NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 2–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 8–14)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which diagram best represents the solution set of $x^2 - 2x - 3 \geq 0$?

A.

B.

C.

D.

2 Given $f(x) = 1 + \sqrt{x}$, what are the domain and range of $f^{-1}(x)$?

A. $x \geq 0, \quad y \geq 0$

B. $x \geq 0, \quad y \geq 1$

C. $x \geq 1, \quad y \geq 0$

D. $x \geq 1, \quad y \geq 1$

3 Which of the following is an anti-derivative of $\dfrac{1}{4x^2 + 1}$?

A. $2\tan^{-1}\left(\dfrac{x}{2}\right) + c$

B. $\dfrac{1}{2}\tan^{-1}\left(\dfrac{x}{2}\right) + c$

C. $2\tan^{-1}(2x) + c$

D. $\dfrac{1}{2}\tan^{-1}(2x) + c$

4 Maria starts at the origin and walks along all of the vector $2\underset{\sim}{i} + 3\underset{\sim}{j}$, then walks along all of the vector $3\underset{\sim}{i} - 2\underset{\sim}{j}$ and finally along all of the vector $4\underset{\sim}{i} - 3\underset{\sim}{j}$.

How far from the origin is she?

A. $\sqrt{77}$

B. $\sqrt{85}$

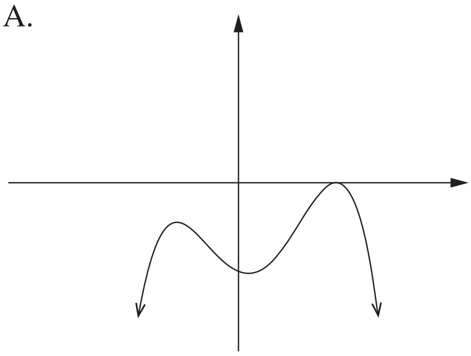
C. $2\sqrt{13} + \sqrt{5}$

D. $\sqrt{5} + \sqrt{7} + \sqrt{13}$

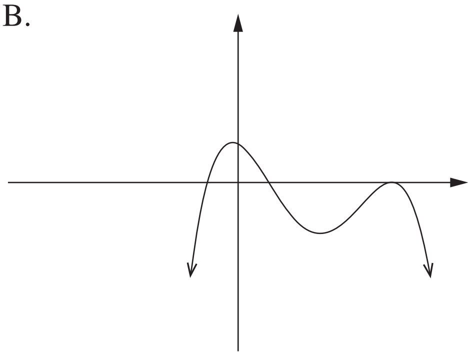
5 A monic polynomial $p(x)$ of degree 4 has one repeated zero of multiplicity 2 and is divisible by $x^2 + x + 1$.

Which of the following could be the graph of $p(x)$?

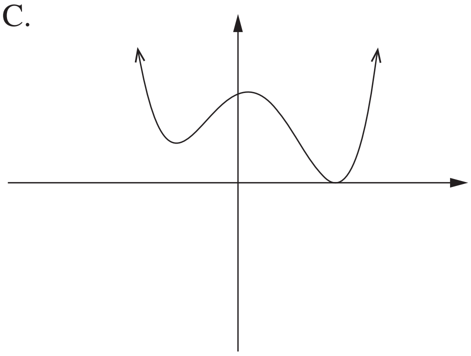
A.



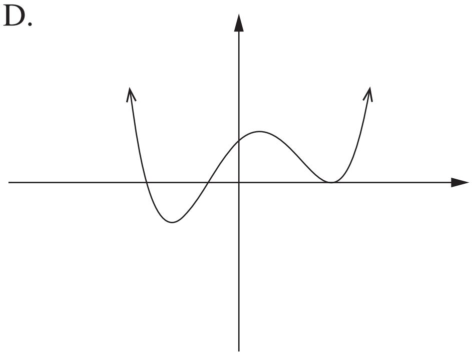
B.



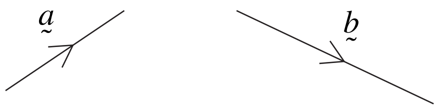
C.



D.

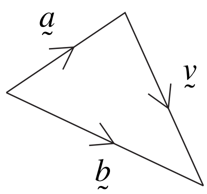


6 The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are shown.

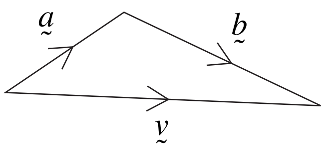


Which diagram below shows the vector $\underset{\sim}{v} = \underset{\sim}{a} - \underset{\sim}{b}$?

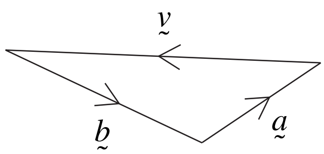
A.



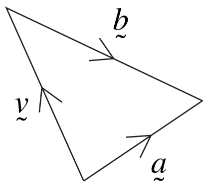
B.



C.

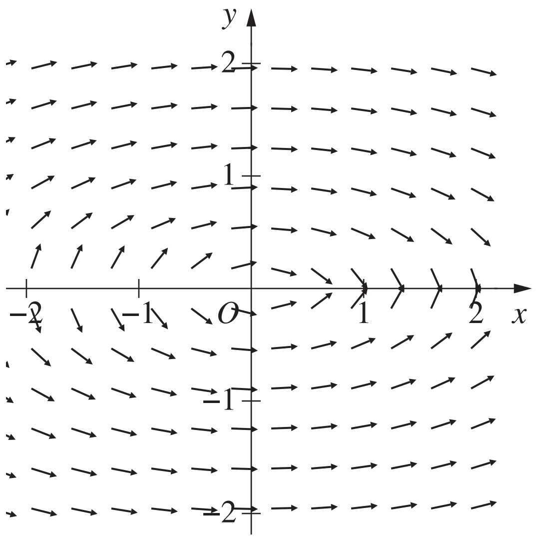


D.

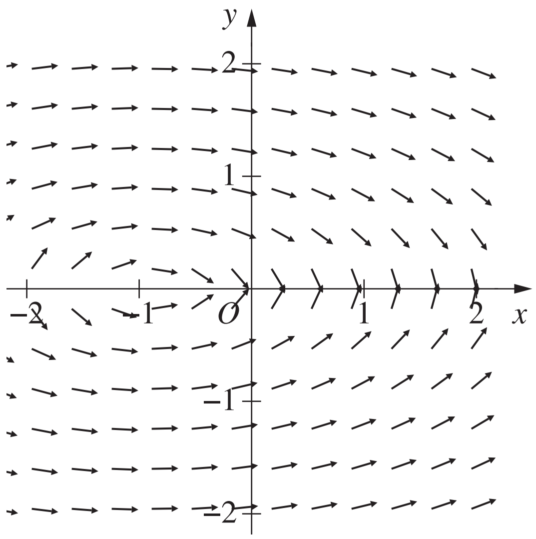


7 Which of the following best represents the direction field for the differential equation $\dfrac{dy}{dx} = -\dfrac{x}{4y}$?

