Student Number

PEM

2021

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- NESA-approved Calculators may be used
- A reference sheet is provided.
- In Questions 11 – 16 show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Questions 1 – 10 **10 marks**

Allow about 15 minutes for this section

Section II Questions 11 – 16 **90 marks**

Allow about 2 hour and 45 minutes for this section

Section 1

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 If $\omega = \text{cis}\left(\frac{\pi}{6}\right)$, then $Im(\omega^4 + 1)$ is:

(A) $\frac{\sqrt{3}}{2}$

(B) $-\frac{1}{2}$

(C) $1 + \frac{\sqrt{3}}{2}$

(D) $\frac{1}{2}$

2 Which expression is equal to $\int \frac{e^x}{\sqrt{1 - e^{2x}}} dx$?

(A) $\cos^{-1}(e^x) + c$

(B) $-\sqrt{1 - e^{2x}} + c$

(C) $\sin^{-1}(e^x) + c$

(D) $-\ln\left(\sqrt{1 - e^{2x}}\right) + c$

3 Which of the following points lies on the line described by the vector equation

$$r = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} ?$$

(A) $\begin{pmatrix} -3 \\ 9 \\ 1 \end{pmatrix}$

(B) $\begin{pmatrix} -3 \\ -8 \\ -3 \end{pmatrix}$

(C) $\begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}$

(D) $\begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$

4 If the vectors $\vec{\mathbf{u}} = \lambda\vec{\mathbf{i}} + \lambda\vec{\mathbf{j}} + 2\vec{\mathbf{k}}$ and $\vec{\mathbf{v}} = \lambda\vec{\mathbf{i}} - 2\vec{\mathbf{j}} - 4\vec{\mathbf{k}}$ are perpendicular, then

(A) $\lambda = -2$ or $\lambda = 4$

(B) $\lambda = -4$ or $\lambda = 2$

(C) $\lambda = -4$ or $\lambda = -2$

(D) $\lambda = 2$ or $\lambda = 4$

5 Consider the statement:

$$P: \text{ for every } x \in \mathbb{Z} \text{ there exists } y \in \mathbb{Z} \text{ such that } y^2 > x.$$

Which of the following statements is $\sim P$?

(A) $\exists x \in \mathbb{Z}\ \forall y \in \mathbb{Z} \quad y^2 > x$

(B) $\forall y \in \mathbb{Z}\ \exists x \in \mathbb{Z} \quad y^2 \leq x$

(C) $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z} \quad y^2 > x$

(D) $\exists x \in \mathbb{Z}\ \forall y \in \mathbb{Z} \quad y^2 \leq x$

6 A quadratic equation with real coefficients has complex roots α, β .

The value of $\arg \alpha + \arg \beta$ could be:

(A) 0

(B) $\dfrac{\pi}{2}$

(C) π

(D) $\dfrac{3\pi}{2}$

7 P, Q and R are points with position vectors $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{r}$.

The points are collinear and P lies between R and Q. If $|QR| = 3|PR|$, then $\underset{\sim}{r}$ has the position vector

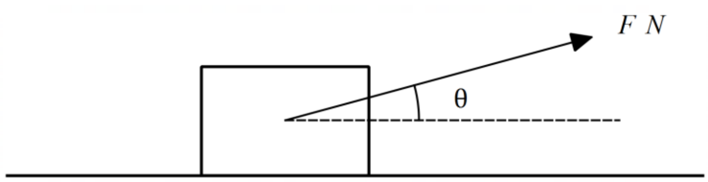
(A) $\underset{\sim}{r} = \frac{1}{2}\underset{\sim}{p} - \frac{3}{2}\underset{\sim}{q}$

(B) $\underset{\sim}{r} = \frac{3}{2}\underset{\sim}{p} - \frac{1}{2}\underset{\sim}{q}$

(C) $\underset{\sim}{r} = \frac{3}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}$

(D) $\underset{\sim}{r} = \frac{3}{2}\underset{\sim}{q} - \frac{1}{2}\underset{\sim}{p}$

8 A block of mass 25 kg is pulled along a smooth horizontal plane by a force F as shown in the diagram below.



Which of the following set of conditions produce the largest acceleration for the mass?

(A) $F = 15,\ \theta = 0°$

(B) $F = 18,\ \theta = 30°$

(C) $F = 18,\ \theta = 45°$

(D) $F = 20,\ \theta = 60°$

9 Consider the following proof:

Proof: Assume *a* and *b* are odd integers. Then $a = 2c + 1$ and $b = 2d + 1$ for some c,d $\in \mathbb{Z}$.

Then $$\begin{aligned} ab^2 &= (2c + 1)(2d + 1)^2 \\ &= 8cd^2 + 8cd + 2c + 4d^2 + 4d + 1 \\ &= 2(4cd^2 + 4cd + c + 2d^2 + 2d) + 1 \end{aligned}$$

Since $4cd^2 + 4cd + c + 2d^2 + 2d \in \mathbb{Z}$, we conclude that ab^2 is odd.

Which of the following statements is being proved?

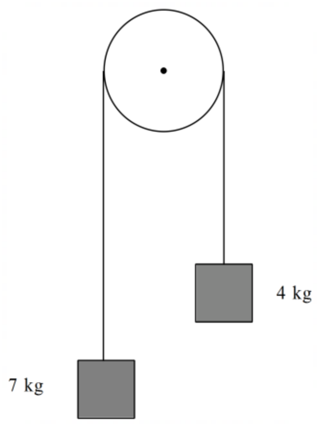
(A) If ab^2 is even, then *a* and *b* are even.

(B) If ab^2 is even, then *a* is even or *b* is even.

(C) If *a* and *b* are even, then ab^2 is even.

(D) If *a* or *b* are even, then ab^2 is even.

10 A light inextensible string passes over a smooth pulley. Attached to each end of the strings are masses of 4 kg and 7 kg, as shown.



The acceleration of the larger mass downwards is

(A) $\dfrac{3g}{11}$

(B) $\dfrac{11g}{3}$

(C) $\dfrac{7g}{11}$

(D) $3g$

END OF SECTION I

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section

Answer the questions in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) Let $z = \dfrac{1+i}{1-i}$ and $w = \dfrac{\sqrt{2}}{1-i}$.

(i) Write each of z and w in modulus-argument form. **2**

(ii) On the same Argand diagram, sketch the points z, w and $z + w$. **2**

(iii) Deduce the exact value of $\tan\left(\dfrac{3\pi}{8}\right)$. **2**

(b) Given $1 < a < 2$, sketch, on an Argand diagram, the region represented by **2**

$$|z - a - i| < 1.$$

(c) Calculate $\displaystyle\int_{-1}^{7} \frac{\sqrt{x+2}}{x+5}\, dx$. **4**

(d) A particle moves along a straight line according to the equation **3**

$$\ddot{x} = -16(x - 2)$$

where x is the displacement in metres and t is the time in seconds.

If the particle is at rest when $x = 7$, find the velocity of the particle when $x = 6$ and the particle is moving towards the origin.

END OF QUESTION 11

Question 12 (14 marks) Use the Question 12 Writing Booklet.

