ZETA
EXAMINATION
MATERIALS

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Total Marks: 70

Section I – 10 marks (pages 2–4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 5–12)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

---

1. Which of these is equal to $\cos(3a + 2b)$?

   A. $\sin 3a \cos 2b - \cos 3a \sin 2b$

   B. $\sin 3a \cos 2b + \cos 3a \sin 2b$

   C. $\cos 3a \cos 2b - \sin 3a \sin 2b$

   D. $\cos 3a \cos 2b + \sin 3a \sin 2b$

2. The roots of the polynomial $P(x) = 2x^3 + 6x^2 - 9$ are α, β and γ.

   What is the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma}$?

   A. $-\frac{2}{3}$

   B. $-\frac{1}{3}$

   C. 0

   D. $\frac{1}{3}$

3. Three collinear points A, B and C on the co-ordinate plane are represented by position vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. Which of the following is definitely true?

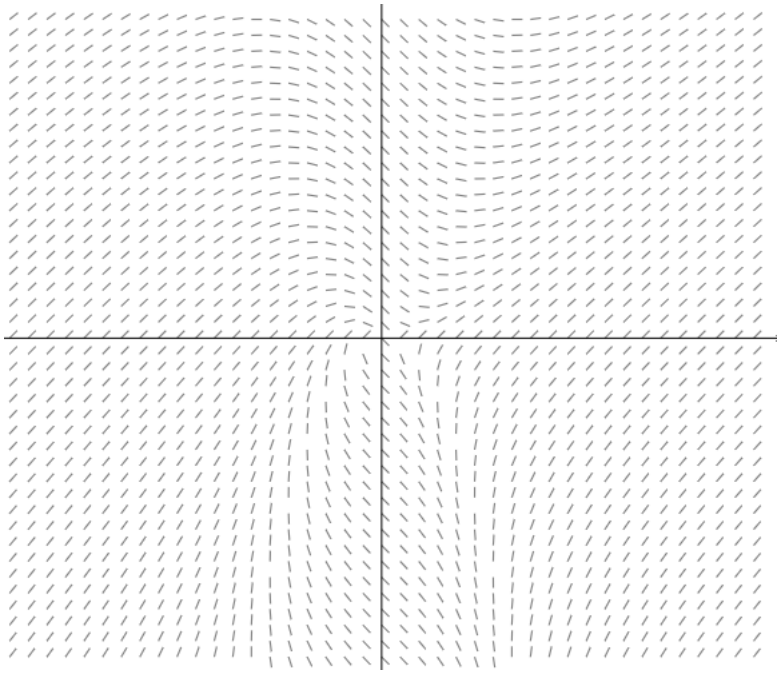
   A. $\boldsymbol{a} + \boldsymbol{b} = \lambda(\boldsymbol{b} + \boldsymbol{c})$ for some real valued λ

   B. $\boldsymbol{a} - \boldsymbol{b} = \lambda(\boldsymbol{b} - \boldsymbol{c})$ for some real valued λ

   C. $\boldsymbol{a} \bullet \boldsymbol{b} = \boldsymbol{b} \bullet \boldsymbol{c}$

   D. $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \boldsymbol{0}$

4. Which of the following differential equations could this direction field correspond to?



A. $\frac{dy}{dx} = \frac{x^2-1}{x^2+1}$

B. $\frac{dy}{dx} = \frac{x-y}{x+y}$

C. $\frac{dy}{dx} = \frac{x^2-y}{x^2+y}$

D. $\frac{dy}{dx} = x^2 y$

5. Which of these is equivalent to $\sin^{-1}(x) + \cos^{-1}(-x)$?

A. $2\cos^{-1} x - \frac{\pi}{2}$

B. $\frac{3\pi}{2} - 2\cos^{-1} x$

C. $\frac{3\pi}{2} - \sin^{-1} x$

D. $\frac{\pi}{2} - 2\sin^{-1} x$

6. What range of values for k does the equation

$$(k-1)\sin\theta + (k+3)\cos\theta = 2\sqrt{10}$$

have solutions?

