Neap

Final Examination 2021

NSW Year 11 Mathematics Extension 1

General Instructions	
	• Reading time – 10 minutes • Working time – 2 hours • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • For questions in Section II, show relevant mathematical reasoning and/or calculations
Total marks: 70	**Section I – 10 marks (pages 2–4)** • Attempt Questions 1–10 • Allow about 15 minutes for this section **Section II – 60 marks (pages 5–8)** • Attempt Questions 11–14 • Allow about 1 hour and 45 minutes for this section

SECTION I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Given $P = 100 + 300e^{-0.2t}$, what value does P approach as t approaches infinity?

A. 100

B. 200

C. 300

D. 400

2 How many three-digit odd numbers can be formed by using the digits 1, 2, 3, 4 and 5 if repetitions are NOT allowed?

A. 24

B. 36

C. 48

D. 60

3 Which of the following expressions is equivalent to $2\sin\left(x + \dfrac{\pi}{3}\right)$?

A. $\sqrt{3}\sin x + \cos x$

B. $\sqrt{3}\sin x - \cos x$

C. $\sin x + \sqrt{3}\cos x$

D $\sin x - \sqrt{3}\cos x$

4 The function $f(x)$ is defined as $f(x) = 3x^3 + 4$.

Which of the following expressions is equal to $f^{-1}(x)$?

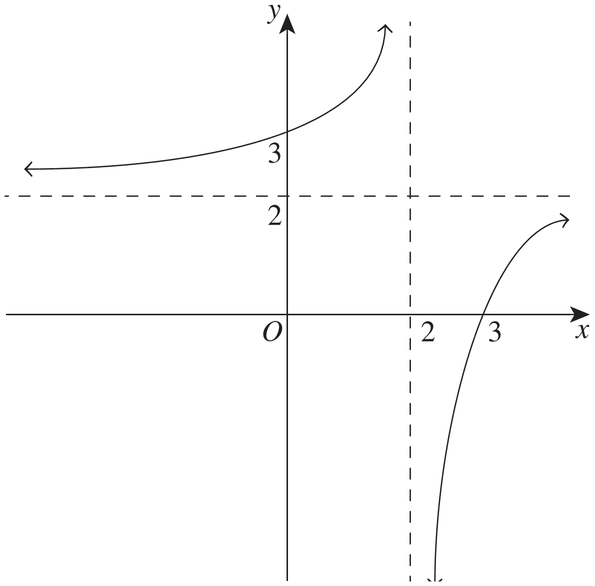
A. $\dfrac{1}{3x^3} + 4$

B. $\dfrac{1}{3x^3 + 4}$

C. $\dfrac{\sqrt[3]{x+3}}{4}$

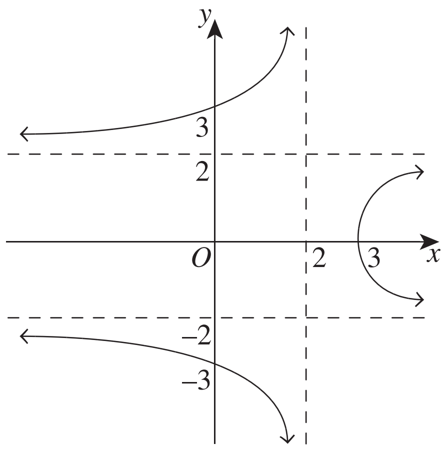
D $\sqrt[3]{\dfrac{x-4}{3}}$

5 The graph of $y = f(x)$ is given below.

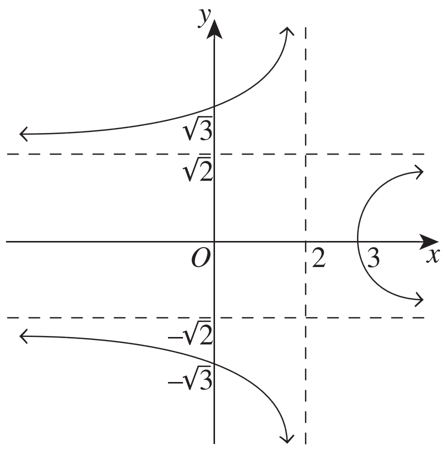


The graph of $y^2 = f(x)$ is

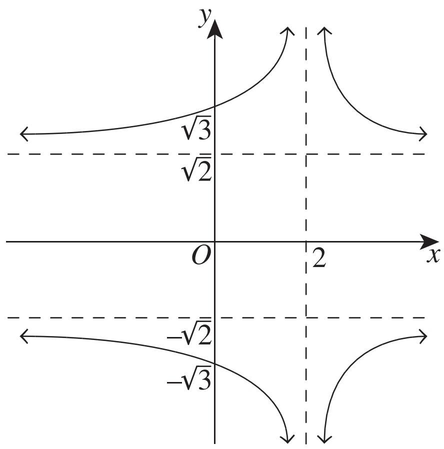
A.



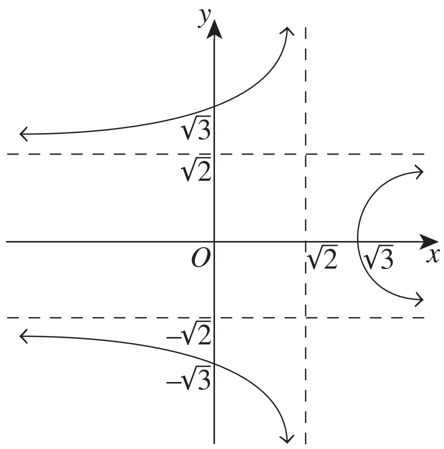
B.



C.



D.



