Student Name:……………………………..

2021 Higher School Certificate
Trial Examination
Mathematics Extension I

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I - 10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II - 60 marks
- Attempt Questions 11-14
- Allow about 1 hour and 45 minutes

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet provided for Questions 1-10.

1 Which one of the following describes a Bernoulli random variable?

(A) The streaming service (Netflix, Disney+ etc..) that a person subscribes to.

(B) The number of points scored in a game by a basketball team.

(C) Picking a strawberry milkshake from a menu that lists strawberry, chocolate and vanilla milkshakes.

(D) The number of hours a person spends on their mobile phone in a week.

2 A function is defined by $f(x) = (x+3)^2 + 1$ for $x \le -3$.
What is the inverse function of $f(x)$?

(A) $f^{-1}(x) = -3 - \sqrt{x-1}$

(B) $f^{-1}(x) = -3 + \sqrt{x-1}$

(C) $f^{-1}(x) = -3 \pm \sqrt{x-1}$

(D) $f^{-1}(x) = \dfrac{1}{(x+3)^2 + 1}$

3 A circle is represented by the parametric equations $x = 2 + 4\cos\theta, \quad y = 3 + 4\sin\theta$.

What are the centre and radius of the circle?

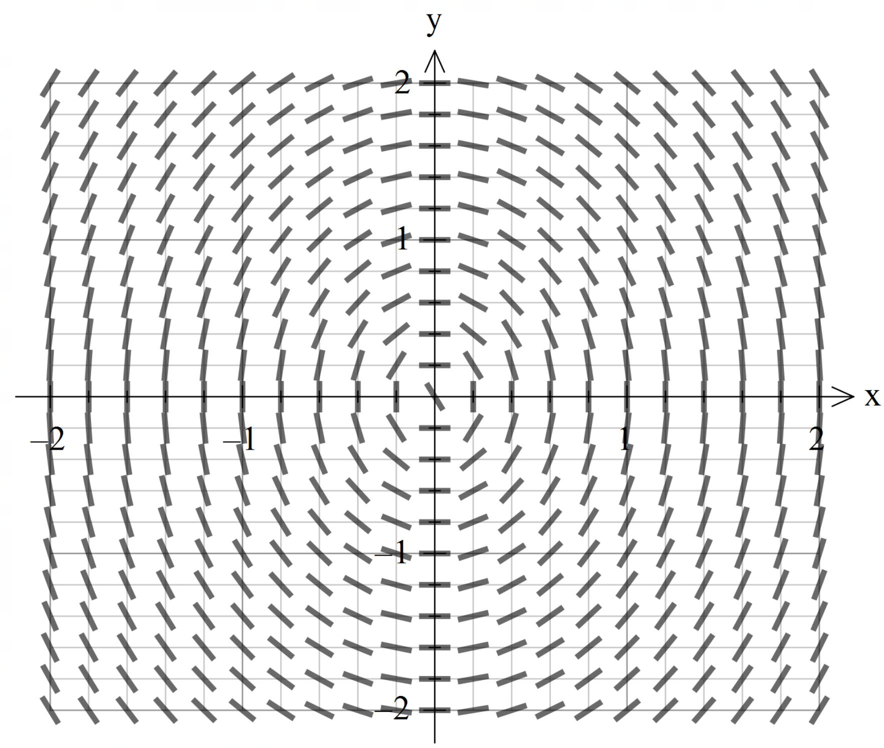
(A) Centre $(-2,-3)$, radius 4.

(B) Centre $(-2,-3)$, radius 16.

(C) Centre $(2,3)$, radius 4.

(D) Centre $(2,3)$, radius 16.

4 The slope field for a first order differential equation is shown.



What could be the differential equation represented?

(A) $\dfrac{dy}{dx} = -\dfrac{2y}{x}$

(B) $\dfrac{dy}{dx} = \dfrac{-2x}{y}$

(C) $\dfrac{dy}{dx} = \dfrac{2y}{x}$

(D) $\dfrac{dy}{dx} = \dfrac{2x}{y}$

5 Given that $\sin x = \dfrac{3}{5}$ what is the exact value of $\sec 2x$?

(A) $\dfrac{25}{24}$

(B) $\pm\dfrac{25}{24}$

(C) $\dfrac{25}{7}$

(D) $\pm\dfrac{25}{7}$

6 What is the derivative of $\cos^{-1} 4x$?

(A) $\dfrac{-1}{\sqrt{4-x^2}}$

(B) $\dfrac{1}{\sqrt{4-x^2}}$

(C) $\dfrac{-4}{\sqrt{1-16x^2}}$

(D) $\dfrac{4}{\sqrt{1-16x^2}}$

7 What is the angle, correct to the nearest degree, between vectors $\underset{\sim}{a} = 4\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$?