A. $k \in (-\infty, -5] \cup [3, \infty)$

B. $k \in (-\infty, -3] \cup [\sqrt{2}, \infty)$

C. $k \in [-5, 3]$

D. $k \in \left[-3, \sqrt{2}\right]$

7. A large flock of 2024 cockatoos arrive to a patch of bushland and sit atop a collection of eucalyptus trees. Every bird in the flock sits on a eucalyptus tree. A local park ranger notices this and concludes that there must be at least one tree with at least 57 cockatoos. He cannot conclude with certainty that any tree holds 58 cockatoos. In his calculations, he has taken into consideration the 3 empty trees he happened to notice.

How many eucalyptus trees are there altogether?

A. 36

B. 37

C. 38

D. 39

8. Twelve cards are labelled with the numbers 1 up to 12 and placed upside down on a table. Three cards are chosen at random and overturned to reveal their numbers. What is the probability that the highest and lowest numbers from the three cards have a difference greater than 4?

A. $\frac{1}{22}$

B. $\frac{3}{11}$

C. $\frac{7}{11}$

D. $\frac{3}{4}$

9. The function $f(x)$ and its inverse function $f^{-1}(x)$ are both defined over all real x. Which of these functions could have an inverse?

   A. $f(|x|)$

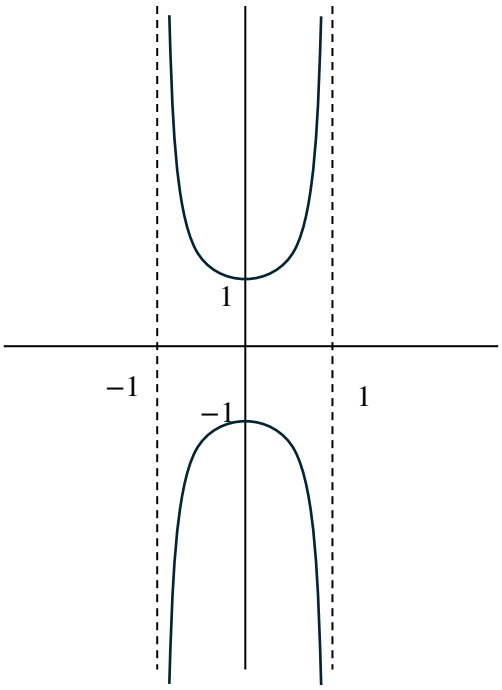
   B. $|f(x)|$

   C. $\frac{1}{f(x)}$

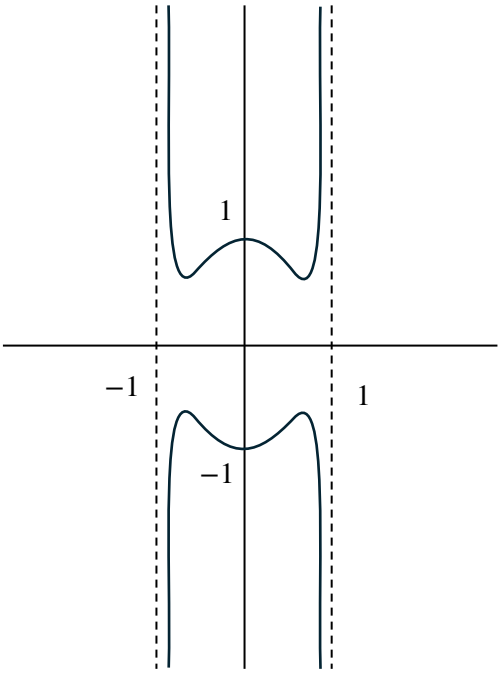
   D. $f(x)\big(1 - f(x)\big)$

10. Which of the following graphs show the relation governed by the parametric equations $x = \cos(t)$ and $y = \operatorname{cosec}(t)$?

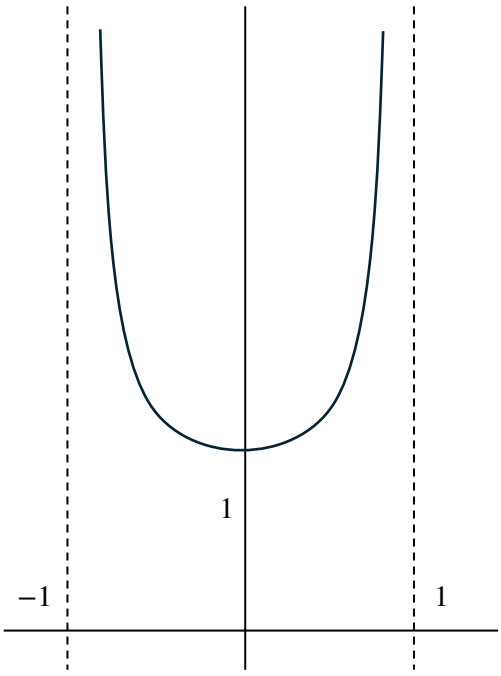
A.



