pi}{4}}^{\frac{\pi}{3}} \frac{\cos\theta}{\sin\theta} \sin\theta\, d\theta$ $= -\int_{\frac{3\pi}{4}}^{\frac{\pi}{3}} \cos\theta\, d\theta$ $= [\sin\theta]_{\frac{\pi}{3}}^{\frac{3\pi}{4}}$ $= \left(\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{2}-\sqrt{3}}{2}$	1 mark: find the correct relationship between dx and $d\theta$, and indicate the new limits correctly. 2 marks: correctly used substitution method to find the definite integral but made some error in working 3 marks: correct answer from full correct working	Mostly well done. Some errors are noteworthy: *writing the $sin\theta$ of the $sin\theta\ d\theta$ in the denominator, making the definite integral to be $-\int_{\frac{3\pi}{4}}^{\frac{\pi}{3}} \frac{\cos\theta}{\sin^2\theta}\, d\theta$ *Another common error was in using that $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{-\pi}{4}$ instead of $\frac{3\pi}{4}$. This is a serious error *Also, this question specifically asks you to the required substitution in the evaluation of the definite integral. The intent of the question is to check your proficiency in using this method. So, correctly doing this question with any other method will not yield any marks. However, if the question was “By using the … substitution, or otherwise”, you could use any method.

(c)	 When $x = -4, y = 1$ (Point R) $\frac{dy}{dx} = \frac{-4+2}{2} = -1$ When $x = -2, y = 1$ (Point Q)	1 mark: correct gradient segments	Well done. Please make sure that the graph actually has a gradient of -1 (some of them looked like -3. This is $-45°$ or a tiny bit of the $y = -x$ line.

	$$\frac{dy}{dx}=\frac{-2+2}{2}=0$$		

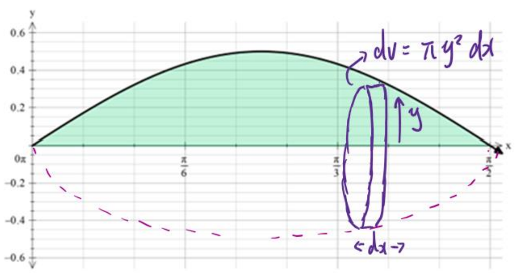
(d)	$\frac{{}^nP_{r-1}}{a}=\frac{{}^nP_r}{b}=\frac{{}^nP_{r+1}}{c}$ $\frac{1}{a}\left(\frac{n!}{(n-r+1)!}\right)=\frac{1}{b}\left(\frac{n!}{(n-r)!}\right)$ $=\frac{1}{c}\left(\frac{n!}{(n-r-1)!}\right)$ From the first two relations, $\frac{b}{a}=\frac{(n-r)!}{(n-r+1)!}=n-r+1$ and from the last two relations, $\frac{c}{b}=\frac{(n-r)!}{(n-r-1)!}=n-r$ Therefore, $\frac{b}{a}-1=\frac{c}{b}$ $\frac{b}{a}=\frac{c+b}{b}$ $b^2=a(b+c)\blacksquare$	1 mark: use the correct formula to expand all three parts of the equation 2 marks: attempted to simplify the equation by making a connection for $\frac{b}{a}$ and $\frac{c}{b}$ 3 marks: correct answers	Most students got 2 out of 3 for this question, struggling to eliminate n and r. Students who have simplified the factorials to linear equations, received 2 marks

(e)	Test for $n=1$ $LHS=\cos 2x$ $RHS=\frac{\sin 4x}{2\sin 2x}$ $=\frac{2\sin 2x\cos 2x}{2\sin 2x}$ $=\cos 2x$ $=LHS$ It is true for $n=1$. Assume it is true for $n=k$, where k is a positive integer. $\cos 2x\cos 2^2x\ldots\cos 2^kx=\frac{\sin 2^{k+1}x}{2^k\sin 2x}\quad(*)$ RTP: When $n=k+1$,	1 mark: show the $n=1$ case is true. 2 marks: show the assumption, and use the assumption in the inductive step, and attempts to prove the result – but where the proof was not complete. 3 marks for a complete induction proof, with correct and clear algebra in the inductive process.	Most students got full marks for this question. To get 2 marks, students need to prove the result for n=1, write te statements for $n=k, n=k+1$, and show some attempt in simplifying this: $\frac{\sin 2^{k+1}x}{2^k\sin 2x}\times\cos 2^{k+1}x$

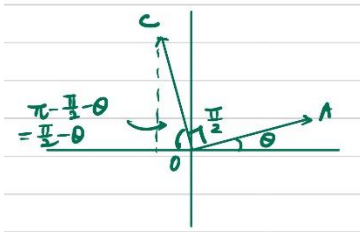
	$= \dfrac{\sin 2^{k+2}x}{2^{k+1}\sin 2x}$ $= RHS$ Since it is true for $n = 1$, assumed it is true for $n = k$, it is proved it is also true for $n = k + 1$, i.e. $n = 2,3,4, \ldots$etc. By the principle of mathematical induction, it is true for all value of integers $(n \geq 1)$.		