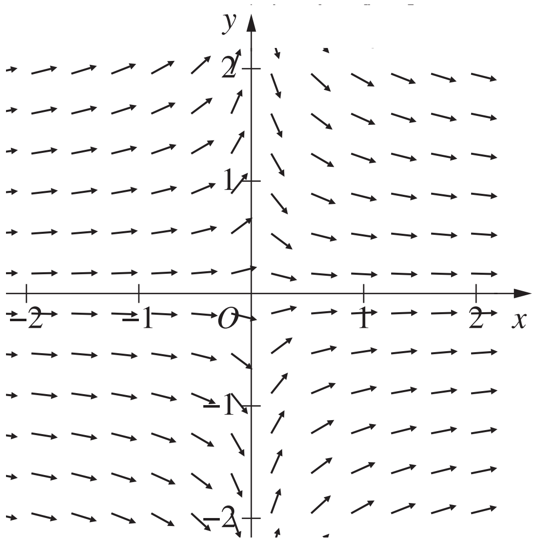
A.



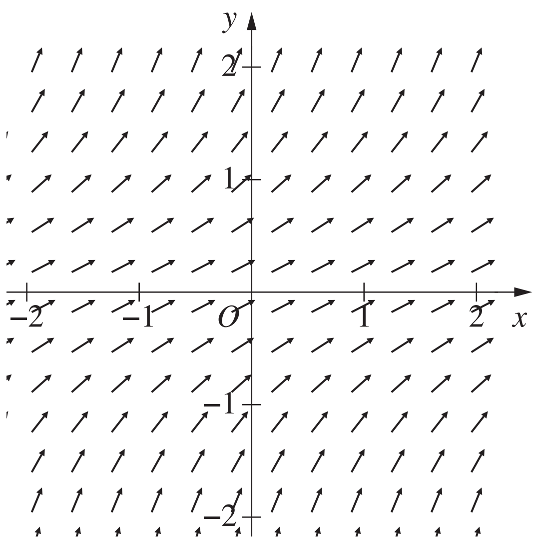
B.



C.



D.



8 Out of 10 contestants, six are to be selected for the final round of a competition. Four of those six will be placed 1st, 2nd, 3rd and 4th.

In how many ways can this process be carried out?

A. $\dfrac{10!}{6!4!}$

B. $\dfrac{10!}{6!}$

C. $\dfrac{10!}{4!2!}$

D. $\dfrac{10!}{4!4!}$

9 The projection of the vector $\begin{pmatrix} 6 \\ 7 \end{pmatrix}$ onto the line $y = 2x$ is $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$.

The point (6, 7) is reflected in the line $y = 2x$ to a point A.

What is the position vector of the point A?

A. $\begin{pmatrix} 6 \\ 12 \end{pmatrix}$

B. $\begin{pmatrix} 2 \\ 9 \end{pmatrix}$

C. $\begin{pmatrix} -6 \\ 7 \end{pmatrix}$

D. $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

10 The quantities P, Q and R are connected by the related rates,

$$\frac{dR}{dt} = -k^2$$

$$\frac{dP}{dt} = -l^2 \times \frac{dR}{dt}$$

$$\frac{dP}{dt} = m^2 \times \frac{dQ}{dt}$$

where k, l and m are non-zero constants.

Which of the following statements is true?

A. P is increasing and Q is increasing

B. P is increasing and Q is decreasing

C. P is decreasing and Q is increasing

D. P is decreasing and Q is decreasing

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

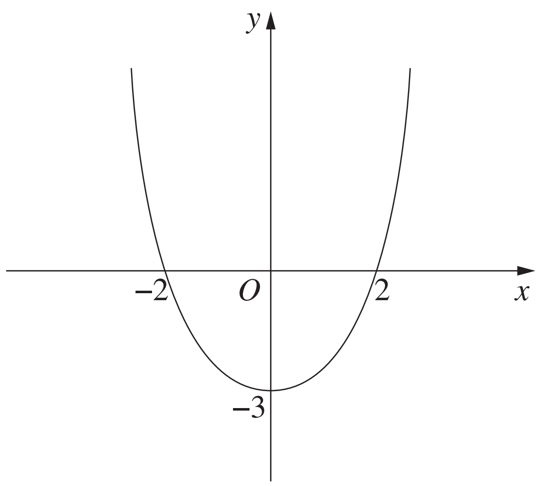
(a) Let $P(x) = x^3 + 3x^2 - 13x + 6$.

(i) Show that $P(2) = 0$. **1**

(ii) Hence, factor the polynomial $P(x)$ as $A(x)B(x)$, where $B(x)$ is a quadratic polynomial. **2**

(b) For what value(s) of a are the vectors $\begin{pmatrix} a \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2a-3 \\ 2 \end{pmatrix}$ perpendicular? **3**

(c) The diagram shows the graph of $y = f(x)$. **3**



Sketch the graph of $y = \dfrac{1}{f(x)}$.

Question 11 continues on page 9

(d) By expressing $\sqrt{3}\sin x + 3\cos x$ in the form $A\sin(x+\alpha)$, solve $\sqrt{3}\sin x + 3\cos x = \sqrt{3}$, for $0 \le x \le 2\pi$. **4**

(e) Solve $\dfrac{dy}{dx} = e^{2y}$, finding x as a function of y. **2**

End of Question 11

Please turn over

Question 12 (14 marks) Use the Question 12 Writing Booklet

(a) Use the principle of mathematical induction to show that for all integers $n \geq 1$, **3**

$$1 \times 2 + 2 \times 5 + 3 \times 8 + \cdots + n(3n-1) = n^2(n+1).$$

(b) When a particular biased coin is tossed, the probability of obtaining a head is $\frac{3}{5}$.

This coin is tossed 100 times.

Let X be the random variable representing the number of heads obtained. This random variable will have a binomial distribution.

(i) Find the expected value, $E(X)$. **1**

(ii) By finding the variance, $\mathrm{Var}(X)$, show that the standard deviation of X is approximately 5. **1**

(iii) By using a normal approximation, find the approximate probability that X is between 55 and 65. **1**

(c) To complete a course, a student must choose and pass exactly three topics. **2**

There are eight topics from which to choose.

Last year 400 students completed the course.

Explain, using the pigeonhole principle, why at least eight students passed exactly the same three topics.

(d) Find $\displaystyle\int_0^{\frac{\pi}{2}} \cos 5x \sin 3x \, dx$. **3**

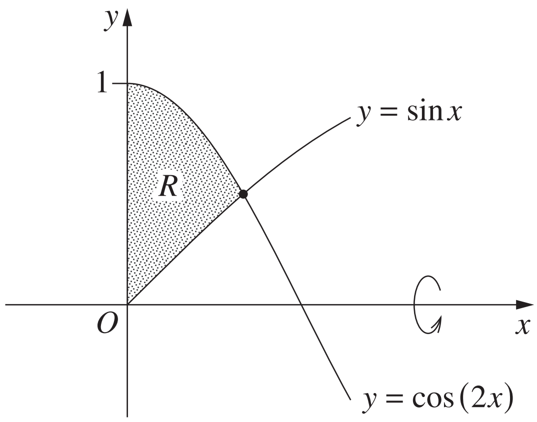
(e) Find the curve which satisfies the differential equation $\dfrac{dy}{dx} = -\dfrac{x}{y}$ and passes through the point $(1, 0)$. **3**

