EPPING BOYS HIGH SCHOOL

2024

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – **10 minutes**
- Working time – **2 hours**

- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided separately

- In Questions 11 to 14, show all relevant mathematical reasoning and/or calculations

Total Marks – 70

Section I – 10 marks (pages 3 to 6)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the Multiple-Choice Answer sheet, provided on page 7.

Section II – 60 marks (pages 8 to 12)
- Attempt Questions 11-14
- Allow about 1 hour and 45 minutes for this section
- Use a separate answer booklet for each question from 11 to 14

STUDENT NESA NUMBER:	

Please tick a box:

	12MX1_71	**Mr Param**
	12MX1_72	**Mr Xu**
	12MX1_73	**Mr McKenzie**

	Section I	**Section II**				**TOTAL**	**%**
	MCQ	**Q11**	**Q12**	**Q13**	**Q14**		
Full Marks	**10**	**15**	**16**	**15**	**14**	**70**	**100**
Marks Awarded							

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Section I – 10 marks

Attempt Questions 1–10.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–10.

1. The parametric equations of a curve are given below.

$$x = 4\cos\theta$$
$$y = 4\sin\theta$$

Where $0 \le \theta \le \pi$.

What is the Cartesian equation of the curve?

(A) $y = 1 - \dfrac{x^2}{16}$

(B) $y = \sqrt{16 - x^2}$

(C) $y = \dfrac{x^2}{16} + 1$

(D) $y^2 = x^2 - 16$

2. Which of the following is equivalent to $2\cos 5x \sin x$?

(A) $\cos 6x + \sin 4x$

(B) $\cos 6x - \sin 4x$

(C) $\sin 6x + \sin 4x$

(D) $\sin 6x - \sin 4x$

3. Maria starts at the origin and walks along all of the vector $2\underset{\sim}{i} + 3\underset{\sim}{j}$, then walks along all of the vector $3\underset{\sim}{i} - 2\underset{\sim}{j}$ and finally along all of the vector $4\underset{\sim}{i} - 3\underset{\sim}{j}$.

How far from the origin is she?

(A) $\sqrt{77}$

(B) $\sqrt{85}$

(C) $2\sqrt{13} + \sqrt{5}$

(D) $\sqrt{5} + \sqrt{7} + \sqrt{13}$

4. If $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are unit vectors and $\left|\underset{\sim}{a}+\underset{\sim}{b}\right| = 1$, then what is the value of $\left|\underset{\sim}{a}-\underset{\sim}{b}\right|$?

(A) $\dfrac{1}{\sqrt{2}}$

(B) $\dfrac{1}{\sqrt{3}}$

(C) $\sqrt{2}$

(D) $\sqrt{3}$

5. A function is defined as $f(x) = tan^{-1}(tan(x))$.

What is the value of $f\left(\dfrac{9\pi}{4}\right)$?

(A) $\dfrac{\pi}{4}$

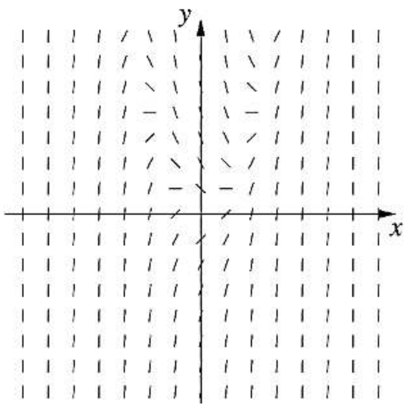
(B) $\dfrac{5\pi}{4}$

(C) $\dfrac{7\pi}{4}$

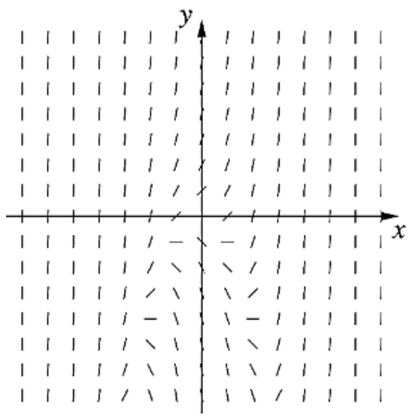
(D) $\dfrac{9\pi}{4}$

6. Which of the following slope fields represent the differential equation $\dfrac{dy}{dx} = x^2 - y$?

(A)



(B)