(f)	$\dfrac{dy}{dx} = \dfrac{xy}{x+2}$ $\dfrac{1}{y}dy = \dfrac{x}{x+2}dx$ $\int \dfrac{1}{y}dy = \int 1 - \dfrac{2}{x+2}dx$ $\ln(\lvert y\rvert) = x - 2\ln(\lvert x+2\rvert) + C$ $\ln(\lvert y(x+2)^2\rvert) = x + C$ $ke^x = \lvert y(x+2)^2\rvert$ $\lvert y\rvert = \dfrac{ke^x}{(x+2)^2}$ $y = \pm\dfrac{ke^x}{(x+2)^2}$ When $x = 0, y = -1$ $\Longrightarrow$ y need to be negative $-1 = \dfrac{ke^0}{(0+2)^2}$ $\therefore k = -4$ $y = \dfrac{-4e^x}{(x+2)^2}$	1 mark: attempted to separate variables and started the integration process, but made errors in integration 2 marks: correctly integrated both sides and attempted to rearrange the equation to make y the subject, but made errors in working 3 marks: correct answer from correct working	Most students got 2 out of 3 for this question. *Students need to completely simplify the final answer (this requires applying the log rules) *Need to show the solution of the absolute value equation $\lvert y\rvert = \dfrac{ke^x}{(x+2)^2}$ $y = \pm\dfrac{ke^x}{(x+2)^2}$, Then use $(0, -1)$ to choose the correct solution

Question 13



13(a)	$\pi\int_0^{\frac{\pi}{2}}(\sin(x)\cos(x))^2\,dx$ $=\pi\left[\int_0^{\frac{\pi}{2}}\frac{1}{4}\sin^2 2x\,dx\right]$ $=\frac{\pi}{8}\int_0^{\frac{\pi}{2}}1-\cos 4x\,dx$ $=\frac{\pi}{8}(x-\frac{\sin 4x}{4})_0^{\frac{\pi}{2}}$ $=\frac{\pi^2}{16}\ units^3$	1 mark: Sketch the diagram and set up the integral $\pi\int_0^{\frac{\pi}{2}}(\sin(x)\cos(x))^2\,dx$ 2 marks: all above and express the integrand in terms of $1-\cos 4x$ 3 marks: correct answer from correct working.	Students didn't do well as expected. Almost all students can set up the correct integral, but took an unnecessary long way to convert $(\sin(x)\cos(x))^2$ into the integrand. Various mistakes were observed in working. 1 mark for setting up the integral correctly. To achieve 2 marks, student must demonstrate attempting to convert the expression to the integrand using correct trig identity but made small mistakes in the middle, leading to incorrect substitution.

13b(i)	$\angle COF = 90° + \theta$ Therefore $\overrightarrow{OC} = 1\times\cos(90°+\theta)\hat{\imath} + 1\times\sin(90°+\theta)\hat{\jmath}$ $\overrightarrow{OC} = -\sin(\theta)\hat{\imath}+\cos(\theta)\hat{\jmath}$	1 mark: correct justification.	Poorly attempted. Lot of students use the orthogonal property of $\overrightarrow{OC}$ and $\overrightarrow{OA}$ but failed to explain why the negative sign must be on the $-\sin(\theta)$. Correct justification must either include a valid diagram as such  to show that $\overrightarrow{OC} = \begin{pmatrix}-\cos(\frac{\pi}{2}-\theta)\\ \sin(\frac{\pi}{2}-\theta)\end{pmatrix} = \begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}$ Or expressed $\angle COF = 90° + \theta$ And justify as the sample answer shown on the left.
(ii)	$\overrightarrow{OC} = \overrightarrow{AB} = (-\sin\theta, \cos\theta)$ $\overrightarrow{OE} = (2,-2)$ $\therefore \overrightarrow{EA} = (\cos\theta - 2, \sin\theta + 2)$	1 mark: Find vectors $\overrightarrow{AB}$ **and** $\overrightarrow{EA}$ or equivalent ($\overrightarrow{AE}$ *or* $\overrightarrow{EB}$ *or* $\overrightarrow{BE}$.	Poorly done. Most students understood that they needed to express $\overrightarrow{EA}$ or $\overrightarrow{EB}$ as a scalar multiple

