CSSA

2021

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

DO NOT REMOVE PAPER FROM EXAM ROOM

Centre Number

Student Number

Mathematics Extension 1

Afternoon Session
Friday, 6 August 2021

General Instructions

- Reading time-10 minutes
- Working time-2 hours
- Write using black pen
- NESA-approved calculators may be used
- A reference sheet is provided
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks-70

Section I Pages 2-6

10 marks

- Attempt Questions 1-10
- Allow 15 minutes for this section

Section II Pages 7-11

60 marks

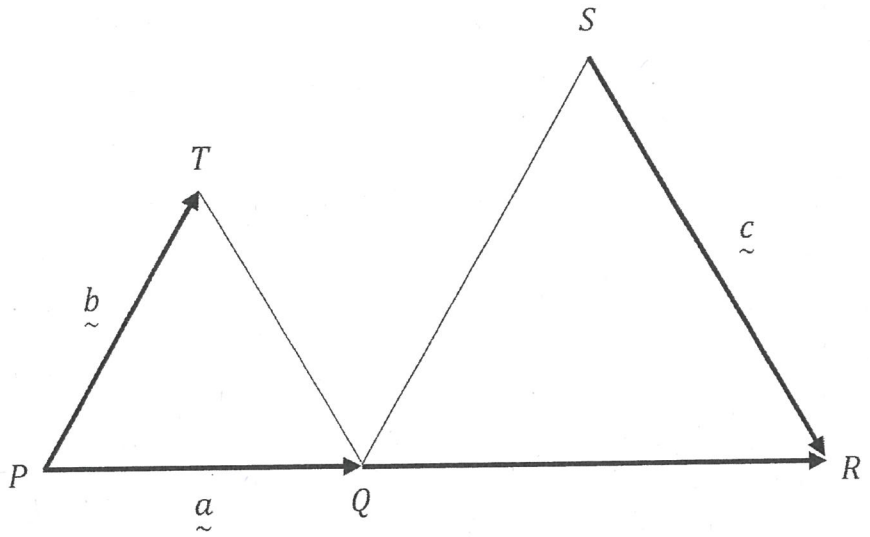
- Attempt Questions 11-14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1-10.

1



The figure shown above consists of two equilateral triangles where P, Q, and R are collinear with $\overrightarrow{PQ} = \underset{\sim}{a}$, $\overrightarrow{PT} = \underset{\sim}{b}$ and $\overrightarrow{SR} = \underset{\sim}{c}$.
Which of the following statement is true?

A. $\underset{\sim}{a} = \underset{\sim}{b}$

B. $\overrightarrow{QR} = \underset{\sim}{c}$

C. $\overrightarrow{SQ} = \underset{\sim}{b}$

D. $|\underset{\sim}{b}| = |\underset{\sim}{a}|$

2 What is the Cartesian equation whose parametric equations are given by: $x = 4\sin\theta - 1$ and $y = 3\cos\theta + 2$?

A. $\dfrac{(x-1)^2}{4} + \dfrac{(y-2)^2}{3} = 1$

B. $\dfrac{(x+1)^2}{4} + \dfrac{(y-2)^2}{3} = 1$

C. $\dfrac{(x-1)^2}{16} + \dfrac{(y+2)^2}{9} = 1$

D. $\dfrac{(x+1)^2}{16} + \dfrac{(y-2)^2}{9} = 1$

3 The polynomial $f(x) = 2x^2 + kx + 4$ can be expressed as $f(x) = (x-2)g(x) + 6$. Which of the following is the correct expression for $g(x)$?

A. $2x - 1$

B. $2x + 1$

C. $2x - 3$

D. $2x + 3$

4 Which of the following expressions is correct?

A. $\sin^{-1} x = \cos^{-1}(-x) - \dfrac{\pi}{2}$

B. $\sin^{-1} x = -\cos^{-1} x - \dfrac{\pi}{2}$

C. $\sin^{-1} x = -\cos^{-1}(-x) - \dfrac{\pi}{2}$

D. $\sin^{-1} x = -\cos^{-1}(-x) + \dfrac{\pi}{2}$

5 Which of the following functions has a domain $[1, 5]$ and range $[1, 4\pi + 1]$?

A. $f(x) = 2\sin^{-1}\left(\frac{x-3}{2}\right)+1$

B. $f(x) = 4\cos^{-1}\left(\frac{x-3}{2}\right)+1$

C. $f(x) = 4\sin^{-1}\left(\frac{x-3}{2}\right)+1$

D. $f(x) = 2\cos^{-1}\left(\frac{x-3}{2}\right)+1$

6 The temperature, T, in degrees Celsius, of a metal bar being heated after t hours is defined as $T = 50 + Be^{kt}$, where B and k are constants. The metal bar's temperature was measured to be $1800°$ C after 2 hours of being heated.
After a further 2 hours its temperature was $2500°$ C. What would the approximate time be when it reaches a temperature of $2200°$ C?

A. 3 hours and 42 minutes

B. 3 hours and 13 minutes

C. 2 hours and 42 minutes

D. 2 hours and 13 minutes

7 Given the points $A\begin{pmatrix}2\\3\end{pmatrix}$, $B\begin{pmatrix}1\\4\end{pmatrix}$ and $C\begin{pmatrix}3\\q\end{pmatrix}$, it is also known that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{BC}$.
What is the value of q?

A. -6

B. -2

C. 2

D. 6

8 A Bernoulli variable, X, has a value of p such that $E(X) = 3\text{Var}(X)$.
Given that $p \neq 0$, what is the value of p?