(A) 1°

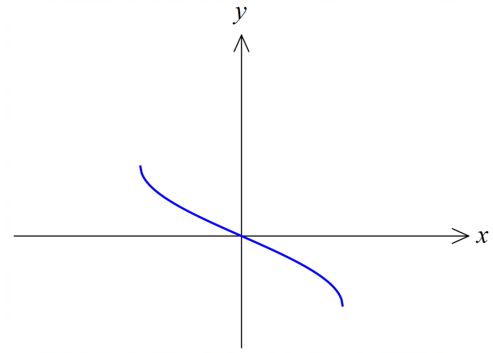
(B) 19°

(C) 20°

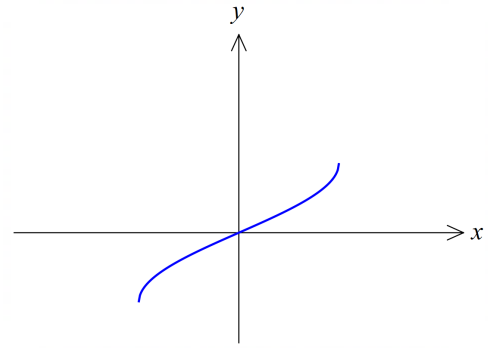
(D) 70°

8 Which graph best represents the function $y = 2\cos^{-1}(-x)$?

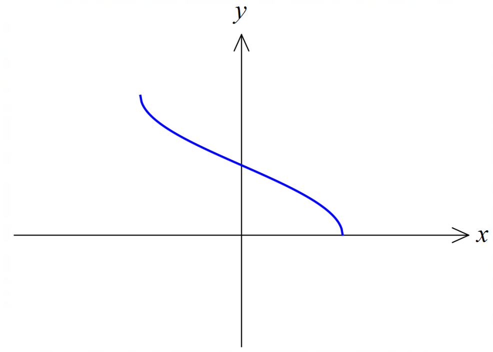
(A)



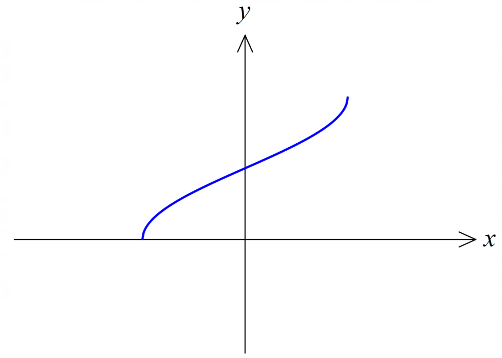
(B)



(C)



(D)



9 What is the projection of $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ on to $\underset{\sim}{b} = \underset{\sim}{i} - 4\underset{\sim}{j}$?

(A) $-\frac{20}{17}\underset{\sim}{i} - \frac{30}{17}\underset{\sim}{j}$

(B) $-\frac{10}{17}\underset{\sim}{i} + \frac{40}{17}\underset{\sim}{j}$

(C) $-\frac{20}{13}\underset{\sim}{i} - \frac{30}{13}\underset{\sim}{j}$

(D) $-\frac{10}{13}\underset{\sim}{i} + \frac{40}{13}\underset{\sim}{j}$

10 What is the value of k such that $\displaystyle\int_0^k \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{\pi}{18}$?

(A) -3

(B) $-\dfrac{1}{3}$

(C) $\dfrac{1}{3}$

(D) 3

Section II

60 marks
Attempt Question 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (14 marks) Use a SEPARATE Writing Booklet.

(a) There are 96 students in a school who have a birthday in October.
What is the minimum number of these students that share the same birthday? **1**

(b) Solve $\dfrac{2}{x-1} \geq -3$ **3**

(c) Points A, B and C have position vectors $\left(3\underset{\sim}{i}+2\underset{\sim}{j}\right)$, $\left(\underset{\sim}{i}-4\underset{\sim}{j}\right)$ and $\left(8\underset{\sim}{i}+5\underset{\sim}{j}\right)$ respectively. **2**

Given that $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{BD}$ find the position vector of D.

(d) Given that $\cos\left(x+\dfrac{\pi}{6}\right)=3\cos\left(x-\dfrac{\pi}{6}\right)$, find the exact value of $\tan x$. **3**

(e) (i) Given that $(x-3)$ and $(x+2)$ are factors of $f(x)=x^3+x^2+px+q$, find the values of p and q. **2**

(ii) Factorise $f(x)$ completely. **2**

(iii) For what values of x is $f(x)<0$?
Give your answer using interval notation. **1**

End of Question 11

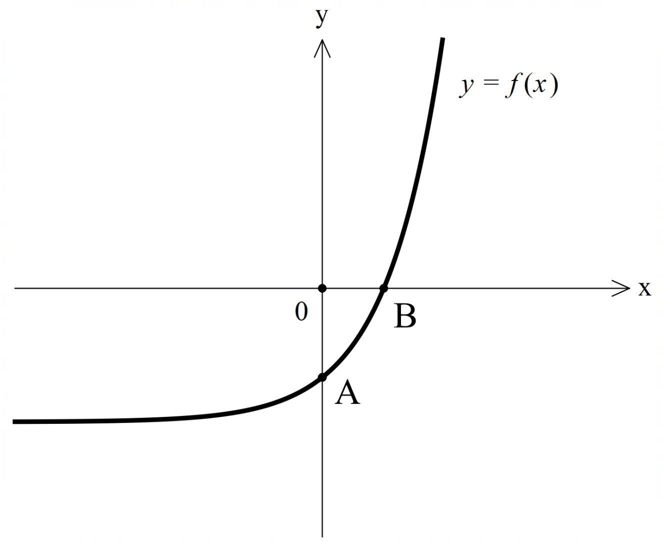
Please turn over

Question 12 (15 marks) Use a SEPARATE Writing Booklet.

(a) Solve the equation $\cos 2x = \sin x$ for $0 \le x < 2\pi$. **3**

(b) The diagram shows a sketch of part of the curve with equation $y = f(x)$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B\left(\frac{1}{2}\ln k, 0\right)$ where $k > 1$.



On separate diagrams, neatly sketch the following curves.

Your diagrams should include the coordinates of any points of intersection with the axes.