Question 13 (16 marks) Use the Question 13 Writing Booklet

(a) (i) Find $\dfrac{d}{d\theta}\left(\sin^3\theta\right)$. **1**

(ii) Use the substitution $x = \tan\theta$ to evaluate $\displaystyle\int_0^1 \frac{x^2}{\left(1+x^2\right)^{\frac{5}{2}}}\,dx$. **4**

(b) The region R is bounded by the y-axis, the graph of $y = \cos(2x)$ and the graph of $y = \sin x$, as shown in the diagram. **4**



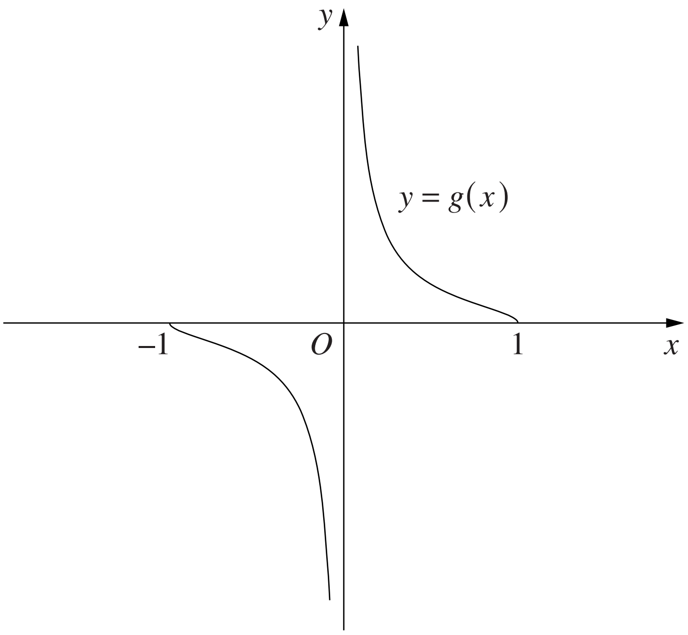
Find the volume of the solid of revolution formed when the region R is rotated about the x-axis.

Question 13 continues on page 12

Question 13 (continued)

(c) Suppose $f(x) = \tan\left(\cos^{-1}(x)\right)$ and $g(x) = \dfrac{\sqrt{1-x^2}}{x}$.

The graph of $y = g(x)$ is given.



(i) Show that $f'(x) = g'(x)$. **4**

(ii) Using part (i), or otherwise, show that $f(x) = g(x)$. **3**

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) (i) Use the identity $(1+x)^{2n} = (1+x)^n(1+x)^n$ **2**

to show that

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2,$$

where n is a positive integer.

(ii) A club has $2n$ members, with n women and n men. **2**

A group consisting of an even number $(0, 2, 4, \ldots, 2n)$ of members is chosen, with the number of men equal to the number of women.

Show, giving reasons, that the number of ways to do this is $\binom{2n}{n}$.

(iii) From the group chosen in part (ii), one of the men and one of the women are selected as leaders. **2**

Show, giving reasons, that the number of ways to choose the even number of people and then the leaders is

$$1^2\binom{n}{1}^2 + 2^2\binom{n}{2}^2 + \cdots + n^2\binom{n}{n}^2.$$

(iv) The process is now reversed so that the leaders, one man and one woman, are chosen first. The rest of the group is then selected, still made up of an equal number of women and men. **2**

By considering this reversed process and using part (ii), find a simple expression for the sum in part (iii).

Question 14 continues on page 14

Question 14 (continued)

(b) (i) Show that $\sin^3\theta - \frac{3}{4}\sin\theta + \frac{\sin(3\theta)}{4} = 0.$ **2**

(ii) By letting $x = 4\sin\theta$ in the cubic equation $x^3 - 12x + 8 = 0.$ **2**

Show that $\sin(3\theta) = \frac{1}{2}.$

(iii) Prove that $\sin^2\frac{\pi}{18} + \sin^2\frac{5\pi}{18} + \sin^2\frac{25\pi}{18} = \frac{3}{2}.$ **3**

End of paper

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2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

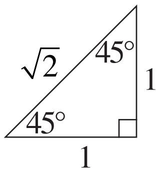
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

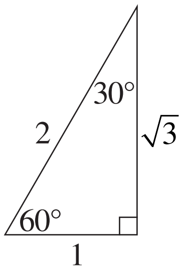
$$A = \frac{1}{2} ab \sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2} r^2 \theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

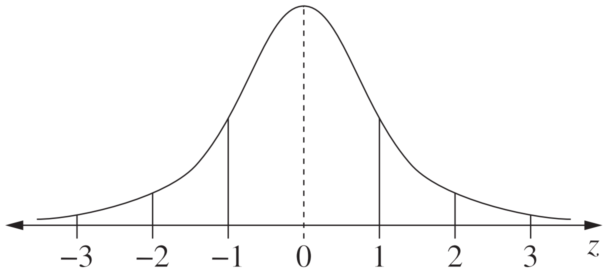
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n - r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n - x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\Big\{f(a) + f(b) + 2\big[f(x_1) + \cdots + f(x_{n-1})\big]\Big\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left| \underset{\sim}{u} \right| = \left| x\underset{\sim}{i} + y\underset{\sim}{j} \right| = \sqrt{x^{2} + y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left| \underset{\sim}{u} \right| \left| \underset{\sim}{v} \right| \cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^{2}(x - c)$$

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2020 HSC Mathematics Extension 1 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	C
3	D
4	B
5	C
6	D
7	A
8	C
9	B
10	A

Section II

Question 11 (a) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\begin{aligned} P(2) &= 2^3 + 3(2)^2 - 13(2) + 6 \\ &= 0 \end{aligned}$$

Question 11 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to divide by $x - 2$, or equivalent merit	1

Sample answer:

$$\begin{array}{r l} & x^2 + 5x - 3 \\ x - 2 \big) & x^3 + 3x^2 - 13x + 6 \\ & \underline{x^3 - 2x^2} \\ & \quad 5x^2 - 13x \\ & \quad \underline{5x^2 - 10x} \\ & \qquad -3x + 6 \\ & \qquad \underline{-3x + 6} \\ & \qquad\qquad 0 \end{array}$$

$$\therefore\ P(x) = (x - 2)(x^2 + 5x - 3)$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	3
• Evaluates the dot product and sets it equal to 0, or equivalent value	2
• Writes a dot product $= 0$, or equivalent merit	1

Sample answer:

Since the vectors are perpendicular,

$$0 = \begin{pmatrix} a \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 2a-3 \\ 2 \end{pmatrix}$$

$$0 = 2a^2 - 3a - 2$$

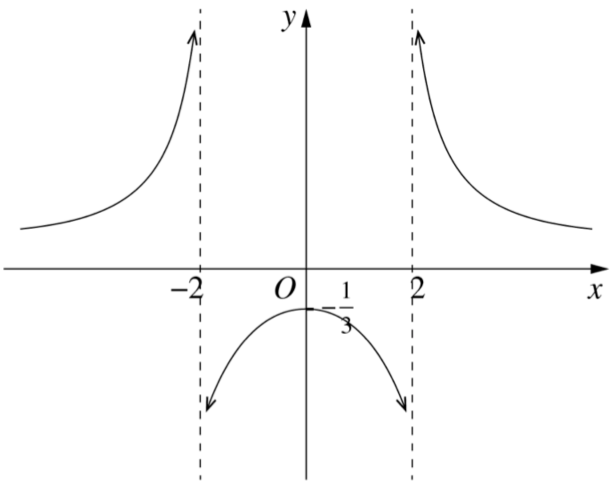
$$0 = (2a+1)(a-2)$$

$$a = -\frac{1}{2} \quad \text{or} \quad 2$$

Question 11 (c)

