



**CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW
2020 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS EXTENSION 2 - MARKING GUIDELINES**

Section I

10 marks

Multiple-choice Answer Key

Question	Answer
1	C
2	B
3	D
4	D
5	B
6	A
7	A
8	D
9	C
10	B

Question 1 (1 mark)

Outcomes Assessed: MEX12-4

Targeted Performance Bands: E2

Solution	Mark
$ z = \sqrt{(1)^2 + (\sqrt{3})^2}$ $= 2$ $\text{Arg}(z) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$ $= \frac{\pi}{3}$ $\therefore z = 2e^{j\frac{\pi}{3}}$ <p>Hence (C)</p>	1

1

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Question 2 (1 mark)**Outcomes Assessed: MEX12-3****Targeted Performance Bands: E2**

Solution	Mark
<p>From the diagram, the distance from the $x-z$ plane is the y coordinate.</p> $\therefore d = 3 \text{ units}$ <p>Hence (B)</p>	1

Question 3 (1 mark)**Outcomes Assessed: MEX12-2****Targeted Performance Bands: E2**

Solution	Mark
<p>The contrapositive of statement $A \Rightarrow B$ is $\sim B \Rightarrow \sim A$</p> <p>$\therefore$ the contrapositive of "if n is a prime number, then n is odd"</p> <p>is "if n is not odd, then n is not prime."</p> <p>Hence (D)</p>	1

Question 4 (1 mark)**Outcomes Assessed: MEX12-5****Targeted Performance Bands: E2-E3**

Solution	Mark
$\int \frac{1}{x(\log_e x)^2} dx = \int \frac{du}{u^2}$ <p style="text-align: center;">let $u = \log_e x \rightarrow du = \frac{1}{x} dx$</p> $= \left[-u^{-1} \right]$ $= \frac{-1}{\log_e x} + c$ <p>Hence (D)</p>	1

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Question 5 (1 mark)*Outcomes Assessed: MEX12-4**Targeted Performance Bands: E3*

Solution	Mark
$\alpha = \sqrt{2} \left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right) \text{ in polar form}$ $\alpha^{10} = 2^5 \left(\cos\left(-\frac{10\pi}{4}\right) + i \sin\left(-\frac{10\pi}{4}\right) \right) \text{ using De Moivre's Theorem}$ $= 32 \left(\cos\left(-\frac{5\pi}{2}\right) + i \sin\left(-\frac{5\pi}{2}\right) \right)$ $= -32i$ <p>$\therefore \alpha^{10}$ is purely imaginary.</p> <p>Hence (B)</p>	1

Question 6 (1 mark)*Outcomes Assessed: 12-2**Targeted Performance Bands: E3*

Solution	Mark
<p>Considering each option:</p> <p>(A) "If a person does not own a pet, then they do not own a cat" does not have a counter-example, as not owning a pet excludes owning any animal.</p> <p>(B) Four points may be joined in a plane in a way that does not form a quadrilateral e.g. joining opposite points to form diagonals.</p> <p>(C) The prime numbers are 2, 3, 5, 7, 11, ... The first prime number is even.</p> <p>(D) Any product of even numbers is even. \therefore if x is even, x^2 is even. \therefore it can be shown simply that if $x = 2 \Rightarrow x^2 = 4$, a counter-example.</p> <p>Hence (A)</p>	1

Question 7 (1 mark)*Outcomes Assessed: MEXI2-6**Targeted Performance Bands: E3-E4*

Solution	Mark
<p>The only unbalanced force is the resistance force in the horizontal direction.</p> $\therefore \ddot{x} = -kv^2$ $\frac{dv}{v^2} = -kdt$ $-\frac{1}{v} = -kt + c$ <p>for $t = 0, v = U$</p> $-\frac{1}{U} = -k(0) + c$ $\therefore -\frac{1}{v} = -kt - \frac{1}{U}$ <p>i.e. $\frac{1}{v} = kt + \frac{1}{U}$</p> <p>Hence (A)</p>	1

Question 8 (1 mark)*Outcomes Assessed: MEXI2-5**Targeted Performance Bands: E3-E4*

Solution	Mark
$\int \frac{dx}{\sqrt{8-2x-x^2}} = \int \frac{dx}{\sqrt{9-(x^2+2x+1)}} \quad \text{by completing the square}$ $= \int \frac{dx}{\sqrt{9-(x+1)^2}}$ $= \sin^{-1}\left(\frac{x+1}{3}\right) + c$ <p>Hence (D)</p>	1

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Question 9 (1 mark)*Outcomes Assessed: MEX12-6**Targeted Performance Bands: E3-E4*

Solution	Mark
<p>The amplitude of motion is $a = 3$</p> <p>Using the identity $v^2 = n^2(a^2 - x^2)$</p> <p>at $x = 0, v = \sqrt{3}$</p> $(\sqrt{3})^2 = n^2((3)^2 - (0)^2)$ $n^2 = \frac{1}{3}$ $\therefore n = \frac{\sqrt{3}}{3}$ $T = \frac{2\pi}{n}$ $= \frac{2\pi}{\frac{\sqrt{3}}{3}}$ $= \frac{6\pi}{\sqrt{3}}$ <p>Hence (C)</p>	1

Question 10 (1 mark)*Outcomes Assessed: MEX12-4**Targeted Performance Bands: E4*

Solution	Mark
<p>The diagram show the locus of z relative to the complex numbers $0 + i$ and $1 + 0i$. The locus is also the line segment joining the two points.</p> <p>This means that the locus is defined by $\arg\left(\frac{z-i}{z-1}\right) = \pm\pi$</p> <p>Hence (B)</p>	1

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Section II
90 marks

Question 11 (15 marks)

11 (a) (i) (2 marks)

Outcomes Assessed: MEX12-4

Targeted Performance Bands: E2

Criteria	Marks
Correct solution	2
Expresses z in modulus-argument form	1

Sample Answer:

$$z = 1 + i$$

$$z = \sqrt{2} \left[\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \right]$$

$$\frac{w}{z} = \frac{1}{\sqrt{2}} \left[\cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{6} - \frac{\pi}{4}\right) \right]$$

$$= \frac{\sqrt{2}}{2} \left[\cos\left(-\frac{\pi}{12}\right) + i \sin\left(-\frac{\pi}{12}\right) \right]$$

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11 (a) (ii) (2 marks)

*Outcomes Assessed: MEX12-4**Targeted Performance Bands: E2*

Criteria	Marks
Correct solution	2
Applies De Moivre's Theorem, or equivalent merit	1

Sample Answer:

$w\bar{z}$ is similar to $\frac{w}{z}$ with a modulus of $\sqrt{2}$:

$$\begin{aligned} w\bar{z} &= \sqrt{2} \left(\cos\left(-\frac{\pi}{12}\right) + i \sin\left(-\frac{\pi}{12}\right) \right) \\ (w\bar{z})^8 &= (\sqrt{2})^8 \left(\cos\left(8 \times -\frac{\pi}{12}\right) + i \sin\left(8 \times -\frac{\pi}{12}\right) \right) \\ &= 16 \left(\cos\left(-\frac{2\pi}{3}\right) + i \sin\left(-\frac{2\pi}{3}\right) \right) \\ &= 16 \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) \\ &= -8 - 8\sqrt{3}i \end{aligned}$$

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11 (b) (2 marks)

*Outcomes Assessed: MEX12-3**Targeted Performance Bands: E2*

Criteria	Marks
Correct solution	2
Finds the correct magnitudes of the vectors or the dot product	1

Sample Answer:

$$\underline{a} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \text{ and } \underline{b} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

The angle between two vectors is given by $\cos\theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}||\underline{b}|}$.

$$\begin{aligned} \underline{a} \cdot \underline{b} &= (2)(-1) + (1)(1) + (3)(2) \\ &= 5 \end{aligned}$$

$$\begin{aligned} |\underline{a}| &= \sqrt{(2)^2 + (1)^2 + (3)^2} \\ &= \sqrt{14} \\ |\underline{b}| &= \sqrt{(-1)^2 + (1)^2 + (2)^2} \\ &= \sqrt{6} \end{aligned}$$

$$\therefore \cos\theta = \frac{5}{\sqrt{14}\sqrt{6}}$$

$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{84}}\right)$$

$$= 56.938\dots$$

$$= 57^\circ \text{ (nearest degree)}$$

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11 (c) (3 marks)

*Outcomes Assessed: MEX12-4**Targeted Performance Bands: E2-E3*

Criteria	Marks
Correct solution	3
Makes some progress towards solution	2
Uses conjugate root theorem	1

Sample Answer:

$$P(z) = z^4 - 4z^3 - 3z^2 + 50z - 52$$

$$\text{let } z = 3 - 2i$$

If $P(z) = 0$ then $P(\bar{z}) = 0$ (conjugate root theorem)

$z = 3 + 2i$ is also a root of $P(z)$

taking the sum and product of the roots:

$$z + \bar{z} = 6$$

$$z\bar{z} = |z|^2$$

$$= 13$$

Let the other roots be α and β .

sum of the roots: $\alpha + \beta + 6 = 4 \Rightarrow \alpha + \beta = -2$

product of the roots: $13\alpha\beta = -52 \Rightarrow \beta = -\frac{4}{\alpha}$

solving simultaneously:

$$\alpha - \frac{4}{\alpha} = -2$$

$$\alpha^2 - 4 = -2\alpha$$

$$\alpha^2 + 2\alpha - 4 = 0$$

$$\therefore \alpha = -1 \pm \sqrt{5}$$

$$\therefore \beta = -2 - (-1 \pm \sqrt{5})$$

$$= -1 \mp \sqrt{5}$$

The solutions of $P(z)$ are $-1 + \sqrt{5}, -1 - \sqrt{5}, 3 - 2i, 3 + 2i$

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11 (d) (i) (2 marks)

Outcomes assessed: MEX12-3**Targeted Performance Bands:** E3

Criteria	Marks
Correct solution	2
Finds \overrightarrow{AB} to use in the general form of a straight line in vector form	1

Sample Answer:

$$\overrightarrow{OA} = \underline{a} = \underline{i} + 3\underline{j} - 2\underline{k}$$

$$\begin{aligned}\overrightarrow{AB} &= \underline{b} = (2-1)\underline{i} + (-1-3)\underline{j} + (5-2)\underline{k} \\ &= \underline{i} - 4\underline{j} + 7\underline{k}\end{aligned}$$

The equation of the line through AB is given by:

$$\underline{r} = \underline{a} + \lambda_i \underline{b} \quad \text{where } \lambda_i \in \mathbb{R}$$

$$= (\underline{i} + 3\underline{j} - 2\underline{k}) + \lambda_i (\underline{i} - 4\underline{j} + 7\underline{k})$$

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11 (d) (ii) (1 mark)

*Outcomes assessed: MEX12-3**Targeted Performance Bands: E3*

Criteria	Mark
Correct solution	1

Sample Answer:

The parametric equations of the line are:

$$x = 1 + \lambda_1$$

$$y = 3 - 4\lambda_1$$

$$z = -2 + 7\lambda_1$$

Testing the point $(3, 4, 9)$:

$$3 = 1 + \lambda_1$$

$$\lambda_1 = 2$$

substitute $\lambda_1 = 2$ into the other parametric equations:

$$y = 3 - 4(2) = -5$$

$$z = -2 + 7(2) = 12$$

\therefore the point $(3, 4, 9)$ does not lie on the line $r = (i + 3j - 2k) + \lambda_1(i - 4j + 7k)$

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11 (d)(iii) (3 marks)

*Outcomes Assessed: MEX12-4**Targeted Performance Bands: E3*

Criteria	Marks
Correct solution	3
Correct substitution without correct geometrical interpretation	2
Progress towards solving the parametric equations	1

Sample Answer:

Equating the parametric equations of the two lines:

$$1 + \lambda_1 = 1 - \lambda_2 \quad (1)$$

$$3 - 4\lambda_1 = 2 + 3\lambda_2 \quad (2)$$

$$-2 + 7\lambda_1 = -1 + \lambda_2 \quad (3)$$

$$\text{rearranging (1)} \Rightarrow \lambda_1 = -\lambda_2$$

substituting into (2):

$$3 + 4\lambda_2 = 2 + 3\lambda_2$$

$$\lambda_2 = -1 \quad (\therefore \lambda_1 = 1)$$

substitute $\lambda_1 = 1$ and $\lambda_2 = -1$ into (3)

$$LHS = -2 + 7(1) = 5$$

$$RHS = -1 + (-1) = -2$$

since $LHS \neq RHS$ the lines do not intersect and are therefore skew.**DISCLAIMER**

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Question 12 (15 marks)

12 (a) (i) (1 mark)

Outcomes Assessed: MEX12-4**Targeted Performance Bands: E2-E3**

Criteria	Mark
Correct solution	1

Sample Answer:

completing the square:

$$\begin{aligned} z^2 - 2(1+2i)z + (1+2i)^2 &= -(1+i) + (1+2i)^2 \\ (z - (1-2i))^2 &= -1 - i + 1 + 4i - 4 \\ \therefore (z - (1-2i))^2 &= -4 + 3i \end{aligned}$$

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12 (a) (ii) (3 marks)

Outcomes Assessed: MEX12-4**Targeted Performance Bands: E3**

Criteria	Marks
Correct solution	3
Correct solutions of $4x^4 + 16x^2 - 9 = 0$	2
Progress towards finding $\sqrt{-4+3i}$	1