(i) $y = |f(x)|$ **2**

(ii) $y = f^{-1}(x)$ **2**

(c) In an orchard, it is known that 10% of the fruit is likely to be spoiled and unfit for consumption. **3**

A farmer picks 30 pieces of fruit. What is the probability that at most two pieces of fruit are spoiled?

Give your answer to three significant figures.

Question 12 continues on page 9

Question 12 (continued)

(d) A cup of water is cooling. Its initial temperature is $100°\text{C}$ and after 3 minutes, the temperature is $80°\text{C}$.

The rate at which the water is cooling follows Newton's law, that is

$$\frac{dT}{dt} = -k\left(T - 25\right)$$

where T represents the temperature of the water at time t minutes.

(i) Show that $T = 25 + Ae^{-kt}$ is a solution of the equation $\frac{dT}{dt} = -k\left(T - 25\right)$. **1**

(ii) What is the temperature of the water after 5 minutes, correct to three significant figures? **3**

(iii) What is the temperature of the water in the long term? **1**

End of Question 12

Please turn over

Question 13 (15 marks) Use a SEPARATE Writing Booklet.

(a) Use the principle of mathematical induction to show that for all integers $n \geq 1$, **3**

$$1+8+27+....+n^3 = \frac{1}{4}n^2(n+1)^2$$

(b) Use the substitution $u=\sqrt{x+2}$ to find the exact value of $\int_{-1}^{7} \frac{x^2}{\sqrt{x+2}}\,dx.$ **3**

(c) In a particular circuit, the current I amperes is given by

$$I = 4\sin\theta - 3\cos\theta$$

where θ, $(\theta>0)$ is an angle related to the voltage.

(i) By expressing I in the form $I = R\sin(\theta-\alpha),\quad R>0,\quad 0 \leq \alpha \leq 360°$, or otherwise, find the greatest value of I. **4**

(ii) Find the value of θ for which this greatest value of I occurs. **1**

(d) A drop of oil is modelled as a circle of radius r at time t such that

$$\frac{dA}{dt} = \frac{A^{3/2}}{t^2} \qquad t>0$$

(i) Given that the area of the drop is 1 at $t=1$, show that $A = \frac{4t^2}{(1+t)^2}$ **3**

(ii) Show that the value of A cannot exceed 4. **1**

Question 14 (16 marks) Use a SEPARATE Writing Booklet.

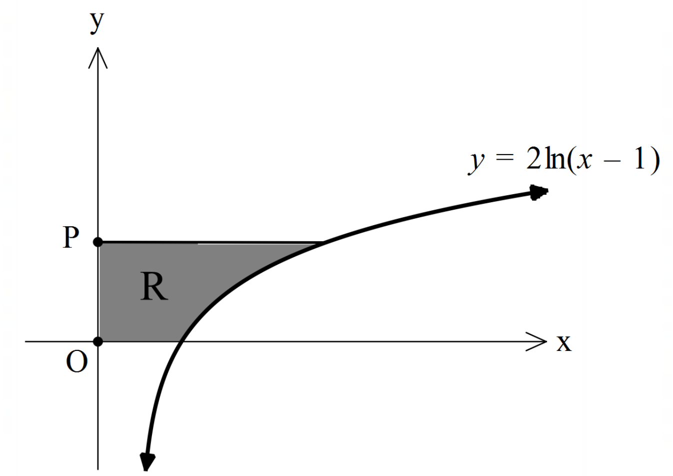
(a) The velocity of a particle, after t seconds of travel, is given by **3**

$$\underset{\sim}{v} = (3\sin 2t)\underset{\sim}{i} + (\cos 2t - 3)\underset{\sim}{j} \ \text{ ms}^{-1}$$

where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors in the horizontal and vertical directions respectively.

Find the magnitude of the acceleration of the particle when $t = \dfrac{\pi}{6}$.

(b) The diagram below shows the curve with equation $y = 2\ln(x-1)$.



The point P has coordinates $(0, p)$. The shaded region R is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$. The units of the axes are centimetres.

The region R is rotated completely about the y-axis to form a solid.

(i) Show that the volume, V cm^3, of the solid is given by **3**

$$V = \pi\left(e^{p} + 4e^{\frac{1}{2}p} + p - 5\right)$$

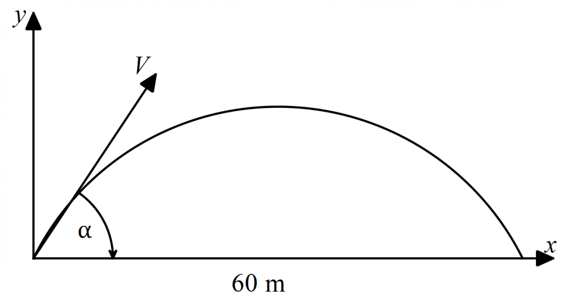
(ii) It is given that the point P is moving in the positive direction along the y-axis at a constant rate of 0.2 cm/min. **2**

Find the rate at which the volume of the solid is increasing at the instant when $p = 4$, giving your answer correct to 2 significant figures.

Question 14 continues on page 12

Question 14 (continued)

(c) A cricket player hits a ball from ground level. The ball is projected under gravity with speed V ms^{-1} in a direction making an angle α with the horizontal. The ball lands on the boundary, which is 60 metres away. **3**



The equations of motion of the ball are $x = Vt\cos\alpha$ and $y = -\frac{1}{2}gt^2 + Vt\sin\alpha$ (do **NOT** prove these), where $g = 9.8\ \text{ms}^{-2}$.

If the angle of flight to the horizontal is $28°$ at the instant the ball leaves the bat, calculate the initial speed of the ball.