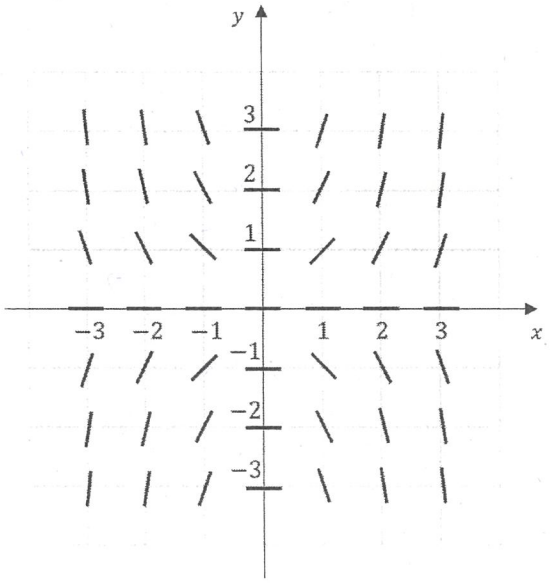
A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

9 Which of the following differential equations is represented by the slope field below?



A. $y \times \frac{dy}{dx} = x$

B. $x \times \frac{dy}{dx} = y$

C. $\frac{dy}{dx} + xy = 0$

D. $\frac{dy}{dx} - xy = 0$

10 There are 26 cards in a bag, each has a different letter of the alphabet written on them.

A game consists of drawing cards one at a time, without replacement, until two consecutive letters of the alphabet have been drawn. A and Z are not consecutive letters.

For example, if B is drawn first and M is drawn second, if the third card is either A, C, L or N the game would stop there as A and B, or B and C, or L and M, or M and N form a consecutive pair of letters. There would be 23 letters left in the bag.

What is the least number of cards that can be left in the bag at the end of the game?

A. 11

B. 12

C. 13

D. 14

End of Section I

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

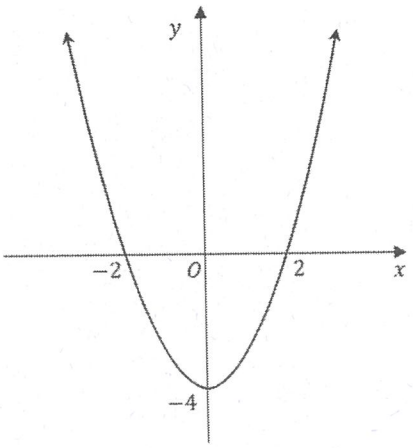
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Your responses in Questions 11-14 should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $\dfrac{3}{x-2} < 4$. **2**

(b) The graph below shows $y = f(x)$ where $f(x) = x^2 - 4$. Sketch the following curves on separate graphs, each at least one-third of a page in size.



(i) $y = -f(x)$. **1**

(ii) $y = |f(x)|$. **1**

(c) How many arrangements of the 8 letters of the word PROBLEMS are possible if the vowels cannot be adjacent to each other? **2**

(d) The polynomial $P(x) = ax^3 + bx^2 + c$ has a double zero at $x = 2$ and has remainder -64 when divided by $x + 2$. Find a, b and c. **3**

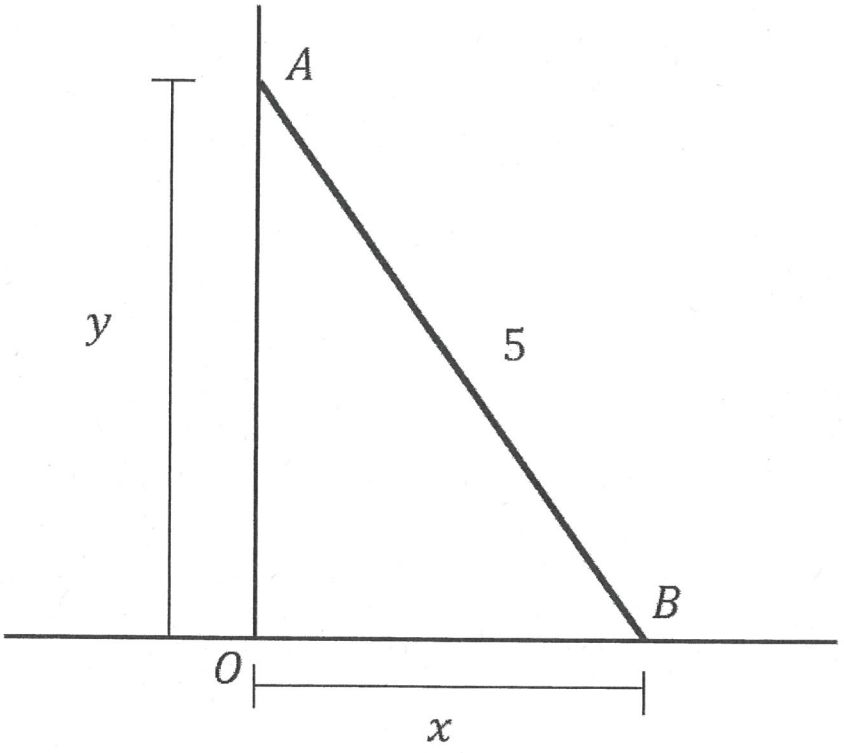
(e) Given that $\cos x - \sin x = \dfrac{1}{3}$ and $0 < x < \dfrac{\pi}{4}$, find the exact value of $\cot 2x$. **3**

(f) Find the value of n if $\int_0^{\frac{2}{3}} \dfrac{dx}{4 + 9x^2} = n\pi$. **3**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Using the substitution $u = 1 + e^x$ or otherwise, find $\int \frac{e^{3x}}{1+e^x} dx$. **3**

(b) A ladder, AB, of length 5 metres, is leaning against a vertical wall OA (y metres), with its foot B, on horizontal ground OB (x metres). The foot of the ladder begins to slide along the ground away from the wall at a constant speed of 1 metre per second. **3**



Find the rate at which the top of the ladder A is moving down the wall at the time when the top of the ladder is 4 metres above the ground.

(c) Prove by mathematical induction that $4^n + 14$ is a multiple of 6 for $n \geq 1$. **3**

Question 12 is continued on page 9

Question 12 continued

(d) The probability that a player hits the bullseye on any one throw of a dart is 70%. Let X be a binomial random variable representing the number of times the player hits the bullseye in 100 consecutive throws.

(i) Find the values of $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. **2**

(ii) Find the probability that they hit the bullseye exactly 70 times from the 100 throws, correct to 3 significant figures. **2**

Let $\hat{p}$ represent the average number of times they hit the bullseye per throw from the 100 consecutive throws.