	If B, A and E are collinear, then there exists a scalar k such that $k\overrightarrow{AB} = \overrightarrow{AE}$ $-k\sin\theta = \cos\theta - 2$ $k\cos\theta = \sin\theta + 2$ $k = \frac{\cos\theta - 2}{-\sin\theta} = \frac{\sin\theta + 2}{\cos\theta}$ $\cos^2\theta - 2\cos\theta = -\sin^2\theta - 2\sin\theta$ $2\cos\theta - 2\sin\theta = \cos^2\theta + \sin^2\theta$ $2\cos\theta - 2\sin\theta = 1$ $\therefore \cos\theta - \sin\theta = \frac{1}{2}$	2 marks: establish two equations relating the x and y components and above. 3 marks: correct working	of $\overrightarrow{BA}$, but made mistakes in vector addition. Some students were attempting to find the value of k, instead of setting a ratio equation.
13(c)	$\frac{dT}{dt} = k(T-6)$ $\int \frac{1}{T-6}dT = \int k\, dt$ Note $T > 6$ $\ln(T-6) = kt + C_1$ $T - 6 = C_2e^{kt}$ C_1 and C_2 are constants. $T = 6 + C_2e^{kt}$ $T(0) = 6 + C_2 = 25$ $\therefore C_2 = 19$ $T = 6 + 19e^{kt}$ $T(30) = 6 + 19e^{30k} = 16$ $19e^{30k} = 10$ $e^{30k} = \frac{10}{19}$ $k = \frac{1}{30}\ln(\frac{10}{19})$	1 mark: Find the constant=19 2 marks: correct answer from correct working.	Mostly done well.
13(d)	Let the number of the students enrolled in the course to be n $p = 0.05, q = 0.95$ $\mu = 0.05n, \sigma = \sqrt{0.05 \times 0.95n}$ The number of absent students each lecture, $N \sim Bin(n, 0.05)$. Let us assume a normal approximation will work here. $Bin(n, 0.05) \approx N(0.05n, 0.05 \times 0.95n)$. **<u>Without using continuity correction</u>** $P(N \leq 20) = 0.90588$ From the z-score table, we know $P(Z \leq 1.32) = 0.9088$ $\therefore \frac{20 - 0.05n}{\sqrt{0.05 \times 0.95n}} = 1.32$ $0.05n + 1.32\sqrt{0.05 \times 0.095n} - 20 = 0$ $n = 300.293$ $n = 300$ *Note: this is a quadratic equation in terms of* $\sqrt{n}$ **<u>Using continuity correction</u>** $P(N \leq 20.5) = 0.90588$	1 mark: Find both expressions for the mean and standard deviation 2 marks: Write an equation involving n and a z-score of 1.32 and above. 3 marks: Calculate the number of students correctly from correct working.	Poorly attempted. Students confused about the formula for mean and standard deviation for sample proportion and normal approximation. Some set up the correct quadratic equation by squaring both sides. Two answers were calculated, must demonstrate that $n = 532$ is not a valid answer as $z = -1.32$