Criteria	Marks
• Provides correct sketch	3
• Provides a sketch with some correct features	2
• Marks asymptotes at $x = -2$ or 2, or marks local maximum at $y = -\frac{1}{3}$, or equivalent merit	1

Sample answer:



Question 11 (d)

Criteria	Marks
• Provides correct solution	4
• Correctly writes $\sqrt{3}\sin x + 3\cos x$ in the form $A\sin(x+\alpha)$ and finds one solution	3
• Finds A and α, or equivalent merit	2
• Finds the value of A, or equivalent merit	1

Sample answer:

$$\begin{aligned}\sqrt{3}\sin x + 3\cos x &= A\sin(x+\alpha)\\ &= A\sin x\cos\alpha + A\cos x\sin\alpha\end{aligned}$$

$$\therefore A\cos\alpha = \sqrt{3} \qquad (1)$$
$$A\sin\alpha = 3 \qquad (2)$$

dividing, $\tan\alpha = \dfrac{3}{\sqrt{3}} = \sqrt{3} \quad \Rightarrow \quad \alpha = \dfrac{\pi}{3}.$

$$\therefore A\cos\frac{\pi}{3} = \sqrt{3} \quad \Rightarrow \quad A = 2\sqrt{3}$$

$$\therefore \sqrt{3}\sin x + 3\cos x = 2\sqrt{3}\sin\left(x+\frac{\pi}{3}\right)$$

$$\begin{aligned}\text{so}\quad 2\sqrt{3}\sin\left(x+\frac{\pi}{3}\right) &= \sqrt{3}\\ \sin\left(x+\frac{\pi}{3}\right) &= \frac{1}{2}\\ x+\frac{\pi}{3} &= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6},\\ x &= \frac{\pi}{2}, \frac{11\pi}{6} \quad \text{given } 0 \le x \le 2\pi.\end{aligned}$$

Question 11 (e)

Criteria	Marks
• Provides correct solution	2
• Rewrites as $\dfrac{dx}{dy} = e^{-2y}$, or equivalent merit	1

Sample answer:

$$\frac{dy}{dx} = e^{2y}$$

$$\frac{dx}{dy} = e^{-2y}$$

$$x = \int e^{-2y}\,dy$$

$$= -\frac{1}{2}e^{-2y} + C$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	3
• Proves inductive step by assuming true for k (or equivalent) and using that assumption to show true for $k + 1$, or equivalent merit	2
• Verifies base case, $n = 1$, or equivalent merit	1

Sample answer:

$$(1\times 2)+(2\times 5)+(3\times 8)+\cdots+n(3n-1)=n^2(n+1)$$

For $n = 1$ LHS $= 1\ (2)$ $= 2$ RHS $= 1\ (2)$ $= 2$

$\therefore$ Statement is true for $n = 1$

Assume statement true for $n = k$, that is,

$$(1\times 2)+(2\times 5)+(3\times 8)+\cdots+k(3k-1)=k^2(k+1)$$

Prove true for $n = k + 1$

That is, we show that

$$(1\times 2)+(2\times 5)+\cdots+k(3k-1)+(k+1)\big(3(k+1)-1\big)=(k+1)^2(k+2)$$

$$\begin{aligned}\text{LHS} &= k^2(k+1)+(k+1)(3k+2)\\ &= \left(k+1\right)\left(k^2+3k+2\right)\\ &= (k+1)(k+1)(k+2)\\ &= (k+1)^2(k+2)\\ &= \text{RHS}\end{aligned}$$

$\therefore$ By the principle of mathematical induction the statement is true for $n \geq 1$.

Question 12 (b) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$E(X) = np$$
$$= 100 \times \frac{3}{5}$$
$$= 60$$

Question 12 (b) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\sigma = \sqrt{np(1-p)}$$
$$= \sqrt{100 \times \frac{3}{5} \times \frac{2}{5}}$$
$$= \sqrt{24}$$
$$\doteqdot 5$$

Question 12 (b) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$P(55 \le X \le 65) = P(-1 \le Z \le 1)$$
$$\simeq 68\%$$

Question 12 (c)

Criteria	Marks
• Provides correct explanation	2
• Evaluates $\binom{8}{3}$, or obtains the expression $\dfrac{400}{\binom{8}{3}}$, or equivalent merit	1

Sample answer:

There are $\binom{8}{3} = 56$ possible choices of 3 topics and $\dfrac{400}{56} = 7.14$.

As there are 56 possible combinations, we can have at most 392 students without exceeding 7 students per combination. But we have 400 students, so at least one combination has 8 or more students.

Question 12 (d)

Criteria	Marks
• Provides correct solution	3
• Finds correct primitive, or equivalent merit	2
• Uses product to sum result, or equivalent merit	1

Sample answer:

$$\int_0^{\frac{\pi}{2}} \cos 5x \sin 3x \, dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}(\sin 8x - \sin 2x)\, dx$$

$$= \left[\frac{1}{2}\left(-\frac{\cos 8x}{8} + \frac{\cos 2x}{2}\right)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left(-\frac{1}{8} - \frac{1}{2}\right) - \frac{1}{2}\left(-\frac{1}{8} + \frac{1}{2}\right)$$

$$= -\frac{1}{2}$$

Question 12 (e)

Criteria	Marks
• Provides correct solution	3
• Obtains $x^2 + y^2 =$ constant, or equivalent merit	2
• Separates the variable, or equivalent merit	1

Sample answer:

$$\frac{dy}{dx} = \frac{-x}{y}$$

Separating variables, $\int y\,dy = \int -x\,dx$

$$\therefore \quad \frac{y^2}{2} = \frac{-x^2}{2} + c$$

$$x^2 + y^2 = d, \text{ where } d = 2c$$

The curve passes through (1, 0)

So $d = 1^2 + 0^2 = 1$

Hence the equation of D is

$$x^2 + y^2 = 1 \qquad \text{(unit circle)}$$

Question 13 (a) (i)

Criteria	Marks
• Provides correct derivative	1

Sample answer:

$$\frac{d}{d\theta}\left(\sin^3\theta\right) = 3\sin^2\theta\cos\theta$$

Question 13 (a) (ii)

Criteria	Marks
• Provides correct solution	4
• Obtains correctly simplified integrand in terms of $\sin\theta$ and $\cos\theta$, or equivalent merit	3
• Correctly substitutes and attempts to simplify (ignoring limits), or equivalent merit	2
• Uses given substitution, or equivalent merit	1

Sample answer:

Let $x = \tan\theta$

$$\frac{dx}{d\theta} = \sec^2\theta$$

At $x = 0, \quad \theta = 0$

at $x = 1, \quad \theta = \frac{\pi}{4}$

$$\therefore \int_0^1 \frac{x^2}{\left(1+x^2\right)^{\frac{5}{2}}}\,dx = \int_0^{\frac{\pi}{4}} \frac{\tan^2\theta}{\left(1+\tan^2\theta\right)^{\frac{5}{2}}}\sec^2\theta\,d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{\tan^2\theta}{\sec^3\theta}\,d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin^2\theta}{\cos^2\theta}\cos^3\theta\,d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sin^2\theta\cos\theta\,d\theta$$

$$= \left[\frac{1}{3}\sin^3\theta\right]_0^{\frac{\pi}{4}} \qquad \text{by part (i)}$$

$$= \frac{1}{3}\times\left(\frac{\sqrt{2}}{2}\right)^3$$

$$= \frac{2\sqrt{2}}{3\times 2^3}$$

$$= \frac{\sqrt{2}}{12}$$

Question 13 (b)

Criteria	Marks
• Provides correct solution	4
• Obtains a correct expression for the volume and uses a double-angle formula, or equivalent merit	3
• Finds point of intersection and writes volume as a difference of volumes OR • Finds point of intersection and finds volume generated by $y = \sin x$, or equivalent merit	2
• Finds point of intersection, or writes volume as a difference of two volumes, or equivalent merit	1

Sample answer:

The curves intersect when

$$\cos 2x = \sin x$$

$$1 - 2\sin^2 x = \sin x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\therefore\ \sin x = \frac{1}{2},\ (\sin x \neq -1).$$

$$\therefore\ x = \frac{\pi}{6}$$

The volume is given by

$$V = \pi \int_0^{\frac{\pi}{6}} \cos^2 2x\, dx - \pi \int_0^{\frac{\pi}{6}} \sin^2 x\, dx$$

$$= \pi \left(\int_0^{\frac{\pi}{6}} \frac{1 + \cos 4x}{2}\, dx - \int_0^{\frac{\pi}{6}} \frac{1 - \cos 2x}{2}\, dx \right)$$

$$= \pi \left[\frac{\sin 4x}{8} + \cancel{\frac{x}{2}} - \cancel{\frac{x}{2}} + \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{6}}$$

$$= \pi \left(\frac{\sin \frac{2\pi}{3}}{8} + \frac{\sin \frac{\pi}{3}}{4} \right)$$

$$= \frac{\pi}{8} \left(\frac{\sqrt{3}}{2} + \sqrt{3} \right)$$

$$= \frac{3\pi\sqrt{3}}{16}$$

Question 13 (c) (i)

Criteria	Marks
• Provides correct solution	4
• Obtains one correct derivative and makes some progress towards obtaining the other, or equivalent merit	3
• Obtains one correct derivative or attempts both, or equivalent merit	2
• Attempts one derivative, or equivalent merit	1

Sample answer:

$$f'(x) = \sec^2\left(\cos^{-1} x\right) \cdot \frac{-1}{\sqrt{1-x^2}}$$

$$= \frac{1}{\cos^2\left(\cos^{-1} x\right)} \cdot \frac{-1}{\sqrt{1-x^2}}$$

$$= \frac{-1}{x^2\sqrt{1-x^2}}$$

$$g'(x) = \frac{-\frac{x}{\sqrt{1-x^2}} \times x - \sqrt{1-x^2} \times 1}{x^2}$$

$$= \frac{-\cancel{x^2} - \left(1 - \cancel{x^2}\right)}{x^2\sqrt{1-x^2}}$$

$$= \frac{-1}{x^2\sqrt{1-x^2}}$$

$\therefore\ f'(x) = g'(x)$

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution for both parts of the domain	3
• Provides a correct solution for one part of the domain, or equivalent merit	2
• Observes that $f(x) - g(x)$ is a constant, or equivalent merit	1

Sample answer:

$$f'(x) = \frac{-1}{x^2\sqrt{1-x^2}} = g'(x)$$

And so $f'(x) - g'(x) = 0$

$$\Rightarrow f(x) - g(x) \text{ is a constant}$$

For $x < 0$, say $x = -1$

$$f(-1) - g(-1) = \tan\left(\cos^{-1}(-1)\right) - \frac{\sqrt{1-(-1)^2}}{(-1)}$$

$$= \tan(\pi) + 0$$

$$= 0$$

So for $x < 0$, $f(x) = g(x)$.

For $x > 0$, say $x = 1$,

$$f(1) - g(1) = \tan\left(\cos^{-1}(1)\right) - \frac{\sqrt{1-(1)^2}}{(1)}$$

$$= \tan(0) + 0$$

$$= 0$$

So for $x > 0$, $f(x) = g(x)$

So for all x in the domain

$f(x) = g(x)$

Question 14 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Identifies coefficient of x^n on the left hand side, or equivalent merit	1

Sample answer:

The left hand side is $\binom{2n}{n}$ which is the coefficient of x^n in the expansion of $(1+x)^{2n}$.

This means that $\binom{n}{0}\binom{n}{0}+\binom{n}{1}\binom{n}{1}+\cdots+\binom{n}{n}\binom{n}{n}$ should be the coefficient of x^n in the expansion of $(1+x)^n(1+x)^n$.

The first term, $\binom{n}{0}$ is the constant term in the expansion of $(1+x)^n$ and so should be multiplied by an x^n term. But $\binom{n}{0}=\binom{n}{n-0}=\binom{n}{n}$ is equal to the coefficient of x^n in the expansion $(1+x)^n$.

Thus $\binom{n}{0}^2$ is the coefficient of the x^n term that comes from the constant in the expansion of $(1+x)^n$ times the coefficient of the x^n term in the expansion of $(1+x)^n$.

Similarly $\binom{n}{1}^2$ is the coefficient of the x^n term that comes from the coefficient of x term in the expansion of $(1+x)^n$ times the coefficient of the x^{n-1} term in the expansion of $(1+x)^n$.

Therefore, the right hand side is the coefficient of the x^n term in the expansion of $(1+x)^n(1+x)^n$.

OR

Question 14 (a) (i) (continued)

The coefficient of x^n in expansion of $(1+x)^{2n}$ is $\binom{2n}{n}$.

The coefficient of x^n in $(1+x)^n(1+x)^n$ is

$$=\binom{n}{0}\binom{n}{n}+\binom{n}{1}\binom{n}{n-1}+\cdots+\binom{n}{k}\binom{n}{n-k}+\cdots+\binom{n}{n}\binom{n}{0}$$

But $\binom{n}{n-k}=\binom{n}{k}$

So the coefficient is $\binom{n}{0}\binom{n}{0}+\binom{n}{1}\binom{n}{1}+\cdots+\binom{n}{n}\binom{n}{n}$

Hence $\binom{2n}{n}=\binom{n}{0}^2+\binom{n}{1}^2+\cdots+\binom{n}{n}^2$

Question 14 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Explains why one of the terms is $\binom{n}{k} \times \binom{n}{k}$, or equivalent merit	1

Sample answer:

We can choose:

0 men and 0 women in $\binom{n}{0}\binom{n}{0} = \binom{n}{0}^2$ ways

or 1 man and 1 woman in $\binom{n}{1}^2$ ways

$\vdots$

or n men and n women in $\binom{n}{n}^2$ ways

Hence the total number of ways is

$$\binom{n}{0}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n} \qquad \text{from part (i)}$$

Question 14 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Explains why one of the terms is $k^2\binom{n}{k}^2$, or equivalent merit	1

Sample answer:

We can choose 1 woman and 1 leader in $\binom{n}{1}\times 1$ ways and similarly for the men. This gives

$\binom{n}{1}\times 1\times\binom{n}{1}\times 1=\binom{n}{1}^2\times 1^2$ for this case.