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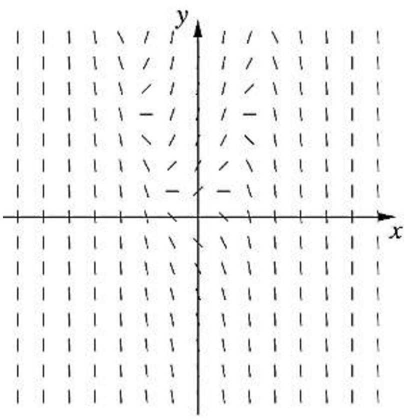
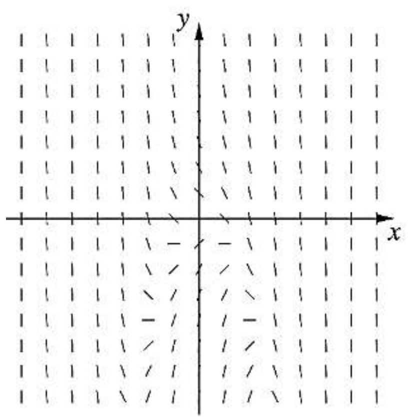
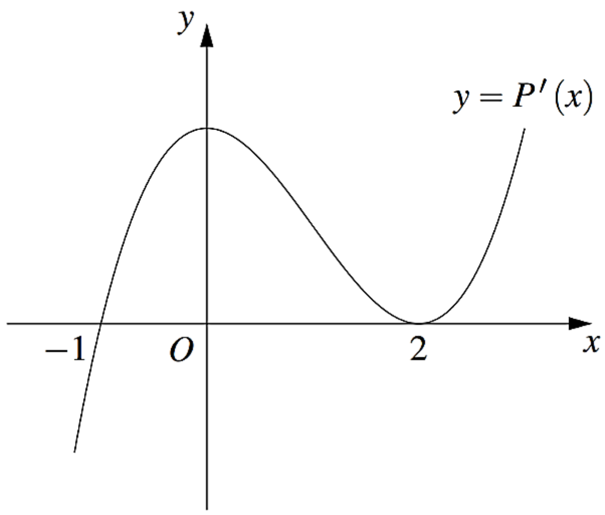


(D)



7. In the graph below, $y = P'(x)$ represents the first derivative of a polynomial $P(x)$ of degree 4, where $P(x)$ has a multiple root.



What can be said about the polynomial $P(x)$?

(A) $x = -1$ is a root of multiplicity 3

(B) $x = 0$ is a root of multiplicity 2

(C) $x = 2$ is root of multiplicity 2

(D) $x = 2$ is a root of multiplicity 3

8. Sixteen people, consisting of eight doubles tennis partners, enter a local tennis competition.

Only four sets of partners are chosen to attend the celebration dinner. They are seated around a circular table.

In how many ways can the doubles partners be selected and seated around the circular table if the partners must sit together?

(A) $\binom{8}{4} \times 3! \times (2!)^3$

(B) $\binom{8}{4} \times 3! \times (2!)^4$

(C) $\binom{16}{8} \times 3! \times (2!)^3$

(D) $\binom{16}{8} \times 3! \times (2!)^4$

9. Consider two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ where $\underset{\sim}{a} \cdot \underset{\sim}{b} < 0$.

It is known $\left|proj_{\underset{\sim}{a}}\underset{\sim}{b}\right| = k$

Which of the following gives the value for $\left|proj_{\underset{\sim}{a}}\left(\underset{\sim}{a} + \underset{\sim}{b}\right)\right|$?

(A) $\left|\underset{\sim}{a} + \underset{\sim}{b}\right| - k$

(B) $\left|\underset{\sim}{a} + \underset{\sim}{b}\right| + k$

(C) $\left|\underset{\sim}{a}\right| - k$

(D) $\left|\underset{\sim}{b}\right| - k$

10. Which statement is always true for the function $f(x) = \sin^{-1}(x^2 + 2x + 1)$?

(A) $f(x)$ has an inverse function in the domain $[-2,0]$

(B) $f(x)$ has an inverse function in the domain $[-1,0]$

(C) $f(x)$ has an inverse function in the domain $[-1,1]$

(D) $f(x)$ has an inverse function in the domain $[-2,2]$

End of Section I

Epping Boys High School Trial HSC Examination 2024
Mathematics Extension 1

NESA Number	Teacher (circle one)			Mark
	Mr Param	Mr Xu	Mr McKenzie	

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ● C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ● B ● C ○ D ○