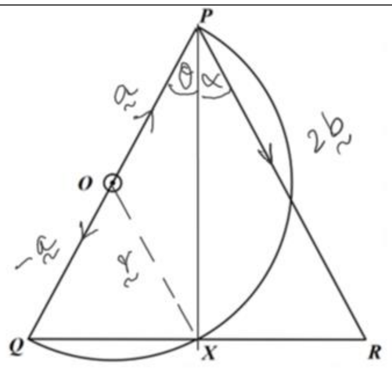
	$\therefore \dfrac{20.5 - 0.05n}{\sqrt{0.05 \times 0.95n}} = 1.32$ $n = 309$ Hence there are either 300 or 309 students enrolled in Mathematics 1A. In fact, if the number of students is 300, then. $np = 300 \times 0.05 = 15 > 10$ $nq = 300 \times 0.95 = 285 > 10$ It is reasonable to use the normal approximation.		
13(e)	$\tan\theta = \dfrac{x}{6}$ $x = 6\tan\theta$ $\frac{dx}{dt} = 6\sec^2\theta\ \frac{d\theta}{dt}$ using the chain rule We need to find $\frac{d\theta}{dt}$ $\dfrac{d\theta}{dt} = \dfrac{1}{6}\cos^2\theta\ \dfrac{dx}{dt}$ Given $x = 8\ m$ and $\frac{dx}{dt} = 1.2\ m/s$ $\tan\theta = \frac{8}{6} = \frac{4}{3}$, $\cos\theta = \frac{3}{5}$ $\dfrac{d\theta}{dt} = \dfrac{1}{6} \times \left(\dfrac{3}{5}\right)^2 \times 1.2 = \dfrac{9}{125}\ rad/s$ $or\ 0.072\ rad/s$	1 mark: establish $x = 6\tan\theta$ and differentiate $\tan\theta$ correctly or equivalent, but fail to apply the chain rule 2 marks: find the correct expression of $\frac{d\theta}{dt}$ 3 marks: correct answer from correct working.	Generally done well. A large number of students convert $x = 6\tan\theta$ To $\theta = \tan^{-1}\left(\frac{x}{6}\right)$ and attempted to find $\frac{d\theta}{dx}$ but made mistakes in differentiation inverse tan, which led to an incorrect answer.

Question 14

(a)	Let S be the no. of sales made at time, t. $\dfrac{dS}{dt} = kS(4000000 - S)$ $\displaystyle\int \frac{ds}{S(4000000 - S)} = \int kdt$ $= \dfrac{1}{4000000}\displaystyle\int \frac{1}{s} + \frac{1}{4000000 - S} = \int kdt$ $= \dfrac{1}{4000000}\displaystyle\int \frac{1}{s} - \frac{-1}{4000000 - S} = \int kdt$ $= \dfrac{1}{4000000}\ln\left\|\dfrac{S}{4000000 - S}\right\| = kt + C$ $= \ln\left\|\dfrac{S}{4000000 - S}\right\| = 4000000(kt + C)$ $= \dfrac{S}{4000000 - S} = Ae^{4000000kt}$	1 mark: Writes the correct differential equation, and makes significant progress toward the general logistic function (i) 2 marks: Writes the correct differential equation, and develops the general logistic function	Students need to practise setting up the differential equation correctly. Some students did not include the constant proportionality constant. While a number of students were able to determine on of the constants $\frac{61}{19}$, they did not find the value of k properly. Some students used t=2 months for 3.5-million sale instead of t=61 days. This has led to a reasonable answer as well.

	$= \dfrac{4000000 - S}{S} = Ae^{-4000000kt}$ $= \dfrac{4000000}{S} - 1 = Ae^{-4000000kt}$ $= \dfrac{4000000}{S} = 1 + Ae^{-4000000kt}$ $\therefore S = \dfrac{4000000}{1 + Ae^{-4000000kt}}$ (1 mark) In this problem $S = 4000000$ Evaluating A T = 0, October 1, S = 950 000 $950000 = \dfrac{4000000}{1 + Ae^{-4000000k\times 0}}$ $950000 = \dfrac{4000000}{1 + A}$ $1 + A = \dfrac{4000000}{950000}$ $A = \dfrac{61}{19}$ 1 mark On December 1, t = 61 $t = 61, \quad S = 3500\,000$ $3500\,000 = \dfrac{4000000}{1 + \frac{61}{19}\, e^{-4000000k\times 61}}$ $1 + \dfrac{61}{19}\, e^{-4000000k\times 61} = \dfrac{4000000}{3500\,000}$ $\dfrac{61}{19}\, e^{-4000000k\times 61} = \dfrac{1}{7}$ $e^{-244\,000\,000k} = \dfrac{19}{427}$ $k = \dfrac{\ln\left(\frac{19}{427}\right)}{-244\,000\,000} = 0.000\,0000127\,....$ Finding t when $S = 2000\,000$ (half the target audience)	1 mark: correctly evaluates A 1 mark: Evaluates the value of k and correctly solves the problem, giving the day sale has hit 2 million mark.	

