DS-GA 3001.001 Special Topics in Data Science: Probabilistic Time Series Analysis

Week 4 EM

In [1]:

```
# Install PyKalman
# pip install pykalman
import numpy as np
import matplotlib.pyplot as plt
from pykalman import KalmanFilter
from scipy.stats import multivariate_normal

# Data Visualiztion
def plot_kalman(x,y,nx,ny,kx=None,ky=None, plot_type="r-", label=None, title=
    """
    Plot the trajectory
    """
    fig, ax = plt.subplots(1,2, figsize=(15,4))
    if kx is not None and ky is not None:
        ax[0].plot(x,y,'g-',nx,ny,'b.',kx,ky, plot_type)
        ax[0].plot(kx[0], ky[0], 'or')
        ax[0].plot(kx[-1], ky[-1], 'xr')

        ax[1].plot(x, kx, '.k', label='latent dim 1')
        ax[1].plot(y, ky, '.', color='grey', label='latent dim 2')
        ax[1].set_xlabel('real latent')
        ax[1].set_ylabel('estimated latent')
        ax[1].legend()
    else:
        ax[0].plot(x,y,'g-',nx,ny,'b.')

        ax[1].plot(x, nx, '.k', label='dim 1')
        ax[1].plot(y, ny, '.', color='grey', label='dim 2')
        ax[1].set_xlabel('latent')
        ax[1].set_ylabel('observed')
        ax[1].legend()

    ax[0].set_xlabel('X position')
    ax[0].set_ylabel('Y position')
    ax[0].set_title(title)
    ax[0].set_aspect(1)
    ax[1].set_aspect(1)

    if kx is not None and ky is not None and label is not None:
        ax[0].legend(('true','measured', label))
    else:
        ax[0].legend(('true','measured'))

    return fig

def visualize_line_plot(data, xlabel, ylabel, title):
```

```
    """
    Function that visualizes a line plot
    """
    plt.plot(data)
    plt.xlabel(xlabel)
    plt.ylabel(ylabel)
    plt.title(title)
    plt.show()

def print_parameters(kf_model, need_params=None, evals=False):
    """
    Function that prints out the parameters for a Kalman Filter
    @param - kf_model : the model object
    @param - need_params : a list of string
    """
    if evals:
        if need_params is None:
            need_params1 = ['transition_matrices', 'transition_covariance', '
            need_params2 = ['observation_matrices', 'initial_state_mean']
        for param in need_params1:
            tmp = np.linalg.eig(getattr(kf_model, param))[0]
            print("{0} = {1}, shape = {2}\n".format(param, tmp, tmp.shape))
        for param in need_params2:
            print("{0} = {1}, shape = {2}\n".format(param, getattr(kf_model, 
    else:
        if need_params is None:
            need_params = ['transition_matrices', 'observation_matrices', 'tr
                           'initial_state_mean', 'initial_state_covariance']
        for param in need_params:
            print("{0} = {1}, shape = {2}\n".format(param, getattr(kf_model, 
```

Kalman

We want to infer the latent variable $z_n$$z_n$ given the observed variable $x_n$$x_n$.

$$P(z_n|x_1,\dots,x_n,x_{n+1},\dots,x_N) \sim N(\hat{\mu_n},\hat{V_n})$$

$$P(z_n|x_1,\dots,x_n,x_{n+1},\dots,x_N) \sim N(\hat{\mu}_n,\hat{V}_n)$$

Forward: Filtering

obtain estimates of latent by running the filtering from $n = 0,\dots. N$$n = 0,\dots. N$

prediction given latent space parameters

<img src='img/LDS_latent.svg', width = 110, height=90>

$$z_n^{pred} \sim N(\mu_n^{pred}, V_n^{pred})$$

$$z_n^{pred} \sim N(\mu_n^{pred}, V_n^{pred})$$

$$\mu_n^{pred} = A\mu_{n-1}$$

$$\mu_n^{pred} = A\mu_{n-1}$$

this is the prediction for z_n obtained simply by taking the expected value of z_{n-1} and projecting it forward one step using the transition probability matrix A