Sample Answer:

$$\text{Consider } (x+iy)^2 = -4+3i \quad \text{where } x, y \in \mathbb{R}$$

$$(x^2 - y^2) + 2ixy = -4 + 3i$$

equating the real and imaginary parts:

$$x^2 - y^2 = -4 \quad (1)$$

$$2xy = 3 \quad \therefore y = \frac{3}{2x} \quad (2)$$

substitute (2) into (1)

$$x^2 - \frac{9}{4x^2} = -4$$

$$4x^4 + 16x^2 - 9 = 0$$

$$(2x^2 + 9)(2x^2 - 1) = 0$$

$$\therefore x = \pm \frac{1}{\sqrt{2}} \quad \text{since } x \in \mathbb{R} \quad \therefore y = \frac{3\sqrt{2}}{2}$$

$$\therefore \sqrt{-4+3i} = \frac{\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i \quad \text{or} \quad -\frac{\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$$

$$\therefore z - (1+2i) = \frac{\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i \quad \text{or} \quad -\frac{\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$$

$$\therefore z = \left(\frac{\sqrt{2}}{2} + 1 \right) + \left(\frac{3\sqrt{2}}{2} + 2 \right)i \quad \text{or} \quad \left(-\frac{\sqrt{2}}{2} + 1 \right) + \left(-\frac{3\sqrt{2}}{2} + 2 \right)i$$

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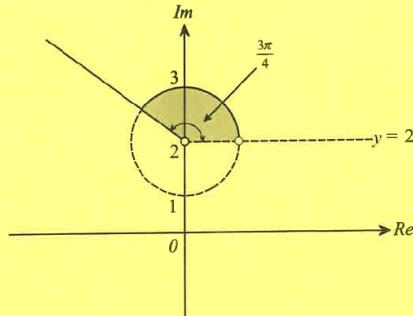
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12 (b) (2 marks)

*Outcomes Assessed: MEXI2-4**Targeted Performance Bands: E2-E3*

Criteria	Marks
Correct solution	2
Correct sketch of one of the lines and curves	1

Sample Answer:

15

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12(c) (4 marks)

Outcomes Assessed: MEXI2-5**Targeted Performance Bands: E2-E3**

Criteria	Marks
Correct solution	4
Definite integral correctly integrated	3
Correct new integral	2
Rewrites integrand using substitution	1

Sample Answer:

$$\text{using } t = \tan \frac{x}{2}$$

$$\text{let } t = \tan \frac{x}{2}$$

$$8\sin x = 8\left(\frac{2t}{1+t^2}\right) \quad \text{and} \quad 6\cos x = 6\left(\frac{1-t^2}{1+t^2}\right)$$

$$\frac{dt}{dx} = \frac{1}{2}\sec^2 \frac{x}{2}$$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{8\sin x + 6\cos x - 10} dx = \int_0^{\frac{1}{\sqrt{3}}} \frac{1}{\frac{16t}{1+t^2} + \frac{6-6t^2}{1+t^2} - 10} \times \frac{2dt}{1+t^2} = \frac{2dt}{1+t^2}$$

$$= \int_0^{\frac{1}{\sqrt{3}}} \frac{2dt}{16t+6-6t^2-10(1+t^2)}$$

$$\text{when } x = 0, t = \tan \frac{0}{2} = 0$$

$$= \int_0^{\frac{1}{\sqrt{3}}} \frac{dt}{-8t^2+8t-2}$$

$$\text{when } x = \frac{\pi}{3}, t = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$= -\frac{1}{2} \int_0^{\frac{1}{\sqrt{3}}} \frac{dt}{4t^2-4t+1}$$

$$= -\frac{1}{2} \int_0^{\frac{1}{\sqrt{3}}} \frac{dt}{(2t-1)^2}$$

$$= -\frac{1}{2} \left[\frac{(2t-1)^{-1}}{-1 \times 2} \right]_{\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}}$$

$$= \frac{1}{4} \left[\frac{1}{2t-1} \right]_0^{\frac{1}{\sqrt{3}}}$$

$$= \frac{1}{4} \left[\frac{1}{\frac{2}{\sqrt{3}}-1} - (-1) \right]$$

$$= \frac{1}{2} \left[\frac{1}{2-\sqrt{3}} \right]$$

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12(d) (3 marks)

*Outcomes Assessed: MEX12-7**Targeted Performance Bands: E3*

Criteria	Marks
Correct solution	3
Attempts to solve for two cases of $5 - x $	2
Correct solution in part of the domain	1

Sample Answer:

$$f(|x|) = \frac{4|x|}{5-|x|}$$

to solve $f(|x|) \leq 2$ we need to solve the inequation:

$$\frac{4|x|}{5-|x|} \leq 2$$

for $5 - |x| > 0$

$$4|x| \leq 2(5 - |x|)$$

$$4|x| \leq 10 - 2|x|$$

$$6|x| \leq 10$$

$$|x| \leq \frac{5}{3}$$

$$\therefore -\frac{5}{3} \leq x \leq \frac{5}{3}$$

Hence the set of solutions for the inequality $f(|x|) \leq 2$ is:

$$x \in (-\infty, -\frac{5}{3}) \cup \left[-\frac{5}{3}, \frac{5}{3}\right] \cup (5, \infty)$$

for $5 - |x| < 0$

$$4|x| \geq 2(5 - |x|)$$

$$4|x| \geq 10 - 2|x|$$

$$6|x| \geq 10$$

$$|x| \geq \frac{5}{3}$$

$$\text{so } x \leq -\frac{5}{3} \text{ or } x \geq \frac{5}{3}$$

but $5 - |x| < 0$ i.e. $|x| > 5$

$$\therefore x < -5 \text{ or } x > 5$$

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12 (e) (2 marks)

Outcomes Assessed: MEX12-2

Targeted Performance Bands: E2-E3

Criteria	Marks
Correct solution	2
Attempts to prove the statement or substitute values to find a counter-example	1

Sample Answer:

for $\frac{1}{p^2} < \frac{1}{q^2}$ then $p^2 > q^2$ i.e. $p > q$

since this was not stated in the conditions, a counter-example would be

$$p = 1, q = 2$$

$$\frac{1}{p^2} = 1 \text{ and } \frac{1}{q^2} = \frac{1}{4}$$

$\therefore \forall p \left(\forall q, \frac{1}{p^2} < \frac{1}{q^2} \right)$ is not true for all p, q real numbers.