(iii) Using a normal approximation to $\hat{p}$, or otherwise, find an approximation to 2 decimal places for the probability that the average number of times they hit the bullseye per throw is less than 65%. **2**

The table below shows a section of the normal distribution table $P(Z < z)$.

z	− 0.00	− 0.01	− 0.02	− 0.03	− 0.04	− 0.05	− 0.06	− 0.07	− 0.08	− 0.09
-1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
-1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
-1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
-0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
-0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
-0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
-0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
-0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
-0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
-0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
-0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
-0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
-0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
z	− 0.00	− 0.01	− 0.02	− 0.03	− 0.04	− 0.05	− 0.06	− 0.07	− 0.08	− 0.09

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

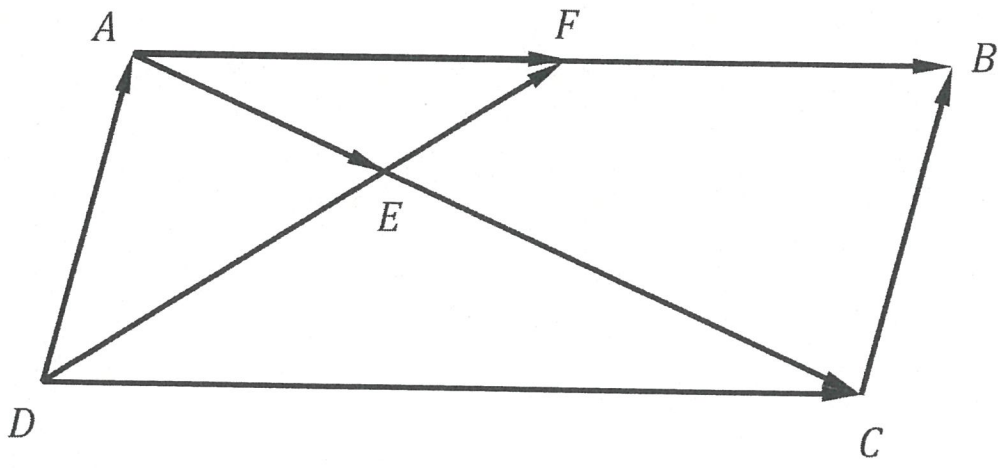
(a) Given the points $A\begin{pmatrix}1\\-1\end{pmatrix}$ and $B\begin{pmatrix}1\\4\end{pmatrix}$, show that $\angle AOB = 121°$ to the nearest degree, where O is the origin. **2**

(b) Find the volume generated when the area bounded by $y = 1 - \cos x$, $x = \frac{\pi}{4}$, $x = \frac{\pi}{2}$ and the x axis is rotated about the x axis. **4**

(c) Solve $3\sin x - 4\cos x = 2.5$ for $[-\pi, \pi]$. **4**

(d) Find the equation of a curve, $y = f(x)$ given that, $f'(x) = \frac{2}{4+x^2}$ and the curve passes through the point $\left(2, \frac{\pi}{2}\right)$. **2**

(e) $ABCD$ is a parallelogram and F is a point on AB such that AC and DF intersect at E. If $\overrightarrow{AE} = \frac{2}{5}\overrightarrow{AC}$, prove that $\overrightarrow{AF} = \frac{2}{3}\overrightarrow{DC}$. Let $\overrightarrow{DA} = \underset{\sim}{u}$ and $\overrightarrow{DC} = \underset{\sim}{v}$ **3**



Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Using the expansion $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$, **2**

show that $2^{n-1} = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots + \binom{n}{n}$ for odd n.

(b) Find the term independent of x in the expansion of **3**

$$\left(5 + \frac{4}{x^3}\right)\left(2x + \frac{3}{x^2}\right)^{15}.$$

(c) A ball is projected upwards from the origin on a horizontal surface with an initial velocity of $\underset{\sim}{v}(0) = \begin{pmatrix} 20 \\ 20\sqrt{3} \end{pmatrix}$ m/s.

Assume $\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -g \end{pmatrix}$, where $g = 10$ m/s^2, and that air resistance is negligible.

(i) Prove that the velocity t seconds after projection is given by **2**

$$\underset{\sim}{v}(t) = \begin{pmatrix} 20 \\ 20\sqrt{3} - 10t \end{pmatrix}.$$

(ii) Find how long the ball is in the air before impact with the horizontal surface. **2**

(iii) Prove that the maximum magnitude of $\underset{\sim}{v}(t)$ at any time during flight is twice its minimum magnitude. **2**

(d) (i) Arrythmia is an abnormal heart condition where the heart beat is irregular. The amplitude, A, of a patient's heart beat varies with time, t as follows **1**

$$\frac{dA}{dt} = \frac{\tan^3 t}{A\cos^4 t}$$

Show that $\dfrac{\tan^3 t}{\cos^4 t} = \sec^2 t(\tan^5 t + \tan^3 t)$.

(ii) Given that $A\left(\dfrac{\pi}{4}\right) = 1$, find the equation of A as a function of time t. **3**

End of examination

02 September 2021

URGENT AND CONFIDENTIAL

MEMORANDUM TO PRINCIPALS / CURRICULUM COORDINATORS / HEADS OF DEPARTMENTS

RE: ERRATA FOR THE 2021 CSSA TRIAL HSC EXAMINATIONS

You have received this Errata via email as you are the ONLY CSSA main contact person for your school/college in the CSSA website, based on entered data by your school/college.

Please ensure that the persons in your school/college responsible for the relevant department receives the following urgent and confidential information today.

Examination: **MATHEMATICS EXTENSION 1**
Date: **TUESDAY 01 SEPTEMBER 2021**
Session: **Afternoon PM**

It is with considerable regret that it is necessary to advise schools/colleges of an error in the Sample Answer of the Marking Guidelines that may impact on the marking of Question 12 (d) (iii).

Question 12 (d) (iii) page 12
should read

Sample Answer:

$$\sigma = \sqrt{\frac{0.7 \times 0.3}{100}} = \sqrt{0.0021}$$

$$z = \frac{0.65 - 0.7}{\sqrt{0.0021}} = -1.09 \text{ (2 dp)}$$

$P(z < -1.09) = 0.1379$ (from z tables)

The probability that $\hat{p} < 65\%$ is 0.14.

Also accept calculations using 0.645 (continuity correction):

$$\sigma = \sqrt{\frac{0.7 \times 0.3}{100}} = \sqrt{0.0021}$$

$$z = \frac{0.645 - 0.7}{\sqrt{0.0021}} = -1.20 \text{ (2 dp)}$$

$P(z < -1.20) = 0.1151$ (from z tables)

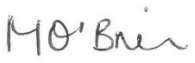
The probability that $\hat{p} < 65\%$ is 0.12.