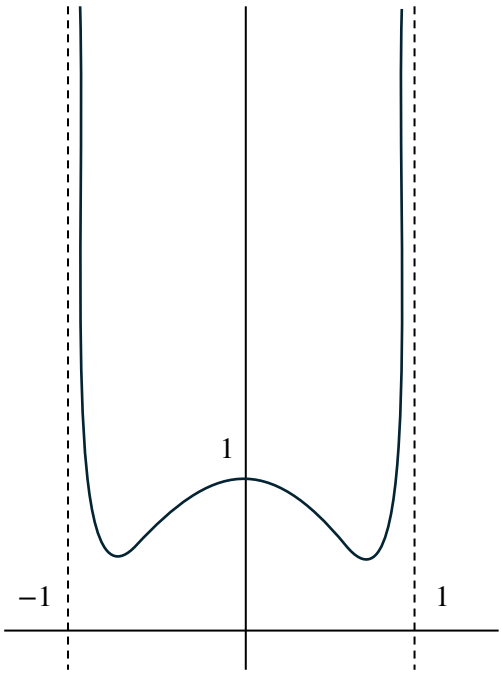
B.



C.



D.



Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use a SEPARATE Writing Book

a) Find the values of p for which the vectors $\begin{pmatrix} p+2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} p \\ 2-p \end{pmatrix}$ are perpendicular **2**

b) How many ways can the letters of the word BARRACKS be arranged in a circle? **2**

c) What is the derivative of $\tan^{-1}(1-e^x)$? **2**

d) Find the specific solution to the differential equation $\frac{dy}{dx} = e^y \sin x$ which passes through the point $\left(\frac{\pi}{3}, 0\right)$. **3**

e) Find all values of x for which $\frac{x+2}{x-5} > 3$ **3**

f) The polynomial $P(x) = x^3 + \left(a^2+3\right)x^2 - 2ax - 12$ has a factor of $x = 1$ and leaves a remainder of 32 when divided by $x - 2$.

(i) Find a **2**

(ii) Factorise $P(x)$ **2**

End of Question 11

Question 12 (14 marks) Use a SEPARATE Writing Book

a) Statistics show that around 22% of Australians speak a language other than English at home.

A sample of 450 Australians are surveyed on their language preferences at home.

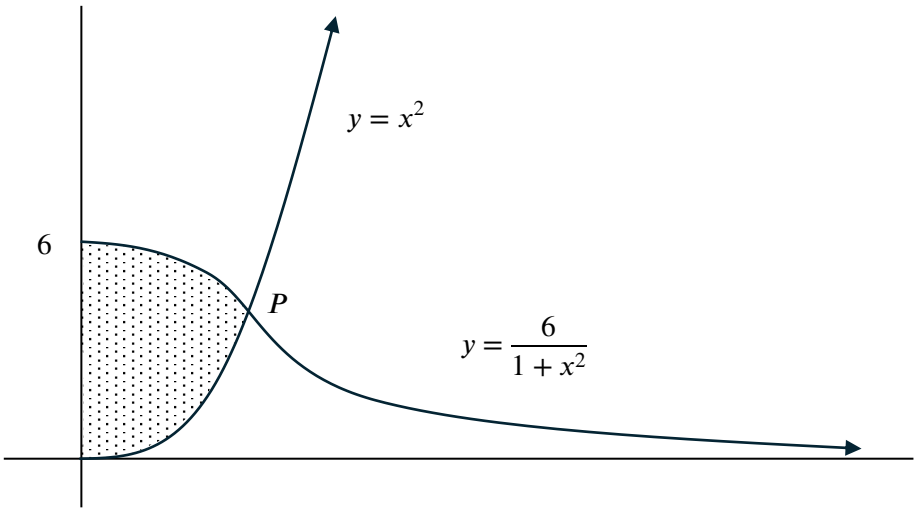
Let $\hat{p}$ represent the proportions of surveyed individuals who spoke a language other than English at home.