6 If $t = \tan\left(\frac{\theta}{2}\right)$, the correct expression for $\dfrac{\text{cosec}^2\theta}{1+\tan^2\theta}$ is

A. $\dfrac{(1+t^2)^2}{(1-t^2)^2}$

B. $\dfrac{1+t^2}{(1-t^2)^2}$

C. $\dfrac{4t^2}{(1-t^2)^2}$

D. $\dfrac{(1-t^2)^2}{4t^2}$

7 The parametric equations of a function are $x = \dfrac{t}{1+t}, y = \dfrac{t}{1-t}$, where $t \neq \pm 1$.

The Cartesian equation is

A. $x - 2xy + y = 0$

B. $x - 2xy - y = 0$

C. $x + 2xy - y = 0$

D. $x + 2xy + y = 0$

8 Which of the following is the range of the function $f(x) = \cos^{-1} x + \sin^{-1} x + \tan^{-1} x$?

A. $\dfrac{\pi}{4} \leq y \leq \dfrac{3\pi}{4}$

B. $\dfrac{\pi}{4} < y < \dfrac{3\pi}{4}$

C. $0 \leq y \leq \pi$

D. $0 < y < \pi$

9 What is the least number of distinct integers that can be chosen from the sequence 1, 3, 5, 7, …, 97, 99 so that it is guaranteed that two of them will have a sum of 102?

A. 24

B. 25

C. 26

D. 27

10 By considering the binomial expansion of $(1+x)^{10}$, what is the value of

$3\binom{10}{1} + 3^2\binom{10}{2} + 3^3\binom{10}{3} + ... + 3^{10}\binom{10}{10}$?

A. 2^{10}

B. 4^{10}

C. $2^{10} - 1$

D. $4^{10} - 1$

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Find the linear factors of $x^3 + 3x^2 - 13x - 15$. **2**

(ii) Hence, solve $x^3 + 3x^2 - 13x - 15 > 0$. **2**

(b) Solve $\dfrac{2x-5}{3x-2} \le 2$. **3**

(c) Find the value of k if ${}^{8}C_{k} = 2 \times {}^{7}C_{k}$. **2**

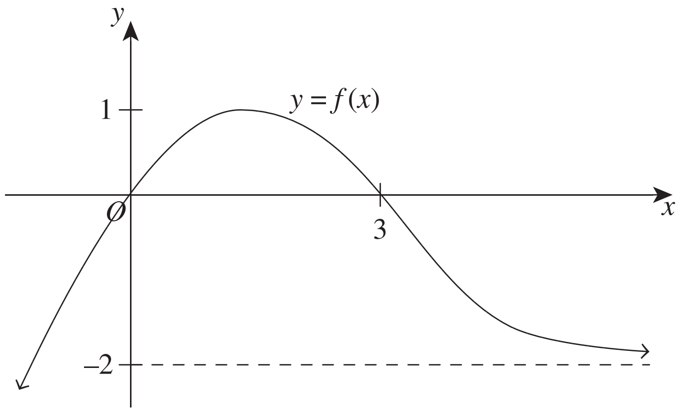
(d) Find the coefficient of x^4 in the expansion of $\left(2 + \dfrac{x}{4}\right)^6$. **2**

(e) Prove that $\tan\theta \tan\dfrac{\theta}{2} = \sec\theta - 1$. **2**

(f) Write $\sin\left(2\cos^{-1}\left(-\dfrac{2}{5}\right)\right)$ in the form $a\sqrt{b}$, where a and b are rational. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The graph $y = f(x)$ is shown.



(i) Sketch the graph of $y = \dfrac{1}{f(x)}$. **2**

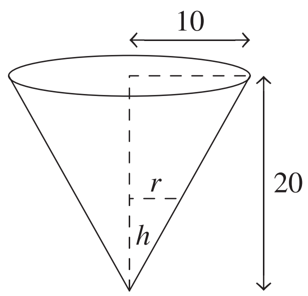
(ii) Sketch the graph of $y = \sqrt{f|x|}$. **2**

(b) (i) Sketch the graph of $f(x) = |x+2| + |x-2|$. **2**

(ii) On the same set of axes, sketch $h(x) = 2x + 6$. **1**

(iii) Hence, or otherwise, solve $f(x) \geq h(x)$. **1**

(c) Sand is being poured into a right conical flask of radius, r, 10 cm and height, h, 20 cm at a rate of 1 cm^3/s. **3**



How fast is the sand level rising when the depth of the sand is 2 cm?

(d) Find the exact value of x so that $\tan^{-1} x = \tan^{-1}\left(\dfrac{1}{2}\right) - \tan^{-1}\left(\dfrac{1}{3}\right)$. **2**

(e) Find the number of words that can be made by all of the letters in the word GEOMETRY so that no vowels are adjacent. **2**

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) The equation $x^4 - 3x^3 - 6x^2 + ax + b = 0$ has a triple root. **3**
Find a and b, and hence all roots of this equation.

(b) Two of the roots of $4x^3 + 7x^2 + kx + 24$ are reciprocals. **3**
Find the THREE roots of this equation AND evaluate k.

(c) Consider the function $f(x) = x^2 - 6x + 10$.

(i) Explain why the inverse of $f(x)$ is NOT a function. **1**

(ii) What is the largest domain of $f(x)$ containing $x = 0$ for which $f^{-1}(x)$ exists? **1**

(iii) Find an expression for $f^{-1}(x)$ using the domain found in part (c) (ii). **2**

(iv) Find the point of intersection where $f(x) = f^{-1}(x)$. **1**

(d) A hand of 13 cards is taken out of a well-shuffled pack of 52 cards. **2**
How many different hands of 13 cards consist of at least THREE aces and THREE kings?

(e) Prove that $\sin 40° + \cos 70° = \cos 10°$. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) An object falls from a skyscraper. The rate of change of its velocity v is given by $\frac{dv}{dt} = -k(v - 100)$, where k is a constant.

(i) Show that $v = 100 - 100e^{-kt}$ is a possible equation to this differential equation. **1**

(ii) The velocity after 10 seconds is 40 m/s. **1**

Find the value of k correct to FOUR decimal places.