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Question 13 (15 marks)

13(a) (i) (2 marks)

*Outcomes Assessed: MEXI2-5**Targeted Performance Bands: E3*

Criteria	Marks
Correct solution	2
Correct value of two of the variables	1

Sample Answer:

$$\frac{-x^2 + 2x + 5}{(x^2 + 2)(1-x)} \equiv \frac{ax + b}{x^2 + 2} + \frac{c}{1-x}$$

$$-x^2 + 2x + 5 \equiv (ax + b)(1-x) + c(x^2 + 2)$$

let $x = 1$

$$-(1)^2 + 2(1) + 5 \equiv 0 + c((1)^2 + 2)$$

$$6 = 3c$$

$$\therefore c = 2$$

let $x = 0$

$$-(0)^2 + 2(0) + 5 \equiv b + 2((0)^2 + 2)$$

$$5 = b + 4$$

$$\therefore b = 1$$

let $x = -1$

$$-(-1)^2 + 2(-1) + 5 \equiv (-a+1)(1-(-1)) + 2((-1)^2 + 2)$$

$$2 = -2a + 2 + 6$$

$$\therefore a = 3$$

$$\therefore \frac{-x^2 + 2x + 5}{(x^2 + 2)(1-x)} \equiv \frac{3x + 1}{x^2 + 2} + \frac{2}{1-x}$$

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13 (a) (ii) (2 marks)

*Outcomes Assessed: MEX12-5**Targeted Performance Bands: E3*

Criteria	Marks
Correct solution	2
One correct integral	1

Sample Answer:

$$\begin{aligned} \int \frac{-x^2 + 2x + 5}{(x^2 + 2)(1-x)} dx &= \int \frac{3x+1}{x^2+2} dx + \int \frac{2}{1-x} dx \\ &= \int \frac{3x}{x^2+2} dx + \int \frac{1}{x^2+2} dx - 2 \int \frac{1}{1-x} dx \\ &= \frac{3}{2} \log_e(x^2+2) + \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - 2 \log_e(1-x) + c \end{aligned}$$

13 (b) (i) (1 mark)

*Outcomes Assessed: MEX12-6**Targeted Performance Bands: E3*

Criteria	Mark
Correct solution	1

Sample Answer:

Since the particle is moving upwards, gravity and air resistance are acting against the direction of motion.

$$\begin{aligned} \therefore \ddot{x} &= -g - 3v \\ &= -(10 + 3v) \end{aligned}$$

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13 (b) (ii) (3 marks)

*Outcomes Assessed: MEX12-6**Targeted Performance Bands: E2-E3*

Criteria	Marks
Correct solution	3
Correctly finds integral	2
Correctly uses $\ddot{x} = v \frac{dv}{dx}$	1

Sample Answer:

using $\ddot{x} = v \frac{dv}{dx}$

$$v \frac{dv}{dx} = -(10 + 3v)$$

$$\frac{v dv}{10 + 3v} = -dx$$

$$\frac{1}{3} \left(\frac{10 + 3v - 10}{10 + 3v} \right) dv = -dx$$

$$\frac{1}{3} \left(1 - \frac{10}{10 + 3v} \right) dv = -dx$$

$$\frac{1}{3} \left(1 - \frac{10}{3} \left(\frac{3}{10 + 3v} \right) \right) dv = -dx$$

integrating both sides:

$$\frac{1}{3} \left(v - \frac{10}{3} \log_e (10 + 3v) \right) = -x + c$$

for $t = 0, x = 0, v = 120 \text{ ms}^{-1}$

$$\frac{1}{3} \left(120 - \frac{10}{3} \log_e (10 + 3(120)) \right) = c$$

$$\therefore c = 40 - \frac{10}{9} \log_e 370$$

$$\therefore x = -\frac{1}{3} \left(v - \frac{10}{3} \log_e (10 + 3v) \right) + 40 - \frac{10}{9} \log_e 370$$

the maximum height is reached when $v = 0$

$$\begin{aligned} x_{\max} &= -\frac{1}{3} \left(-\frac{10}{3} \log_e (10) \right) + 40 - \frac{10}{9} \log_e 370 \\ &= \frac{10}{9} \log_e \left(\frac{1}{37} \right) + 40 \end{aligned}$$

 $\approx 36 \text{ metres (nearest metre)}$ **DISCLAIMER**

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13 (b) (iii) (2 marks)

*Outcomes Assessed: MEXI2-6**Targeted Performance Bands: E3*

Criteria	Marks
Correct solution	2
Uses $\ddot{x} = \frac{dv}{dt}$ to find integral relating x and t	1

Sample Answer:

$$\text{using } \ddot{x} = \frac{dv}{dt}$$

$$\frac{dv}{dt} = -(10 + 3v)$$

$$\frac{dv}{10 + 3v} = -dt$$

integrating both sides:

$$\frac{1}{3} \log_e(10 + 3v) = -t + c$$

when $t = 0, v = 120 \text{ ms}^{-1}$

$$\therefore \frac{1}{3} \log_e(370) = c$$

$$\therefore \frac{1}{3} \log_e(10 + 3v) = -t + \frac{1}{3} \log_e(370)$$

the maximum height is reached when $v = 0$

$$\frac{1}{3} \log_e(10) = -t + \frac{1}{3} \log_e(370)$$

$$\therefore t = \frac{1}{3} \log_e\left(\frac{370}{10}\right)$$

$$\approx 1.2 \text{ seconds (1 decimal place)}$$

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13 (c) (i) (2 marks)

Outcomes Assessed: MEX12-3

Targeted Performance Bands: E3

Criteria	Marks
Correct solution	2
Finds the radius of the sphere	1

Sample Answer:

The radius of the sphere is given by

$$\begin{aligned} r &= \sqrt{|x - 2|} \\ &= \sqrt{2^2 + 2^2 + 2^2} \\ &= \sqrt{12} \end{aligned}$$

The sphere has equation:

$$(x - 2)^2 + (y - 2)^2 + (z - 2)^2 = 12$$

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13 (c) (ii) (3 marks)

Outcomes Assessed: MEX12-4**Targeted Performance Bands: E2-E3**

Criteria	Marks
Correct solution	3
Finds correct equation of circle without geometrical description of centre and radius	2
Finds the value of z for which the spheres intersect	1

Sample Answer:

Solving simultaneously:

$$(x-2)^2 + (y-2)^2 + (z-2)^2 = 12$$

$$(x-2)^2 + (y-2)^2 + (z-5)^2 = 1$$

subtracting equation (2) from (1)

$$(z-2)^2 - (z-5)^2 = 12 - 1$$

$$(z^2 - 4z + 4) - (z^2 - 10z + 25) = 11$$

$$\therefore z = \frac{32}{6} = \frac{16}{3}$$

substitute $z = \frac{16}{3}$ into eq (1)

$$(x-2)^2 + (y-2)^2 + \left(\frac{16}{3} - 2\right)^2 = 12$$

 $\therefore (x-2)^2 + (y-2)^2 = \frac{8}{9}$ is the equation of the circle.
This circle has centre $\left(2, 2, \frac{16}{3}\right)$ and radius $r = \frac{\sqrt{8}}{3}$.**DISCLAIMER**