(i) Find the value of $E(\hat{p})$ and $\text{Var}(\hat{p})$ **2**

(ii) Using a normal approximation to the binomial distribution, find $P(\hat{p} > 0.26)$. You may refer to the table on page 12 in answering this question. **2**

b) Use mathematical induction to prove that $4^{2n+1} + 7^{2n+1}$ is divisible by 11 for all $n \geq 0$ **3**

c) The diagram below shows the graphs of $y = x^2$ and $y = \frac{6}{1+x^2}$

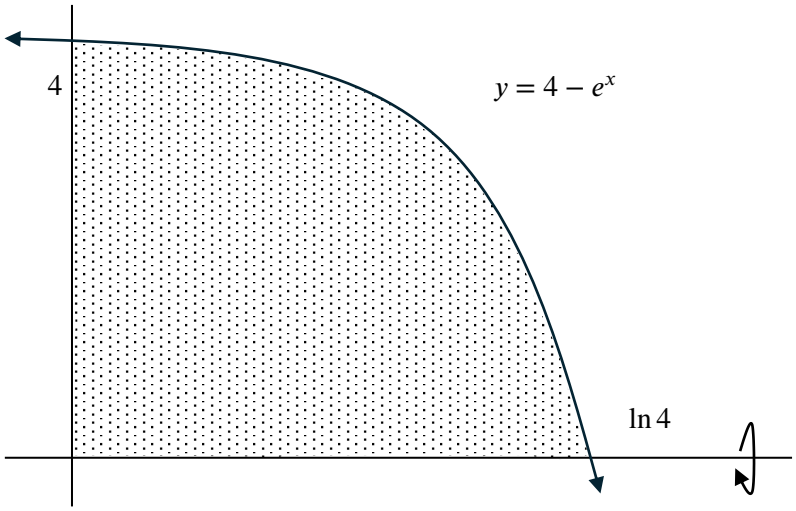


(i) Find the point of intersection P **2**

(ii) Hence find the shaded area **2**

Question 12 continues on the next page

d) The region bound by the co-ordinate axes and the graph $y = 4 - e^x$ is rotated about the x-axis to form a solid. **3**



Find the volume of this solid, leaving your answer in exact form

End of Question 12

Question 13 (15 marks) Use a SEPARATE Writing Book

a) Define the vectors $\boldsymbol{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$. A vector $\boldsymbol{x}$ has magnitude $\sqrt{130}$, and the projections of $\boldsymbol{x}$ onto $\boldsymbol{u}$ and $\boldsymbol{v}$ have equal magnitude. Find the vector $\boldsymbol{x}$, given that it is the position vector of a point in the first quadrant. **3**

b) The local library has 10 study rooms for public use. When a room is occupied, it is assumed that it will remain occupied for the remainder of the day Each room has a 70% chance of being unoccupied on any particular day.

(i) Write down an expression for the probability that at least one room is unoccupied on a given day. **1**

Jonathan occasionally goes to this library to study. On any given day, the probability of him visiting the library is 0.4.

(ii) Write down an expression for the probability that Jonathan visits the library on 3 days of a given week, and that he finds an empty study room on only two of those days. **2**

c) Let $I = \int_0^{\frac{\pi}{8}} \cos^3 x \sin^3 x \, dx$

(i) Using the substitution $u = \cos(2x)$ or otherwise, find **2**

$$\int \sin 2x \cos^2 2x \, dx$$

(ii) By considering a double angle formula or otherwise, evaluate I in exact form **3**

Question 13 continues on the next page

d) A tub initially contains 100L of fresh water. It is fed a solution of 0.08kg/L hydrochloric acid, at a rate of 5L/min. It also has a leakage pipe through which the contents of the tub exit at a rate of 5L/min.