(iii) Find the velocity after a further 15 seconds. **1**

(iv) Find the limiting velocity. **1**

(v) Sketch the graph of v versus t. **1**

(b) State the domain and range of $f(x) = 3\cos^{-1}(5 - 2x)$ and, hence, sketch the graph of $y = f(x)$. **3**

(c) By expanding both sides of the identity $(1+x)^{15} = (1+x)^{12}(1+x)^3$, prove that **3**

$$\binom{15}{4} = \binom{12}{4} + 3\binom{12}{3} + 3\binom{12}{2} + \binom{12}{1}.$$

(d) (i) Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **2**

(ii) Hence, or otherwise, show that $\sin 18° = \frac{\sqrt{5}-1}{4}$. **2**

End of paper

MATHEMATICS ADVANCED
MATHEMATICS EXTENSION 1
MATHEMATICS EXTENSION 2
REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3} Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\dfrac{d}{a}$

Relations

$$\left(x - h\right)^2 + \left(y - k\right)^2 = r^2$$

Financial Mathematics

$$A = P\left(1 + r\right)^n$$

Sequences and series

$$T_n = a + \left(n - 1\right)d$$

$$S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right] = \frac{n}{2}\left(a + l\right)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r} = \frac{a\left(r^n - 1\right)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ \left|r\right| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

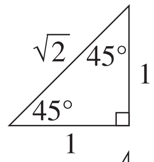
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

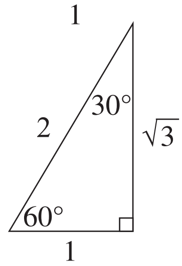
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

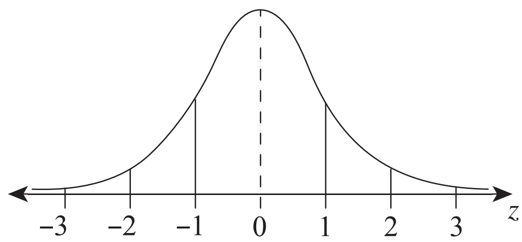
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 - 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

TEN_Y11_MEx1_QA_2021

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1+\left[f(x)\right]^2}}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 - \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \ldots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^r + \ldots + a^n$$

Vectors

$$|\underset{\sim}{u}| = \left|x\underset{\sim}{i} + x\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

Neap

Final Examination 2021

NSW Year 11 Mathematics Extension 1

Section II Writing Booklet

Question Number

Student Name/Number: ____________________

Instructions

Use a separate writing booklet for each question in Section II.

Write the number of this booklet and the total number of booklets that you have used for this question (e.g. $\boxed{1}$ of $\boxed{3}$) ⇒ ☐ of ☐

this booklet — number of booklets for this question

Write in black or blue pen (black is recommended).

You may ask for an extra writing booklet if you need more space.

If you have not attempted the question(s), you must still hand in a writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover.

You may NOT take any writing booklets, used or unused, from the examination room.

Tick this box if you have continued this answer in another writing booklet. ☐

Neap

Final Examination 2021

NSW Year 11 Mathematics Extension 1

DIRECTIONS:

Write your name in the space provided.

Write your student number in the boxes provided below. Then, in the columns of digits below each box, fill in the oval which has the same number as you have written in the box. Fill in **one** oval only in each column.

Read each question and its suggested answers. Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely, using blue or black pen. Mark only **one oval** per question.

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and draw an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

STUDENT NAME: ____________________

STUDENT NUMBER:

1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9
0	0	0	0	0	0	0	0	0

SECTION I

MULTIPLE-CHOICE ANSWER SHEET

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

STUDENTS SHOULD NOW CONTINUE WITH SECTION II

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Final Examination 2021

NSW Year 11 Mathematics Extension 1

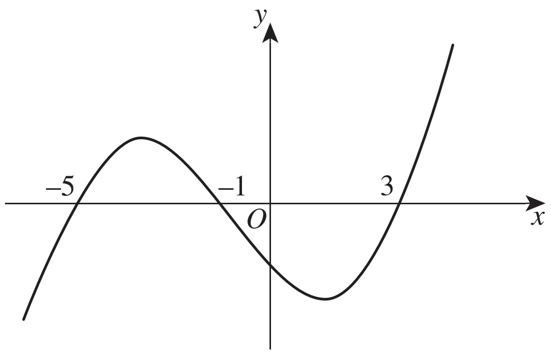
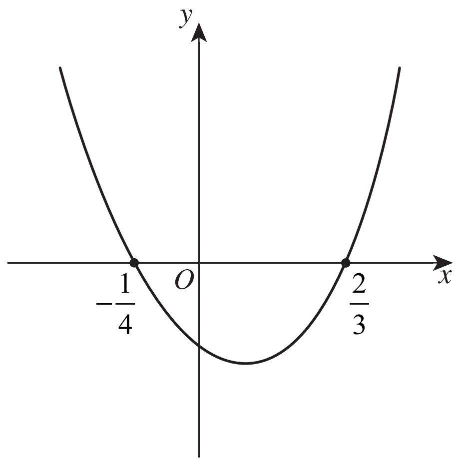
Solutions and marking guidelines