(d) In a certain school, 23% of Year 12 students study Mathematics Extension 1.

(i) If X represents the number of students who study Mathematics Extension 1, describe the skewness of the binomial distribution for $P(X = x)$. You must give reasons for your answer. **1**

(ii) The Principal meets with a random sample of 60 Year 12 students, to discuss their Year 12 HSC courses. **4**

What is the probability that more than 30% of the students that meet with the Principal study Mathematics Extension 1?

(You may assume that the sample of students is approximately normally distributed, and make reference to this extract below from a table of z scores).

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

End of Paper

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$\text{A}= \frac{\theta}{360} \times \pi r^2$$

$$\text{A}= \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \quad r \neq 1$$

$$S = \frac{a}{1-r}, \quad |r| < 1$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

$$(x-h)^2+(y-k)^2=r^2$$

Trigonometric Functions

$$\sin A=\frac{\text{opp}}{\text{hyp}},\quad \cos A=\frac{\text{adj}}{\text{hyp}},\quad \tan A=\frac{\text{opp}}{\text{adj}}$$

$$A=\frac{1}{2}ab\sin C$$

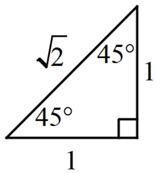
$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

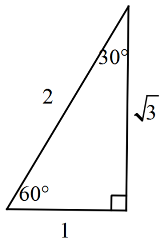
$$c^2=a^2+b^2-2ab\cos C$$

$$\cos C=\frac{a^2+b^2-c^2}{2ab}$$

$$l=r\theta$$

$$A=\frac{1}{2}r^2\theta$$





Trigonometric Identities

$$\sec A=\frac{1}{\cos A},\quad \cos A\neq 0$$

$$\operatorname{cosec}A=\frac{1}{\sin A},\quad \sin A\neq 0$$

$$\cot A=\frac{\cos A}{\sin A},\quad \sin A\neq 0$$

$$\cos^2 x+\sin^2 x=1$$

Compound Angles

$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

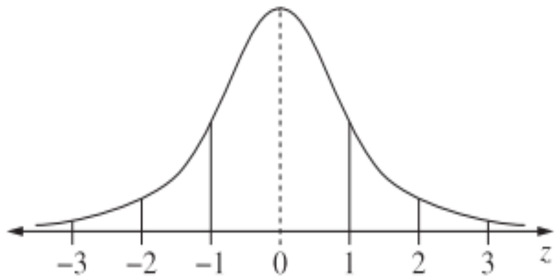
If $t=\tan\frac{A}{2}$, then

Statistical Analysis

$$z=\frac{x-\mu}{\sigma}$$

An outlier is a score less than $Q_1-1.5\times IQR$, or more than $Q_3+1.5\times IQR$

Normal distribution



- approximately 68% of scores have z-scores between -1 and 1
- approximately 95% of scores have z-scores between -2 and 2
- approximately 99.7% of scores have z-scores between -3 and 3

$$E(X)=\mu$$

$$\operatorname{Var}(X)=E\left[(X-\mu)^2\right]=E\left(X^2\right)-\mu^2$$

Probability

$$P(A\cap B)=P(A)P(B)$$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$P(A|B)=\frac{P(A\cap B)}{P(B)},\quad P(B)\neq 0$$

Continuous random variables

$$P(X\le r)=\int_a^r f(x)dx$$

$$\sin A = \frac{2t}{1+t^2}$$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

Compound Angles

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B)-\sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1-\cos 2nx)$$

$$\cos^2 nx = \frac{1}{2}(1+\cos 2nx)$$

$$P(a < X < b) = \int_a^b f(x)dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0,1,\ldots,\text{n}$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	**Derivative**
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$y=\tan f(x)$	$\dfrac{dy}{dx}=f'(x)\sec^2 f(x)$	$\int f'(x)a^{f(x)}dx=\dfrac{a^{f(x)}}{\ln a}+c$
$y=e^{f(x)}$	$\dfrac{dy}{dx}=f'(x)e^{f(x)}$	$\int \dfrac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}\,dx=\sin^{-1}\dfrac{f(x)}{a}+c$
$y=\ln f(x)$	$\dfrac{dy}{dx}=\dfrac{f'(x)}{f(x)}$	$\int \dfrac{f'(x)}{a^2+\left[f(x)\right]^2}\,dx=\dfrac{1}{a}\tan^{-1}\dfrac{f(x)}{a}+c$
$y=a^{f(x)}$	$\dfrac{dy}{dx}=(\ln a)f'(x)a^{f(x)}$	$\int u\dfrac{dv}{dx}dx=uv-\int v\dfrac{du}{dx}dx$
$y=\log_a f(x)$	$\dfrac{dy}{dx}=\dfrac{f'(x)}{(\ln a)f(x)}$	$\int_a^b f(x)dx$ $=\dfrac{b-a}{2n}\left\{f(a)+f(b)+2\left[f(x_1)+\ldots+f(x_{n-1})\right]\right\}$
$y=\sin^{-1} f(x)$	$\dfrac{dy}{dx}=\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$	Where $a=x_0$ and $b=x_n$
$y=\cos^{-1} f(x)$	$\dfrac{dy}{dx}=-\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$	
$y=\tan^{-1} f(x)$	$\dfrac{dy}{dx}=\dfrac{f'(x)}{1+\left[f(x)\right]^2}$	

Combinatorics

$$^nP_r=\frac{n!}{(n-r)!}$$

$$\binom{n}{r}={}^nC_r=\frac{n!}{r!(n-r)!}$$

$$(x+a)^n=x^n+\binom{n}{1}x^{n-1}a+\ldots+\binom{n}{r}x^{n-r}a^r+\ldots+a^n$$

Vectors

$$\left|\underset{\sim}{u}\right|=\left|x\underset{\sim}{i}+y\underset{\sim}{j}\right|=\sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v}=\left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta=x_1x_2+y_1y_2$$

where $\underset{\sim}{u} = x_1 \underset{\sim}{i} + y_1 \underset{\sim}{j}$

and $\underset{\sim}{v} = x_2 \underset{\sim}{i} + y_2 \underset{\sim}{j}$

$\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{b}$

Student Name:……………………………..