$$V_n^{pred} = AV_{n-1}A^T + \Gamma$$

$$V_n^{pred} = AV_{n-1}A^T + \Gamma$$

same for the covariance taking into account the noise covariance Γ

correction (innovation) from observation

<img src='img/LDS_observed.svg', width = 40, height=80>

project to observational space:

$$x_n^{pred} \sim N(C\mu_n^{pred}, CV_n^{pred}C^T + \Sigma)$$

$$x_n^{pred} \sim N(C\mu_n^{pred}, CV_n^{pred}C^T + \Sigma)$$

correct prediction by actual data:

$$z_n^{innov} \sim N(\mu_n^{innov}, V_n^{innov})$$

$$z_n^{innov} \sim N(\mu_n^{innov}, V_n^{innov})$$

$$\mu_n^{innov} = \mu_n^{pred} + K_n(x_n - C\mu_n^{pred})$$

$$\mu_n^{innov} = \mu_n^{pred} + K_n(x_n - C\mu_n^{pred})$$

$$V_n^{innov} = (I - K_nC)V_n^{pred}$$

$$V_n^{innov} = (I - K_nC)V_n^{pred}$$

Kalman gain matrix:

$$K_n = V_n^{pred}C^T(CV_n^{pred}C^T + \Sigma)^{-1}$$

$$K_n = V_n^{pred}C^T(CV_n^{pred}C^T + \Sigma)^{-1}$$

we use the latent-only prediction to project it to the observational space and compute a correction proportional to the error $x_n - CAz_{n-1}$ between prediction and data, *coefficient of this correction is the Kalman gain matrix*

<img src='img/Kfilter_Bishop.png', width = 600, height=600> from Bishop (2006), chapter 13.3

if measurement noise is small and dynamics are fast -> estimation will depend mostly on observed data

Backward: Smoothing

<img src='img/LDS_smooth.svg', width = 110, height=100>

obtain estimates by propagating from x_n back to x_1 using results of forward pass ($\mu_n^{innov}, V_n^{innov}, V_n^{pred}$)

$$N(z_n|\mu_n^{smooth}, V_n^{smooth})$$

$$N(z_n|\mu_n^{smooth}, V_n^{smooth})$$

$$\mu_n^{smooth} = \mu_n^{innov} + J_n(\mu_{n+1}^{smooth} - A\mu_n^{innov})$$

$$\mu_n^{smooth} = \mu_n^{innov} + J_n(\mu_{n+1}^{smooth} - A\mu_n^{innov})$$

$$V_n^{smooth} = V_n^{innov} + J_n(V_{n+1}^{smooth} - V_{n+1}^{pred})J_n^T$$

$$V_n^{smooth} = V_n^{innov} + J_n(V_{n+1}^{smooth} - V_{n+1}^{pred})J_n^T$$

$$J_N = V_n^{innov}A^T(V_{n+1}^{pred})^{-1}$$

$$J_N = V_n^{innov}A^T(V_{n+1}^{pred})^{-1}$$

This gives us the final estimate for z_n.

$$\hat{\mu_n} = \mu_n^{smooth}$$

$$\hat{\mu}_n = \mu_n^{smooth}$$

$$\hat{V_n} = V_n^{smooth}$$

$$\hat{V}_n = V_n^{smooth}$$

EM algorithm

- want to maximize $logp(x|\theta)$
- need to marginalize out latent *(which is not tractable)*

$$log\left(p(x|\theta)\right) = log\left(\int p(x,z|\theta)dz\right)$$

$$log(p(x|\theta)) = log\left(\int p(x,z|\theta)dz\right)$$

- add a probability distribution $q(z)$ which will approximate the latent distribution

$$= \int_z q(z)logp(x|\theta)dz$$

$$= \int_z q(z)logp(x|\theta)dz$$

- can be rewritten as

$$= \mathcal{L}(q,\theta) + KL\left(q(z)||p(z|x),\theta\right)$$

$$= \mathcal{L}(q,\theta) + KL(q(z)||p(z|x),\theta)$$

- $\mathcal{L}(q,\theta)$ contains the joint distribution of x and z
- $KL(q||p)$ contains the conditional distribution of $z|x$