SECTION I

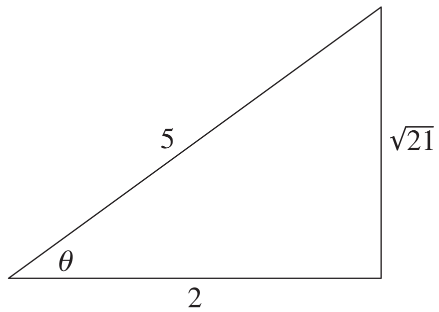
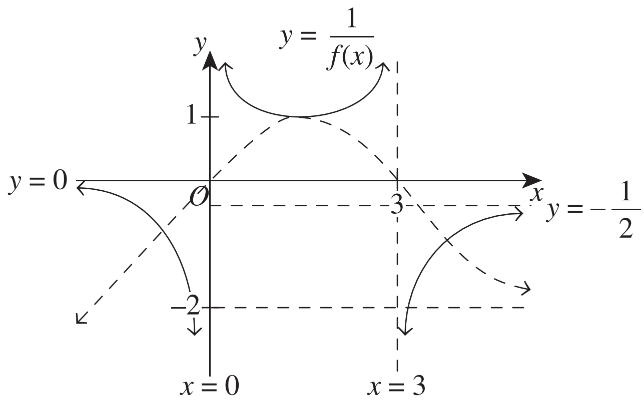
Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 A $t \to \infty,\ P = 100 + 300e^{-0.2t} \to 100^+$	ME–C1 Rates of Change ME11–4 Band E2
Question 2 B 1, 2, 3, 4, 5 $4 \times 3 \times 3$ since last digit can only be a 1, 2 or 5. Therefore, there are 36 options.	ME–A1 Working with Combinatorics ME11–5 Band E2
Question 3 C $2\sin\left(x+\frac{\pi}{3}\right) = 2\sin x\cos\frac{\pi}{3} + 2\cos x\sin\frac{\pi}{3}$ $= 2\sin x \times \frac{1}{2} + 2\cos x \times \frac{\sqrt{3}}{2}$ $= \sin x + \sqrt{3}\cos x$	ME–T2 Further Trigonometric Identities ME11–3 Band E2
Question 4 D $y = 3x^3 + 4$ Interchanging x and y, $x = 3y^3 + 4$ $3y^3 = x - 4$ $y = \sqrt[3]{\frac{x-4}{3}}$	ME–F1 Further Work with Functions ME11–2 Band E3
Question 5 B $y^2 = f(x) \Rightarrow y = \pm\sqrt{f(x)}$ y-intercept should be $\pm\sqrt{3} \Rightarrow$ **A** is incorrect. For $2 < x < 3$, y is undefined $\Rightarrow$ **C** is incorrect. x-intercept should be 3 and vertical asymptote at $x = 2$ $\Rightarrow$ **D** is incorrect.	ME–F1 Further Work with Functions ME11–2 Band E2
Question 6 D $\frac{\text{cosec}^2\theta}{1+\tan^2\theta} = \frac{\text{cosec}^2\theta}{\sec^2\theta}$ $= \frac{\cos^2\theta}{\sin^2\theta}$ $= \cot^2\theta$ $= \left(\frac{1-t^2}{2t}\right)^2$ $= \frac{(1-t^2)^2}{4t^2}$	ME–T2 Further Trigonometric Identities ME11–3 Band E3

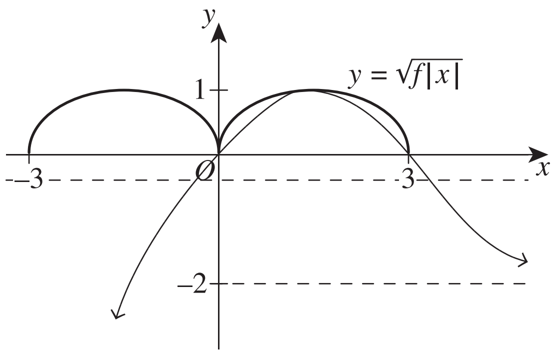
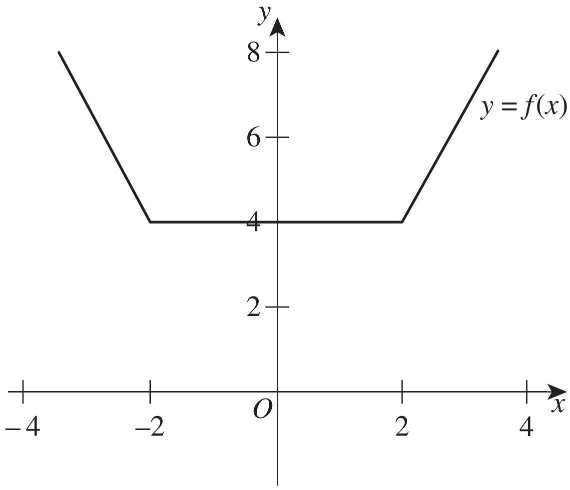
Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 7 C $x=\dfrac{t}{1+t}$ $\quad y=\dfrac{t}{1-t}$ $x(1+x)=t \quad y(1-y)=t$ $t(1-x)=x \quad t(1+y)=y$ $t=\dfrac{x}{1-x} \quad t=\dfrac{y}{1+y}$ $\dfrac{x}{1-x}=\dfrac{y}{1+y}$ $x(1+y)=y(x-1)$ $x+xy=y-xy$ $x+2xy-y=0$	ME–F1 Further Work with Functions ME11–3 Band E3
Question 8 A $f(x)=\cos^{-1}x+\sin^{-1}x+\tan^{-1}x$ Domain: [–1,1] $\cos^{-1}x+\sin^{-1}x=\dfrac{\pi}{2}$ $f(x)=\dfrac{\pi}{2}+\tan^{-1}x$ $x=-1$ $f(-1)=\dfrac{\pi}{2}+\tan^{-1}(-1)=\dfrac{\pi}{4}$ $x=1$ $f(1)=\dfrac{\pi}{2}+\tan^{-1}(1)=\dfrac{3\pi}{4}$ $\therefore \dfrac{\pi}{4}\le y\le\dfrac{3\pi}{4}$	ME–T1 Inverse Trigonometric Functions ME11–3 Band E3
Question 9 D 1, 3, 5, 7, …, 97, 99 (3, 99), (5, 97), (7, 95), …, (49, 53) $\Rightarrow$ 24 pairs + number 1 + middle number (51) + extra = 24 + 1 + 1 + 1 = 27	ME–A1 Working with Combinatorics ME11–5 Bands E3–E4
Question 10 D $(1+x)^{10}=\binom{10}{0}+\binom{10}{1}x+\binom{10}{2}x^2+...+\binom{10}{10}x^{10}$ Let $x=3$. $4^{10}=\binom{10}{0}+3\binom{10}{1}+3^2\binom{10}{2}+3^3\binom{10}{3}+...+3^{10}\binom{10}{10}$ $4^{10}-1=3\binom{10}{1}+3^2\binom{10}{2}+3^3\binom{10}{3}+...+3^{10}\binom{10}{10}$	ME–A1 Working with Combinatorics ME11–5 Band E4