MATHEMATICS EXTENSION 1 – MULTIPLE-CHOICE ANSWER SHEET

ATTEMPT ALL QUESTIONS

Question	**1**	A ○	B ○	C ○	D ○
	2	A ○	B ○	C ○	D ○
	3	A ○	B ○	C ○	D ○
	4	A ○	B ○	C ○	D ○
	5	A ○	B ○	C ○	D ○
	6	A ○	B ○	C ○	D ○
	7	A ○	B ○	C ○	D ○
	8	A ○	B ○	C ○	D ○
	9	A ○	B ○	C ○	D ○
	10	A ○	B ○	C ○	D ○

QATs
Quality Assessment Tasks

2021 Higher School Certificate
Trial Examination

Mathematics Extension1
Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	C
2	A
3	C
4	B
5	C
6	C
7	D
8	D
9	B
10	C

Section II

Question 11 (a)

Criteria	Mark
• Provides correct solution	1

Sample answer

$$\frac{96}{31} = 3.09677...$$

i.e. 4 people

Question 11 (b)

Criteria	Marks
• Provides correct solution	3
• Factorises a relevant quadratic, or equivalent merit	2
• Observes that $x \neq 1$ or equivalent merit	1

Sample answer

$x \neq 1$.

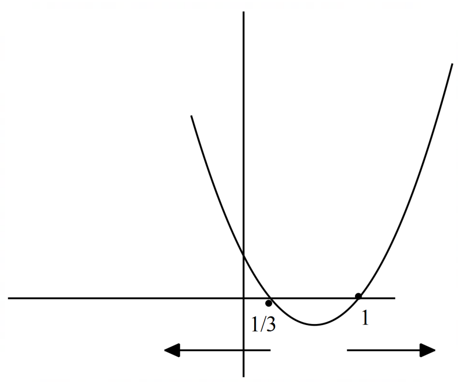
$$\frac{2}{x-1} \geq -3$$

$$(x-1)^2\left(\frac{2}{x-1}\right) \geq -3(x-1)^2$$

$$2(x-1) \geq -3(x^2 - 2x + 1)$$

$$3x^2 - 4x + 1 \geq 0$$

$$(3x-1)(x-1) \geq 0$$



$$x \leq \frac{1}{3}, \quad x > 1$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	**2**
• Finds $\overrightarrow{BD}$, or equivalent merit	**1**

Sample answer

$$\overrightarrow{AB} = -2\underset{\sim}{i} - 6\underset{\sim}{j}$$
$$\overrightarrow{BC} = 7\underset{\sim}{i} + 9\underset{\sim}{j}$$
$$\overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} = 5\underset{\sim}{i} + 3\underset{\sim}{j}$$

Therefore,

$$\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$$
$$\overrightarrow{OD} = \underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{i} + 3\underset{\sim}{j}$$
$$\overrightarrow{OD} = 6\underset{\sim}{i} - \underset{\sim}{j}$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	**3**
• Correctly expands both sides of equation, and uses correct values for $\sin\frac{\pi}{6}$ and $\cos\frac{\pi}{6}$, or equivalent merit	**2**
• Expands $\cos(A+B)$, or equivalent merit	**1**

Sample answer

$$\cos\left(x+\frac{\pi}{6}\right) = 3\cos\left(x-\frac{\pi}{6}\right)$$
$$\cos x\cos\frac{\pi}{6} - \sin x\sin\frac{\pi}{6} = 3\left(\cos x\cos\frac{\pi}{6} + \sin x\sin\frac{\pi}{6}\right)$$
$$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \frac{3\sqrt{3}}{2}\cos x + \frac{3}{2}\sin x$$
$$-\sqrt{3}\cos x = 2\sin x$$
$$\tan x = -\frac{\sqrt{3}}{2}$$

Question 11 (e) (i)

Criteria	Marks
• Provides correct solution	2
• Correctly finds one linear equation, or equivalent merit	1

Sample answer

$$f(3) = 0 \qquad\qquad f(-2) = 0$$

$$27 + 9 + 3p + q = 0 \qquad\qquad -8 + 4 - 2p + q = 0$$

$$3p + q = -36 \qquad\qquad 2p - q = -4$$

Solve simultaneously,

$$5p = -40 \qquad\qquad q = -12$$

$$p = -8$$

Question 11 (e) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts polynomial long division, or equivalent merit	1

Sample answer

$$\begin{array}{r} x+2 \\ x^2 - x - 6\overline{\smash{\big)}\,x^3 + x^2 - 8x - 12} \\ \underline{x^3 - x^2 - 6x} \quad\quad \\ 2x^2 - 2x - 12 \\ \underline{2x^2 - 2x - 12} \\ 0 \end{array}$$

Therefore,

$f(x) = (x-3)(x+2)^2$.