(i) Write down a differential equation to model the problem, and then solve it to show that the concentration of hydrochloric acid in the tub at any given time is **3**

$$C(t) = 0.08 - 0.08e^{-\frac{1}{20}t}$$

(ii) Find when the concentration reaches 0.05kg/L **1**

End of Question 13

Question 14 (15 marks) Use a SEPARATE Writing Book

a) A particle is launched with initial speed 50m/s at an angle of θ to the horizontal. It moves under the influence of gravity, which results in an acceleration of 10m/s^2. The motion of the particle is governed by the position vector:

$$r = \begin{pmatrix} 50t\cos\theta \\ 50t\sin\theta - 5t^2 \end{pmatrix}$$

The range of the particle's motion is $R = 500\sin\theta\cos\theta$.

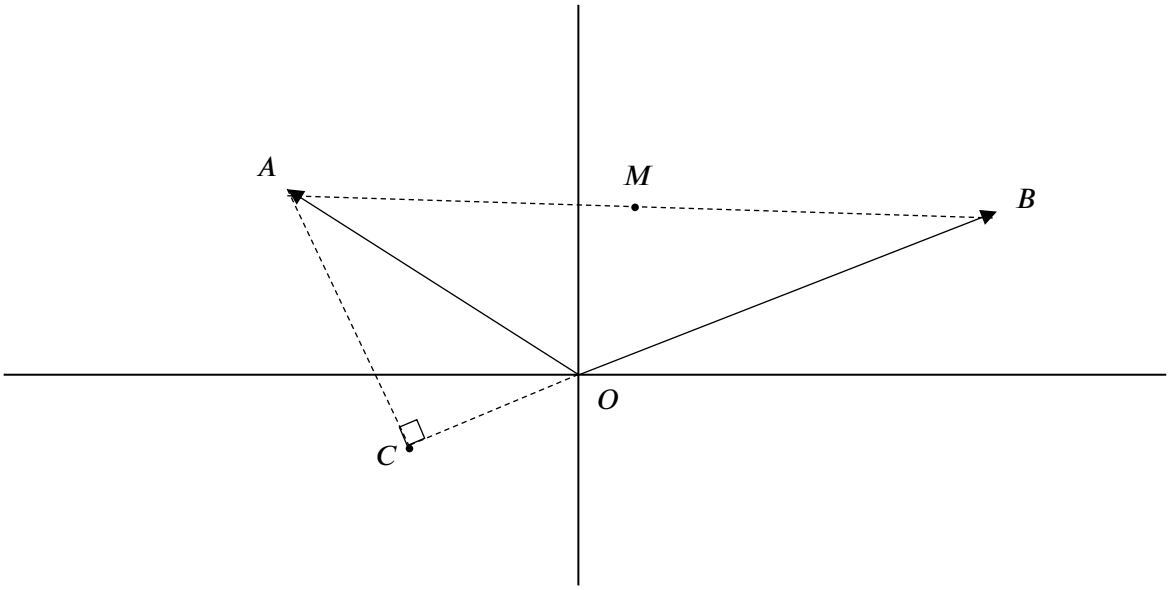
(i) Show that the maximum height reached by the particle is $H = \frac{125}{2} - \frac{125}{2}\cos 2\theta$ **2**

(ii) Find all possible values of theta for which the sum of the maximum height H and the range R exceeds 300m. Leave your answer in degrees, correct to the nearest minute. **4**

b) A rare disease is detected in a population of kangaroos. After testing a total of 800 individuals, 38 are found to have the disease. Researchers claim that such a high presence of the disease in a sample of this size had only a 0.32% chance of occurring. **4**

Based on this information, and using a normal approximation to the binomial distribution, find the probability that any given kangaroo has this disease, correct to 2 decimal places.

c) The points A and B are represented by the position vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively. The point O is the origin, M is the midpoint of AB, and C is a point such that OC is parallel to OB and AC is perpendicular to CB.



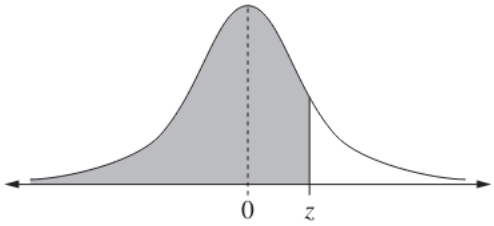
(i) Write down an expression for the position vector of the point C in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ **1**

(ii) Hence or otherwise show that $|MC| = |AM| = |MB|$ **4**

End of Exam

Use the following information to answer questions 12a and 14b (Credit: NESA)

Table of values $P(Z \leq z)$ for the normal distribution $N(0, 1)$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995