(a)

(i) Find numbers a, b and c such that **2**

$$\frac{9x - 6}{x^3 + 8} \equiv \frac{a}{x + 2} + \frac{bx + c}{x^2 - 2x + 4}.$$

(ii) Hence evaluate $\displaystyle\int_0^1 \frac{9x - 6}{x^3 + 8}\, dx$. **3**

(b)

(i) For a complex number z, prove that $|z|^2 = z\bar{z}$ **1**

(ii) Prove that, for complex number z, w **2**

$$|z - w|^2 + |z + w|^2 = 2(|z|^2 + |w|^2).$$

The vertices of triangle $P_1\, P_2\, P_3$ are the complex numbers z_1, z_2, z_3 where

$|z_{\underset{\sim}{i}}| = r > 0$ for $\underset{\sim}{i} = 1, 2, 3$ and $z_1 + z_2 + z_3 = 0$.

(iii) Show that $|z_1 - z_2|^2 = 3r^2$. **2**

(iv) Deduce that $P_1\, P_2\, P_3$ is equilateral. **2**

(c) Given that $|\underset{\sim}{a}| = 3$, $|\underset{\sim}{b}| = 2$ and $\underset{\sim}{a} \cdot \underset{\sim}{b} = 4$, calculate the length of $2\underset{\sim}{a} - 3\underset{\sim}{b}$. **2**

END OF QUESTION 12

Question 13 (14 marks) Use the Question 13 Writing Booklet.

(a)

(i) If $z = e^{i\theta}$, show that $z^n + z^{-n} = 2\cos(n\theta)$. **1**

(ii) Hence, or otherwise, determine the values of θ, where $0 \leq \theta < 2\pi$ **4**

such that

$$|e^{4i\theta} + 1| = \sqrt{3}\ .$$

(b) A mass of 1 kg moves along a straight line with velocity $v\ ms^{-1}$. It encounters a resistance of $v + v^3$. The particle has initial velocity U, where $U > 0$ and starts from the origin.

At time t the particle has velocity v and displacement x.

(i) Show that the equation of motion is $\ddot{x} = -v(1 + v^2)$. **1**

(ii) Show that $x = \tan^{-1}\left(\dfrac{U - v}{1 + Uv}\right)$. **3**

(iii) Show that $v^2 = \dfrac{U^2}{(1 + U^2)\,e^{2t} - U^2}$. **3**

(iv) Describe the motion of the particle as $t \to \infty$. **2**

END OF QUESTION 13

Question 14 (16 marks) Use the Question 14 Writing Booklet.

(a) By rewriting the equation in the form $a^2 + b^2 = 0$, or otherwise, disprove the statement: **3**

$$\exists x \in \mathbb{R} \text{ such that } x^6 + x^4 + 1 = 2x^2.$$

(b) A particle is projected from the origin with an initial speed V at an angle of θ. The particle is affected both by gravity and an air resistance proportional to the velocity such that the components of acceleration are

$$\ddot{x} = -k\dot{x}$$
$$\ddot{y} = -k\dot{y} - g$$

where k is a constant and $g\ ms^{-2}$ is the acceleration due to gravity.

(i) Using integration, show that **3**

$$\dot{x} = V\cos\theta\ e^{-kt}$$
$$\dot{y} = \frac{1}{k}(g + kV\sin\theta)\,e^{-kt} - \frac{g}{k}$$

(ii) Show that the particle reaches maximum height at time T where **2**

$$T = \frac{1}{k}\ln\left(\frac{g + kV\sin\theta}{g}\right).$$

(iii) At the instant the particle attains maximum height, determine the horizontal displacement. **3**

(c) Relative to the origin O, the points A, B, C and D have position vectors given respectively by $-4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $4\mathbf{i} + \lambda\mathbf{j} + 6\mathbf{k}$, $4\mathbf{i} - \mathbf{j} - \mathbf{k}$ and $2\mathbf{j} - 6\mathbf{k}$.

(i) Given that the line AC is perpendicular to the line BD, determine the value of λ. **2**

(ii) Hence find the position vector of F, the point of intersection of the lines AC and BD. **3**

END OF QUESTION 14

Question 15 (16 marks) Use the Question 15 Writing Booklet.

(a) D is the midpoint of the side BC or triangle ABC. **4**

Using vectors, show that $|\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2 = 2(|\overrightarrow{AD}|^2 + |\overrightarrow{BD}|^2)$.

(b)

(i) Using de Moivre's Theorem, show that **2**

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(ii) Show that $\sin\frac{\pi}{18}$ is a solution of $8x^3 - 6x + 1 = 0$. **2**

(iii) Deduce that $\sin\frac{\pi}{18}$ is irrational. **3**

(c) Let $I_n = \int \frac{\sin(nx)}{\sin x}\,dx$, where $n \geq 1$.

(i) Prove that, for $n \geq 3$, **2**

$$I_n - I_{n-2} = 2\int \cos(n-1)x\,dx.$$

(ii) Hence determine the exact value of **3**

$$\int_{\pi/6}^{\pi/3} \frac{\sin 5x}{\sin x}\,dx$$

END OF QUESTION 15

Question 16 (15 marks) Use the Question 16 Writing Booklet.

(a) Using integration by parts, calculate $\int (1 + 2x^2)\, e^{x^2}\, dx$. **3**

(b) Let P_n be the proposition that

$$\sqrt[n]{(a_1\, a_2\, \ldots\, a_n)} \leq \frac{a_1 + a_2 + \ldots + a_n}{n}$$

where each $a_i > 0$.

(i) Show that P_2 is true. **1**

(ii) Deduce that $P_n \Rightarrow P_{2n}$. **2**

(iii) By letting $a_n = \dfrac{a_1 + a_2 + \ldots + a_{n-1}}{n - 1}$, show that $P_n \Rightarrow P_{n-1}$ **2**

(iv) Using parts (i), (ii) and (iii) above, explain why P_n is true for $n \geq 2$. **2**

(v) Let $a_1 = a_2 = \ldots = a_p = x$ and $a_{p+1} = a_{p+2} = \ldots = a_{p+q} = y$. **2**

Show that $x^p\, y^q \leq \left(\dfrac{px + qy}{p + q}\right)^{p+q}$.

(vi) Hence determine the maximum value of **3**

$$\sin^{2p}\theta \cos^{2q}\theta .$$

End of paper

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TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

Multiple-Choice Answer Sheet

Select the alternative A, B, C, or D that best answers the question by placing a **X** in the box.