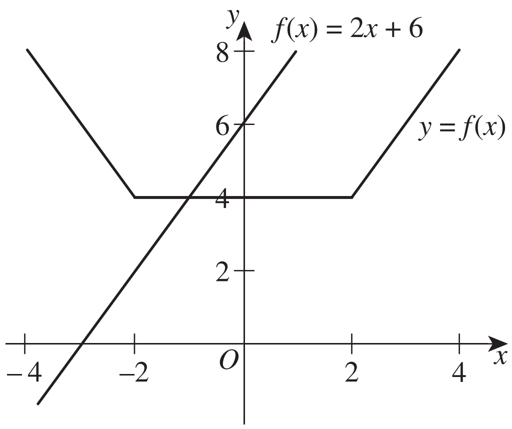
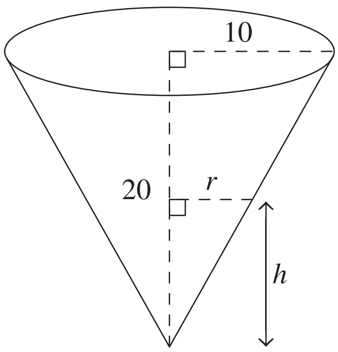
SECTION II

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
(a) (i) Let $P(x)=x^3+3x^2-13x-15$. $P(-1)=(-1)^3+3(-1)^2-13(-1)-15$ $=-1+3+13-15$ $=0$ $\therefore x+1$ is a factor of $P(x)$. $P(x)=x^3+3x^2-13x-15$ $=(x+1)(x^2+2x-15)$ (By inspection) $=(x+1)(x+5)(x-3)$	ME–F2 Polynomials ME11–2 Band E2 • Gives the correct solution . . . . 2 • Attempts to factorise the polynomial expression using the root $x=-1,-5$ or 3 OR equivalent merit 1
(ii)  $x^3+3x^2-13x-15>0$ $-5<x<-1, x>3$ $\left(\text{or } (-5,-1)\cup(3,\infty)\right)$	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Gives the correct solution . . . . 2 • Attempts to apply result from part (a)(i) OR equivalent merit 1
(b) $\left(\dfrac{2x-5}{3x-2}\le 2\right)(3x-2)^2, x\ne\dfrac{2}{3}$ $(2x-5)(3x-2)\le 2(3x-2)^2$ $2(3x-2)^2-(2x-5)(3x-2)\ge 0$ $(3x-2)\left[2(3x-2)-(2x-5)\right]\ge 0$ $(3x-2)(4x+1)\ge 0$  $x\le -\dfrac{1}{4}, x>\dfrac{2}{3}$ since $x\ne\dfrac{2}{3}$	ME–F1 Further Work with Functions ME11–4 Bands E2–E3 • Gives the correct solution . . . . 3 • Gives the solution but concludes $x\le -\dfrac{1}{4}, x\ge\dfrac{2}{3}$ . . . . . . 2 • Shows some understanding of the problem 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) $^{8}C_k = 2\times {}^{7}C_k$ $\dfrac{8!}{(8-k)!k!} = 2\times\dfrac{7!}{(7-k)!k!}$ $\dfrac{8\times 7!}{(8-k)(7-k)!k!} = 2\times\dfrac{7!}{(7-k)!k!}$ $4 = 8-k$ $k=4$	ME–A1 Working with Combinatorics ME11–5 Bands E2–E3 • Gives the correct solution .... 2 • Shows some understanding of the problem 1
(d) $\left(2+\dfrac{x}{4}\right)^6 = \binom{6}{0}2^6 + \binom{6}{1}2^5\left(\dfrac{x}{4}\right) + \binom{6}{2}2^4\left(\dfrac{x}{4}\right)^2 + \binom{6}{3}2^3\left(\dfrac{x}{4}\right)^3 + \binom{6}{4}2^2\left(\dfrac{x}{4}\right)^4 + \binom{6}{5}2^1\left(\dfrac{x}{4}\right)^5 + \binom{6}{6}\left(\dfrac{x}{4}\right)^6$ The coefficient of $x^4 = \binom{6}{4}2^2\left(\dfrac{1}{4}\right)^4$. $\binom{6}{4}2^2\left(\dfrac{1}{4}\right)^4 = 15\times 2^2\times\dfrac{1}{2^8}$ $=\dfrac{15}{64}$	ME–A1 Working with Combinatorics ME11–5 Band E3 • Gives the correct solution .... 2 • Uses the binomial theorem OR equivalent merit 1
(e) Prove $\tan\theta\tan\dfrac{\theta}{2} = \sec\theta - 1$. Let $t=\tan\dfrac{\theta}{2}$, $\tan\theta = \dfrac{2t}{1-t^2}$, $\sec\theta = \dfrac{1+t^2}{1-t^2}$. $\text{LHS} = \tan\theta\tan\dfrac{\theta}{2} = \dfrac{2t}{1-t^2}\times t = \dfrac{2t^2}{1-t^2}$ $\text{RHS} = \sec\theta - 1 = \dfrac{1+t^2}{1-t^2}-1 = \dfrac{1+t^2-(1-t^2)}{1-t^2} = \dfrac{2t^2}{1-t^2}$ $\therefore \text{LHS}=\text{RHS}$	ME–T2 Further Trigonometric Identities ME11–3 Bands E2–E3 • Gives the correct solution .... 