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Question 14 (15 marks)

14 (a) (3 marks)

*Outcomes Assessed: MEX12-2**Targeted Performance Bands: E3-E4*

Criteria	Marks
Correct proof	3
Deduces that p is divisible by 5	2
Establishes assumption to prove by contradiction	1

Sample Answer:

Assume $\frac{1+\sqrt{5}}{2}$ is rational, i.e. $\sqrt{5}$ is rational (since 1, 2 $\in \mathbb{Q}$)

let $\sqrt{5} = \frac{p}{q}$, where p and q are integers with no common factor, $q \neq 0$.

$$(\sqrt{5})^2 = \left(\frac{p}{q}\right)^2$$

$$5 = \frac{p^2}{q^2}$$

$$5q^2 = p^2$$

$\therefore p$ is divisible by 5 i.e. $p = 5k$, $k \in \mathbb{Z}$

$$\therefore 5q^2 = (5k)^2$$

$$q^2 = 5k^2$$

$\therefore q$ is also divisible by 5, a contradiction since p and q have no common factor

$\therefore \sqrt{5}$ is irrational

$\therefore \frac{1+\sqrt{5}}{2}$ is irrational.

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14 (b) (i) (3 marks)

*Outcomes Assessed: MEX12-6**Targeted Performance Bands: E3*

Criteria	Marks
Correct solution	3
Correct amplitude or period	2
Correct transformation of trigonometric function	1

Sample Answer:

Express $\sin \frac{\pi}{7}t + \cos \frac{\pi}{7}t$ in the form $r \sin\left(\frac{\pi}{7}t + \alpha\right)$

$$\sin \frac{\pi}{7}t + \cos \frac{\pi}{7}t = r \left(\sin\left(\frac{\pi}{7}t\right) \sin \alpha + \cos\left(\frac{\pi}{7}t\right) \cos \alpha \right)$$

equating the coefficients of $\sin \frac{\pi}{7}t$ and $\cos \frac{\pi}{7}t$

$$r \sin \alpha = 1$$

$$r \cos \alpha = 1$$

$$\therefore r = \sqrt{2} \text{ and } \alpha = \frac{\pi}{4}$$

$$\begin{aligned} \therefore p &= 1.5 + \frac{1}{2} \sin \frac{\pi}{7}t + \frac{1}{2} \cos \frac{\pi}{7}t \\ &= 1.5 + \frac{\sqrt{2}}{2} \sin\left(\frac{\pi}{7}t + \frac{\pi}{4}\right) \end{aligned}$$

$$\text{amplitude: } a = \frac{\sqrt{2}}{2}$$

$$\text{period: } T = \frac{2\pi}{\frac{\pi}{7}} = 14 \text{ days}$$

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14 (b) (ii) (1 mark)

Outcomes Assessed: MEX12-6

Targeted Performance Bands: E3

Criteria	Mark
Correct solution	1

Sample Answer:

12pm on Monday is 12 hours after midnight Sunday.

$$\therefore t = 0.5 \text{ days}$$

$$p = 1.5 + \frac{\sqrt{2}}{2} \sin\left(\frac{\pi}{7} \times 0.5 + \frac{\pi}{4}\right)$$

$$= 2.0987\dots$$

$$= \$2.10 \text{ (nearest cent)}$$

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14 (b) (iii) (3 marks)

*Outcomes Assessed: MEX12-6**Targeted Performance Bands: E3*

Criteria	Marks
Correct solution	3
Correct values of t	2
Correct differentiation	1

Sample Answer:

$$\frac{dp}{dt} = \frac{\sqrt{2}}{2} \times \frac{\pi}{7} \cos\left(\frac{\pi}{7}t + \frac{\pi}{4}\right)$$

let $\frac{dp}{dt} = 0$ to find stationary points

$$\frac{\sqrt{2}}{2} \times \frac{\pi}{7} \cos\left(\frac{\pi}{7}t + \frac{\pi}{4}\right) = 0$$

$$\therefore \frac{\pi}{7}t + \frac{\pi}{4} = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

$$\therefore t = \frac{7}{4}, \frac{35}{4}, \dots$$

$$\frac{d^2p}{dt^2} = -\frac{\sqrt{2}}{2} \times \left(\frac{\pi}{7}\right)^2 \sin\left(\frac{\pi}{7}t + \frac{\pi}{4}\right)$$

for $t = \frac{7}{4}$, $\frac{d^2p}{dt^2} = -\frac{\sqrt{2}}{2} \times \left(\frac{\pi}{7}\right)^2$ i.e. price is max when $t = \frac{7}{4}$ for $t = \frac{35}{4}$, $\frac{d^2p}{dt^2} = \frac{\sqrt{2}}{2} \times \left(\frac{\pi}{7}\right)^2$ i.e. price is min when $t = \frac{35}{4}$ \therefore the fuel price is first at a minimum at $t = 8.75$ daysi.e. $t = 8$ days, 18 hours after midnight Sunday \therefore fuel price is first at a minimum at 6:00pm on Tuesday week.**DISCLAIMER**

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14 (c) (i) (2 marks)

*Outcomes Assessed: MEX12-6**Targeted Performance Bands: E3*

Criteria	Marks
Correct solution	2
Correctly uses $\ddot{x} = v \frac{dv}{dx}$	1

Sample Answer:

$$\frac{d\left(\frac{1}{2}v^2\right)}{dx} = 2x^3 + 6x^2 + 4x$$

$$\int d\left(\frac{1}{2}v^2\right) = \int (2x^3 + 6x^2 + 4x) dx$$

$$\frac{1}{2}v^2 = \frac{x^4}{2} + 2x^3 + 2x^2 + c$$

$$v^2 = x^4 + 4x^3 + 4x^2 + c$$

when $t = 0, x = 1, v = -3$

$$(-3)^2 = (1)^4 + 4(1)^3 + 4(1)^2 + c$$

$$9 = 9 + c$$

$$\therefore c = 0$$

$$\therefore v^2 = x^4 + 4x^3 + 4x^2$$

$$= x^2(x^2 + 4x + 4)$$

$$= x^2(x+2)^2$$

$$v = \pm \sqrt{x^2(x+2)^2}$$

$$v = -x(x+2) \quad \text{since initially } v = -3$$

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14 (c) (ii) (2 marks)