We can choose 2 women and 1 leader in $\binom{n}{2}\binom{2}{1}=\binom{n}{2}\times 2$ ways and similarly for the men.

This gives $\binom{n}{2}\times 2\times\binom{n}{2}\times 2=\binom{n}{2}^2\times 2^2$ for this case.

$$\vdots$$

We can choose n women and 1 leader in $\binom{n}{n}\binom{n}{1}=\binom{n}{n}\times n$ ways and similarly for the men.

This gives $\binom{n}{n}\times n\times\binom{n}{n}\times n=\binom{n}{n}^2\times n^2$ for this case.

And so the total is $1^2\binom{n}{1}^2+2^2\binom{n}{2}^2+\cdots+n^2\binom{n}{n}^2$.

Question 14 (a) (iv)

Criteria	Marks
• Provides correct solution	2
• Recognises that choosing the leaders first reduces the problem to using part (ii) with $n - 1$ women and $n - 1$ men, or equivalent merit	1

Sample answer:

There are n ways to choose a woman leader, leaving $(n - 1)$ women from which to choose.

There are n ways to choose a man leader, leaving $(n - 1)$ men from which to choose.

By part (ii) there are $\binom{2(n-1)}{(n-1)}$ ways to choose the $(n - 1)$ women and $(n - 1)$ men.

Hence $1^2\binom{n}{1}^2 + 2^2\binom{n}{2}^2 + \cdots + n^2\binom{n}{n}^2 = n^2\binom{2n-2}{n-1}$.

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Expands $\sin(2\theta + \theta)$, or equivalent merit	1

Sample answer:

$$\begin{aligned}\sin 3\theta &= \sin(2\theta + \theta)\\ &= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta\\ &= 2\sin\theta \cos^2\theta + \left(1 - 2\sin^2\theta\right)\sin\theta\\ &= 2\sin\theta\left(1 - \sin^2\theta\right) + \sin\theta - 2\sin^3\theta\\ &= 3\sin\theta - 4\sin^3\theta\end{aligned}$$

$$\therefore 4\sin^3\theta - 3\sin\theta + \sin(3\theta) = 0$$

$$\therefore \sin^3\theta - \frac{3}{4}\sin\theta + \frac{\sin(3\theta)}{4} = 0$$

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Obtains $\sin^3\theta - \frac{3}{4}\sin\theta + \frac{8}{64} = 0$, or equivalent merit	1

Sample answer:

$$x^3 - 12x + 8 = 0$$

$$x = 4\sin\theta, \text{ so}$$

$$64\sin^3\theta - 48\sin\theta + 8 = 0$$

$$\therefore \sin^3\theta - \frac{3}{4}\sin\theta + \frac{8}{64} = 0$$

comparing with (i),

$$\frac{8}{64} = \frac{\sin 3\theta}{4} \quad \Rightarrow \quad \sin 3\theta = \frac{32}{64} = \frac{1}{2}.$$

Question 14 (b) (iii)

Criteria	Marks
• Provides correct proof	3
• Obtains all the roots of the cubic, or equivalent merit	2
• Obtains $4\sin\frac{\pi}{18}$ as a solution of the cubic in part (b) (ii), or equivalent merit	1

Sample answer:

From $\quad \sin 3\theta = \frac{1}{2}$

we have $\quad 3\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{6} + 2\pi, \frac{5\pi}{6} + 2\pi, \frac{\pi}{6} + 4\pi, \ldots$

$$\theta = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \ldots$$

So $4\sin\frac{\pi}{18},\ 4\sin\frac{5\pi}{18},\ 4\sin\frac{13\pi}{18}$ etc are roots of the cubic, but they are not all distinct.

The angles $\frac{\pi}{18}$ and $\frac{5\pi}{18}$ are distinct angles in the first quadrant and so have different positive sine values. The angle $\frac{25\pi}{18}$ is in the third quadrant and so has a negative sine value. Hence we can take $\alpha = 4\sin\frac{\pi}{18},\ \beta = 4\sin\frac{5\pi}{18},\ \gamma = 4\sin\frac{25\pi}{18}$ as 3 distinct roots,

$$\begin{aligned}\text{Now} \quad \alpha^2 + \beta^2 + \gamma^2 &= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma) \\ &= 0^2 + 2 \times 12 \\ &= 24\end{aligned}$$

$$\therefore\ 16\left(\sin^2\frac{\pi}{18} + \sin^2\frac{5\pi}{18} + \sin^2\frac{25\pi}{18}\right) = 24$$

$$\therefore\ \sin^2\frac{\pi}{18} + \sin^2\frac{5\pi}{18} + \sin^2\frac{25\pi}{18} = \frac{24}{16} = \frac{3}{2}.$$

2020 HSC Mathematics Extension 1 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	ME-F1 Further Work with Functions	ME11–2
2	1	ME-F1 Further Work with Functions	ME11–1
3	1	ME-C2 Further Calculus Skills	ME12–1
4	1	ME-V1 Introduction to Vectors	ME12–2
5	1	ME-F2 Polynomials	ME11–1
6	1	ME-V1 Introduction to Vectors	ME12–2
7	1	ME-C3 Applications of Calculus	ME12–4
8	1	ME-A1 Working with Combinatorics	ME11–5
9	1	ME-V1 Introduction to Vectors	ME12–2
10	1	ME-C1 Rates of Change	ME11–4

Section II

Question	Marks	Content	Syllabus outcomes
11 (a) (i)	1	ME-F2 Polynomials	ME11–2
11 (a) (ii)	2	ME-F2 Polynomials	ME11–2
11 (b)	3	ME-V1 Introduction to Vectors	ME12–2
11 (c)	3	ME-F1 Further Work with Functions	ME11–2
11 (d)	4	ME-T3 Trigonometric Equations	ME12–3
11 (e)	2	ME-C3 Applications of Calculus	ME12–4
12 (a)	3	ME-P1 Proof by Mathematical Induction	ME12–1
12 (b) (i)	1	ME-S1 The Binomial Distribution	ME12–5
12 (b) (ii)	1	ME-S1 The Binomial Distribution	ME12–5
12 (b) (iii)	1	ME-S1 The Binomial Distribution	ME12–5
12 (c)	2	ME-A1 Working with Combinatorics	ME11–5, ME12-7
12 (d)	3	ME-T2 Further Trigonometric Identities ME-C2 Further Calculus Skills	ME12–1
12 (e)	3	ME-C3 Applications of Calculus	ME12–4
13 (a) (i)	1	ME-C2 Further Calculus Skills	ME12–1

Question	Marks	Content	Syllabus outcomes
13 (a) (ii)	4	ME-C2 Further Calculus Skills	ME12–1
13 (b)	4	ME-C3 Applications of Calculus	ME12–4
13 (c) (i)	4	ME-C2 Further Calculus Skills	ME12–4
13 (c) (ii)	3	ME-C2 Further Calculus Skills	ME12–4, ME 12 -7
14 (a) (i)	2	ME-A1 Working with Combinatorics	ME11–5
14 (a) (ii)	2	ME-A1 Working with Combinatorics	ME11–5
14 (a) (iii)	2	ME-A1 Working with Combinatorics	ME11–5
14 (a) (iv)	2	ME-A1 Working with Combinatorics	ME11–5
14 (b) (i)	2	ME-T3 Trigonometric Equations	ME12–3
14 (b) (ii)	2	ME-T3 Trigonometric Equations	ME12–3
14 (b) (iii)	3	ME-F2 Polynomials ME-T3 Trigonometric Equations	ME12–3

Mathematics Extension 1

HSC Marking Feedback 2020

Question 11

Part (a) (i)

Students should:

- apply their knowledge of function notation to substitute and simplify the expression, noting the value of the question to realise only one step was required.