2 • Correctly substitutes to form an expression in terms of t OR equivalent merit.................. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(f) $\sin\left(2\cos^{-1}\left(-\frac{2}{5}\right)\right)$ Let $\theta = \cos^{-1}\left(-\frac{2}{5}\right)$. $\cos\theta = -\frac{2}{5}$ (second quadrant)  $\sin\theta = -\frac{\sqrt{21}}{5}$ $\sin 2\theta = 2\sin\theta\cos\theta$ $\sin\left(2\cos^{-1}\left(-\frac{2}{5}\right)\right) = 2 \times \frac{\sqrt{21}}{5} \times \left(-\frac{2}{5}\right)$ $= -\frac{4\sqrt{21}}{25}$	ME–T1 Inverse Trigonometric Functions ME11–3 Band E3 • Gives the correct solution .... 2 • Shows some understanding of the problem 1
Question 12	
(a) (i)  	ME–F1 Further Work with Functions ME11–2 Band E2 • Gives the correct sketch........... 2 • Gives some correct features of the graph OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) 	ME–F1 Further Work with Functions ME11–2 Band E3 • Gives the correct sketch. . . . . 2 • Graphs a square-root graph based on the given graph, or $y = f\left\|x\right\|$ based on the given graph. OR • Gives some correct features of the graph OR equivalent merit 1
(b) (i) The graph of $f(x) = \left\|x+2\right\| + \left\|x-2\right\|$:  $\left\|x+2\right\| = \begin{cases} x+2, & x \ge -2 \\ -x-2, & x < -2 \end{cases}$ $\left\|x-2\right\| = \begin{cases} x-2, & x \ge 2 \\ -x+2, & x < 2 \end{cases}$ $f(x) = \begin{cases} 2x, & x \ge 2 \\ 4, & -2 \le x \le 2 \\ -2x, & x < -2 \end{cases}$	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Gives the correct sketch. . . . . 2 • Gives some correct features of the graph OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) 	ME–F1 Further Work with Functions ME11–2 Band E2 • Gives the correct sketch. 1
(iii) For $f(x) = h(x)$, $2x + 6 = 4$ $x = -1$ $f(x) \geq h(x) \Rightarrow x \leq -1$	ME–F1 Further Work with Functions ME11–3 Bands E2–E3 • Gives the correct solution 1
(c)  Given: $\dfrac{dV}{dt} = 1$ cm^3/s $\dfrac{dh}{dt} = \dfrac{dV}{dt} \times \dfrac{dh}{dV}$ $= 1 \times \dfrac{4}{\pi h^2}$ $h = 2,\ \dfrac{dh}{dt} = \dfrac{4}{\pi 2^2}$ $= \dfrac{1}{\pi}$ $V = \dfrac{\pi}{3} r^2 h$ Similar triangles $\Rightarrow \dfrac{r}{h} = \dfrac{10}{20}$ $r = \dfrac{h}{2}$ $V = \dfrac{\pi}{3}\left(\dfrac{h}{2}\right)^2 h$ $= \dfrac{\pi h^3}{12}$ $\dfrac{dV}{dh} = \dfrac{\pi h^2}{4}$ $\therefore$ The sand level is rising at $\dfrac{1}{\pi}$ cm/s when it is 2 cm deep.	ME–C1 Rates of Change ME11–4 Band E3 • Gives the correct solution . . . . 3 • Makes some correct use of the chain rule to attempt to find the rate of change. 2 • Establishes that $V = \dfrac{\pi h^3}{12}$ OR equivalent merit 1