*Outcomes Assessed: MEX12-6**Targeted Performance Bands: E3*

Criteria	Marks
Correct solution	2
Correctly integrates to relate x and t	1

Sample Answer:

$$\frac{dx}{dt} = -x(x+2)$$

$$\frac{dx}{x(x+2)} = -dt$$

using partial fraction decomposition:

$$\frac{1}{2} \left(\frac{1}{x} - \frac{1}{x+2} \right) dx = -dt$$

integrating both sides:

$$\frac{1}{2} (\ln x - \ln(x+2)) = -t + c$$

$$\frac{1}{2} \ln \left(\frac{x}{x+2} \right) = -t + c$$

when $t = 0, x = 1$

$$\frac{1}{2} \ln \left(\frac{1}{1+2} \right) = 0 + c$$

$$\therefore c = -\frac{1}{2} \ln 3$$

$$\frac{1}{2} \ln \left(\frac{x}{x+2} \right) = -t - \frac{1}{2} \ln 3$$

$$\frac{1}{2} \ln \left(\frac{x}{x+2} \right) + \frac{1}{2} \ln 3 = -t$$

$$\frac{1}{2} \ln \left(\frac{3x}{x+2} \right) = -t$$

$$\ln \left(\frac{x+2}{3x} \right) = 2t$$

$$\ln \left(\frac{x}{3x} + \frac{2}{3x} \right) = 2t$$

$$\frac{1}{3} + \frac{2}{3x} = e^{2t}$$

$$\frac{2}{3x} = \frac{3e^{2t} - 1}{3}$$

$$\therefore x = \frac{2}{3e^{2t} - 1}$$

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14 (c) (iii) (1 mark)

Outcomes Assessed: MEXI2-6

Targeted Performance Bands: E3

Criteria	Mark
Correct solution	1

Sample Answer:

for $t \rightarrow \infty$

$$x \rightarrow \lim_{t \rightarrow \infty} \frac{2}{3e^{2t} - 1}$$

$$\rightarrow \frac{2}{\infty}$$

$$\rightarrow 0$$

∴ the limiting position of the particle is $x = 0$.

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Question 15 (15 marks)

15 (a) (3 marks)

Outcomes Assessed: MEXI2-5**Targeted Performance Bands: E3**

Criteria	Marks
Correct solution, or equivalent merit	3
Correctly uses integration by parts and applies it again	2
Attempts to use integration by parts, or equivalent merit	1

Sample Answer:

$$I = \int e^{-x} \sin(-x) dx$$

using by parts integration:

$$u = e^{-x} \quad dv = \sin(-x) dx$$

$$du = -e^{-x} dx \quad v = \cos(-x)$$

$$\therefore I = e^{-x} \cos(-x) - \int -e^{-x} \cos(-x) dx$$

$$= e^{-x} \cos(-x) + \int e^{-x} \cos(-x) dx$$

using a second application of by parts integration:

$$I = e^{-x} \cos(-x) + \int e^{-x} \cos(-x) dx$$

$$= e^{-x} \cos(-x) - e^{-x} \sin(-x) - \int (-e^{-x}) \times (-\sin(-x)) dx$$

$$= e^{-x} \cos x + e^{-x} \sin x - \underbrace{\int e^{-x} \sin(-x) dx}_{\gamma}$$

$$2I = e^{-x} \cos x + e^{-x} \sin x$$

$$I = \frac{e^{-x} \cos x + e^{-x} \sin x}{2}$$

$$\therefore \int e^{-x} \sin(-x) dx = \frac{1}{2} e^{-x} (\cos x + \sin x) + c$$

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15 (b) (i) (2 marks)

Outcomes Assessed: MEX12-6**Targeted Performance Bands: E3-E4**

Criteria	Marks
Correct solution	2
Correct use of vertical component	1

Sample Answer:

$$\begin{aligned}
 y &= \frac{10 + 0.4 \times 10\sqrt{3} \times \sin 60^\circ}{(0.4)^2} (1 - e^{-0.4t}) - \frac{10t}{0.4} \\
 &= \frac{10 + 4\sqrt{3} \times \frac{\sqrt{3}}{2}}{4} (1 - e^{-0.4t}) - 25t \\
 &= \frac{25}{4} (10 + 6)(1 - e^{-0.4t}) - 25t \\
 &= 100 - 100e^{-0.4t} - 25t \\
 \dot{y} &= 40e^{-0.4t} - 25
 \end{aligned}$$

the greatest height is reached when $\dot{y} = 0$:

$$\begin{aligned}
 0 &= 40e^{-0.4t} - 25 \\
 e^{-0.4t} &= \frac{25}{40} \\
 -0.4t &= \log_e \left(\frac{5}{8} \right) \\
 t &= -\frac{5}{2} \log_e \left(\frac{5}{8} \right) \\
 &= 1.175... \\
 &= 1.18 \text{ seconds (2 decimal places)}
 \end{aligned}$$

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15 (b) (ii) (3 marks)

*Outcomes Assessed: MEX12-6**Targeted Performance Bands: E3-E4*

Criteria	Marks
Correct solution	3
Correctly finds the magnitude or direction of the velocity	2
Correct value of horizontal component of velocity	1

*Sample Answer:*for $t = 2.6$

$$\dot{y} = 40e^{-0.4 \times 2.6} - 25$$

$$= -10.86 \text{ ms}^{-1} \text{ (2 decimal places)}$$

$$\text{also } x = \frac{u \cos \theta}{k} (1 - e^{-kt})$$

$$= \frac{10\sqrt{3} \cos 60^\circ}{0.4} (1 - e^{-0.4t})$$

$$= \frac{25\sqrt{3}}{2} (1 - e^{-0.4t})$$

$$\dot{x} = 5\sqrt{3}e^{-0.4t}$$

for $t = 2.6$

$$\dot{x} = 5\sqrt{3}e^{-0.4 \times 2.6}$$

$$= 3.06 \text{ ms}^{-1} \text{ (2 decimal places)}$$

 \therefore the magnitude of the velocity of the particle is:

$$|v| = \sqrt{(10.86)^2 + (3.06)^2}$$

$$= 11.28 \text{ ms}^{-1}$$

the direction of the particle is $\tan^{-1}\left(\frac{10.86}{3.06}\right) = 74^\circ$ below the horizontal (to the nearest degree).**DISCLAIMER**

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15 (c) (i) (2 marks)