Note: Published CSSA Trial HSC Examination Marking Guidelines (marking criteria) are accurate however the Sample Answer requires updating to accommodate this Errata.

At this time, schools/colleges are reminded:

- of the importance in maintaining the security of the CSSA Trial HSC Examinations and associated documents, and Marking Guidelines for the integrity of the CSSA Trial HSC Examination program
- to **collect and retain CSSA Trial HSC Examinations and associated documents and with students' scripts** until the end of the nominated CSSA Trial HSC Examination security period, which is **8am, Monday 6 September 2021**.
- CSSA Trial HSC Examinations and associated documents and students' scripts can be given back **from 8.01am, Monday 6 September 2021**.

Yours sincerely



Monica O'Brien
Head of CSSA Exams
Catholic Schools NSW

CSSA

2021

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MARKING GUIDELINES

Mathematics Extension I

Section I
10 marks
Multiple Choice Answer Key

Question	Answer
1	D
2	D
3	B
4	A
5	B
6	B
7	D
8	C
9	D
10	B

Question 1 (1 mark)
Outcomes Assessed: ME-V1.1/ME12-2
Targeted Performance Bands: E2-E3

Solution	Mark
$\underset{\sim}{a} \neq \underset{\sim}{b}$ so not (A) $\overrightarrow{QR} \neq \underset{\sim}{c}$ so not (B) $\overrightarrow{SQ} \neq \underset{\sim}{b}$ so not (C) $\lvert\underset{\sim}{b}\rvert = \lvert\underset{\sim}{a}\rvert$ is true Hence (D)	1

Question 2 (1 mark)
Outcomes Assessed: ME-F1.4/ME11-1
Targeted Performance Bands: E2-E3

Solution	Mark
$x = 4\sin\theta - 1 \Rightarrow x + 1 = 4\sin\theta$ $\sin\theta = \frac{x+1}{4}$ $y = 3\cos\theta + 2 \Rightarrow y - 2 = 3\cos\theta$ $\cos\theta = \frac{y-2}{3}$ Using $\sin^2\theta + \cos^2\theta = 1$ $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{9} = 1$ Hence (D)	1

Question 3 (1 mark)
Outcomes Assessed: ME-F2.1/ME11-2
Targeted Performance Bands: E2-E3

Solution	Mark
$f(x) = (x-2)g(x) + 6$ then $f(2) = 6$ $f(2) = 2\times 2^2 + k\times 2 + 4 = 6$ $12 + 2k = 6$ $2k = -6$ $k = -3$ $2x^2 - 3x + 4 = (x-2)g(x) + 6$ $2x^2 - 3x - 2 = (x-2)g(x)$ $(x-2)(2x+1) = (x-2)g(x)$ $g(x) = (2x+1)$ Hence (B)	1

Question 4 (1 mark)
Outcomes Assessed: ME-T1/ME11-3
Targeted Performance Bands: E2-E3

Solution	Mark
Since $\cos^{-1}(-x) = \pi - \cos^{-1} x$ and $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ $\Rightarrow \cos^{-1} x = \pi - \cos^{-1}(-x)$ $\Rightarrow \frac{\pi}{2} - \sin^{-1} x = \pi - \cos^{-1}(-x)$ $\therefore \sin^{-1} x = \cos^{-1}(-x) - \frac{\pi}{2}$ Hence (A)	1

Question 5 (1 mark)
Outcomes Assessed: ME-T1/ME11-3
Targeted Performance Bands: E2-E3

Solution	Mark
All options have the correct domain of [1,5]. The correct range can be found by dilating $y = \cos^{-1}\left(\frac{x-3}{2}\right)$ vertically by a factor of 4 and translating the curve up 1 unit, so $y = 4\cos^{-1}\left(\frac{x-3}{2}\right) + 1$ Hence (B)	1

Question 6 (1 mark)
Outcomes Assessed: ME-C1.2/ME11-4
Targeted Performance Bands: E3-E4

Solution	Mark
$T = 50 + Be^{kt}$ $t = 2, T = 1800 \Rightarrow 1800 = 50 + Be^{2k}$ $1750 = Be^{2k} \quad (1)$ $t = 4, T = 2500 \Rightarrow 2500 = 50 + Be^{4k}$ $2450 = Be^{4k} \quad (2)$ $(2) \div (1) \quad e^{2k} = \frac{2450}{1750} = \frac{7}{5}$ $\Rightarrow \quad k = \frac{1}{2}\ln\left(\frac{7}{5}\right), \; B = 1250$ $T = 2200 \quad \Rightarrow 2200 = 50 + 1250e^{\frac{1}{2}\ln\left(\frac{7}{5}\right)t}$ $\therefore \; t \approx 3.2236$ hours $\approx$ 3 hours 13 minutes Hence (B)	1

Question 7 (1 mark)
Outcomes Assessed: ME-V1.2/ME12-2
Targeted Performance Bands: E3-E4

Solution	Mark
$\overrightarrow{AB}.\overrightarrow{BC} = 0$ $\begin{pmatrix}1-2\\4-3\end{pmatrix}.\begin{pmatrix}3-1\\q-4\end{pmatrix} = 0$ $\begin{pmatrix}-1\\1\end{pmatrix}.\begin{pmatrix}2\\q-4\end{pmatrix} = 0$ $-2 + q - 4 = 0$ $q - 6 = 0$ $q = 6$ Hence (D)	1

Question 8 (1 mark)
Outcomes Assessed: ME-S1.1/ME12-5
Targeted Performance Bands: E3-E4

Solution	Mark
$p = 3p(1-p)$ $p = 3p - 3p^2$ $3p^2 - 2p = 0$ $p(3p-2) = 0$ $\therefore\ p = \frac{2}{3}$ since $p \neq 0$ Hence (C)	1

Question 9 (1 mark)
Outcomes Assessed: ME-C3.2/ME12-4
Targeted Performance Bands: E3-E4

Solution	Mark
Rearranging we get: (A) $\frac{dy}{dx} = \frac{x}{y}$ which would be undefined when $y = 0$ which is untrue so not (A) (B) $\frac{dy}{dx} = \frac{y}{x}$ which would be undefined when $x = 0$ which is untrue so not (B) (C) $\frac{dy}{dx} = -xy$ which would be negative in the first quadrant where $x, y > 0$ which is untrue so not (C) (D) $\frac{dy}{dx} = xy$ which would be positive in the first and third quadrants, negative in second and fourth, and zero on either axes which is true. Hence (D)	1