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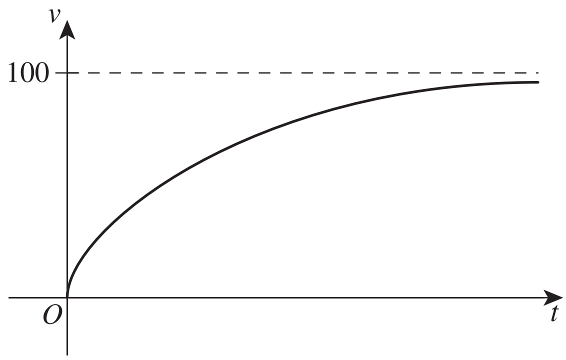
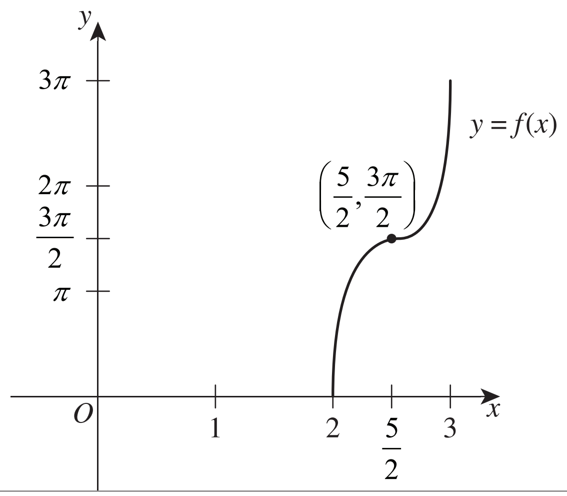
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(d) $\tan^{-1}x = \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{3}\right)$ $x = \tan\left(\tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{3}\right)\right)$ $= \dfrac{\tan\left(\tan^{-1}\left(\frac{1}{2}\right)\right) - \tan\left(\tan^{-1}\left(\frac{1}{3}\right)\right)}{1 + \tan\left(\tan^{-1}\left(\frac{1}{2}\right)\right) \times \tan\left(\tan^{-1}\left(\frac{1}{3}\right)\right)}$ $= \dfrac{\frac{1}{2} - \frac{1}{3}}{1 + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$ $= \dfrac{\frac{1}{6}}{\frac{7}{6}}$ $= \dfrac{1}{7}$	ME–F1 Further Work with Functions ME–T2 Further Trigonometric Identities ME11–2, 11–3 Band E3 • Gives the correct solution .... 2 • Shows some understanding of the problem 1
(e) Total arrangement = $\dfrac{8!}{2!}$ Number of arrangements when all the vowels are together: $\dfrac{3!6!}{2!}$ $\therefore$ Number of arrangements when all the vowels are not together: $\dfrac{8!}{2!} - \dfrac{3!6!}{2!} = 18\ 000$	ME–A1 Working with Combinatorics ME11–5 Band E3 • Gives the correct solution .... 2 • Finds the number of arrangements where all the vowels are together OR equivalent merit.................. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 13	
(a) Let $P(x) = x^4 - 3x^3 - 6x^2 + ax + b$ $P'(x) = 4x^3 - 9x^2 - 12x + a$ $P''(x) = 12x^2 - 18x - 12 = 0$ (triple root) $2x^2 - 3x - 2 = 0$ $(2x+1)(x-2) = 0$ $x = -\frac{1}{2},\ x = 2$ $\therefore\ x = 2$ is the triple root since $P(x)$ is MONIC. $P'(2) = 0 \Rightarrow 4(2)^3 - 9(2)^2 - 12(2) + a = 0$ $-28 + a = 0$ $a = 28$ $P(2) \Rightarrow 2^4 - 3(2)^3 - 6(2)^2 + 2a + b = 0$ $-32 + 2a + b = 0$ Substitute $a = 28 \Rightarrow -32 + 2(28) + b = 0$ $b = -24$	ME–F2 Polynomials ME11–2 Bands E2–E3 • Gives the correct solution .... 3 • Obtains a correct value for a or b OR equivalent merit .... 2 • Obtains ONE correct equation OR equivalent merit 1
(b) $4x^3 + 7x^2 + kx + 24 = 0$ Let $\alpha, \frac{1}{\alpha}$ and β be the roots. $\alpha\left(\frac{1}{\alpha}\right)\beta = -6$ $\beta = -6$ $\alpha + \frac{1}{\alpha} + \beta = -\frac{7}{4}$ Substitute $\beta = -6 \Rightarrow \alpha + \frac{1}{\alpha} - 6 = -\frac{7}{4}$ $\alpha + \frac{1}{\alpha} - \frac{17}{4} = 0$ $4\alpha^2 - 17\alpha + 4 = 0$ $(4\alpha - 1)(\alpha - 4) = 0$ $\therefore \alpha = \frac{1}{4}$ or $\alpha = 4$ Hence, the three roots are $\frac{1}{4}, 4$ and -6. $\left(\frac{1}{4}\right)(4) + \left(\frac{1}{4}\right)(-6) + 4(-6) = \frac{k}{4}$ $-24\frac{1}{2} = \frac{k}{4}$ $k = -98$	ME–F2 Polynomials ME11–2 Bands E2–E3 • Gives the correct solution .... 3 • Obtains THREE correct roots. OR • Obtains ONE correct root AND k value. OR • Equivalent merit .... 2 • Obtains ONE correct root. OR • Obtains TWO correct equations. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) (i) The inverse of $f(x)$ is NOT a function since $f(x)$ is NOT a one-to-one function.	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Gives the correct solution 1
(ii) $f(x)=x^2-6x+10$ $=x^2-6x+9+1$ $=(x-3)^2+1$ The largest domain of $f(x)$ containing $x=0$ for which $f^{-1}(x)$ exists is $x\le 3$ or $(-\infty,3]$.	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Gives the correct solution 1
(iii) Let $y=(x-3)^2+1$. Domain $f(x)$: $(-\infty,3]\Rightarrow$ Range $f^{-1}(x):(-\infty,3]$ Range $f(x)$: $[1,\infty)\Rightarrow$ Domain $f^{-1}(x):[1,\infty)$ Interchanging *x* and *y*: $x=(y-3)^2+1$ $(y-3)^2=x-1$ $y=3\pm\sqrt{x-1}$ $\therefore f^{-1}(x)=3-\sqrt{x-1}$ since range $f^{-1}(x):(-\infty,3]$	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Gives the correct solution .... 2 • Shows some understanding of the problem 1
(iv) For $f(x)=f^{-1}(x)$ $\Rightarrow f(x)=f^{-1}(x)=x$ So $x^2-6x+10=x$ $x^2-7x+10=0$ $(x-2)(x-5)=0$ $x=2,5$ $\Rightarrow x=2$ since $x\le 3$ $\therefore$ The point of intersection is $(2,2)$.	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Gives the correct solution 1
(d) 13 cards from 52 Combinations: (see table below) Number of ways $=\binom{4}{3}\binom{4}{3}\binom{44}{7}+2\times\binom{4}{3}\binom{4}{4}\binom{44}{6}+\binom{4}{4}\binom{4}{4}\binom{44}{5}$ $=670\,687\,512$	ME–A1 Working with Combinatorics ME11–5 Band E3 • Gives the correct solution .... 2 • Obtains at least TWO correct combinations OR equivalent merit 1

