Name:……………………………………..

Student Number	

Teacher:…………………………………..

Mathematics Extension 1

HSC Trial Examination

Term 3 2024

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks 70

SECTION 1 – 10 marks (pages 1-4)

- Attempt Questions 1-10
- Allow about 15 minutes for this section
- Answer each question on the multiple-choice answer sheet provided in the answer booklet.

SECTION II – 60 marks (pages 5-11)

- Attempt Questions 11-14
- Allow about 1 hours and 45 minutes for this section
- Answer each question in the appropriate space in the Answer Booklet. Extra writing pages are included at the end of each question.

SECTION I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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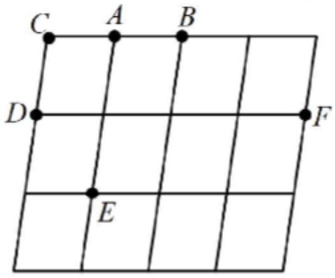
1. The grid shown is made up of identical parallelograms.

Let $\underset{\sim}{a} = \overrightarrow{AB}$ and $\underset{\sim}{c} = \overrightarrow{CD}$. What is the vector $\overrightarrow{EF}$ is equal to?



(A) $\underset{\sim}{a} + 3\underset{\sim}{c}$

(B) $-3\underset{\sim}{a} + \underset{\sim}{c}$

(C) $-3\underset{\sim}{a} - \underset{\sim}{c}$

(D) $3\underset{\sim}{a} - \underset{\sim}{c}$

2. Four female and four male students are to be seated around a circular table.
In how many ways can this be done if the males and females must alternate?

(A) $4! \times 4!$

(B) $3! \times 4!$

(C) $3! \times 3!$

(D) $2 \times 3! \times 3!$

3. Which of the following is equivalent to $\dfrac{d}{dx}\left(2\sin^{-1}\dfrac{x}{2}\right)$?

(A) $\dfrac{1}{\sqrt{1-x^2}}$

(B) $\dfrac{2}{\sqrt{1-x^2}}$

(C) $\dfrac{2}{\sqrt{4-x^2}}$

(D) $\dfrac{1}{2\sqrt{4-x^2}}$

4. Which expression is identical to $2\sin 3x \sin 5x$?

(A) $-\cos 2x - \cos 8x$

(B) $\cos 8x - \cos 2x$

(C) $\cos 2x - \cos 8x$

(D) $\cos 2x + \cos 8x$

5. If $\sin A = t$ and $\cos B = t$, where $\frac{\pi}{2} < A < \pi$ and $0 < B < \frac{\pi}{2}$, then what is $\cos(B + A)$ equal to?

(A) 0

(B) $\sqrt{1-t^2}$

(C) $1-2t^2$

(D) $-2t\sqrt{1-t^2}$

6. Let R be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. Which expression gives the volume of the solid obtained by revolving R about the x-axis?

(A) $2\pi \int_0^{\frac{\pi}{2}} x \sin x \, dx$

(B) $\pi \int_0^{\frac{\pi}{2}} (1 - \sin x)^2 \, dx$

(C) $\pi \int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

(D) $\pi \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$

7. A spherical balloon is being inflated at a constant rate of $200\pi\,\text{cm}^3\text{s}^{-1}$. At what rate is the radius of the balloon increasing when the radius is 10 cm?

(A) 0.25 cm s^{-1}

(B) 0.5 cm s^{-1}

(C) 1 cm s^{-1}

(D) 2 cm s^{-1}

8. Which of the following is the domain of $y = 5\cos^{-1}\left(\frac{2-x}{3}\right)$?

(A) $x \in [1,5]$

(B) $x \in [-1,5]$

(C) $x \in [-5,1]$

(D) $x \in [-5,-1]$

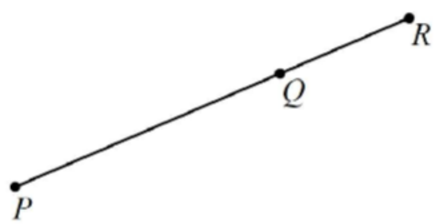
9. PQR is a straight line and $PQ = 2QR$.
If $\vec{OQ} = 3\underset{\sim}{i} - 2\underset{\sim}{j}$ and $\vec{OR} = \underset{\sim}{i} + 3\underset{\sim}{j}$, where O is the origin, then what is $\vec{OP}$?

(A) $-\underset{\sim}{i} + 8\underset{\sim}{j}$

(B) $7\underset{\sim}{i} - 12\underset{\sim}{j}$

(C) $4\underset{\sim}{i} - 10\underset{\sim}{j}$

(D) $-4\underset{\sim}{i} + 10\underset{\sim}{j}$



10. The inverse function of $f(x) = \ln(x-1)$ is $g(x)$.
Which one of these statements must be true for all x in the domain of $g(x)$?