Expectation step

- parameters are kept fixed
- find a good approximation $q(z)$: maximize lower bound $\mathcal{L}(q,\theta)$ with respect to $q(z)$
- (already implemented Kalman filter+smoother)

Maximization step

- keep distribution $q(z)$ fixed
- change parameters to maximize the lower bound $\mathcal{L}(q,\theta)$

M-step

(see Bishop, chapter 13.3.2 Learning in LDS)

Update parameters of the probability distribution

Initial parameters

$$\mu_0^{new} = E(z_1)$$

$$\mu_0^{new} = E(z_1)$$

$$\Gamma_0^{new} = E(z_1 z_1^T) - E(z_1)E(z_1^T)$$

$$\Gamma_0^{new} = E(z_1 z_1^T) - E(z_1)E(z_1^T)$$

Latent parameters

$$A^{new} = \left(\sum_{n=2}^{N} E(z_n z_{n-1}^T)\right)\left(\sum_{n=2}^{N} E(z_{n-1} z_{n-1}^T)\right)^{-1}$$

$$A^{new} = \left(\sum_{n=2}^{N} E(z_n z_{n-1}^T)\right)\left(\sum_{n=2}^{N} E(z_{n-1} z_{n-1}^T)\right)^{-1}$$

$$\Gamma^{new} = \frac{1}{N-1}\sum_{n=2}^{N} E(z_n z_n^T) - A^{new}E(z_{n-1} z_n^T) - E(z_n z_{n-1}^T)A^{new} + A^{new}E(z_{n-1} z_{n-1}^T)(A$$

$$\Gamma^{new} = \frac{1}{N-1}\sum_{n=2}^{N} E(z_n z_n^T) - A^{new}E(z_{n-1} z_n^T) - E(z_n z_{n-1}^T)A^{new} + A^{new}E(z_{n-1} z_{n-1}^T)(A^{new})^T$$

Observable space parameters

$$C^{new} = \left(\sum_{n=1}^{N} x_n E(z_n^T)\right)\left(\sum_{n=1}^{N} E(z_n z_n^T)\right)^{-1}$$

$$C^{new} = \left(\sum_{n=1}^{N} x_n E(z_n^T)\right)\left(\sum_{n=1}^{N} E(z_n z_n^T)\right)^{-1}$$

$$\Sigma^{new} = \frac{1}{N}\sum_{n=1}^{N} x_n x_n^T - C^{new}E(z_n)x_n^T - x_n E(z_n^T)C^{new} + C^{new}E(z_n z_n^T)C_{new}$$

$$\Sigma^{new} = \frac{1}{N}\sum_{n=1}^{N} x_n x_n^T - C^{new}E(z_n)x_n^T - x_n E(z_n^T)C^{new} + C^{new}E(z_n z_n^T)C_{new}$$

For the updates in the M-step we will need the following posterior marginals obtained from the Kalman smoothing results $\hat{\mu}_n, \hat{V}_n \hat{\mu}_n, \hat{V}_n$

$$E(z_n) = \hat{\mu}_n$$

$$E(z_n) = \hat{\mu}_n$$

$$E(z_n z_{n-1}^T) = J_{n-1}\hat{V}_n + \hat{\mu}_n \hat{\mu}_{n-1}^T$$

$$E(z_n z_{n-1}^T) = J_{n-1}\hat{V}_n + \hat{\mu}_n \hat{\mu}_{n-1}^T$$

$$E(z_n z_n^T) = \hat{V}_n + \hat{\mu}_n \hat{\mu}_n^T$$

$$E(z_n z_n^T) = \hat{V}_n + \hat{\mu}_n \hat{\mu}_n^T$$

Kalman + EM Implementation

In this part of the exercise, you will implement the EM algorithm, building up on the exercises from last week.