	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

SUGGESTED SOLUTIONS PEM 2021 Mathematics Extension 2 Trial HSC Examination

Section 1

Multiple Choice Answer Key

Question	Answer
1	A
2	C
3	C
4	A
5	D
6	A
7	B
8	B
9	B
10	A

Section II

Question 11 (ai)

Criteria	Marks
• Provides correct solution	**2**
• Correctly calculates at least $1+i$ or $1-i$ in modulus-argument form	**1**

Sample answer:

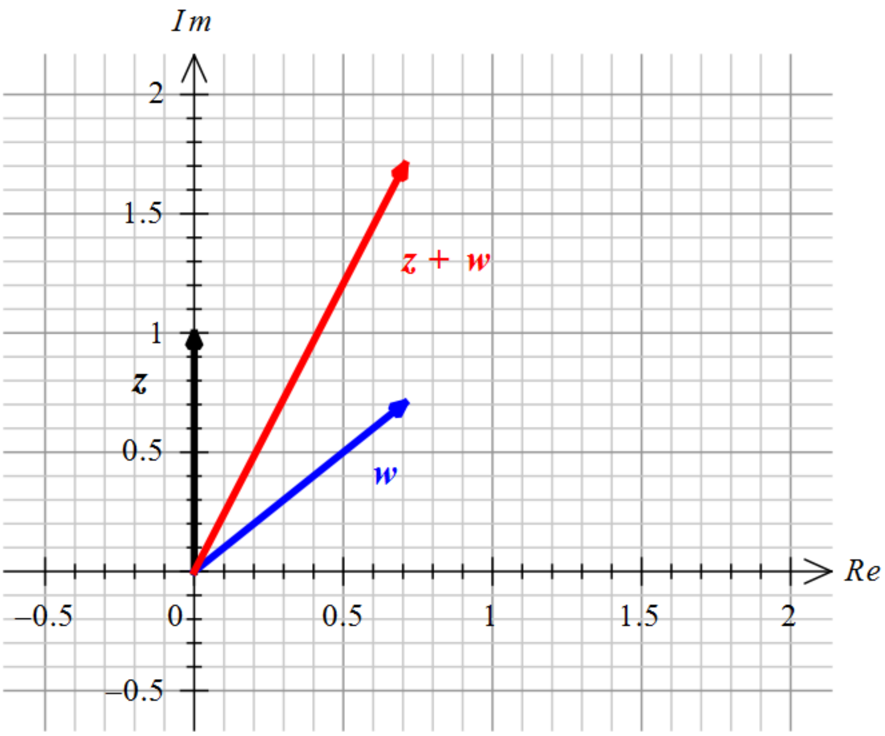
$$z=\frac{\sqrt{2}cis\left(\pi/4\right)}{\sqrt{2}cis\left(-\pi/4\right)}=cis\left(\pi/4-\left(-\pi/4\right)\right)=cis\left(\pi/2\right)$$

$$w=\frac{\sqrt{2}}{\sqrt{2}cis\left(-\pi/4\right)}=cis\left(\pi/4\right)$$

Question 11 (a) (ii)

Criteria	Marks
• Provides correct solution	**2**
• Sketches z and w accurately	**1**

Sample answer:



Question 11(a) (iii)

Criteria	Marks
• Provides correct solution	**2**
• Indicates that $\arg(z+w) = 3\pi/8$	**1**

Sample answer:

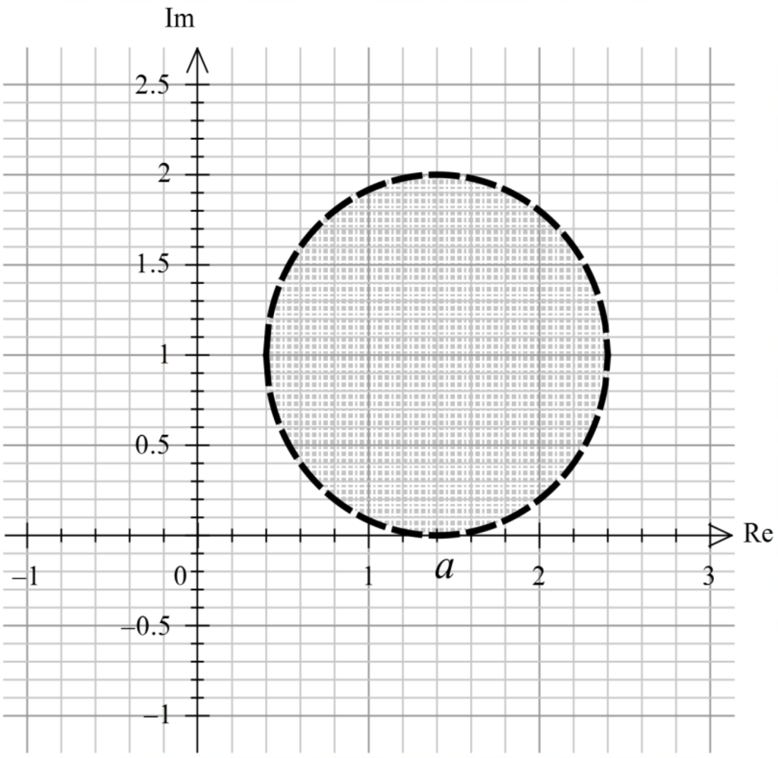
By the parallelogram rule of addition, $\arg(z+w) = \arg w + \frac{1}{2}(\arg z - \arg w) = \frac{3\pi}{8}$

$$\therefore \tan\left(\frac{3\pi}{8}\right) = \frac{1 + 1/\sqrt{2}}{1/\sqrt{2}} = \sqrt{2} + 1$$

Question 11(b)

Criteria	Marks
• Provides correct solution	**2**
• Sketches interior of correctly centred disc but includes boundary OR • Sketches interior of incorrectly centred disc	**1**

Sample answer:



Question 11 (c)

Criteria	**Marks**
• Provides correct solution	**4**
• Makes significant progress towards calculating the integral after substation	**3**
• Correctly makes the substitution including limits	**2**
• Determines an accurate substitution	**1**

Sample answer:

Let $u^2 = x+2, u \geq 0$

Then: $x=-1 \Rightarrow u=1$ and $x=7 \Rightarrow u=3$

and $2u\dfrac{du}{dx}=1 \Rightarrow dx = 2udu$

$$\int_{-1}^{7} \frac{\sqrt{x+2}}{x+5}\,dx = \int_{1}^{3} \frac{u}{u^2+3} 2u\,du$$

$$= 2\int^{3} \frac{u^2+3-3}{u^2+3}\,du$$

$$= 2\int_{1}^{3} \left(1-\frac{3}{u^2+3}\right)du$$

$$= 2\left[u-\sqrt{3}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right)\right]_{1}^{3}$$

$$= 4-\frac{\pi\sqrt{3}}{3}$$

Question 11(d)

Criteria	**Marks**
• Provides correct solution	**3**
• Correctly calculates speed only	**2**
• Determines amplitude	**1**

Sample answer:

$\ddot{x} = -16(x-2) \Rightarrow n=4$ and $x=7, v=0 \Rightarrow a=5$

From $v^2 = n^2\left(a^2-(x-2)^2\right) = 16\left(25-(x-2)^2\right)$

when $x=6$, $\; v^2 = 16\left(25-4^2\right) = 144 \Rightarrow v = \pm 12$

Since moving towards O, $v=-12\ ms^{-1}$

Question 12(a) (i)

Criteria	Marks
• Provides correct solution	**2**
• Makes some progress towards finding *a*, *b*, *c* either through substitution and/or equating coefficients	**1**

Sample answer:

$9x-6=a\left(x^2-2x+4\right)+\left(bx+c\right)\left(x+2\right)$

$x=-2 \Rightarrow -24=12a \quad \therefore \underline{a=-2}$

Coefficients of x^2 : $\quad 0=a+b \Rightarrow \underline{b=2}$

Constant terms: $\quad -6=4(-2)+2c \Rightarrow \underline{c=1}$

Question 12(a) (ii)

Criteria	Marks
• Provides correct solution	**3**
• Makes some progress towards calculating integral with quadratic denominator	**2**
• Correctly calculates integral with linear denominator	**1**