Question 11 (e) (iii)

Criteria	Mark
• Provides correct solution	1

Sample answer

$x \in (-\infty, -2) \cup (-2, 3)$

Question 12 (a)

Criteria	Marks
• Provides correct solution	**3**
• Partial solution for some values of x	**2**
• Correct use of identity for $\cos 2x$ or equivalent merit	**1**

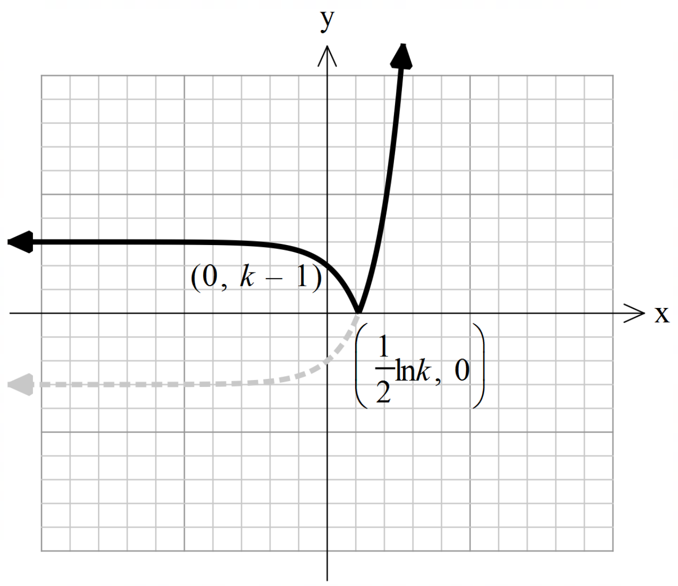
Sample answer

$\cos 2x = \sin x$ $1 - 2\sin^2 x = \sin x$ $2\sin^2 x + \sin x - 1 = 0$ $(2\sin x - 1)(\sin x + 1) = 0$	$\sin x = \frac{1}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$	$\sin x = -1$ $x = \frac{3\pi}{2}$

Question 12 (b) (i)

Criteria	Marks
• Correct sketch, including points of intersection	**2**
• Sketches graph of $y = \lvert f(x) \rvert$, or equivalent merit	**1**

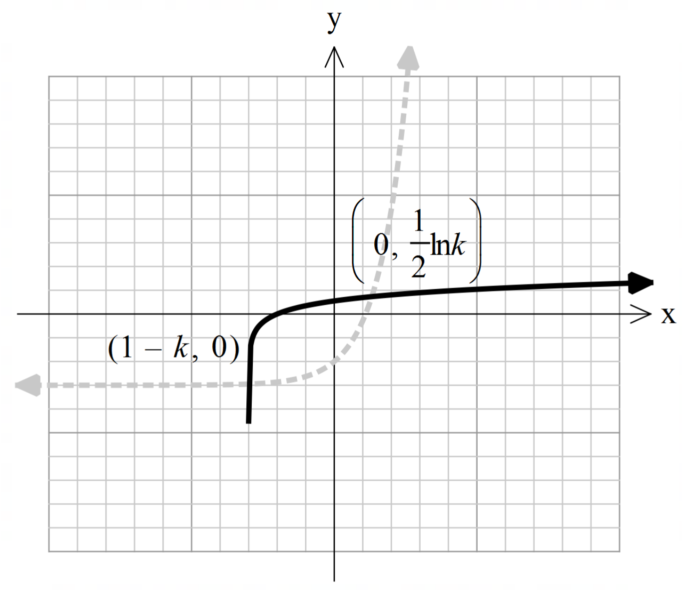
Sample answer



Question 12 (b) (ii)

Criteria	Marks
• Correct sketch, including points of intersection	2
• Sketches graph of $y = f^{-1}(x)$, or equivalent merit	1

Sample answer



Question 12 (c)

Criteria	Marks
• Provides correct solution	3
• Correctly finds one probability, or equivalent merit	2
• Attempts to use binomial probability, or equivalent merit	1

Sample answer

Use $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ with $n = 30,\ p = 0.1$

$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$
$$= \binom{30}{0}(0.9)^{30}(0.1)^0 + \binom{30}{1}(0.9)^{29}(0.1)^1 + \binom{30}{2}(0.9)^{28}(0.1)^2$$
$$= 0.04239 + 0.14130 + 0.22766$$
$$= 0.41135$$

=0.411 to 3sf.

Question 12 (d) (i)

Criteria	Mark
• Provides correct solution	1

Sample answer

$$T = 25 + ae^{-kt}$$
$$\frac{dT}{dt} = -k \times ae^{-kt}$$
$$\frac{dT}{dt} = -k\left(T - 25\right), \quad \text{since } ae^{-kt} = T - 25.$$

Question 12 (d) (ii)

Criteria	Marks
• Provides correct solution	3
• Solves for $k \approx 0.1034$, or equivalent merit	2
• Finds the value of a, or equivalent merit	1

Sample answer

Substituting initial conditions in;

$$100 = 25 + ae^{-k\times 0}$$
$$100 = 25 + a$$
$$a = 75$$

Let $x = \sin A$, then

$$80 = 25 + 75e^{-3k}$$
$$55 = 75e^{-3k}$$
$$e^{3k} = \frac{75}{55}$$
$$k = \frac{1}{3}\ln\left(\frac{75}{55}\right)$$
$$k \approx 0.1034$$

Therefore, if $t = 5$,

$$T = 25 + 75e^{-0.1034\times 5}$$
$$T = 69.726..$$
$$T = 69.7°\text{C}, \text{ correct to 3 sf.}$$

Question 12 (d) (iii)

Criteria	Mark
• Provides correct solution	1

Sample answer

$$T = 25°\text{C}$$

Question 13 (a)

Criteria	Marks
• Provides correct solution	**3**
• Establishes the inductive step, or equivalent merit	**2**
• Establishes the $n=1$ case, or equivalent merit	**1**

Sample answer

$n=1$ LHS=1

$$\begin{aligned}\text{RHS} &= \frac{1}{4}(1)^2(1+1)^2 \\ &= \frac{1}{4}\times 1\times 4 \\ &= 1 \\ &= \text{LHS}\end{aligned}$$

True when $n=1$

Suppose true for $n=k$

So, $1+8+27+\ldots+k^3=\frac{1}{4}k^2(k+1)^2$.