In [2]:

```
class MyKalmanFilter:
    """
    Class that implements the Kalman Filter
    """
    def __init__(self, n_dim_state=2, n_dim_obs=2):
        """
        @param n_dim_state: dimension of the laten variables
        @param n_dim_obs: dimension of the observed variables
        """
        self.n_dim_state = n_dim_state
        self.n_dim_obs = n_dim_obs
        self.transition_matrices = np.eye(n_dim_state)
        self.transition_covariance = np.eye(n_dim_state)
        self.observation_matrices = np.eye(n_dim_obs, n_dim_state)
        self.observation_covariance = np.eye(n_dim_obs)
        self.initial_state_mean = np.zeros(n_dim_state)
        self.initial_state_covariance = np.eye(n_dim_state)


    def sample(self, n_timesteps, initial_state=None, random_seed=None):
        """
        Method that gives samples
        @param initial_state: numpy array whose length == self.n_dim_state
        @param random_seed: an integer, for test purpose
        @output state: a 2d numpy array with dimension [n_timesteps, self.n_d
        @output observation: a 2d numpy array with dimension [n_timesteps, se
        """

        latent_state = np.zeros([n_timesteps, self.n_dim_state])
        observed_state = np.zeros([n_timesteps, self.n_dim_obs])
```

```
        if random_seed is not None:
            np.random.seed(random_seed)

        ################
        ##### TODO #####
        ################
        current_latent_state = initial_state
        for t in range(n_timesteps):
            if t == 0:
                latent_state[t] = (current_latent_state)
            else:
                latent_state[t] = (np.dot(self.transition_matrices, current_la
                             np.random.multivariate_normal(np.zeros(self.n_di
                current_latent_state = latent_state[t]
            observed_state[t] = (np.dot(self.observation_matrices, current_la
                              np.random.multivariate_normal(np.zeros(self.n_


        return latent_state, observed_state


    def filter(self, X):
        """
        Method that performs Kalman filtering
        @param X: a numpy 2D array whose dimension is [n_example, self.n_dim_
        @output: filtered_state_means: a numpy 2D array whose dimension is [n
        @output: filtered_state_covariances: a numpy 3D array whose dimension
        """

        # validate inputs
        n_example, observed_dim = X.shape
        assert observed_dim==self.n_dim_obs

        # create holders for outputs
        filtered_state_means = np.zeros( [n_example, self.n_dim_state] )
        filtered_state_covariances = np.zeros( [n_example, self.n_dim_state, 

        ################################################################
        ################## insert your own filter here ################
        ################################################################
        # below: this is an alternative if you do not have an implementation
        kf = KalmanFilter(n_dim_state=self.n_dim_state, n_dim_obs=self.n_dim_
        need_params = ['transition_matrices', 'observation_matrices', 'transi
          'observation_covariance', 'initial_state_mean', 'initial_state_cova
        for param in need_params:
            setattr(kf, param, getattr(self, param))
        filtered_state_means, filtered_state_covariances = kf.filter(X)
        ################################################################

        return filtered_state_means, filtered_state_covariances

    def smooth(self, X):
        """
        Method that performs the Kalman Smoothing
        @param X: a numpy 2D array whose dimension is [n_example, self.n_dim_
        @output: smoothed_state_means: a numpy 2D array whose dimension is [n
        @output: smoothed_state_covariances: a numpy 3D array whose dimension
```

```
        """

        # validate inputs
        n_example, observed_dim = X.shape
        assert observed_dim==self.n_dim_obs

        # run the forward path
        mu_list, v_list = self.filter(X)

        # create holders for outputs
        smoothed_state_means = np.zeros( (n_example, self.n_dim_state) )
        smoothed_state_covariances = np.zeros( (n_example, self.n_dim_state, 

        ###########################################################################
        ################## insert your own smoother here ########################
        ###########################################################################
        A = self.transition_matrices
        C = self.observation_matrices
        mu_pre = mu_list[-1]
        v_pre = v_list[-1]
        self.J = []

        for i in range(n_example-2, -1, -1):

            V_pred = np.matmul(np.matmul(A, v_list[i]), A.T) + self.transitio
            J_N = np.matmul(np.matmul(v_list[i], A.T), np.linalg.inv(V_pred))


            mean_smooth = mu_list[i] + np.matmul(J_N, (mu_pre - np.matmul(A, 