Aces	*Kings*	*Others*
3	3	7
4	3	6
3	4	6
4	4	5

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(e) Prove $\sin 40° + \cos 70° = \cos 10°$. $\text{LHS} = \sin 40° + \cos 70°$ $= \sin 40° + \sin 20°$ $= 2\sin\left(\dfrac{40° + 20°}{2}\right)\cos\left(\dfrac{40° - 20°}{2}\right)$ $= 2\sin 30° \cos 10°$ $= 2 \times \dfrac{1}{2} \times \cos 10°$ $= \cos 10°$ $= \text{RHS}$	ME–T2 Further Trigonometric Identities ME11–3 Band E3 • Gives the correct solution .... 2 • Uses the sum to produce identities 1
Question 14	
(a) (i) $v = 100 - 100e^{-kt}$ $\dfrac{dv}{dt} = -100e^{-kt} \times -k$ (chain rule) $= -k(100 - 100e^{-kt} - 100)$ $= -k(v - 100)$ $\therefore\ v = 100 - 100e^{-kt}$ is a possible equation to this differential equation.	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Gives the correct proof............ 1
(ii) $v = 100 - 100e^{-kt}$ $t = 10$ s, $v = 40$ m/s $\Rightarrow 40 = 100 - 100e^{-10k}$ $100e^{-10k} = 60$ $e^{-10k} = 0.6$ $k = \dfrac{\ln(0.6)}{-10}$ $= 0.0511$ (correct to two decimal places)	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Gives the correct solution 1
(iii) $v = 100 - 100e^{-0.0511t}$ $t = 25$ s $\Rightarrow v = 100 - 100e^{-0.0511(25)}$ $= 72.13$ m/s (correct to four decimal places)	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Gives the correct solution 1
(iv) $t \to \infty,\ 100e^{-0.0511t} \to 0$ $v = 100 - 100e^{-0.0511t} \to 100^-$ $\therefore v = 100$ m/s	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Gives the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(v) 	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Gives the correct sketch. 1
(b) $f(x) = 3\cos^{-1}(5-2x)$ Domain: $-1 \le 5-2x \le 1$ $-6 \le -2x \le -4$ $3 \ge x \ge 2$ $\therefore 2 \le x \le 3$ Range: $0 \le \dfrac{y}{3} \le \pi$ $\therefore 0 \le y \le 3\pi$ When $x = 2$, $f(2) = 3\cos^{-1}(1) = 0$ 	ME–T1 Inverse Trigonometric Functions ME11–3 Bands E2–E3 • Gives the correct solution . . . . 3 ——— • Obtains correct domain and range OR equivalent merit . . . . 2 ——— • Obtains correct domain and range 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) $(1+x)^{15}=(1+x)^{12}(1+x)^{3}$ LHS $=(1+x)^{15}$ $=\binom{15}{0}+\binom{15}{1}x+\binom{15}{2}x^{2}+$ $\binom{15}{3}x^{3}+\binom{15}{4}x^{4}+\ldots+\binom{15}{15}x^{15}$ $\binom{15}{4}$ is the coefficient of x^4. RHS $=(1+x)^{12}(1+x)^{3}$ $(1+x)^{12}=\binom{12}{0}+\binom{12}{1}x+\binom{12}{2}x^{2}+$ $\binom{12}{3}x^{3}+\binom{12}{4}x^{4}+\ldots+\binom{12}{15}x^{15}$ $(1+x)^{3}=\binom{3}{0}+\binom{3}{1}x+\binom{3}{2}x^{2}+\binom{3}{3}x^{3}$ Coefficient of $x^4=\binom{3}{0}\binom{12}{4}+\binom{3}{1}\binom{12}{3}+$ $\binom{3}{2}\binom{12}{2}+\binom{3}{3}\binom{12}{1}$ $\therefore\binom{15}{4}=\binom{12}{4}+\binom{3}{1}\binom{12}{3}+\binom{3}{2}\binom{12}{2}+\binom{12}{1}$	ME–A1 Working with Combinatorics ME11–5 Bands E3–E4 • Gives the correct solution .... 3 • Makes substantial progress towards finding the coefficient of x^4 in the expansion of $(1+x)^{12}(1+x)^{3}$ OR equivalent merit.................. 2 • Gives correct explanation that $\binom{15}{4}$ is the coefficient of x^4 1
(d) (i) Prove $\cos 3\theta=4\cos^{3}\theta-3\cos\theta.$ $\cos 3\theta=\cos(2\theta+\theta)$ $=\cos 2\theta\cos\theta-\sin 2\theta\sin\theta$ $=(2\cos^{2}\theta-1)\cos\theta-(2\sin\theta\cos\theta)\sin\theta$ $=2\cos^{3}\theta-\cos\theta-2\cos\theta\sin^{2}\theta$ $=2\cos^{3}\theta-\cos\theta-2\cos\theta(1-\cos^{2}\theta)$ $=2\cos^{3}\theta-\cos\theta-2\cos\theta+2\cos^{3}\theta$ $=4\cos^{3}\theta-3\cos\theta$	ME–T1 Inverse Trigonometric Functions ME11–3 Band E3 • Gives the correct solution .... 2 • Makes substantial progress in expanding $\cos 3\theta$ 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) $\sin 2\theta = \cos(90° - 2\theta)$ Let $\theta = 18°$. $\Rightarrow \sin(2\times 18°) = \cos(90° - (2\times 18°))$ $= \cos(3\times 18°)$ $2\sin 18° \cos 18° = 4\cos^3 18° - 3\cos 18°$ (from part (d)(i)) $2\sin 18° = 4\cos^2 18° - 3$ $2\sin 18° = 4(1 - \sin^2 18°) - 3$ $4\sin^2 18° + 2\sin 18° - 1 = 0$ $\sin 18° = \dfrac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2(4)}$ $= \dfrac{-2 \pm \sqrt{20}}{8}$ $= \dfrac{-2 \pm 2\sqrt{5}}{8}$ $= \dfrac{-1 \pm \sqrt{5}}{4}$ $\therefore \sin 18° = \dfrac{\sqrt{5} - 1}{4}$ since $\sin 18° > 0$ as it is in the first quadrant.	ME–T1 Inverse Trigonometric Functions ME11–3 Band E4 • Gives the correct solution .... 2 • Attempts to apply answer from part (d)(i) OR equivalent merit 1