	$2000\,000 = \dfrac{4000000}{1+\dfrac{61}{19}\,e^{-4\times 10^{6}\times 1.275\ldots\times 10^{-7}\times t}}$ $1+\dfrac{61}{19}\,e^{-0.05102\ldots t} = 2$ $e^{-0.05102\ldots t} = \dfrac{19}{61}$ $t = \dfrac{\ln\left(\frac{19}{61}\right)}{-0.05102\ldots.} = 22.861\ldots$ So, t = 23, this is October 23. 1 mark		
	A little more concise solution is given below:		
	$\dfrac{1}{x(a-x)} = \dfrac{1}{ax} + \dfrac{1}{a(a-x)}$ $\dfrac{dS}{dt} = kS(4-S)$ $\dfrac{dS}{S(4-S)} = kdt$ When t = 0, S= 0.95 million $\int_{0.95}^{S} \dfrac{dS}{S(4-S)} = \int_{0}^{t} kdt$ $\int_{0.95}^{S} \dfrac{1}{4S} + \dfrac{1}{4(4-S)}\,dS = \int_{0}^{t} kdt$ 1 mark: $\dfrac{1}{4}\int_{0.95}^{S} \dfrac{1}{S} - \dfrac{-1}{(4-S)}\,dS = \int_{0}^{t} kdt$ $\dfrac{1}{4}\left[\ln\left\|\dfrac{S}{4-S}\right\|\right]_{0.95}^{S} = kt$ $\dfrac{1}{4}\left\{\ln\left\|\dfrac{S}{4-S}\right\| - \ln\left\|\dfrac{0.95}{3.05}\right\|\right\} = kt$	1mark: correct differential equation	

	$\frac{1}{4}\left\{\ln\frac{\left\lvert\frac{S}{4-S}\right\rvert}{\left\lvert\frac{0.95}{3.05}\right\rvert}\right\} = kt$ $\frac{1}{4}\ln\left\lvert\frac{61}{19}\left(\frac{S}{4-S}\right)\right\rvert = kt$ $t = 61, S = 3.5$ $\frac{1}{4}\ln\left\lvert\frac{61}{19}\left(\frac{3.5}{0.5}\right)\right\rvert = 61k$ $k = \frac{1}{61\times 4}\ln\left\lvert\frac{61}{19}\left(\frac{3.5}{0.5}\right)\right\rvert$ $S = 2, \quad t = \frac{1}{4k}\ln\left\lvert\frac{61}{19}\left(\frac{2}{4-2}\right)\right\rvert$ $= 22.86\ldots$ $\approx 23\ days$ This is October 23.	1 mark: correct k 2 marks: sub S = 2 gives the correct date	
(b)	 We need to prove that $\overrightarrow{PX} \perp \overrightarrow{QR}$ Let $\overrightarrow{OX} = \underset{\sim}{r}$ $\overrightarrow{PX}\cdot\overrightarrow{QX} = \left(\underset{\sim}{r}-\underset{\sim}{a}\right)\cdot\left(\underset{\sim}{r}+\underset{\sim}{a}\right)$ $\underset{\sim}{r}\cdot\underset{\sim}{r}+\underset{\sim}{r}\cdot\underset{\sim}{a}-\underset{\sim}{a}\cdot\underset{\sim}{r}-\underset{\sim}{a}\cdot\underset{\sim}{a}$ $\left\lvert\underset{\sim}{r}\right\rvert^2-\left\lvert\underset{\sim}{a}\right\rvert^2$ $\left\lvert\underset{\sim}{r}\right\rvert = \left\lvert\underset{\sim}{a}\right\rvert$ Hence, $\overrightarrow{PX}\cdot\overrightarrow{QX} = 0$ Hence, $\overrightarrow{PX} \perp \overrightarrow{QR}$ That is, PX is the altitude of ΔPQR. Hence, $Scal\ Proj_{\overrightarrow{PX}}\overrightarrow{PQ} = Scal\ Proj_{\overrightarrow{PX}}\overrightarrow{PR} = \left\lvert\overrightarrow{PX}\right\rvert$	1 mark: proves $\overrightarrow{PX} \perp \overrightarrow{QR}$ 2 marks: Correctly proves the result 1 mark: correct logic and makes progress toward the proof	Most students have not attempted this part properly. Only a small number of students proved $\overrightarrow{PX} \perp \overrightarrow{QR}$ using vectors, but they did not continue their proof using vectors. This was not penalised. Giving reasons such as congruence and other geometric properties would not earn any mark.