            V_smooth = v_list[i] + np.matmul(np.matmul(J_N, ( v_pre - V_pred)

            smoothed_state_means[i] = mean_smooth
            smoothed_state_covariances[i] = V_smooth
            mu_pre = mean_smooth
            v_pre = V_smooth
            self.J.append(J_N)

        smoothed_state_means[-1] = mu_list[-1]
        smoothed_state_covariances[-1] = v_list[-1]
        p_N = np.matmul(np.matmul(self.transition_matrices, v_list[-1,:,:]), 
        J_N = np.matmul(
            np.matmul(v_list[-1,:,:], self.transition_matrices.T),
            np.linalg.inv(p_N))
        self.J = list(reversed(self.J))
        self.J.append(J_N)

        ###########################################################################

        return smoothed_state_means, smoothed_state_covariances

    def em(self, X, max_iter=10):
        """
        Method that perform the EM algorithm to update the model parameters
        Note that in this exercise we ignore offsets
        @param X: a numpy 2D array whose dimension is [n_example, self.n_dim_
        @param max_iter: an integer indicating how many iterations to run
        """
```

```python
        # validate inputs have right dimensions
        n_example, observed_dim = X.shape
        assert observed_dim==self.n_dim_obs

        # keep track of log posterior (use function calculate_posterior below
        self.avg_em_log_posterior = np.zeros(max_iter)*np.nan

        ##############################
        #### TODO: EM iterations ####
        ##############################
        self.avg_em_log_posterior = []
        for iter_num in range(max_iter):
            self.Ezn = []
            self.Ezn_znminus = []
            self.Ezn_zn = []
            smoothed_state_means, smoothed_state_covariances = self.smooth(X)
            self.avg_em_log_posterior.append(np.nanmean(self.calculate_poster

            for i in range(n_example):
                if i != 0:
                    self.Ezn_znminus.append(np.matmul(self.J[i-1], smoothed_s
                self.Ezn_zn.append(smoothed_state_covariances[i] + np.outer(sm
                self.Ezn.append(smoothed_state_means[i])

            self.initial_state_mean = smoothed_state_means[0, :]
            self.initial_state_covariance = smoothed_state_covariances[0, :]


            Ezy = np.sum(np.array(self.Ezn_znminus), 0)
            Ezz = np.sum(np.array(self.Ezn_zn), 0)
            Ezz_minus_n = Ezz - self.Ezn_zn[-1]
            Ezz_minus_1 = Ezz - self.Ezn_zn[0]
            self.transition_matrices = np.matmul(Ezy, np.linalg.inv(Ezz_minus_
            Ezy_a = np.matmul(Ezy, self.transition_matrices.T)
            self.transition_covariance = (Ezz_minus_1 - Ezy_a - Ezy_a.T +
                                          np.matmul(np.matmul(self.transitio

            Xzn =np.dot( X.T, np.array(self.Ezn))
            self.observation_matrices = np.matmul(Xzn, np.linalg.inv(Ezz.T))
            self.observation_covariance = np.zeros((self.n_dim_obs, self.n_di
            for j in range(n_example):
                error = (
                    X[j]
                    - np.dot(self.observation_matrices, smoothed_state_means[
                )
                self.observation_covariance += (np.outer(error, error) + np.d
                        np.dot(smoothed_state_covariances[j],
                               self.observation_matrices.T)))
            self.observation_covariance /= n_example



    def import_param(self, kf_model):
        """
        Method that copies parameters from a trained Kalman Model
        @param kf_model: a Pykalman object
        """
```

```
        need_params = ['transition_matrices', 'observation_matrices', 'transi
                       'observation_covariance', 'initial_state_mean', 'initial_st
        for param in need_params:
            setattr(self, param, getattr(kf_model, param))


    def calculate_posterior(self, X, state_mean, v_n=None):
        """
        Method that calculates the log posterior
        @param X: a numpy 2D array whose dimension is [n_example, self.n_dim_
        @param state_mean: a numpy 2D array whose dimension is [n_example, se
        @output: a numpy 1D array whose dimension is [n_example]
        """

        if v_n is None:
            _, v_n = self.filter(X)
        llh = []
        for i in range(1,len(state_mean)):
            normal_mean = np.dot(self.observation_matrices, np.dot(self.trans
            p_n = self.transition_matrices.dot(v_n[i].dot(self.transition_mat
            #normal_cov = np.matmul(self.observation_matrices, np.matmul(self
            normal_cov = np.matmul(self.observation_matrices, np.matmul(p_n, 
            pdf_val = multivariate_normal.pdf(X[i], normal_mean, normal_cov)
            # replace 0 to prevent numerical underflow
            if pdf_val < 1e-10:
                pdf_val = 1e-10
            llh.append(np.log(pdf_val))
        return np.array(llh)
```