Sample answer:

$$\begin{aligned}
\int_0^1 \frac{9x-6}{x^3+8}\,dx &= \int_0^1 \left(\frac{-2}{x+2}+\frac{2x+1}{x^2-2x+4}\right)dx \\
&= \int_0^1 \left(\frac{-2}{x+2}+\frac{2x-2}{x^2-2x+4}+\frac{3}{\left(x-1\right)^2+3}\right)dx \\
&= \left[-2\ln\left|x+2\right|+\ln\left(x^2-2x+4\right)+\sqrt{3}\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)\right]_0^1 \\
&= -2\ln 3+\ln 3+0-\left(-2\ln 2+\ln 4+\sqrt{3}\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right) \\
&= \frac{\sqrt{3}\pi}{6}-\ln 3
\end{aligned}$$

Question 12(b) (i)

Criteria	Marks
• Provides correct solution	**1**

Sample answer:

Let $z = x + iy$, then

$$|z|^2 = \left(\sqrt{x^2 + y^2}\right)^2 = x^2 + y^2 = (x + iy)(x - iy) = z\bar{z}$$

Question 12(b) (ii)

Criteria	Marks
• Provides correct solution	**2**
• Makes some correct use of conjugate results	**1**

Sample answer:

$$\begin{aligned} |z-w|^2 + |z+w|^2 &= (z-w)\overline{(z-w)} + (z+w)\overline{(z+w)} \\ &= (z-w)(\bar{z} - \bar{w}) + (z+w)(\bar{z} + \bar{w}) \\ &= z\bar{z} - \cancel{z\bar{w}} - \cancel{w\bar{z}} + w\bar{w} + z\bar{z} + \cancel{z\bar{w}} + \cancel{w\bar{z}} + w\bar{w} \\ &= 2\left(z\bar{z} + w\bar{w}\right) \\ &= 2\left(|z|^2 + |w|^2\right) \end{aligned}$$

Question 12(b) (iii)

Criteria	Marks
• Provides correct solution	**2**
• Substitutes $\lvert z_1 \rvert = \lvert z_2 \rvert = r$ into the result from part (i) OR • correctly uses $\lvert z_1 + z_2 \rvert = \lvert -z_3 \rvert$	**1**

Sample answer:

$z_1 + z_2 = -z_3 \Rightarrow |z_1 + z_2| = |z_3| = r$

$\therefore\ |z_1 - z_2|^2 + r^2 = 2\left(r^2 + r^2\right) = 4r^2$

i.e. $|z_1 - z_2|^2 = 3r^2$

Question 12(b) (iv)

Criteria	Marks
• Provides correct solution	**2**
• Makes a similar statement for $\lvert z_2 - z_3 \rvert$ or $\lvert z_3 - z_1 \rvert$	**1**

Sample answer:

Similar to part (ii), $\lvert z_2 - z_3 \rvert^2 = 3r^2$ and $\lvert z_3 - z_1 \rvert^2 = 3r^2$

$\therefore \lvert z_1 - z_2 \rvert = \lvert z_2 - z_3 \rvert = \lvert z_3 - z_1 \rvert = \sqrt{3}r > 0$

i.e. the sides of $P_1P_2P_3$ are all equal, that is, $P_1P_2P_3$ is an equilateral triangle.

Question 12(c)

Criteria	Marks
• Provides correct solution	**2**
• Makes some correct use of the result $\lvert \underset{\sim}{v} \rvert^2 = \underset{\sim}{v} \cdot \underset{\sim}{v}$	**1**

Sample answer:

$$\begin{aligned}
\lvert 2\underset{\sim}{a} - 3\underset{\sim}{b} \rvert^2 &= \left(2\underset{\sim}{a} - 3\underset{\sim}{b}\right) \cdot \left(2\underset{\sim}{a} - 3\underset{\sim}{b}\right) \\
&= 4\underset{\sim}{a} \cdot \underset{\sim}{a} - 6\underset{\sim}{a} \cdot \underset{\sim}{b} - 6\underset{\sim}{b} \cdot \underset{\sim}{a} + 9\underset{\sim}{b} \cdot \underset{\sim}{b} \\
&= 4\lvert \underset{\sim}{a} \rvert^2 - 12\underset{\sim}{a} \cdot \underset{\sim}{b} + 9\lvert \underset{\sim}{b} \rvert^2 \\
&= 4 \times 3^2 - 12 \times 4 + 9 \times 2^2 \\
&= 24 \\
&\therefore\ 2\underset{\sim}{a} - 3\underset{\sim}{b} \text{ has length } 2\sqrt{6}
\end{aligned}$$

Question 13(a) (i)

Criteria	Marks
• Provides correct solution	**1**

Sample answer:

$$\begin{aligned} z^n + z^{-n} &= (\cos n\theta + i\sin n\theta) + (\cos(-n\theta) + i\sin(-n\theta)) \\ &= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta \\ &= 2\cos n\theta \end{aligned}$$

Question 13(a) (ii)

Criteria	Marks
• Provides correct solution	**2**
• Some progress towards the result.	**1**

Sample answer:

$$\begin{aligned} \sqrt{3} &= \left| e^{2i\theta}\left(e^{2i\theta} + e^{-2i\theta}\right) \right| \\ &= \left|e^{2i\theta}\right| \left|e^{2i\theta} + e^{-2i\theta}\right| \\ &= \left|2\cos 2\theta\right| \end{aligned}$$

$$\Rightarrow \cos 2\theta = \pm\frac{\sqrt{3}}{2} \quad \text{for } 0 \le 2\theta < 4\pi$$

$$\text{i.e. } 2\theta = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6} \text{ or } \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}$$

$$\text{i.e. } \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12} \text{ or } \frac{23\pi}{12}$$

Question 13(b) (i)

Criteria	Marks
• Provides correct solution	**1**

Sample answer:

$$m\ddot{x} = -\left(v + v^3\right) \Rightarrow \ddot{x} = -v\left(1 + v^2\right)$$

Question 13(b) (ii)

Criteria	Marks
• Provides correct solution	**3**
• Correctly derives $x = \tan^{-1} U - \tan^{-1} v$	**2**
• Makes some use of $\ddot{x} = v\dfrac{dv}{dx}$	**1**

Sample answer:

$$v\frac{dv}{dx} = -v\left(1+v^2\right) \Rightarrow \frac{dx}{dv} = -\frac{1}{1+v^2}$$

$$\therefore x = -\tan^{-1} v + c$$

$$x = 0, v = U \Rightarrow c = \tan^{-1} U$$

$$\begin{aligned} x &= \tan^{-1} U - \tan^{-1} v \\ &= \tan^{-1}\left(\tan\left(\tan^{-1} U - \tan^{-1} v\right)\right) \\ &= \tan^{-1}\left(\frac{\tan\left(\tan^{-1} U\right) - \tan\left(\tan^{-1} v\right)}{1+\tan\left(\tan^{-1} U\right)\tan\left(\tan^{-1} v\right)}\right) \\ &= \tan^{-1}\left(\frac{U-v}{1+Uv}\right) \end{aligned}$$

Question 13(b) (iii)

Criteria	Marks
• Provides correct solution	**3**
• Correctly calculates *t* as a function of *v*	**2**
• Correctly determines the partial fraction decomposition	**1**