1.	**A**	○	**B**	○	**C**	○	**D**	○
2.	**A**	○	**B**	○	**C**	○	**D**	○
3.	**A**	○	**B**	○	**C**	○	**D**	○
4.	**A**	○	**B**	○	**C**	○	**D**	○
5.	**A**	○	**B**	○	**C**	○	**D**	○
6.	**A**	○	**B**	○	**C**	○	**D**	○
7.	**A**	○	**B**	○	**C**	○	**D**	○
8.	**A**	○	**B**	○	**C**	○	**D**	○
9.	**A**	○	**B**	○	**C**	○	**D**	○
10.	**A**	○	**B**	○	**C**	○	**D**	○

Section II – 60 marks

Attempt Questions 11-14.
Allow about 1 hour 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

Use a SEPARATE writing booklet.

a. A student representative committee of 7 people is to be chosen from a group of 10 men and 11 women. **2**

Find the probability that the committee is made of 3 men and 4 women.

b. The polynomial $P(x) = x^3 - qx^2 + 32$ has real roots α, α and β **3**

Find the value of q.

c. Express $3\sin\theta + 4\cos\theta$ in the form $Rcos\ (\theta - \alpha)$, where $R > 0$ and $0° \leq \alpha \leq 90°$, correct to one decimal place. **2**

d. Solve $\dfrac{x^2 - 6}{x} \leq 1$. **3**

e. Find $\displaystyle\int \frac{3}{1 + 4x^2}\,dx$ **2**

f. Let $f(x)$ be a function where $f(2) = -11$ and $f'(2) = 8$. **3**

Find the equation of the tangent to the graph of $y = f^{-1}(x)$ at the point where $x = -11$.

End of Question 11

Question 12 (16 marks)

Use a SEPARATE writing booklet.

a. Use mathematical induction to prove that **3**

$$2^3 + 4^3 + 6^3 + \cdots + (2n)^3 = 2n^2(n+1)^2$$

for all integers $n \geq 1$.

b. Evaluate $\int_3^{18} \frac{x}{\sqrt{x-2}}\,dx$ using the substitution $u = \sqrt{x-2}$. **3**

c. **(i)** Sketch the graph of $f(x) = 2\cos^{-1}(x-1)$ **2**

(ii) Hence, explain why **1**

$$\int_0^2 [2\cos^{-1}(x-1) - \pi]dx = 0$$

d. Let $S = 1 + 2\cos\theta + 2\cos 2\theta + 2\cos 3\theta$

(i) Prove that $S \times sin\frac{\theta}{2} = sin\frac{7\theta}{2}$ **2**

(ii) Hence, show that if $\theta = \frac{2\pi}{7}$, then $1 + 2\cos\theta + 2\cos 2\theta + 2\cos 3\theta = 0$ **1**

(iii) Hence, or otherwise, show that $cos\frac{2\pi}{7}$ is a solution to the equation: **4**

$8x^3 + 4x^2 - 4x - 1 = 0.$

End of Question 12

Question 13 (15 marks)

Use a SEPARATE writing booklet.

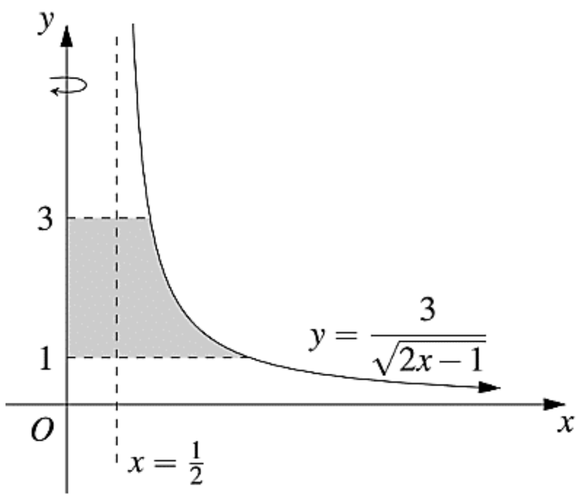
a. The wind chill index, W, measures the apparent temperature by taking into account the speed of the wind, v km/h in a location.

A meteorologist suggests that the wind chill index in the region where his laboratory is located is given by the equation $W = 19.5 - 7.4v^{0.17}$.