Sampling

In [3]:

```
# Sampling
n_dim_state = 2
n_dim_obs = 2
kf = KalmanFilter(n_dim_state=n_dim_state, n_dim_obs=n_dim_obs)
# set paramters
kf.transition_matrices = np.eye(kf.n_dim_state)*.5
kf.transition_covariance = np.eye(kf.n_dim_obs)
kf.observation_matrices = np.eye(kf.n_dim_state)
kf.observation_covariance = np.eye(kf.n_dim_obs)*.1
kf.initial_state_mean = np.zeros(kf.n_dim_state)
kf.initial_state_covariance = np.eye(kf.n_dim_state)*.1
# import to your own kalman object
my_kf = MyKalmanFilter(n_dim_state=n_dim_state, n_dim_obs=n_dim_obs)
my_kf.import_param(kf)
# print the parameters
print_parameters(my_kf, evals=True)
```

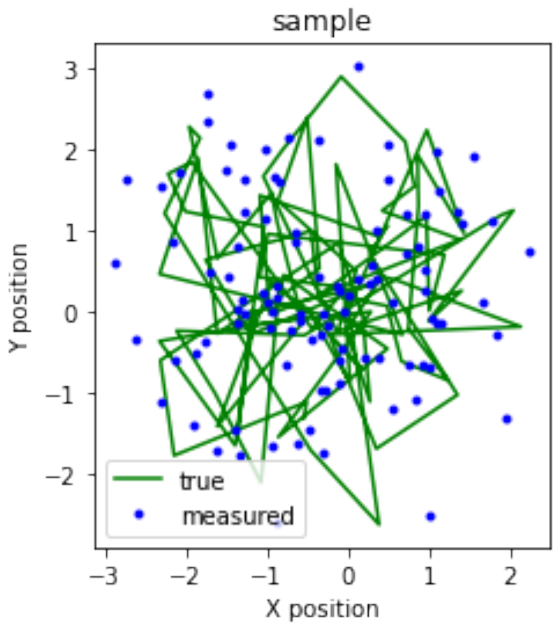
```
transition_matrices = [0.5 0.5], shape = (2,)

transition_covariance = [1. 1.], shape = (2,)

observation_covariance = [0.1 0.1], shape = (2,)

initial_state_covariance = [0.1 0.1], shape = (2,)

observation_matrices = [[1. 0.]
 [0. 1.]], shape = (2, 2)

initial_state_mean = [0. 0.], shape = (2,)
```