In better responses, students were able to:

- obtain the result efficiently.

Areas for students to improve include:

- substituting into algebraic expressions
- demonstrating their knowledge or understanding of function notation.

Part (a) (ii)

Students should:

- use the result given in part (i) and their knowledge of the factor theorem to find the quadratic factor. This could be achieved by either long division, inspection or applying knowledge of the relationships between coefficients and roots of an equation.

In better responses, students were able to:

- recognise the linear factor and perform a long division to obtain the quadratic factor, or
- factorise the expression by inspection by using the given statement in part (i).

Areas for students to improve include:

- displaying their understanding of the factor theorem
- using their skills with long division of polynomials.

Part (b)

Students should:

- define, calculate and use the dot product of two vectors to examine the properties of perpendicular vectors. Alternatively, use the gradients of position vectors to determine properties of the two vectors.

In better responses, students were able to:

- state the property of perpendicular vectors
- apply the dot product formula from the Reference Sheet and solve the resultant quadratic equation
- state the relationship between the gradients of perpendicular position vectors and solve the resultant equation.

Areas for students to improve include:

- using their knowledge and understanding of vectors
- demonstrating the use and meaning of vector notation
- knowing how to apply the dot product or gradient formula to determine if vectors are perpendicular
- understanding methods of solving quadratic equations.

Part (c)

Students should:

- examine the relationship between $y = f(x)$ and $y = \frac{1}{f(x)}$, and recognise features to assist with graphing the functions.

In better responses, students were able to:

- initially recognise asymptotes and the y-intercept of $y = \frac{1}{f(x)}$
- use these features to graph the function $y = \frac{1}{f(x)}$.

Areas for students to improve include:

- understanding the relationship between functions and their reciprocal function
- avoiding the attempt to find the equation of the function and its reciprocal
- stating the important features of the function and hence its reciprocal
- knowing the difference between reciprocal and inverse functions.

Part (d)

Students should:

- convert an equation of the form $a\cos x + b\sin x$ to $A\sin(x + \alpha)$
- apply this to solving an equation of the form $a\cos x + b\sin x = c$.

In better responses, students were able to:

- efficiently find values for A and α in exact form
- obtain the solutions to the equation within the given domain.

Areas for students to improve include:

- using the Reference Sheet to expand $A\sin(x + \alpha)$ and finding exact values
- solving simultaneous equations

- developing their skills in solving trigonometric equations in a given domain.

Part (e)

Students should:

- solve a differential equation of the form $\frac{dy}{dx} = f(x)$.

In better responses, students were able to:

- manipulate the differential equation into a form that can be integrated
- integrate an exponential function
- realise there was no need to rewrite with y as the subject.

Areas for students to improve include:

- considering the mark value of the question and using it as a guide to the amount of working required
- rewriting a differential equation by separating the variables
- improving their knowledge of the difference between integration and differentiation of exponential functions.

Question 12

Part (a)

Students should:

- show their substitution when verifying the base case
- understand that only one term of the series is needed for $n = 1$
- take care when working with brackets
- show all steps in the proof.

In better responses, students were able to:

- deal with the LHS and RHS separately for base case $n = 1$ and for $n = k + 1$
- clearly articulate the inductive hypothesis for $n = k$
- state what they were required to prove when $n = k + 1$
- use factorisation rather than expansion when proving true for $n = k + 1$.

Areas for students to improve include:

- factorising algebraic expressions
- exercising care when working with brackets
- ensuring that each step of the proof is algebraically correct
- correctly use sigma notation, should they choose to use it
- setting out proofs in a logical sequence
- working from one side to the other when attempting proofs, and not manipulate both sides simultaneously.

Part (b)(i)

Students should:

- understand the parameters associated with binomial distributions, ie $X \sim \mathrm{Bin}(100, 0.6)$
- identify correct formula from Reference Sheet.

In better responses, students were able to:

- use the correct formula to arrive at the answer.

Areas for students to improve include:

- familiarising themselves with the Reference Sheet.

Part (b)(ii)

Students should:

- understand the relationship between variance and standard deviation
- calculate the variance and hence, the standard deviation.

In better responses, students were able to:

- substitute correctly in the formula for variance
- show that the standard deviation is $\sqrt{24}$.

Areas for students to improve include:

- focusing on using the Reference Sheet correctly.

Part (b)(iii)

Students should:

- link the probability to the standardised z-scores in normal distributions
- read-off the probability from the Reference Sheet.

In better responses, students were able to:

- use the empirical rule found in the Reference Sheet rather than attempting to calculate probabilities from z-scores.

Areas for students to improve include:

- reading the question carefully to ascertain when and how normal distribution approximation should be used.

Part (c)

Students should:

- evaluate the correct expression for ${}^{n}C_{r}$
- identify and quantify the pigeons and pigeonholes

- be prepared to answer questions on the Year 11 content in the HSC.

In better responses, students were able to:

- work out that a minimum of 393 students are needed to ensure that at least eight students pass exactly the same three topics
- establish that 392 pigeons are needed to equally fill each pigeonhole with 7 pigeons and the 8 remaining pigeons could be added to any one or more of the pigeonholes.

Areas for students to improve include:

- clearly explaining the pigeonhole principle
- understanding the ceiling function (or why we round up) when using the pigeonhole principle
- solving the problem using logic and sound reasoning, instead of simply using a formula with the hope of getting the correct answer
- practicing interpreting worded problems
- thoroughly revising Year 11 content.

Part (d)

Students should:

- be familiar with trigonometric identities on the Reference Sheet
- know the rules of integration: the integral of products is not always equal to the product of the integrals
- check results when integrating trigonometric functions, paying particular attention to the signs and fractional coefficients involved.

In better responses, students were able to:

- use the appropriate transformation and provide the correct solution in concise manner.

Areas for students to improve include:

- recognising when to use trigonometric identities
- ensuring accuracy when working with negatives and fractions.

Part (e)

Students should:

- understand the need and process of separation of variables when dealing with differential equations
- not assume that the subject of the equation must be y, or that the function must be linear.

In better responses, students were able to:

- use definite integrals to arrive at the solution, thereby obviating the need to work with constants of integration
- use only one constant of integration
- realise that the solution does not have to be a function.

Areas for students to improve include:

- understanding that differential equations should not be integrated where there are multiple variables on one side
- avoiding placing constants of integration on both sides as this can cause confusion when evaluating the constant of the primitive
- focusing on algebraic skills when dealing with negative numbers, particularly when substituting and solving equations.

Question 13

Part (a) (i)

Students should:

- be able to differentiate $3\sin^3\theta$, using the chain rule.

In better responses, students were able to:

- correctly use the chain rule to differentiate and arrive at the correct answer.