(i) Find $\dfrac{dW}{dv}$ **1**

(ii) Find the rate of change of W, given that the wind speed is 10 km/h and is increasing at a rate of 5 km/h per hour. Give your answer correct to two decimal places. **2**

b. The region between the curve $y = \dfrac{3}{\sqrt{2x-1}}$ and the y-axis between $y = 1$ and $y = 3$ is shaded in the diagram. **4**



Calculate the exact volume generated when the shaded region is rotated about the y-axis.

c. The points A, B and C have position vectors $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{c}$, respectively. Point D lies on the line going through AB, and has position vector $\underset{\sim}{d}$.

λ and μ are non-zero numbers such that $\lambda\underset{\sim}{a} + \mu\underset{\sim}{b} - \underset{\sim}{c} = 0$ and $\lambda + \mu = 1$.

(i) Show that the points A, B and C are collinear. **2**

(ii) It is known that $\left|\underset{\sim}{a}\right| = 2$, the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is acute and, the area of triangle OAB is k units². Show that $\left(\underset{\sim}{a} \cdot \underset{\sim}{b}\right)^2 = 4\left(\left|\underset{\sim}{b}\right|^2 - k^2\right)$ **3**

(iii) Given that $k = 6$, $\left|\underset{\sim}{b}\right| = 10$ and $\angle AOD = 90°$, find $\underset{\sim}{d}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ **3**

End of Question 13

Question 14 (14 marks)

Use a SEPARATE writing booklet.

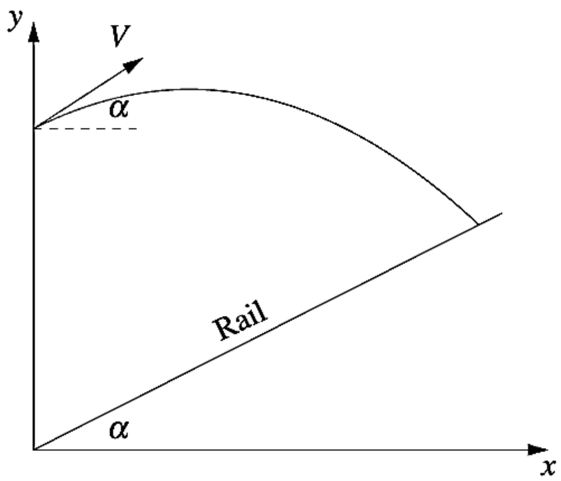
a. Given that $\dfrac{dy}{dx} = \dfrac{y+2}{x-1}$ and $y(3) = 6$, find the value of $y(-1)$ **3**

b. The population of foxes is released into the wild to control the population of rabbits. The population of foxes is modelled by: **3**

$$f(t) = \frac{3000}{50 + 24\sin t + 7\cos t}$$

Where t is time in months. After how many months is the population of foxes at a maximum? Leave your answer to 2 decimal places.

c. A scene in an action movie involves an archer shooting an arrow with initial velocity V m/s at an angle of projection α to the horizontal from a platform h metres above the ground.



The target travels on a straight rail with constant velocity V m/s, starting from ground level, with incline angle equal to the angle of projection of the arrow. The target and arrow are projected simultaneously and meet at time T seconds.

The position vector of the arrow, A, t seconds after it is projected is given by:

$$\boldsymbol{r_A} = \begin{pmatrix} Vt\cos\alpha \\ -\dfrac{gt^2}{2} + Vt\sin\alpha + h \end{pmatrix} \text{(Do NOT prove this.)}$$

(i) Show that the arrow will hit the target at $T = \sqrt{\dfrac{2h}{g}}$ seconds. **2**

Question 14c continues on the next page

The film director believes the best visual effect will occur if the impact occurs when the arrow is at maximum height.

(ii) Show that, if the impact occurs at the maximum height of the arrow's trajectory, that the length of the rail on which the target travels does not need to exceed a length of $\dfrac{V^2}{g}$ metres. **3**

(iii) Show that, for the impact to occur at the maximum height of the trajectory, the height of the platform needs to be exactly half the maximum height of the arrow. **2**

(iv) Hence, or otherwise, show that if the platform cannot be raised above 10 metres in height, then the initial velocity of the arrow must exceed $14ms^{-1}$ for the impact to occur at the maximum height of the trajectory. **1**

End of Exam