Want to show it is then true for $n=k+1$,

That is, $1+8+27+\ldots+k^3+(k+1)^3=\frac{1}{4}(k+1)^2(k+2)^2$.

$$\begin{aligned}\text{LHS} &= 1+8+27+\ldots+k^3+(k+1)^3 \\ &= \frac{1}{4}k^2(k+1)^2+(k+1)^3 && \text{Using assumption for } n=k \\ &= \frac{1}{4}(k+1)^2\left(k^2+4(k+1)\right) \\ &= \frac{1}{4}(k+1)^2\left(k^2+4k+4\right) \\ &= \frac{1}{4}(k+1)^2(k+2)^2 \\ &= \text{RHS}\end{aligned}$$

If true for $n=k$ then true for $n=k+1$

Hence, by mathematical induction, true for $n\ge 1$.

Question 13 (b)

Criteria	Marks
• Provides correct solution	**3**
• Uses correct limits and expression for dx, or equivalent merit	**2**
• Substitutes for $u=\sqrt{x+2}$, or equivalent merit	**1**

Sample answer

$x=-1, \rightarrow u=1$

$x=7, \rightarrow u=3$

$$u^2 = x+2$$

$$x = u^2 - 2$$

$$\frac{dx}{du} = 2u$$

$$dx = 2u\ du$$

$$\int_1^3 \frac{\left(u^2-2\right)^2}{u} 2u\ du = 2\int_1^3 \left(u^4 - 4u^2 + 4\right) du$$

$$= 2\left[\frac{u^5}{5} - \frac{4u^3}{3} + 4u\right]_1^3$$

$$= 2\left[\left(\frac{3^5}{5} - \frac{4.3^3}{3} + 12\right) - \left(\frac{1}{5} - \frac{4}{5} + 4\right)\right]$$

$$= \frac{652}{15}$$

$$= 43\frac{7}{15}$$

Question 13 (c) (i)

Criteria	Marks
• Provides correct solution	**4**
• Correct expression for $I = R\sin(\theta - \alpha)$	**3**
• Correctly finds α or equivalent merit	**2**
• Correctly finds R, or equivalent merit	**1**

Sample answer

$$4\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$$
$$= R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$$

$$\therefore R\cos\alpha = 4$$
$$R\sin\alpha = 3$$

$$\tan\alpha = \frac{3}{4}$$
$$\alpha = 36.87°$$

$$R^2(\cos^2\alpha + \sin^2\alpha) = 16 + 9$$
$$R^2 = 25$$
$$R = 5 \quad (R > 0)$$

$$\therefore 2\sqrt{3}\sin x - 2\cos x = 5\sin(\theta - 36.87°)$$

So, greatest value of I is 5 amperes.

Question 13 (c) (ii)

Criteria	Mark
• Provides correct solution	**1**

Sample answer

Max value is 5, occurs when,

$$\sin(\theta - 36.87) = 1$$
$$\theta - 36.87 = 90$$
$$\theta = 126.87°$$

Question 13 (d) (i)

Criteria	Marks
• Provides correct solution	**3**
• Progress towards making A the subject of the equation	**2**
• Obtains correct integral for $\int A^{-3/2}dA$ or equivalent merit	**1**

Sample answer

$$\int A^{-3/2}dA = \int t^{-2}dt$$

$$-2A^{-1/2} = -\frac{1}{t} + c$$

When $A = 1,\ t = 1$

$$\therefore -2 = -1 + c$$

$$c = -1$$

$$-2A^{-1/2} = -\frac{1}{t} - 1$$

$$\frac{-2}{A^{1/2}} = \frac{-(1+t)}{t}$$

$$A^{1/2} = \frac{2t}{(1+t)}$$

$$A = \frac{4t^2}{(1+t)^2}$$

Question 13 (d) (ii)

Criteria	Mark
• Provides correct solution	**1**

Sample answer

Since $\frac{t^2}{(1+t)^2} < 1$ for $t > 0,$ then A cannot exceed 4.

Question 14 (a) (i)

Criteria	Marks
• Provides correct solution	3
• Substitutes $t = \frac{\pi}{6}$ correctly, or equivalent merit	2
• Some progress to find $\frac{d\underset{\sim}{v}}{dt}$ or equivalent merit	1

Sample answer

$$\underset{\sim}{a} = (6\cos 2t)\underset{\sim}{i} - (2\sin 2t)\underset{\sim}{j}$$

$$\underset{\sim}{a} = 3\underset{\sim}{i} - \sqrt{3}\underset{\sim}{j}, \text{ when } t = \frac{\pi}{6}$$

$$\begin{aligned} |\underset{\sim}{a}| &= \sqrt{9+3} \\ &= \sqrt{12} \\ &\approx 3.46 \text{ ms}^{-2}. \end{aligned}$$

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	3
• Correct primitive function for V, or equivalent merit	2
• Correctly expresses x as a function of y, or equivalent merit	1