Question 10 (1 mark)
Outcomes Assessed: ME-A1.1/ME11-5
Targeted Performance Bands: E3-E4

Solution	Mark
Break the alphabet into 13 consecutive pairs: *AB*, *CD*, … *YZ*. There are 13 pigeonholes. There can be up to 14 cards drawn before a consecutive pair is made, leaving 12 cards in the bag. Hence (B)	1

Section II
60 marks

Question 11 (15 marks)

11(a) (2 marks)
Outcomes Assessed: ME-F1.2/ME11-1
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Correctly identifies 2 and $\frac{11}{4}$ as important, or equivalent merit	1

Sample Answer:

$$\frac{3}{x-2} < 4$$

$$(x-2)^2 \times \frac{3}{(x-2)} < 4 \times (x-2)^2$$

$$3(x-2) < 4(x-2)^2$$

$$(x-2)(3-4x+8) < 0$$

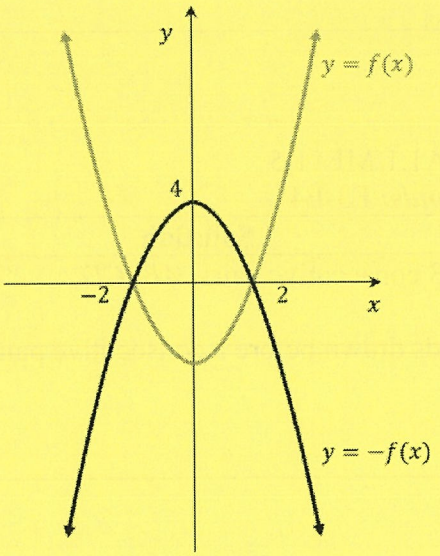
$$(x-2)(11-4x) < 0$$

Hence, $(-\infty, 2) \cup (\frac{11}{4}, \infty)$.

11 (b) (i) (1 mark)
Outcomes assessed: ME-F1.1/ME11-1
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct sketch	1

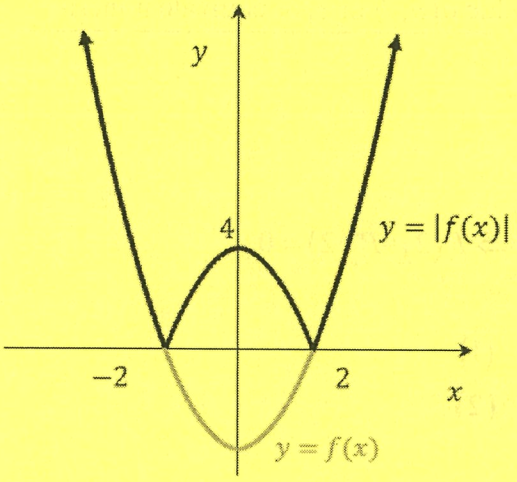
Sample Answer:



11 (b) (ii) (1 mark)
Outcomes assessed: ME-F1.1/ME11-1
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct sketch	1

Sample Answer:



11(c) (2 marks)
Outcomes assessed: ME-A1.1/ME11-5
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Arranges the letters with the vowels not adjacent, but missing some arrangements OR • Find the number of arrangements where the vowels are adjacent	1

Sample Answer:

Method 1:
Arrange the 6 consonants in a line in 6! ways.
Arrange the vowels in the 7 positions between and beside the consonants in ${}^7P_2$ ways.
Total arrangements $= 6! \times {}^7P_2 = 30\ 240$

Method 2:
Find the number of arrangements with the vowels together:
Arrange the vowels in a bubble in 2! ways
There are now 7 bubbles which can be arranged in 7! ways.
There are $2! \times 7!$ arrangements with the vowels together.
There are 8! arrangements without restrictions.
Total arrangements $= 8! - 2! \times 7! = 30\ 240$

11(d) (3 marks)

Outcomes Assessed: ME-F2.2/ME11-2

Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	3
• Correctly find any two values of a, b and c, or equivalent merit	2
• Correctly find one value of a, b or c, or equivalent merit	1

Sample Answer:

$P(x) = ax^3 + bx^2 + c$

$P'(x) = 3ax^2 + 2bx$

Since double root at $x = 2 \Rightarrow P(2) = P'(2) = 0$

$P'(2) = 12a + 4b = 0$

$\Rightarrow \quad 3a + b = 0 \qquad (1)$

$P(2) = 8a + 4b + c = 0 \qquad (2)$

Since $P(-2) = -64$

$\Rightarrow \quad -8a + 4b + c = -64 \qquad (3)$

$(2) - (3) \Rightarrow \quad 16a = 64$

$a = 4$

Sub $a = 4$ into (1)

$\Rightarrow \quad 12 + b = 0$

$b = -12$

Sub $a = 4$ and $b = -12$ into (2)

$\Rightarrow \quad 32 - 48 + c = 0$

$c = 16$

11 (e) (3 marks)

Outcomes Assessed: ME-T2/ME11-3

Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	3
• Obtains correct value for $\sin 2x$, or equivalent merit	2
• Attempts to arrive at a double angle, or equivalent merit	1

Sample Answer:

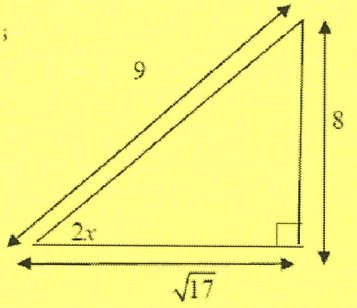
From $\cos x - \sin x = \frac{1}{3}$, squaring both sides

$\cos^2 x - 2\cos x \sin x + \sin^2 x = \frac{1}{9}$

$\sin 2x = 1 - \frac{1}{9}$

$\sin 2x = \frac{8}{9}$

$\cot 2x = \frac{\sqrt{17}}{8}$



11(f) (3 marks)

Outcomes Assessed: ME-C2/ME12-1

Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	3
• Attempts to evaluate correct integral, or equivalent merit	2
• Attempts to obtain $\tan^{-1}\left(\frac{3x}{2}\right)$, or equivalent merit	1

Sample Answer:

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = \frac{1}{9}\int_0^{\frac{2}{3}} \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2}$$

$$= \frac{1}{9} \times \frac{3}{2}\left[\tan^{-1}\left(\frac{3x}{2}\right)\right]_0^{\frac{2}{3}}$$

$$= \frac{1}{6}\left(\tan^{-1} 1 - \tan^{-1} 0\right)$$

$$= \frac{\pi}{24}$$

$$\therefore\ n = \frac{1}{24}$$

Question 12 (15 marks)

12 (a) (3 marks)
Outcomes Assessed: ME-C2/ME12-1
Targeted Performance Bands: E2-E4

Criteria	Marks
• Provides correct solution	3
• Provides a correct primitive in terms of u, or equivalent merit	2
• Attempts to use given substitution, or equivalent merit	1

Sample Answer:

$$u = 1 + e^x$$

$$\frac{du}{dx} = e^x$$

$$du = e^x dx$$

Also $u = 1 + e^x$ or $e^x = u - 1$

$$\int \frac{e^{3x}}{1+e^x}\, dx = \int \frac{e^{2x} \times e^x dx}{1+e^x}$$

$$= \int \frac{(u-1)^2 \times du}{1+u-1}$$

$$= \int \frac{u^2 - 2u + 1}{u}\, du$$

$$= \int \left(u - 2 + \frac{1}{u}\right) du$$

$$= \frac{u^2}{2} - 2u + \ln|u| + C$$

$$= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln|1+e^x| + C$$

12 (b) (3 marks)

Outcomes Assessed: ME-C1.3/ME11-4

Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	3
• Correctly finds $\frac{dy}{dt}$, or equivalent merit	2
• Correctly finds $\frac{dy}{dx}$, or equivalent merit	1

Sample Answer:

Using $x^2 + y^2 = 25$

$y = \sqrt{25 - x^2} \quad (y > 0)$

$\frac{dy}{dx} = \frac{-x}{\sqrt{25 - x^2}}$

Now $\frac{dx}{dt} = 1$, $\frac{dy}{dt} = \frac{dy}{dx} . \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-x}{\sqrt{25 - x^2}} \times 1$

Now when $y = 4, \ x = 3$

$\frac{dy}{dt} = \frac{-3}{\sqrt{25 - 3^2}} \times 1$

$= -\frac{3}{4}$ (i.e. $\frac{3}{4}$ metres per second down the wall)

12 (c) (3 marks)

Outcomes assessed: ME-P1/ME12-1

Targeted Performance Bands: E2-E4

Criteria	Marks
• Provides correct solution	3
• Provides inductive step by assuming k (or equivalent) and using assumption to show true for $k+1$	2
• Verifies base case, $n=1$ or equivalent	1

Sample Answer:

$4^1+14=18$, 18 is divisible by 6, therefore true for $n=1$.

Assume true for $n=k$

ie $4^k+14=6M$, M is an integer

$$4^k=6M-14$$

Prove true for $n=k+1$

$4^{k+1}+14=6Q$, Q is an integer

$4^k.4+14$

$=(6M-14)4+14$

$=24M-56+14$

$=24M-42$

$=6(4M-7)$

$=6Q$, since $4M-7$ is an integer

By the principle of mathematical induction the statement is true for $n\geq 1$

12 (d) (i) (2 marks)

Outcomes Assessed: ME-S1.1/ME12-5

Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Obtains correctly either $\mathrm{E}(X)$ or $\mathrm{Var}(X)$	1

Sample Answer:

$\mathrm{E}(X)=np=100\times 0.7=70$

$\mathrm{Var}(X)=np(1-p)=100\times 0.7\times 0.3=21$

12 (d) (ii) (2 marks)
Outcomes Assessed: ME-S1.1/ME12-5
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Evaluates an incorrect expression correctly, where the indices are in the incorrect order, or equivalent merit OR • Obtains correct expression but does not evaluate it	1

Sample Answer:

$$P(X = 70) = \binom{100}{70} 0.7^{70} \times 0.3^{30}$$
$$= 0.086783...$$
$$= 0.0868 \ (3 \text{ sf})$$

12 (d) (iii) (2 marks)
Outcomes Assessed: ME-S1.2/ME12-5
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	2
• Obtains correct z value OR • Obtains correct answer from incorrect z value	1

Sample Answer:

$\sigma = \sqrt{0.7 \times 0.3} = \sqrt{0.21}$
$z = \dfrac{0.65 - 0.7}{\sqrt{0.21}} = -0.11 \text{ (2 dp)}$
$P(z < -0.11) = 0.4562$ (from z tables)
The probability that $\hat{p} < 65\%$ is 0.46.

Also accept calculations using 0.645 (continuity correction):
$\sigma = \sqrt{0.7 \times 0.3} = \sqrt{0.21}$
$z = \dfrac{0.645 - 0.7}{\sqrt{0.21}} = -0.12 \text{ (2 dp)}$
$P(z < -0.12) = 0.4522$ (from z tables)
The probability that $\hat{p} < 65\%$ is 0.45.

Question 13 (15 marks)

13 (a) (2 marks)
Outcomes Assessed: ME-V1.2/ME12-2
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Obtains correct magnitudes of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ OR • Obtains angle using one or two incorrect magnitudes OR • Obtains an answer using sine instead of cosine with no further mistakes	1

Sample Answer:

$$\left|\overrightarrow{OA}\right| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\left|\overrightarrow{OB}\right| = \sqrt{1^2 + 4^2} = \sqrt{17}$$

$$\cos \angle AOB = \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix}}{\sqrt{2} \times \sqrt{17}}$$

$$= \frac{1 - 4}{\sqrt{34}}$$

$$= -\frac{3}{\sqrt{34}}$$

$$\angle AOB = 120.963\ldots$$

$$= 121^\circ \text{ (nearest degree)}$$

13 (b) (4 marks)

Outcomes Assessed: ME-C2/ME12-1 ME-C3.1/ME12-4

Targeted Performance Bands: E2-E4

Criteria	Marks
• Provides correct solution	4
• Attempts to evaluate correct integral, or equivalent	3
• Attempts to integrate the volume expression with correct primitive for $\cos^2 x$, or equivalent merit	2
• Obtains correct expression for the volume, or equivalent merit	1