Sample answer:

$$\frac{dv}{dt} = -v\left(1+v^2\right) \Rightarrow \frac{dt}{dv} = -\frac{1}{v\left(1+v^2\right)} \equiv \frac{a}{v} + \frac{bv+c}{1+v^2}$$

$$\therefore -1 = a\left(1+v^2\right) + v\left(bv+c\right) = \left(a+b\right)v^2 + cv + a$$

Equating constants $\Rightarrow a = -1$

Equating coefficients of $v \Rightarrow c = 0$

Equating coefficients of $v^2 \Rightarrow a+b = 0 \Rightarrow b = 1$

$$\therefore \frac{dt}{dv} = -\frac{1}{v} + \frac{v}{1+v^2} \Rightarrow t = -\ln v + \frac{1}{2}\ln\left(1+v^2\right) + c$$

$$t = 0, v = U \Rightarrow c = \ln U - \frac{1}{2}\ln\left(1+U^2\right)$$

$$\therefore t = \ln U - \frac{1}{2}\ln\left(1+U^2\right) - \ln v + \frac{1}{2}\ln\left(1+v^2\right) = \frac{1}{2}\ln\left(\frac{U^2\left(1+v^2\right)}{v^2\left(1+U^2\right)}\right)$$

$$\Rightarrow e^{2t} = \frac{U^2\left(1+v^2\right)}{v^2\left(1+U^2\right)}$$

$$v^2\left(1+U^2\right)e^{2t} = U^2 + U^2v^2$$

$$v^2\left[\left(1+U^2\right)e^{2t} - U^2\right] = U^2$$

$$\Rightarrow v^2 = \frac{U^2}{\left(1+U^2\right)e^{2t} - U^2}$$

Question 13(b) (iv)

Criteria	Marks
• Provides correct solution	**2**
• Correctly describes value of v or x as $t \to \infty$	**1**

Sample answer:

As $t \to \infty,\ v^2 \to 0$ i.e. $v \to 0$ hence $x \to \tan^{-1} U$
Since $v \neq 0,$ and the particle is initially moving in the positive direction,it continues moving in the direction, but slows down towards zero velocity as it approachs the position $\tan^{-1} U$

Question 14(a)

Criteria	Marks
• Provides correct solution	**3**
• Correctly writes equation as $\left(x^3\right)^2 + \left(x^2 - 1\right)^2 = 0$ and solves for x	**2**
• Attempts to rewrite the equation in form $\left(x^3\right)^2 + \left(x^2 - 1\right)^2 = 0$	**1**

Sample answer:

Suppose that there is a solution. Then
$\left(x^3\right)^2 + \left(x^2 - 1\right)^2 = 0 \Rightarrow x^3 = 0$ AND $x^2 = 1$ simultaneously
This is a contradiction, hence the equation has no real solution.

ALTERNATIVELY

Using the AM-GM inequality, $a^2+b^2\geq 2ab$:

$x^4+1\geq 2x^2$ with equality when $x^2=1$

$\therefore \underbrace{x^6+x^4+1\geq x^6+2x^2}_{\text{equality iff } x^2=1}=2x^2$ iff $x^6=0$

$\therefore x^6+x^4+1=2x^2$ requires both $x^2=1$ AND $x^6=0$

$\Rightarrow$ the equation has no real solution

Question 14(b) (i)

Criteria	Marks
• Provides correct solution	**3**
• Determines $\dot{x}$ and makes some progress towards $\dot{y}$	**2**
• Correctly determines $\dot{x}$	**1**

Sample answer:

$$\frac{d\dot{x}}{dt}=-k\dot{x}\Rightarrow \dot{x}=Ae^{-kt}$$

$$t=0,\dot{x}=V\cos\theta\Rightarrow \underline{\dot{x}=V\cos\theta\, e^{-kt}}$$

$$\frac{d\dot{y}}{dt}=-\left(k\dot{y}+g\right)\Rightarrow \frac{dt}{d\dot{y}}=-\frac{1}{k\dot{y}+g}\Rightarrow t=-\frac{1}{k}\ln\left(k\dot{y}+g\right)+c$$

$$y=0,\dot{y}=V\sin\theta\Rightarrow c=\frac{1}{k}\ln\left(kV\sin\theta+g\right)$$

$$\therefore t=\frac{1}{k}\ln\left(kV\sin\theta+g\right)-\frac{1}{k}\ln\left(k\dot{y}+g\right)$$

$$\Rightarrow kt=\ln\left(\frac{kV\sin\theta+g}{k\dot{y}+g}\right)$$

$$\Rightarrow \frac{k\dot{y}+g}{kV\sin\theta+g}=e^{-kt}$$

$$\Rightarrow k\dot{y}+g=\left(kV\sin\theta+g\right)e^{-kt}$$

$$\Rightarrow \dot{y}=\frac{1}{k}\left(kV\sin\theta+g\right)e^{-kt}-\frac{g}{k}$$

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	**2**
• States that $\dot{y} = 0$ for maximum height	**1**

Sample answer:

$$\dot{y} = 0 \Rightarrow \frac{1}{k}\left(g + kV\sin\theta\right)e^{-kT} = \frac{g}{k}$$

$$\Rightarrow e^{kT} = \frac{g + kV\sin\theta}{g}$$

$$\Rightarrow T = \frac{1}{k}\ln\left(\frac{g + kV\sin\theta}{g}\right)$$

Question 14(b) (iii)

Criteria	Marks
• Provides correct solution	**3**
• Correctly determines x as a function of t	**2**
• Makes some progress towards integrating $\dot{x}$	**1**

Sample answer:

$$\frac{dx}{dt} = V\cos\theta\, e^{-kt} \Rightarrow x = -\frac{V\cos\theta}{k}e^{-kt} + c$$

$$t = 0, x = 0 \Rightarrow c = \frac{V\cos\theta}{k}$$

$$\therefore x = \frac{V\cos\theta}{k}\left(1 - e^{-kt}\right)$$

At $t = T,\ x = \dfrac{V\cos\theta}{k}\left(1 - \dfrac{g}{g + kV\sin\theta}\right) = \dfrac{V^2\sin\theta\cos\theta}{g + kV\sin\theta}$

Question 14(c) (i)

Criteria	Marks
• Provides correct solution	**2**
• Calculates at least one of the vectors $\overrightarrow{AC}$ or $\overrightarrow{BD}$	**1**

Sample answer:

$\overrightarrow{AC} = 8\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}$

$\overrightarrow{BD} = -4\mathbf{i} + (2-\lambda)\mathbf{j} - 12\mathbf{k}$

$\overrightarrow{AC} \perp \overrightarrow{BD} \Rightarrow \left(8\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}\right)\cdot\left(-4\mathbf{i} + (2-\lambda)\mathbf{j} - 12\mathbf{k}\right) = 0$

i.e. $-32 - 4\left(2-\lambda\right) + 48 = 0 \Rightarrow \underline{\lambda = -2}$

Question 14(c) (ii)

Criteria	Marks
• Provides correct solution	**3**
• Forms both vector equations and makes some progress towards F	**2**
• Correctly forms at least one of the two lines in parametric form	**1**

Sample answer:

Line through A, C is:

$\mathbf{r}_{AC} = -4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + m\left(8\mathbf{i} - 4\mathbf{j} - 4\mathbf{k}\right) = \left(8m-4\right)\mathbf{i} + \left(3-4m\right)\mathbf{j} + \left(3-4m\right)\mathbf{k}$

Line through B, D is:

$\mathbf{r}_{BD} = 4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k} + n\left(-4\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}\right) = \left(4-4n\right)\mathbf{i} + \left(4n-2\right)\mathbf{j} + \left(6-12n\right)\mathbf{k}$

Since the $\mathbf{j}$ and $\mathbf{k}$ components of $\mathbf{r}_{AC}$ are always equal, intersection requires

$4n - 2 = 6 - 12n \Rightarrow n = \dfrac{1}{2}$, which gives the point $2\mathbf{i}+0\mathbf{j}+0\mathbf{k} = 2\mathbf{i}$ on $\mathbf{r}_{BD}$

Checking $\mathbf{r}_{AC}$: when $8m - 4 = 2$, i.e. when $m = \dfrac{3}{4}, 3 - 4m = 0,$

which also gives the point $2\mathbf{i}+0\mathbf{j}+0\mathbf{k} = 2\mathbf{i}$ on $\mathbf{r}_{AC}$

Since $2\mathbf{i}$ lies on both lines, F is the point $2\mathbf{i}$

Question 15(a)

Criteria	Marks
• Provides correct solution	**4**
• Makes significant progress towards proving the relationship	**3**
• Determines $D, \overrightarrow{AD}$ and attempts to use $\lvert \underset{\sim}{a} \rvert^2 = \underset{\sim}{a} \cdot \underset{\sim}{a}$	**2**
• Correctly determines at least one of $\overrightarrow{AB}, \overrightarrow{AC}$	**1**

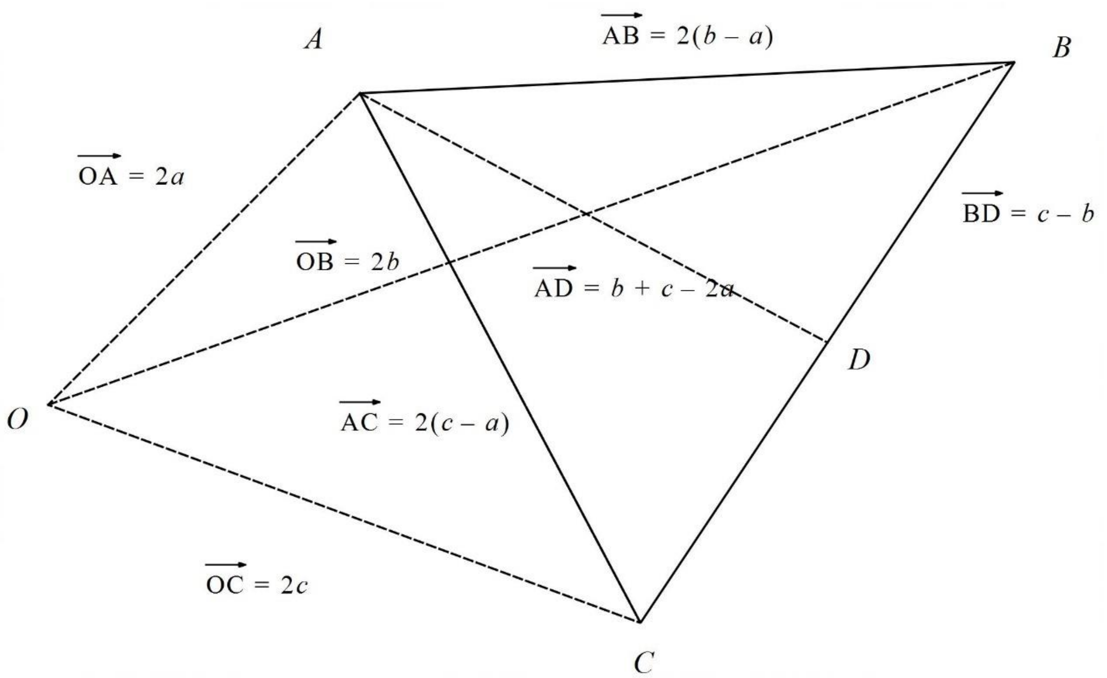
Sample answer:

Let O be the origin and A, B, C have position vectors $2a, 2b, 2c$ respectively.
[Note: this choice of A, B, C is designed to make the calculations easier to follow]

$\overrightarrow{AB} = 2b - 2a = 2(b-a), \overrightarrow{AC} = 2(c-a)$

$\overrightarrow{BD} = \frac{1}{2}(2c - 2b) = c - b \Rightarrow \overrightarrow{OD} = 2b + (c-b) = b + c$

$\Rightarrow \overrightarrow{AD} = b + c - 2a$



$$
\begin{aligned}
\left|\overrightarrow{AB}\right|^2+\left|\overrightarrow{AC}\right|^2 &= 2(b-a)\cdot 2(b-a)+2(c-a)\cdot 2(c-a) \\
&= 4\{b\cdot b-2b\cdot a+a\cdot a+c\cdot c-2c\cdot a+a\cdot a\} \\
&= 2\{(b\cdot b+2b\cdot c+c\cdot c)-4(b+c)\cdot a+4a\cdot a+(c\cdot c-2c\cdot b+b\cdot b)\} \\
&= 2\{(b+c)\cdot(b+c)-2(b+c)\cdot 2a+2a\cdot 2a+(c-b)\cdot(c-b)\} \\
&= 2\{(b+c-2a)\cdot(b+c-2a)+(c-b)\cdot(c-b)\} \\
&= 2\left(\left|\overrightarrow{AD}\right|^2+\left|\overrightarrow{BD}\right|^2\right)
\end{aligned}
$$

Question 15(b) (i)

Criteria	Marks
• Provides correct solution	**2**
• Correctly expands using Binomial Theorem	**1**

Sample answer:

$$
\begin{aligned}
\cos 3\theta + i\sin 3\theta &= (\cos\theta + i\sin\theta)^3 \\
&= \cos^3\theta - 3\cos\theta\sin^2\theta + i\left(3\cos^2\theta\sin\theta - \sin^3\theta\right)
\end{aligned}
$$

Imaginary parts $\Rightarrow \sin 3\theta = 3\left(1-\sin^2\theta\right)\sin\theta - \sin^3\theta = 3\sin\theta - 4\sin^3\theta$

Question 15(b) (ii)

Criteria	Marks
• Provides correct solution	**2**
• Attempts to substitute $\theta = \dfrac{\pi}{18}$ into the previous result	**1**

Sample answer:

$$3\sin\frac{\pi}{18} - 4\sin^3\frac{\pi}{18} = \sin\left(3\times\frac{\pi}{18}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$

$\therefore\ x = \sin\dfrac{\pi}{18}$ is a solution of

$3x - 4x^3 = \dfrac{1}{2} \Rightarrow 6x - 8x^3 = 1$ or $8x^3 - 6x + 1 = 0$

Question 15(b) (iii)

Criteria	Marks
• Provides correct solution	**3**
• Makes some progress towards proving a contradiction	**2**
• Makes a statement assuming rational form of $\sin\frac{\pi}{18}$, including 'no common factor'	**1**

Sample answer:

Suppose $x = \frac{a}{b}$ is a rational solution of $8x^3 - 6x + 1 = 0$

where $a, b \in \mathbb{Z}$ with no common factor and $a > 0$.