Outcomes Assessed: MEX12-4**Targeted Performance Bands: E3**

Criteria	Marks
Correct solution	2
Correct method in finding roots of unity	1

Sample Answer:

$$\text{let } z = r(\cos\theta + i\sin\theta)$$

$z^5 = 1$ can be written in mod-arg form:

$$(r(\cos\theta + i\sin\theta))^5 = \cos 0 + i\sin 0$$

$r^5(\cos 5\theta + i\sin 5\theta) = \cos 0 + i\sin 0$ using De Moivre's Theorem

$$\therefore r = 1 \text{ and } 5\theta = 2k\pi \Rightarrow \theta = \frac{2k\pi}{5}$$

for $k = 0$, $\theta = 0$

$$\text{for } k = 1, \theta = \frac{2\pi}{5}$$

$$\text{for } k = 2, \theta = \frac{4\pi}{5}$$

$$\text{for } k = -1, \theta = -\frac{2\pi}{5}$$

$$\text{for } k = -2, \theta = -\frac{4\pi}{5}$$

\therefore the roots of $z^5 = 1$ are:

$$z_1 = \cos\left(\frac{2\pi}{5}\right) + i\sin\left(\frac{2\pi}{5}\right)$$

$$z_2 = \cos\left(\frac{4\pi}{5}\right) + i\sin\left(\frac{4\pi}{5}\right)$$

$$z_3 = \cos\left(-\frac{2\pi}{5}\right) + i\sin\left(-\frac{2\pi}{5}\right)$$

$$z_4 = \cos\left(-\frac{4\pi}{5}\right) + i\sin\left(-\frac{4\pi}{5}\right)$$

$$z_5 = 1$$

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15 (c) (ii) (2 marks)

*Outcomes Assessed: MEX12-4**Targeted Performance Bands: E3-E4*

Criteria	Marks
Correct solution	2
Correct use of the conjugates to simplify complex factors	1

Sample Answer:

From part (i) it can be seen that:

$$z_4 = \bar{z}_1 \text{ and } z_3 = \bar{z}_2$$

using the fundamental theorem of algebra:

$$\begin{aligned}z^5 - 1 &= (z - 1)(z - z_1)(z - z_2)(z - z_3)(z - z_4) \\&= (z - 1)(z - z_1)(z - z_2)(z - \bar{z}_2)(z - \bar{z}_1)\end{aligned}$$

using the property $(x - \alpha)(x - \bar{\alpha}) = x^2 - 2\operatorname{Re}(\alpha)x + |\alpha|^2$

$$z^5 - 1 = (z - 1) \left(z^2 - 2\cos\left(\frac{2\pi}{5}\right) + 1 \right) \left(z^2 - 2\cos\left(\frac{4\pi}{5}\right) + 1 \right)$$

$$\therefore \frac{z^5 - 1}{z - 1} = \left(z^2 - 2\cos\left(\frac{2\pi}{5}\right) + 1 \right) \left(z^2 - 2\cos\left(\frac{4\pi}{5}\right) + 1 \right)$$

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15 (c) (iii) (3 marks)

Outcomes Assessed: MEX12-4**Targeted Performance Bands: E3-E4**

Criteria	Marks
Correct solution	3
Progress towards solution	2
Attempts to establish $\cos\left(\frac{2\pi}{5}\right)$ and $\cos\left(\frac{4\pi}{5}\right)$ as roots of a quadratic equation using part (ii)	1

Sample Answer:

The sum of the roots is 0.

$$\therefore z_1 + z_2 + z_3 + z_4 + z_5 = 0$$

$$z_1 + z_2 + \bar{z}_2 + \bar{z}_1 + 1 = 0$$

$$2\cos\left(\frac{2\pi}{5}\right) + 2\cos\left(\frac{4\pi}{5}\right) = -1$$

$$\therefore \cos\left(\frac{2\pi}{5}\right) + \cos\left(\frac{4\pi}{5}\right) = -\frac{1}{2} \quad (1)$$

also, from part (ii):

$$\frac{z^5 - 1}{z - 1} = \left(z^2 - 2\cos\left(\frac{2\pi}{5}\right) + 1\right) \left(z^2 - 2\cos\left(\frac{4\pi}{5}\right) + 1\right) \quad \text{as } \frac{z^5 - 1}{z - 1} \text{ is the sum of a GP with } a=1, r=z, n=5$$

$$z^4 + z^3 + z^2 + z + 1 = \left(z^2 - 2\cos\left(\frac{2\pi}{5}\right) + 1\right) \left(z^2 - 2\cos\left(\frac{4\pi}{5}\right) + 1\right)$$

considering the z^2 terms on both sides :

$$LHS = z^2$$

$$RHS = z^2 + 4\cos\left(\frac{2\pi}{5}\right)\cos\left(\frac{4\pi}{5}\right) + z^2 = z^2 \left(2 + 4\cos\left(\frac{2\pi}{5}\right)\cos\left(\frac{4\pi}{5}\right)\right)$$

equating the coefficients:

$$2 + 4\cos\left(\frac{2\pi}{5}\right)\cos\left(\frac{4\pi}{5}\right) = 1$$

$$\cos\left(\frac{2\pi}{5}\right)\cos\left(\frac{4\pi}{5}\right) = -\frac{1}{4} \quad (2)$$

\therefore from (1) and (2), $\cos\left(\frac{2\pi}{5}\right)$ and $\cos\left(\frac{4\pi}{5}\right)$ are the roots of a quadratic of the form:

$$x^2 + \frac{1}{2}x - \frac{1}{4} = 0$$

$$4x^2 + 2x - 1 = 0$$

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Question 16 (15 marks)

16 (a) (3 marks)

*Outcomes Assessed: MEXI2-2**Targeted Performance Bands: E3-E4*

Criteria	Marks
Correct solution	3
Significant progress toward completing the inductive step	2
Shows statement is true for $n = 3$ and assumes true for $n = k$	1

Sample Answer:Show the statement is true for $n = 1, 2, 3$:

$$u_1 = (1-1)5^1 = 0$$

$$u_2 = (2-1)5^2 = 25$$

$$\begin{aligned} u_3 &= 10u_2 - 25u_1 & u_3 &= (3-1) \times 5^3 \\ &= 10 \times 25 - 25 \times 0 & &= 250 \\ &= 250 & & \end{aligned}$$

hence the statement is true for $n = 1, 2, 3$.Assume the statements are true for $n = k$, that is:

$$u_k = (k-1) \times 5^k \text{ and } u_k = 10u_{k-1} - 25u_{k-2}$$

Prove that the statement is true for $n = k+1$, that is, required to prove $u_{k+1} = k \times 5^{k+1}$

$$\begin{aligned} u_{k+1} &= 10u_k - 25u_{k-1} \\ &= 10(k-1) \times 5^k - 25(k-2) \times 5^{k-1} \\ &= 2(k-1) \times 5^{k+1} - (k-2) \times 5^{k+1} \\ &= (2k-2-k+2) \times 5^{k+1} \\ &= k \times 5^{k+1} \end{aligned}$$

hence the statement is true for $n = k+1$.since the statement is true for $n = 1$, $n = k$ and $n = k+1$ it is true by mathematical induction for all positive integers $n \geq 1$.**DISCLAIMER**

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16 (b) (i) (4 marks)

Outcomes Assessed: MEX12-5**Targeted Performance Bands: E3-E4**

Criteria	Marks
Correct solution	4
Correctly recognises the equivalent integral on the RHS	3
Correctly uses one application of integration by parts	2
Attempts to use integration by parts	1

Sample Answer:

Let $I_n = \int_0^{\frac{\pi}{2}} \cos^{n-1} x \cos x \sin^2 x dx$

let $u = \cos^{n-1} x \quad dv = \cos x \sin^2 x dx$

$du = (n-1)\cos^{n-2} x (-\sin x) \quad v = \frac{1}{3} \sin^3 x$

using by parts integration:

$$\begin{aligned} I_n &= uv - \int v du \\ &= \left[\frac{1}{3} \sin^3 x \cos^{n-1} x \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{1}{3} \sin^3 x (n-1) \cos^{n-2} x (-\sin x) dx \\ &= 0 + \frac{n-1}{3} \int_0^{\frac{\pi}{2}} \sin^3 x \cos^{n-2} x \sin x dx \\ &= \frac{n-1}{3} \int_0^{\frac{\pi}{2}} \sin^2 x \cos^{n-2} x \sin^2 x dx \\ &= \frac{n-1}{3} \int_0^{\frac{\pi}{2}} \sin^2 x \cos^{n-2} x (1 - \cos^2 x) dx \\ &= \frac{n-1}{3} \int_0^{\frac{\pi}{2}} \sin^2 x \cos^{n-2} x dx - \frac{n-1}{3} \int_0^{\frac{\pi}{2}} \sin^2 x \cos^n x dx \\ &= \frac{n-1}{3} I_{n-2} - \frac{n-1}{3} I_n \\ 3I_n &= (n-1)I_{n-2} - (n-1)I_n \\ 3I_n + (n-1)I_n &= (n-1)I_{n-2} \\ \therefore I_n &= \frac{n-1}{n+2} I_{n-2} \end{aligned}$$

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16 (b) (ii) (1 mark)

*Outcomes Assessed: MEX12-5**Targeted Performance Bands: E3-E4*

Criteria	Mark
Correctly applies recurrence relation	1

Sample Answer:

$$\begin{aligned}
 I_2 &= \frac{2-1}{2+2} I_0 \\
 &= \frac{1}{4} \int_0^{\frac{\pi}{2}} \cos^0 x \sin^2 x dx \\
 &= \frac{1}{4} \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 - \cos 2x) dx \\
 &= \frac{1}{8} \left[x - \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{2}} \\
 &= \frac{1}{8} \left[\left(\frac{\pi}{2} - \frac{1}{2} \sin \pi \right) - \left(0 - \frac{1}{2} \sin 0 \right) \right] \\
 &= \frac{\pi}{16}
 \end{aligned}$$

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16 (c) (i) (1 mark)

Outcomes Assessed: MEX12-2

Targeted Performance Bands: E3-E4

Criteria	Mark
Correct solution	1

Sample Answer:

For $x > 1$:

$$x^3 > x^2 > x > 0$$

$$\text{so } x\sqrt{x} > x > \sqrt{x} > 0$$

by taking the reciprocals:

$$0 < \frac{1}{x\sqrt{x}} < \frac{1}{x} < \frac{1}{\sqrt{x}}$$

16 (c)(ii) (1 mark)

Outcomes Assessed: MEX12-2

Targeted Performance Bands: E3-E4

Criteria	Mark
Correct solution	1

Sample Answer:

For $0 < x < 1$:

$$x > x^2 > x^3 > 0$$

$$\text{so } \sqrt{x} > x > x\sqrt{x} > 0$$

by taking reciprocals:

$$0 < \frac{1}{\sqrt{x}} < \frac{1}{x} < \frac{1}{x\sqrt{x}}$$

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16 (c)(iii) (2 marks)

*Outcomes Assessed: MEX12-2**Targeted Performance Bands: E3-E4*

Criteria	Marks
Correct solution	2
Correctly integrates the terms of the inequation	1

Sample Answer:

For $x \geq 1$, $\frac{1}{x\sqrt{x}} \leq \frac{1}{x} \leq \frac{1}{\sqrt{x}}$ with equality when $x = 1$.

Integrating from $x = 1$ to $x = t$

$$\begin{aligned} \int_1^t \frac{1}{x\sqrt{x}} dx &\leq \int_1^t \frac{1}{x} dx \leq \int_1^t \frac{1}{\sqrt{x}} dx \\ \left[-2x^{\frac{1}{2}} \right]_1^t &\leq [\ln x]_1^t \leq \left[2x^{\frac{1}{2}} \right]_1^t \\ -2\left[\frac{1}{\sqrt{t}} - \frac{1}{\sqrt{1}} \right] &\leq \ln t - \ln 1 \leq 2\left(\sqrt{t} - 1 \right) \\ \therefore \frac{2(\sqrt{t}-1)}{\sqrt{t}} &\leq \ln t \leq 2(\sqrt{t}-1). \end{aligned}$$

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16 (c)(iv) (3 marks)

*Outcomes Assessed: MEX12-2**Targeted Performance Bands: E3-E4*

Criteria	Marks
Correct solution	3
One limit correctly evaluated	2
Attempts to use the squeeze law of limits or equivalent	1

Sample Answer:

from part (ii) $\frac{2(\sqrt{t}-1)}{\sqrt{t}} \leq \ln t \leq 2(\sqrt{t}-1)$.

multiplying through by t gives:

$$2\sqrt{t}(\sqrt{t}-1) \leq t \ln t \leq 2t(\sqrt{t}-1)$$

as $t \rightarrow 0$

$$2\sqrt{t}(\sqrt{t}-1) \rightarrow 0$$

$$\text{and } 2t(\sqrt{t}-1) \rightarrow 0$$

$$\therefore 0 \leq \lim_{t \rightarrow 0} (t \ln t) \leq 0$$

$$\therefore \lim_{t \rightarrow 0} (t \ln t) = 0$$

from part (ii) $\frac{2(\sqrt{t}-1)}{\sqrt{t}} \leq \ln t \leq 2(\sqrt{t}-1)$.

dividing through by t gives:

$$\frac{2(\sqrt{t}-1)}{t\sqrt{t}} \leq \frac{\ln t}{t} \leq \frac{2(\sqrt{t}-1)}{t}$$

$$\frac{2}{t} - \frac{2}{t\sqrt{t}} \leq \frac{\ln t}{t} \leq \frac{2}{\sqrt{t}} - \frac{2}{t}$$

as $t \rightarrow \infty$

$$\frac{2}{t} - \frac{2}{t\sqrt{t}} \rightarrow 0$$

$$\text{and } \frac{2}{\sqrt{t}} - \frac{2}{t} \rightarrow 0$$

$$\therefore 0 \leq \lim_{t \rightarrow \infty} \left(\frac{\ln t}{t} \right) \leq 0$$

$$\therefore \lim_{t \rightarrow \infty} \left(\frac{\ln t}{t} \right) = 0 \quad \text{and} \quad \lim_{t \rightarrow 0} (t \ln t) = 0$$

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