Areas for students to improve include:

- enhancing their knowledge and application of the chain rule.

Part (a) (ii)

Students should:

- find the limits of integration in terms of θ
- substitute the trigonometric ratios correctly, obtaining a simplified integrand
- integrate the trigonometric expression, and substitute the limits of integration to obtain a simplified numerical solution
- follow the instruction in the question to substitute
- recognise the connection to part (i) of the question.

In better responses, students were able to:

- find the new limits of integration and substitute correctly
- successfully integrate and evaluate the limits of integration to arrive at the correct numerical solution
- simplify the resulting equation into a form which allowed the substitution of the result from part (i).

Areas for students to improve include:

- realising the need for finding the derivative of $\tan\theta$ to replace dx
- finding limits in terms of θ integrating trigonometric ratios correctly
- relating part (ii) to part (i) to obtain a simplified integrand
- substituting the limits of integration into the required trigonometric ratios to obtain a simplified numerical expression, ie correct algebraic manipulation.

Part (b)

Students should:

- find the point of intersection of the two functions
- write the required volume as a difference of the volumes of the two functions
- use the double angle formula of $\cos 2\theta$ correctly
- substitute the limits of integration into the required trigonometric ratios to obtain a simplified numerical expression.

In better responses, students were able to:

- find and chose the correct the point of intersection
- correctly use the double angle formula
- correctly integrate the functions, and correctly manipulate the limits of integration to obtain the correct numerical solution.

Areas for students to improve include:

- realising the regions of the functions and hence writing the correct order of the difference of the volumes
- using the Reference Sheet to determine the correct forms to use for both $sin^2(nx)$ and $cos^2(nx)$
- knowing that the volume of the solid of revolution is the difference of the squares of each function, rather than the difference of the functions all squared.

Part (c) (i)

Students should:

- differentiate $\tan(\cos^{-1}(x))$ using the chain rule, and $\frac{\sqrt{1-x^2}}{x}$ using the quotient rule
- either use a drawing of a right-angled triangle, or simplify $\sec^2(\cos^{-1} x)$, to arrive at $\frac{1}{x^2}$
- manipulate the quotient rule to create a simplified fraction involving a square root expression in the denominator.

In better responses, students were able to:

- find both derivatives correctly
- correctly manipulate the fraction involving surds
- find the derivative of $\tan(\cos^{-1}(x))$
- arrive at the correct derivatives and equate them.

Areas for students to improve include:

- reading the question carefully to understand that they had to equate the derivatives of both functions by manipulating and further simplifying the derived functions
- taking time to build the answer step by step
- manipulating surds as fractions
- taking care with algebraic processes to ensure that the expressions from one line to the next remain equivalent.

Part (c) (ii)

Students should:

- recognise the need to use part (i) to find the functions of each derivative
- observe that the difference of both functions will be a constant, through the process of integration
- recognise that the difference of the derivatives should be 0
- recognise there is a restricted domain
- test a point on each side of the domain of both functions to arrive at the same answer.

In better responses, students were able to:

- recognise the integrals of the two functions are equal to a constant
- equate the derivative of both functions
- acknowledge there is a domain restriction to both functions
- test on both sides of the domain to prove the functions are equal.

Areas for students to improve include:

- focusing on the setting out of formal proofs
- recognising and considering the natural domain restrictions of trigonometric as well as rational functions
- testing both sides of the domain for each function to enable them to arrive at a correct conclusion
- recognising the constant of integration.

Question 14

Part (a) (i)

Students should:

- recognise that the coefficients of terms in each of these expansions will be equal
- note that the term on the LHS of the required result is the coefficient of x^n in the expansion of $(1+x)^{2n}$.

In better responses, students were able to:

- clearly state that the coefficient of x^n in expansion of $(1+x)^n(1+x)^n$ is
$$\binom{n}{0}\binom{n}{n}+\binom{n}{1}\binom{n}{n-1}+\binom{n}{k}\binom{n}{n-k}+\cdots+\binom{n}{n}\binom{n}{0}$$
- explain why $\binom{n}{n}=\binom{n}{0}$ etc, using $\binom{n}{k}=\binom{n}{n-k}$.

Areas for students to improve include:

- demonstrating knowledge of the binomial expansions of these expressions.
- using the Pascal triangle relationship represented by $\binom{n}{k}=\binom{n}{n-k}$.

Part (a) (ii)

Students should:

- ensure that they fully justify how they are deriving the (supplied) answer.

In better responses, students were able to:

- explain, using a pattern, how each case could be generated
- add these cases together to derive the required result.

Areas for students to improve include:

- reading the question carefully to gain a full understanding of the situation
- taking time to build the answer step by step
- knowing when cases should be multiplied and added.

Part (a) (iii)

Students should:

- ensure that they fully justify how they are deriving the (supplied) answer.

In better responses, students were able to:

- explain, using a pattern, how each case could be generated
- clearly differentiate between the leader and the group formations
- add these cases together to derive the required result.

Areas for students to improve include:

- reading the question carefully to gain a full understanding of the situation
- taking time to build the answer step by step
- knowing when cases should be multiplied and added.

Part (a) (iv)

Students should:

- note that the choices for the leadership group will be the same in all cases
- recognise that the number of men and women from which to choose the groups has now been reduced by one
- recognise that the reversed method will, ultimately, generate an equivalent number of choices as the method in part (iii).

In better responses, students were able to:

- express the choices for leaders as $\binom{n}{1}^2 = n^2$
- express the choices for the members of the groups in the form $\binom{n-1}{k}^2$
- multiply these parts to generate an expression for each individual case
- add these cases together to derive a correct result

- use the result from part (i) to simplify this summation.

Areas for students to improve include:

- reading the question carefully to gain a full understanding of the situation
- taking time to build the answer step by step
- knowing when cases should be multiplied and added
- realising that parts of questions are often connected, finding these connections and using them.

Part (b) (i)

Students should:

- recognise the need to apply trigonometric identities.

In better responses, students were able to:

- apply relevant trigonometric identities correctly in a manner such that the expression is simplified
- outline their proof with clear and correct algebraic steps.

Areas for students to improve include:

- focusing on the setting out of formal proofs
- taking care with algebraic processes to ensure that the expressions from one line to the next remain equivalent.

Part (b) (ii)

Students should:

- follow the instruction in the question to substitute
- recognise the connection to part (i) of the question.

In better responses, students were able to:

- correctly make the suggested substitution
- simplify the resulting equation into a form which allows the substitution of the result from part (i)
- correctly simplify this equation to obtain the required result.

Areas for students to improve include:

- demonstrating skills in substitution
- demonstrating skills in algebraic simplification.

Part (b) (iii)

Students should:

- recognise a question involving sum and product of roots results
- recognise connections to previous parts of questions.

In better responses, students were able to:

- correctly solve the $\sin(3\theta) = \frac{1}{2}$ equation from part (ii)
- use their solutions to generate roots of the cubic equation in the form $x = 4\sin\theta$
- correctly justify why the three (supplied) roots are distinct
- apply the sum and product of roots results to achieve the required result.

Areas for students to improve include:

- recognising that the first step of this part is to continue from the result in part (ii) of the question
- understanding how to determine which roots are repeated and which are distinct
- showing the ability to generate the $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ result.