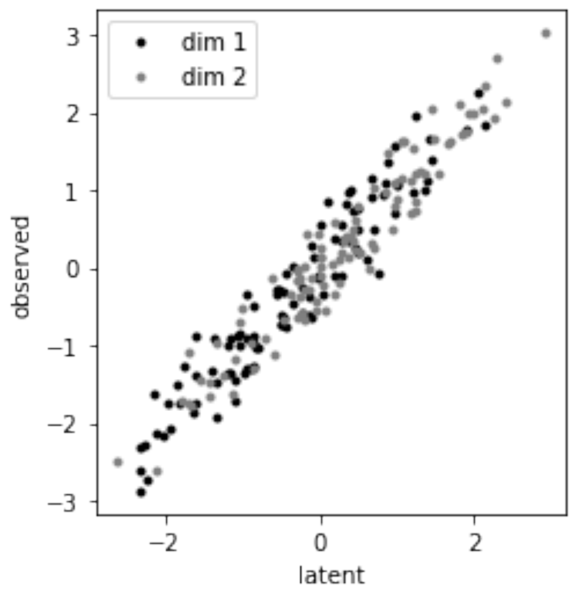
test that your sampling works:

In [4]:

```python
sampled_states, sampled_observations = kf.sample(100, initial_state=kf.initia
sampled_states_impl, sampled_observations_impl = my_kf.sample(100, initial_st
print('sampled states pykalman at t=2: ', sampled_states[2,:])
print('sampled states own implementation at t=2: ', sampled_states_impl[2,:])
fig = plot_kalman(sampled_states_impl[:,0],sampled_states_impl[:,1],sampled_o
```

```
sampled states pykalman at t=2:  [1.43945741 0.96908939]
sampled states own implementation at t=2:  [1.43945741 0.96908939]
```