	That is $\left\|-2\underset{\sim}{a}\right\|\cos\theta = \left\|2\underset{\sim}{b}\right\|\cos\alpha = \left\|\overrightarrow{PX}\right\|$ $\left\|-2\underset{\sim}{a}\right\| = \left\|2\underset{\sim}{b}\right\|$ (PQ = PR, given) Hence, $\cos\theta = \cos\alpha$ Hence $\theta = \alpha$ $\left\|-2\underset{\sim}{a}\right\|\sin\theta = \left\|2\underset{\sim}{b}\right\|\sin\alpha$ That is $\left\|\overrightarrow{QX}\right\| = \left\|\overrightarrow{RX}\right\|$ That is X is the midpoint of QR		
(c)(i)	Mean $\mu_{\hat{p}} = p = 0.7$ Standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ $= \sqrt{\frac{0.7(1-0.7)}{1012}}$ $= 0.0144052$	1 mark each	Some students struggled with sample proportion. A common mistake is $\mu_{\hat{p}} = 0.67$. Students should learn 0.67 is the sample proportion that is used to estimate the population mean 0.70. Some students used np to calculate the mean. However this is not a binomial distribution. Some students made similar mistakes when calculating the standard deviation. **A major mistake** $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$ $= \sqrt{\frac{0.67(1-0.67)}{1012}}$ Only 1 mark was given if the student gave mean as 0.67 (Error Carried Forward)
(ii)	Sample proportion $(\hat{p}) = 0.67$ Testing whether normal approximation can be applied. $np = 1012 \times 0.7$ $= 708.4 > 10$	1 mark: tests the appropriateness of normal approximation	Students did not test the appropriateness of the normal distribution. Most students could find the z score, but a common mistake is :

	$n(1-p) = 1012 \times 0.3$ $= 303.6 > 10$ Hence, normal approximation can be applied. The probability that fewer or 67% are choosing to drink the cereal milk is: $P(\hat{p} \leq 0.67)$ $P\left(\frac{\hat{p}-0.7}{0.0144052} \leq \frac{0.67-0.7}{0.0144052}\right)$ $P(Z \leq -2.08)$ $= 0.0188 \approx 1.9\%$ This probability is less than 0.05. Thus, there is doubt on the occurrence of this event. Very unlikely.	2 marks: calculates the correct probability of $P(\hat{p} \leq 0.67)$ and gives the correct decision. 1 mark: makes significant progress toward the solution – must give $P\left(\frac{\hat{p}-0.7}{0.0144052} \leq \frac{0.67-0.7}{0.0144052}\right)$	$\frac{0.7-0.67}{0.0144052}$

(d)	At B, for the particle P, $-\frac{gt_p^2}{2} + u\sin\theta\, t_p = 0$ $t_p \neq 0$ $t_p = \frac{2u\sin\theta}{g}$ The time taken by Q to reach B, $-\frac{gt_Q^2}{2} + u\sin 2\theta\, t_Q = 0$ So, $t_Q = \frac{4u\sin 2\theta}{g}$ We know, the time taken by P is 0.6 seconds. Hence, $t_p = \frac{2u\sin\theta}{g} = 0.6$ $t_Q = \frac{2 \times 4u\sin\theta\cos\theta}{g}$ $= 4 \times 0.6 \times \cos\theta \qquad (1)$ 1 mark We need to find $\cos\theta$. The Horizontal distances travelled by P and Q are equal. $u\cos\theta\, t_p = 2u\cos 2\theta\, t_Q$	1 mark: find expressions for t_p and t_Q, and gives $t_Q = 4 \times 0.6 \times \cos\theta$ 1 mark: equates the horizontal range and proves $\cos 2\theta = \frac{1}{8}$ 1 mark: finds the value of $\cos\theta$, and correctly evaluates t_Q	Students had trouble with the question, not knowing which pronumeral can be eliminated. For example, u can be cancelled. Another issue is that some students used same pronumeral t to represent the flight time of both particle p and particle q. Most students could not figure out $\cos\theta$ and $\cos 2\theta$

	$$u\cos\theta \times \frac{2u\sin\theta}{g} = 2u\cos 2\theta \times \frac{4u\sin 2\theta}{g}$$ $$\sin 2\theta = 8\sin 2\theta \cos 2\theta$$ Hence, $$\cos 2\theta = \frac{1}{8}$$ 1 mark We need to find the values of $\cos\theta$ $$\cos 2\theta = \frac{1}{8}$$ $$2\cos^2\theta - 1 = \frac{1}{8}$$ $$\cos^2\theta = \frac{9}{16}$$ θ is acute , so, $$\cos\theta = \frac{3}{4}$$ Hence, $$t_Q = 4 \times 0.6 \times \cos\theta$$ $$t_Q = 2.4 \times \frac{3}{4} = 1.8 \text{ seconds}$$		