(A) $g(x) < 0$

(B) $g'(x) < 0$

(C) $g''(x) > 0$

(D) $g''(x) < 0$

SECTION II

60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section.

Answer these questions in the Answer Book provided.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Find the term independent of x in the expansion of $\left(3x^2 + \dfrac{2}{x}\right)^{12}$. **2**

(b) Mrs Munro needs to decide the order in which to schedule 8 exams. **2**
Two of these exams are Mathematics Extension 1 and Mathematics Extension 2.

Find the number of different ways that Mrs Munro can schedule the 8 exams so that Mathematics Extension 1 and Mathematics Extension 2 are NOT consecutive.

(c) Solve $\dfrac{x^2+10}{x} \le 7$. **3**

(d) The polynomial $P(x) = 8x^4 - 38x^3 + 9x^2 + ax + b$ has a double root at $x = 3$. **3**
Find the values of a and b, where a and b are real numbers.

(e) Evaluate $\displaystyle\int_0^2 \frac{dx}{\sqrt{16-x^2}}$. **2**

(f) Use mathematical induction to prove that $8n^3 - 2n$ is divisible by 3 for all integers $n \ge 1$. **3**

End of Question 11

Question 12 (14 marks) **Marks**

(a) By using the substitution $t = \tan\frac{x}{2}$, solve **3**

$$\cos x - \sqrt{3}\sin x + 1 = 0 \quad \text{for } 0 \le x \le 2\pi.$$

(b) Use the substitution $u = x - 4$ to find the following integral: **3**

$$\int x\sqrt{x-4}\,dx$$

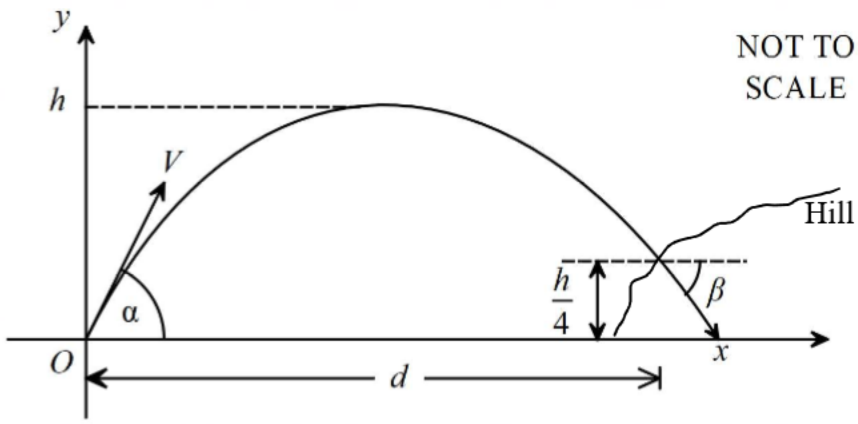
(c) When the polynomial $P(x)$ is divided by $9x^2 - 1$ the remainder is $3x + 7$. **2**

What is the remainder when $P(x)$ is divided by $3x + 1$?

Question 12 continues on page 7

Question 12 continued. **Marks**

(d) Julie kicks a soccer ball from the origin O, which is on level ground, with velocity V ms^{-1}, at an angle of α to the horizontal. The ball rises to a maximum height h and lands on a hill at distance d metres from the origin, with a height of $\frac{h}{4}$ metres, making an angle of β with the horizontal as shown.



Use the axes as shown and assume there is no air resistance.

The position vector of the ball, t seconds after being kicked, where g is acceleration due to gravity, is given by

$$\underset{\sim}{r}(t) = (Vt\cos\alpha)\underset{\sim}{i} + \left(Vt\sin\alpha - \frac{g}{2}t^2\right)\underset{\sim}{j}$$