Sample Answer:

$$V = \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1-\cos x)^2 \, dx$$

$$= \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(1-2\cos x+\cos^2 x\right) dx$$

$$= \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(1-2\cos x+\frac{1+\cos 2x}{2}\right) dx$$

$$= \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\frac{3}{2}-2\cos x+\frac{1}{2}\cos 2x\right) dx$$

$$= \pi \left[\frac{3x}{2}-2\sin x+\frac{1}{4}\sin 2x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \pi\left(\frac{3\pi}{4}-2-\left(\frac{3\pi}{8}-\sqrt{2}+\frac{1}{4}\right)\right)$$

$$= \pi\left(\frac{3\pi}{8}-\frac{9}{4}+\sqrt{2}\right) u^3.$$

13 (c) (4 marks)

Outcomes Assessed: ME-T3/ME12-3

Targeted Performance Bands: E2-E4

Criteria	Marks
• Provides correct solution	4
• Correctly writes $3\sin x - 4\cos x$ in the form $R\sin(x+\alpha)$ and finds one solution, or equivalent merit	3
• Finds R and α, or equivalent merit	2
• Finds the value of R or α, or equivalent merit	1

Sample Answer:

$$R = \sqrt{3^2 + 4^2} = 5, R > 0$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\therefore 3\sin x - 4\cos x = 5\sin\left(x - \tan^{-1}\left(\frac{4}{3}\right)\right)$$

$$5\sin\left(x - \tan^{-1}\left(\frac{4}{3}\right)\right) = 2.5$$

$$\sin\left(x - \tan^{-1}\left(\frac{4}{3}\right)\right) = \frac{1}{2}$$

$$x - \tan^{-1}\left(\frac{4}{3}\right) = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{\pi}{6} + \tan^{-1}\left(\frac{4}{3}\right), \frac{5\pi}{6} + \tan^{-1}\left(\frac{4}{3}\right)$$

$$= 1.451^c, 3.545^c \text{ (3 dp)}$$

$$= 1.451^c, -2.738^c \text{ since } -\pi \le x \le \pi$$

13 (d) (2 marks)

Outcomes Assessed: ME-C2/ME12-1

Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Obtains the correct primitive	1

Sample Answer:

$$f(x) = \int \frac{2}{4+x^2}\,dx$$

$$= 2 \times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$\text{Given } \left(2, \frac{\pi}{2}\right) \Rightarrow \frac{\pi}{2} = \tan^{-1}(1) + C$$

$$C = \frac{\pi}{4}$$

$$\therefore f(x) = \tan^{-1}\left(\frac{x}{2}\right) + \frac{\pi}{4}$$

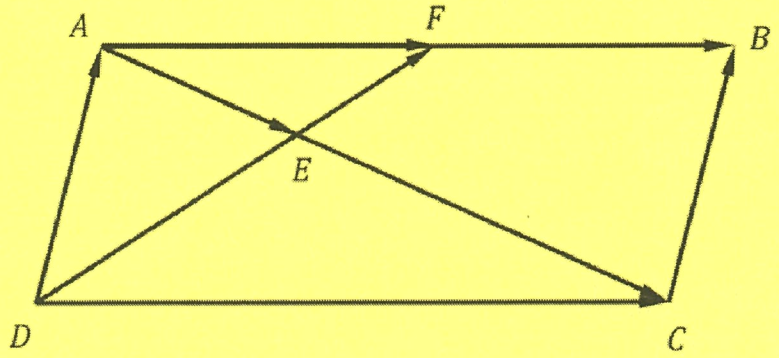
13 (e) (3 marks)

Outcomes Assessed: ME-V1.2/ME12-2

Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	3
• Express $\overrightarrow{AE}$ in terms of $\overrightarrow{DA}$ and $\overrightarrow{DC}$ and $\overrightarrow{AF}$ in terms of $\overrightarrow{DA}$ and $\overrightarrow{DC}$	2
• Express $\overrightarrow{AE}$ in terms of $\overrightarrow{DA}$ and $\overrightarrow{DC}$ or $\overrightarrow{AF}$ in terms of $\overrightarrow{DA}$ and $\overrightarrow{DC}$	1

Sample Answer:



Let $\overrightarrow{DA} = \underset{\sim}{u}$ and $\overrightarrow{DC} = \underset{\sim}{v}$

In ΔADC, $\overrightarrow{DA} + \overrightarrow{AC} = \overrightarrow{DC}$ $\therefore$ $\overrightarrow{AC} = \overrightarrow{DC} - \overrightarrow{DA} = \underset{\sim}{v} - \underset{\sim}{u}$

Given $\overrightarrow{AE} = \frac{2}{5}\overrightarrow{AC}$ $\therefore$ $\overrightarrow{AE} = \frac{2}{5}(\underset{\sim}{v} - \underset{\sim}{u})$

In ΔAED, $\overrightarrow{DA} + \overrightarrow{AE} = \overrightarrow{DE}$ $\therefore$ $\overrightarrow{DE} = \underset{\sim}{u} + \frac{2}{5}(\underset{\sim}{v} - \underset{\sim}{u}) = \frac{2}{5}\underset{\sim}{v} + \frac{3}{5}\underset{\sim}{u}$

Let $\overrightarrow{DF} = \alpha\overrightarrow{DE}$ and $\overrightarrow{AF} = \mu\overrightarrow{AB} = \mu\overrightarrow{DC} = \mu\underset{\sim}{v}$

In ΔDAF, $\overrightarrow{DA} + \overrightarrow{AF} = \overrightarrow{DF} = \alpha\overrightarrow{DE}$ $\therefore$ $\underset{\sim}{u} + \mu\underset{\sim}{v} = \alpha\left(\frac{2}{5}\underset{\sim}{v} + \frac{3}{5}\underset{\sim}{u}\right)$

Hence $\frac{3}{5}\alpha = 1$ and $\mu = \frac{2}{5}\alpha$ $\therefore$ $\alpha = \frac{5}{3}$ and $\mu = \frac{2}{3}$

$\therefore$ $\overrightarrow{AF} = \frac{2}{3}\overrightarrow{DC}$.