Sample answer

$$\begin{aligned} y &= 2\ln(x-1) \\ \frac{1}{2}y &= \ln(x-1) \\ x-1 &= e^{\frac{1}{2}y} \\ x &= e^{\frac{1}{2}y} + 1 \end{aligned}$$

$$\begin{aligned} V &= \pi \int x^2 dy \\ &= \pi \int_0^p \left(e^y + 2e^{\frac{1}{2}y} + 1 \right) dy \\ &= \pi \left[e^y + 4e^{\frac{1}{2}y} + y \right]_0^p \\ &= \pi \left[\left(e^p + 4e^{\frac{1}{2}p} + p \right) - (1+4+0) \right] \\ &= \pi \left(e^p + 4e^{\frac{1}{2}p} + p - 5 \right) \end{aligned}$$

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts an answer	1

Sample answer

$$V = \pi\left(e^p + 4e^{\frac{1}{2}p} + p - 5\right)$$

$$\frac{dV}{dp} = \pi\left(e^p + 2e^{\frac{1}{2}p} + 1\right)$$

When $p = 4$,

$$\frac{dV}{dp} = \pi\left(e^4 + 2e^2 + 1\right)$$

Using the chain rule,

$$\frac{dV}{dt} = \frac{dp}{dt} \times \frac{dV}{dp}$$

$$\frac{dV}{dt} = 0.2 \times \pi\left(e^4 + 2e^2 + 1\right)$$

$$\frac{dV}{dt} = 44 \text{ cm}^3\text{s}^{-1} \quad \text{correct to 2sf.}$$

Question 14 (c)

Criteria	Marks
• Provides correct solution	3
• Some progress to substitute for t into y, or equivalent merit	2
• Correct expression for t, or equivalent merit	1

Sample answer

$$x = Vt\cos\alpha$$

$$t = \frac{60}{V\cos 28}$$

$$y = -\frac{1}{2}gt^2 + Vt\sin\alpha$$

$$0 = -\frac{1}{2}g\left(\frac{60}{V\cos 28}\right)^2 + V\left(\frac{60}{V\cos 28}\right)\sin 28$$

$$\frac{g}{2}\left(\frac{60}{V\cos 28}\right)^2 = 60\tan 28$$

$$\left(\frac{60}{V\cos 28}\right)^2 = \frac{120\tan 28}{g}$$

$$V^2\cos^2 28 = \frac{3600g}{120\tan 28}$$

$$V^2 = 709.256\ldots$$

$$V = 26.6 \text{ ms}^{-1}.$$

Question 14 (d) (i)

Criteria	Mark
• Provides correct solution	1

Sample answer

Since $p = 0.15$, where p is the probability of a student studying Mathematics Extension 1, then the distribution is skewed to the right, i.e. positive skewness.

Question 14 (d) (ii)

Criteria	Marks
• Provides correct solution	4
• Correct value of z, or equivalent merit	3
• Correct value of σ, or equivalent merit	2
• Correct value of μ, or equivalent merit	1

Sample answer

$\mu = p = 0.23$	$\sigma^2 = \dfrac{p(1-p)}{n}$ $\sigma^2 = \dfrac{0.23 \times 0.77}{60}$ $\sigma^2 = 0.0029516..$ $\sigma = 0.054329...$	$z = \dfrac{x-\mu}{\sigma}$ $z = \dfrac{0.3-0.23}{0.054329}$ $z = 1.28844...$ $z = 1.29$

$P(z < 1.29) = 0.9015$

Therefore,

$$
\begin{aligned}
P(z > 1.29) &= 1 - 0.9015 \\
&= 0.0985
\end{aligned}
$$

2021 Higher School Certificate
Mathematics Extension 1
Mapping Grid

Section I

Question	Marks	Content	Syllabus Outcomes
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2	1	F1.3	ME11-1
3	1	F1.4	ME11-2
4	1	C3.2	ME12-1
5	1	T2	ME11-3
6	1	C2	ME12-1
7	1	V1.2	ME12-2
8	1	T1	ME11-3
9	1	V1.2	ME12-2
10	1	C2	ME12-1

Section II

Question	Marks	Content	Syllabus Outcomes
11 (a)	1	A1.1	ME11-5
11 (b)	3	F1.2	ME11-3
11 (c)	2	V1.1	ME12-2
11 (d)	3	T2	ME11-3
11 (e) (i)	2	F2.1	ME11-2
11 (e) (ii)	2	F2.1	ME11-2
11 (e) (iii)	1	F1.2	ME11-2
12 (a)	3	T3	ME12-3
12 (b) (i)	2	F1.1	ME11-2
12 (b) (ii)	2	F1.1	ME11-1
12 (c)	3	S1.1	ME12-5
12 (d) (i)	1	C1.2	ME11-4
12 (d) (ii)	3	C1.2	ME11-4
12 (d) (iii)	1	C1.2	ME11-4
13 (a)	3	P1	ME12-1
13 (b)	3	C2	ME12-1
13 (c) (i)	4	T3	ME12-3
13 (c) (ii)	1	T3	ME12-3
13 (d) (i)	3	C3.2	ME12-4
13 (d) (ii)	1	C3.2	ME12-4

Question	Marks	Content	Syllabus Outcomes
14 (a)	3	V1.2	ME12-2
14 (b) (i)	3	C3.1	ME12-4
14 (b) (ii)	2	C1.3	ME11-4
14 (c)	3	V1.3	ME12-2
14 (d) (i)	1	S1.2	ME12-5
14 (d) (ii)	4	S1.2	ME12-5