DO NOT prove this

(i) Show that the maximum height h reached by the ball is **2**

$$h = \frac{V^2\sin^2\alpha}{2g}.$$

(ii) Show that the time taken for the ball to land on the hill is **3**

$$t = \frac{\left(2+\sqrt{3}\right)V\sin\alpha}{2g} \text{ seconds.}$$

(iii) Calculate the horizontal distance d travelled by the ball. **1**

End of Question 12

Question 13 (15 marks) **Marks**

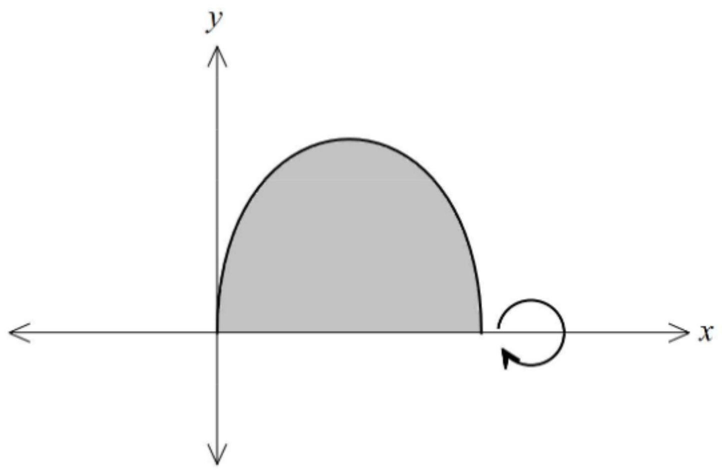
(a) Given the equation $f(x) = x\sin^{-1}\left(\dfrac{x}{2}\right), \quad -2 \le x \le 2$

(i) show that $f(x)$ is an even function **1**

(ii) sketch the graph of $y = f(x)$ showing all features including intercept(s) and endpoints. **2**

(b) After t minutes the temperature $T°C$ of water in a jug is given by $T = 20 + 80e^{-0.2t}$. What is the rate at which the water is cooling when its temperature has fallen to half its initial value? **2**

(c) Part of the graph of $y = \sqrt{\cos(3x)\sin(2x)}$ is shown in the diagram.



(i) By solving $\cos(3x)\sin(2x) = 0$, show that the smallest positive solution is $x = \dfrac{\pi}{6}$. **2**

(ii) Hence, find the volume of the solid of revolution formed when the shaded region is rotated around the x-axis. **3**

Question 13 continues on page 9

Question 13 continued. **Marks**

(d) (i) Sketch the curve $y = -\tan^{-1} x$ and label the point where $x = 1$. **2**

(ii) Find the area bounded by the curve, the x-axis and the line $x = 1$. **3**

End of Question 13

Question 14 (16 marks) **Marks**

(a) A bug moves such that its acceleration is given by $\frac{dv}{dt} = \sqrt{v+1}\ ms^{-2}$. **3**

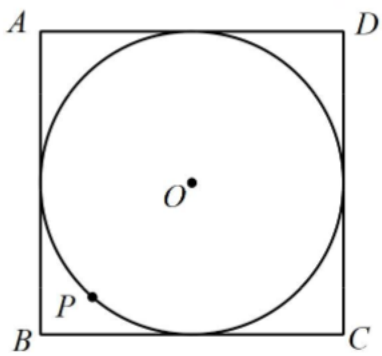
Initially the bug is at rest. Find its velocity after 1 second.

(b) A population of penguins on an island satisfies $\frac{dP}{dt} = 0.001P(400-P)$ where P is the number of penguins and t is measured in years. Initially there are 50 penguins.

(i) What is the carrying capacity of the island? **1**

(ii) Given that $\frac{1}{P(400-P)} = \frac{1}{400}\left(\frac{1}{P} + \frac{1}{400-P}\right)$, calculate when the population of penguins will reach 50% of the carrying capacity. **3**

(c) In the figure, the circle has centre O and radius r. The circle is inscribed in a square $ABCD$, and P is any point on the circle.



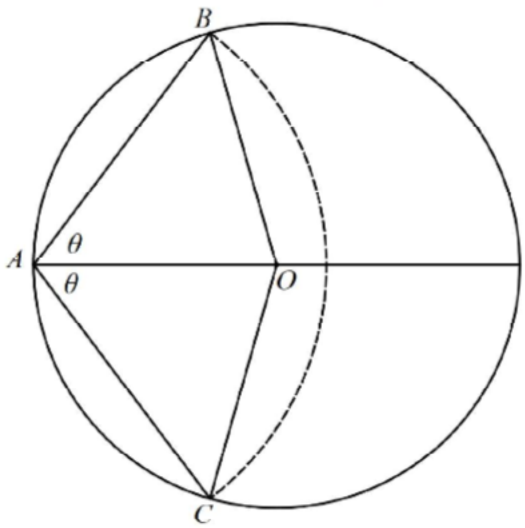
(i) Show that $\overrightarrow{AP} \cdot \overrightarrow{AP} = 3r^2 - 2\overrightarrow{OP} \cdot \overrightarrow{OA}$. **2**

(ii) Hence find $AP^2 + BP^2 + CP^2 + DP^2$ in terms of r. **2**

Question 14 continues on page 11

Question 14 continued. **Marks**

(d) A is the point on the circumference of a circle with centre O and radius a. With A as the centre, an arc of radius r is drawn which meets the circle at two points B and C, and $r < 2a$. Arc length $BC = \ell$ and $\angle BAC = 2\theta$.



(i) Show that $r = 2a\cos\theta$ and $\ell = 4a\theta\cos\theta$. **2**

(ii) Hence, show that ℓ is a maximum when $\theta = \cot\theta$. **3**

END OF PAPER