Then, $8\left(\frac{a}{b}\right)^3 - 6\frac{a}{b} + 1 = 0 \Rightarrow 8a^3 - 6ab^2 + b^3 = 0$ (*)

Rewrite $(*)$ as: $b^3 = a\left(6b^2 - 8a^2\right) \Rightarrow a = 1$ or a divides b

Since a and b have no common factors and $a > 0$, $a = 1$

Then, $(*)$ becomes $b^3 - 6b^2 + 8 = 0$ or $b\left(6b - b^2\right) = 8 \Rightarrow b$ is a divisor of 8

Hence possible values of b are $\pm 1, \pm 2, \pm 4, \pm 8$

Checking these 8 values in $b^3 - 6b^2 + 8 = 0$ shows that none of these values are solutions, hence $(*)$ has no rational solutions

$\Rightarrow$ the solution $x = \sin\frac{\pi}{18}$ is irrational.

Question 15(c) (i)

Criteria	Marks
• Provides correct solution	**2**
• Makes some progress towards $\sin nx - \sin(n-2)x = 2\sin x\cos(n-1)x$	**1**

Sample answer:

$$I_n - I_{n-2} = \int \frac{\sin nx - \sin(n-2)x}{\sin x}\,dx$$

$$\sin(A+B) - \sin(A-B) = 2\sin B\cos A$$

$$\therefore \begin{cases} A + B = nx \\ A - B = (n-2)x \end{cases} \Rightarrow \begin{cases} A = (n-1)x \\ B = x \end{cases}$$

$$\therefore \sin nx - \sin(n-2)x = 2\sin x\cos(n-1)x$$

$$\Rightarrow I_n - I_{n-2} = \int \frac{2\sin x\cos(n-1)x}{\sin x}\,dx = 2\int \cos(n-1)x\,dx$$

Question 15(c) (ii)

Criteria	Marks
• Provides correct solution	**3**
• Expresses I_5 as an indefinite integral based on $I_n - I_{n-2}$	**2**
• Forms some sequence of recurrences	**1**

Sample answer:

$$I_5 = \int \frac{\sin 5x}{\sin x}\,dx$$

$$I_5 - I_3 = 2\int \cos 4x\,dx \qquad (1)$$

$$I_3 - I_1 = 2\int \cos 2x\,dx \qquad (2)$$

$$I_1 = \int \frac{\sin x}{\sin x}\,dx = \int 1\,dx \qquad (3)$$

$$I_5 = (I_5 - I_3) + (I_3 - I_1) + I_1$$

$$= \int (2\cos 4x + 2\cos 2x + 1)\,dx$$

$$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{\sin 5x}{\sin x}\,dx = \int_{\pi/6}^{\pi/3} (2\cos 4x + 2\cos 2x + 1)\,dx$$

$$= \left[\frac{1}{2}\sin 4x + \sin 2x + x\right]_{\pi/6}^{\pi/3}$$

$$= \frac{1}{2}\sin\frac{4\pi}{3} + \sin\frac{2\pi}{3} + \frac{\pi}{3} - \left(\frac{1}{2}\sin\frac{2\pi}{3} + \sin\frac{\pi}{3} + \frac{\pi}{6}\right)$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{2}$$

Question 16(a)

Criteria	Marks
• Provides correct solution	**3**
• Correctly performs the integration by parts by hasn't rearranged correctly.	**2**
• Correctly determines u and v'	**1**

Sample answer:

$$\text{Consider } I = \int 2x^2 e^{x^2}\,dx$$

$$= \int \underset{u}{x}\left(\underset{v'}{2xe^{x^2}}\right)dx \qquad u = x, u' = 1; v' = 2xe^{x^2}, v = e^{x^2}$$

$$= xe^{x^2} - \int 1e^{x^2}\,dx$$

$$\Rightarrow \int 2x^2 e^{x^2}\,dx = xe^{x^2} + c - \int e^{x^2}\,dx$$

$$\Rightarrow \int e^{x^2}\,dx + \int 2x^2 e^{x^2}\,dx = xe^{x^2} + c$$

i.e. $\int \left(1 + 2x^2\right) e^{x^2}\,dx = xe^{x^2} + c$

Question 16(b) (i)

Criteria	Marks
• Provides correct solution	**1**

Sample answer:

Since $\left(\sqrt{a_1} - \sqrt{a_2}\right)^2 \geq 0$

$$a_1 - 2\sqrt{a_1 a_2} + a_2 \geq 0$$

$$a_1 + a_2 \geq 2\sqrt{a_1 a_2}$$

$$\frac{a_1 + a_2}{2} \geq \sqrt{a_1 a_2}, \text{ i.e. } P_2 \text{ is true}$$

Question 16(b) (ii)

Criteria	Marks
• Provides correct solution	**2**
• Correctly uses P_n on two sets of n positive numbers	**1**

Sample answer:

$$P_n \Rightarrow \frac{a_1+a_2+\cdots+a_n}{n} \geq \sqrt[n]{a_1a_2\cdots a_n} \quad (*)$$

and $\dfrac{a_{n+1}+a_{n+2}+\cdots+a_{2n}}{n} \geq \sqrt[n]{a_{n+1}a_{n+2}\cdots a_{2n}} \quad (**)$

Then P_2, using $(*)$ and $(**)$ gives

$$\frac{\left(\frac{a_1+a_2+\cdots+a_n}{n}\right)+\left(\frac{a_{n+1}+a_{n+2}+\cdots+a_{2n}}{n}\right)}{2} \geq \sqrt{\left(\sqrt[n]{a_1a_2\cdots a_n}\right)\left(\sqrt[n]{a_{n+1}a_{n+2}\cdots a_{2n}}\right)}$$

$$\frac{(a_1+a_2+\cdots+a_n)+(a_{n+1}+a_{n+2}+\cdots+a_{2n})}{2n} \geq \left(\left((a_1a_2\cdots a_n)(a_{n+1}a_{n+2}\cdots a_{2n})\right)^{1/n}\right)^{1/2}$$

$$\frac{a_1+a_2+\cdots+a_{2n}}{2n} \geq (a_1a_2\cdots a_{2n})^{1/2n} = \sqrt[2n]{a_1a_2\cdots a_{2n}}, \text{ which is } P_{2n}$$

Question 16(b) (iii)

Criteria	Marks
• Provides correct solution	**2**
• Makes some progress in substituting to a_n	**1**

Sample answer:

With $a_n = \dfrac{a_1+a_2+\cdots+a_{n-1}}{n-1}$

$$P_n \Rightarrow \frac{a_1+a_2+\cdots+\frac{a_1+a_2+\cdots+a_{n-1}}{n-1}}{n} \geq \sqrt[n]{a_1a_2\cdots a_{n-1}\frac{a_1+a_2+\cdots+a_{n-1}}{n-1}}$$

$$\Rightarrow \frac{(a_1+a_2+\cdots+a_{n-1})\left(1+\frac{1}{n-1}\right)}{n} \geq \sqrt[n]{a_1a_2\cdots a_{n-1}\frac{a_1+a_2+\cdots+a_{n-1}}{n-1}}$$

$$\Rightarrow \frac{a_1+a_2+\cdots+a_{n-1}}{n-1} \geq \sqrt[n]{a_1a_2\cdots a_{n-1}\frac{a_1+a_2+\cdots+a_{n-1}}{n-1}}$$

$$\Rightarrow \left(\frac{a_1+a_2+\cdots+a_{n-1}}{n-1}\right)^n \geq a_1a_2\cdots a_{n-1}\frac{a_1+a_2+\cdots+a_{n-1}}{n-1}$$

$$\Rightarrow \left(\frac{a_1+a_2+\cdots+a_{n-1}}{n-1}\right)^{n-1} \geq a_1a_2\cdots a_{n-1}$$

$$\Rightarrow \frac{a_1+a_2+\cdots+a_{n-1}}{n-1} \geq \sqrt[n-1]{a_1a_2\cdots a_{n-1}} \quad \text{i.e. } P_n \Rightarrow P_{n-1}$$

Question 16(b) (iv)

Criteria	Marks
• Provides correct solution	**2**
• Makes some progress towards an inductive argument	**1**

Sample answer:

Part (i) gives the base for an induction argument, i.e. P_2 is true

Part (ii) extends P_2 to all $n = 2^k$, ['leap-forward induction']
i.e. $P_2 \Rightarrow P_4 \Rightarrow P_8 \Rightarrow P_{16} \Rightarrow \cdots$

Part (iii) backwards fills the P_n from $n = 2^k$ back to $n = 2^{k-1}+1$, ['backward-fill induction']
For example, once we know P_8 and P_{16} is true then
$P_n \Rightarrow P_{n-1}$ gives $P_{16} \Rightarrow P_{15} \Rightarrow P_{14} \Rightarrow \cdots \Rightarrow P_{10} \Rightarrow P_9$

This 'backward-fill' occurs for all values of n between powers of 2, hence P_n is true for all $n \geq 2$

Question 16(b) (v)

Criteria	Marks
• Provides correct solution	**2**
• Substitutes into P_{p+q}	**1**

Sample answer:

$$P_{p+q} \Rightarrow \frac{\left(a_1 + a_2 + \cdots + a_p\right) + \left(a_{p+1} + a_{p+2} + \cdots + a_{p+q}\right)}{p+q} \geq \sqrt[p+q]{\left(a_1 a_2 \cdots a_p\right)\left(a_{p+1} a_{p+2} \cdots a_{p+q}\right)}$$

With $a_1 = a_2 = \cdots = a_p = x$ and $a_{p+1} = a_{p+2} = \cdots = a_{p+q} = y$, then

$$\frac{(px)+(qy)}{p+q} \geq \sqrt[p+q]{\left(x^p\right)\left(y^q\right)} \Rightarrow \left(\frac{px+qy}{p+q}\right)^{p+q} \geq x^p y^q$$

Question 16(b) (vi)

Criteria	Marks
• Provides correct solution	**3**
• Attempts to use correct substitution of $x=\dfrac{\sin^2\theta}{p}, y=\dfrac{\cos^2\theta}{q}$	**2**
• Attempts to use incorrect substitution of $x=\sin^2\theta, y=\cos^2\theta$	**1**

Sample answer:

Let

$$x=\frac{\sin^2\theta}{p}, y=\frac{\cos^2\theta}{q} \Rightarrow px+qy=\sin^2\theta+\cos^2\theta=1$$

Then $\left(\dfrac{\sin^2\theta}{p}\right)^p\left(\dfrac{\cos^2\theta}{q}\right)^q \le \left(\dfrac{1}{p+q}\right)^{p+q}$

$$\Rightarrow \frac{\sin^{2p}\theta\cos^{2q}\theta}{p^p q^q} \le \frac{1}{(p+q)^{p+q}}$$

$$\Rightarrow \sin^{2p}\theta\cos^{2q}\theta \le \frac{p^p q^q}{(p+q)^{p+q}}$$

Mathematics Extension 2

PEM Trial HSC Mapping Grid

Section I

Question	Marks	Content	Syllabus Outcomes
1	1	MEX-N1 Introduction to Complex Numbers	MEX12-4
2	1	MEX-C1 Further Integration	MEX12-5
3	1	MEX-V1 Further Work with Vectors	MEX12-3
4	1	MEX-V1 Further Work with Vectors	MEX12-3
5	1	MEX-P1 The Nature of Proof	MEX12-2
6	1	MEX-N1 Introduction to Complex Numbers	MEX12-4
7	1	MEX-V1 Further Work with Vectors	MEX12-3
8	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
9	1	MEX-P1 The Nature of Proof	MEX12-2
10	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6

Section II

Question	Marks	Content	Syllabus Outcomes
11 (a) (i)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4
11 (a) (ii)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4
11 (a) (iii)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4
11 (b)	2	MEX-N2 Using Complex Numbers	MEX12-4
11 (c)	4	MEX-C1 Further Integration	MEX12-5
11 (d)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
12 (a) (i)	2	MEX-C1 Further Integration	MEX12-5
12 (a) (ii)	3	MEX-C1 Further Integration	MEX12-5
12 (b) (i)	1	MEX-N1 Introduction to Complex Numbers	MEX12-4
12 (b) (ii)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4
12 (b) (iii)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4
12 (b) (iv)	2	MEX-N1 Introduction to Complex Numbers	MEX12-2 MEX12-4
12 (c)	2	MEX-V1 Further Work with Vectors	MEX12-3
13 (a) (i)	1	MEX-N1 Introduction to Complex Numbers	MEX12-4
13 (a) (ii)	4	MEX-N2 Using Complex Numbers	MEX12-4
13 (b) (i)	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
13 (b) (ii)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
13 (b) (iii)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
13 (b) (iv)	2	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
14 (a)	3	MEX-P1 The Nature of Proof	MEX12-2
14 (b) (i)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
14 (b) (ii)	2	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
14 (b) (iii)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
14 (c) (i)	2	MEX-V1 Further Work with Vectors	MEX12-3
14 (c) (ii)	3	MEX-V1 Further Work with Vectors	MEX12-3

Question	Marks	Content	Syllabus Outcomes
15 (a)	4	MEX-V1 Further Work with Vectors	MEX12-3
15 (b) (i)	2	MEX-N2 Using Complex Numbers	MEX12-4
15 (b) (ii)	2	MEX-N2 Using Complex Numbers	MEX12-4
15 (b) (iii)	3	MEX-P1 The Nature of Proof	MEX12-2 MEX12-8
15 (c) (i)	2	MEX-C1 Further Integration	MEX12-5
15 (c) (ii)	3	MEX-C1 Further Integration	MEX12-5 MEX12-8
16 (a)	3	MEX-C1 Further Integration	MEX12-5 MEX12-8
16 (b) (i)	1	MEX-P1 The Nature of Proof	MEX12-2
16 (b) (ii)	2	MEX-P1 The Nature of Proof	MEX12-2
16 (b) (iii)	2	MEX-P1 The Nature of Proof	MEX12-2
16 (b) (iv)	2	MEX-P2 Further Proof by Mathematical Induction	MEX12-8
16 (b) (v)	2	MEX-P1 The Nature of Proof	MEX12-2
16 (b) (vi)	3	MEX-P1 The Nature of Proof	MEX12-2