Question 14 (15 marks)

14 (a) (2 marks)

Outcomes Assessed: ME-A1.2/ME11-5

Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	2
• Obtains correct substitution for $x = 1$ or $x = -1$	1

Sample Answer:

Using the expansion

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n$$

When $x = 1$

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots + \binom{n}{n} \qquad (1)$$

When $x = -1$

$$0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} + \ldots - \binom{n}{n} \qquad (2)$$

$(1) - (2)$:

$$2^n = 2\left(\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \ldots + \binom{n}{n}\right)$$

$$\therefore \quad 2^{n-1} = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \ldots + \binom{n}{n}$$

14 (b) (3 marks)

Outcomes Assessed: ME-A1.2/ME11-5

Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	3
• Obtains $C_1 = 5\times\binom{15}{5}2^{10}3^5$ or $C_2 = 4\times\binom{15}{4}2^{11}3^4$	2
• Obtains the general term for the expression $\left(2x+\frac{3}{x^2}\right)^{15}$ and attempts to find the independent terms in the expansion, or equivalent merit	1

Sample Answer:

General term for $\left(2x+\frac{3}{x^2}\right)^{15} \Rightarrow T_{k+1} = \binom{15}{k}(2x)^{15-k}\left(\frac{3}{x^2}\right)^k$

$$= \binom{15}{k}2^{15-k}3^k x^{15-3k}$$

There are two terms which are independent of x in the expansion of $\left(5+\frac{4}{x^3}\right)\left(2x+\frac{3}{x^2}\right)^{15}$.

$5\times x^0$ when $k=5$ $(15-3k=0) \Rightarrow C_1 = 5\times\binom{15}{5}2^{10}3^5$

and

$\frac{4}{x^3}\times x^3$ when $k=4$ $(15-3k=3) \Rightarrow C_2 = 4\times\binom{15}{4}2^{11}3^4$

Therefore the term independent of x in the expansion of $\left(5+\frac{4}{x^3}\right)\left(2x+\frac{3}{x^2}\right)^{15}$

is $C_1 + C_2 = 5\times\binom{15}{5}2^{10}3^5 + 4\times\binom{15}{4}2^{11}3^4 = 4\ 641\ 960\ 960$

14 (c) (i) (3 marks)

Outcomes Assessed: ME-V1.3/ME12-2

Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Obtains expression for $\underset{\sim}{v}(t)$ with constants OR • Obtains expressions for $\dot{x}$ and $\dot{y}$ with constants	1

Sample Answer:

$$\underset{\sim}{a}(t) = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$$

$$\underset{\sim}{v}(t) = \begin{pmatrix} c_1 \\ -10t + c_2 \end{pmatrix}$$

Let $t = 0$, $\underset{\sim}{v}(0) = \begin{pmatrix} 20 \\ 20\sqrt{3} \end{pmatrix}$

$$\therefore \begin{pmatrix} c_1 \\ -10(0) + c_2 \end{pmatrix} = \begin{pmatrix} 20 \\ 20\sqrt{3} \end{pmatrix}$$

$$\therefore c_1 = 20,\ c_2 = 20\sqrt{3}$$

$$\therefore \underset{\sim}{v}(t) = \begin{pmatrix} 20 \\ 20\sqrt{3} - 10t \end{pmatrix}$$

14 (c) (ii) (2 marks)
Outcomes Assessed: ME-V1.3/ME12-2
Targeted Performance Bands: E2-E3

Criteria	Marks
• Provides correct solution	2
• Obtains the time to the top of the flight but doesn't double the answer, or equivalent merit	1

Sample Answer:

At the top of the flight $\underset{\sim}{v}(t) = \begin{pmatrix} 20 \\ 0 \end{pmatrix}$, so

$20\sqrt{3} - 10t = 0$

$t = 2\sqrt{3}$

$\therefore$ time of flight $= 2\left(2\sqrt{3}\right) = 4\sqrt{3}$ seconds.

14 (c) (iii) (2 marks)
Outcomes Assessed: ME-V1.3/ME12-2
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	2
• Finds either the maximum or minimum magnitude	1

Sample Answer:

The maximum magnitude of velocity occurs at the point of projection and impact, while the minimum occurs at the top of the flight.

$\left|\underset{\sim}{v}\right|_{\max} = \sqrt{20^2 + \left(20\sqrt{3}\right)^2} = 40$ m/s

$\left|\underset{\sim}{v}\right|_{\min} = 20$ m/s (the horizontal velocity is constant and the vertical velocity is zero)

$\therefore \left|\underset{\sim}{v}\right|_{\max} = 2\left|\underset{\sim}{v}\right|_{\min}$

14 (d) (i) (1 mark)
Outcomes Assessed: ME-C3.2/ME12-4
Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	1

Sample Answer:

$$LHS = \frac{\tan^3 t}{\cos^4 t}$$
$$= \sec^2 t \sec^2 t \tan^3 t$$
$$= \sec^2 t\left(1 + \tan^2 t\right)\tan^3 t$$
$$= \sec^2 t\left(\tan^3 t + \tan^5 t\right)$$
$$= RHS$$

14(d) (ii) (3 marks)

Outcomes Assessed: ME-C3.2/ME12-4

Targeted Performance Bands: E3-E4

Criteria	Marks
• Provides correct solution	3
• Obtains correct expression of y^2 as a function of x	2
• Separates the variable and integrate for x or y	1

Sample Answer:

$$\frac{dA}{dt} = \frac{\tan^3 t}{A\cos^4 t}$$

$$\int_1^A A\ dA = \int_{\frac{\pi}{4}}^{t} \frac{\tan^3 t}{\cos^4 t}\, dt$$

$$\frac{1}{2}\left[A^2\frac{2}{2}\right]_1^A = \int_{\frac{\pi}{4}}^{t} \sec^2 t\,(\tan^5 t + \tan^3 t)\, dt$$

$$\frac{1}{2}(A^2 - 1) = \left[\frac{\tan^6 t}{6} + \frac{\tan^4 t}{4}\right]_{\frac{\pi}{4}}^{t}$$

$$A^2 - 1 = 2\left(\left(\frac{\tan^6 t}{6} + \frac{\tan^4 t}{4}\right) - \left(\frac{1}{6} + \frac{1}{4}\right)\right)$$

$$A^2 = \frac{\tan^6 t}{3} + \frac{\tan^4 t}{2} + \frac{1}{6}$$

$$A = \sqrt{\frac{2\tan^6 t + 3\tan^4 t + 1}{6}} \quad \text{since } A \geq 0$$

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