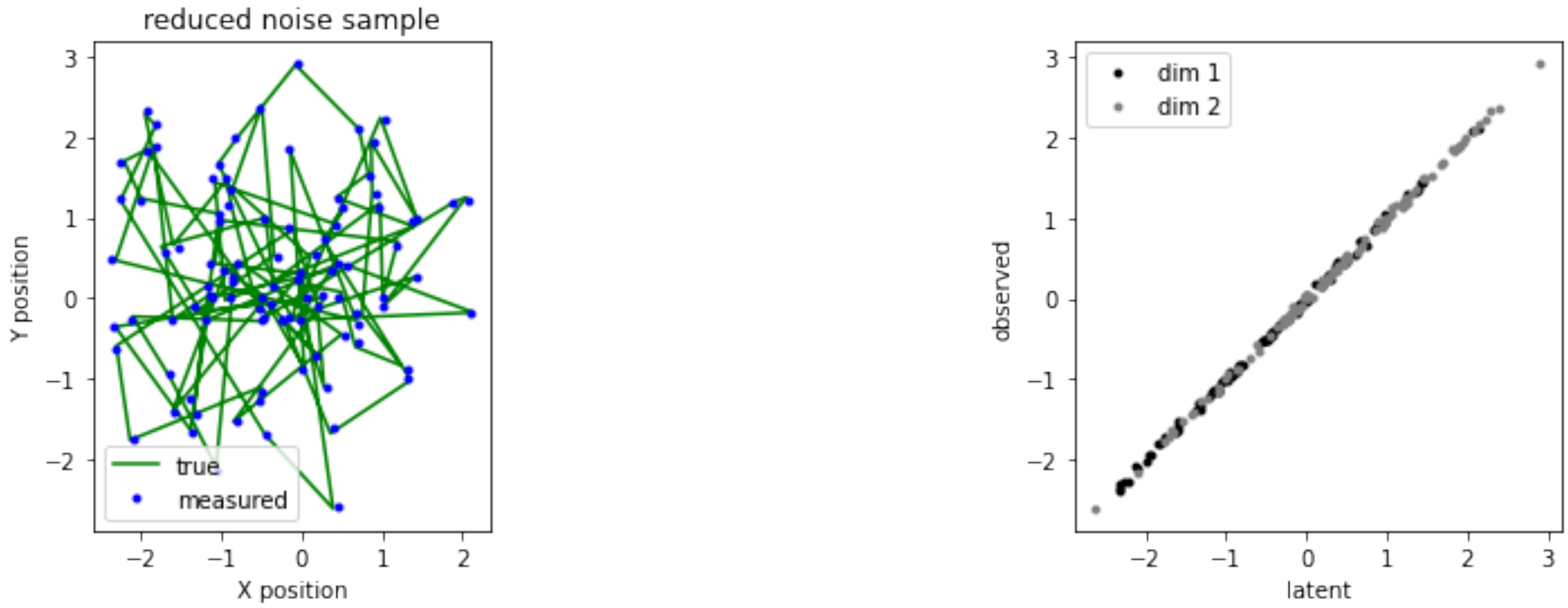


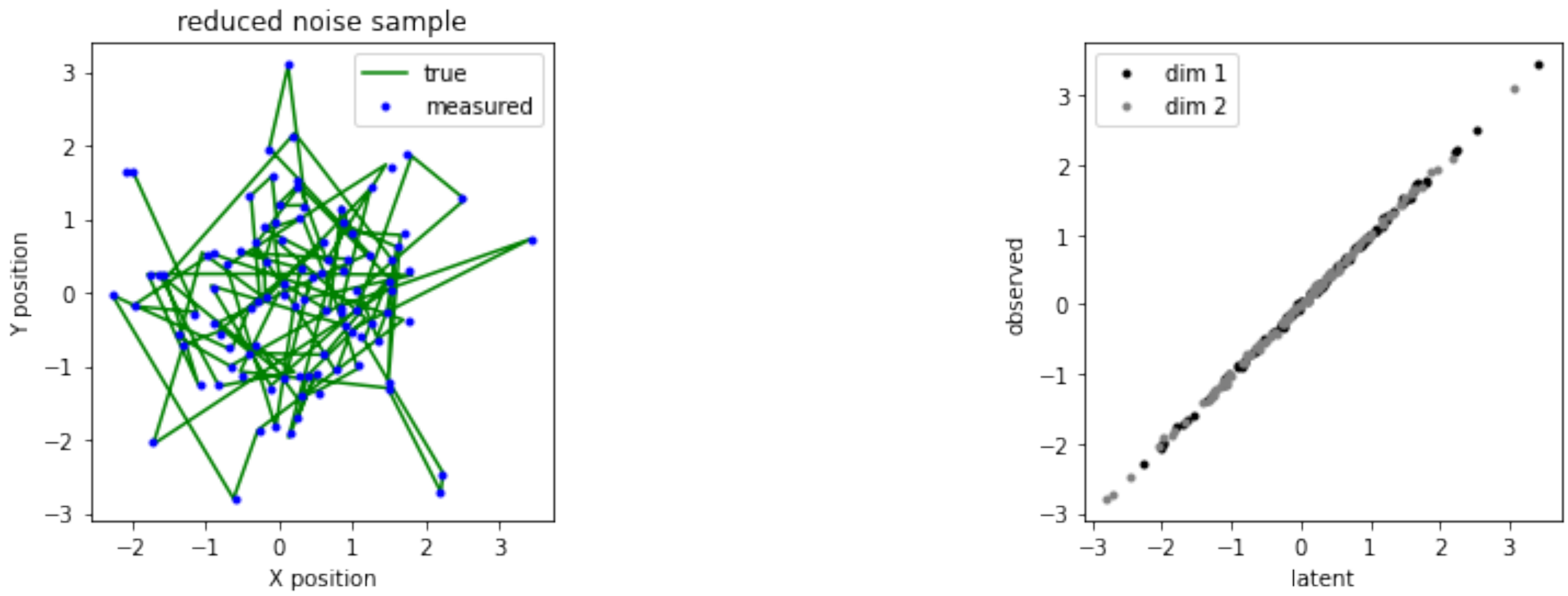
reduce observation noise

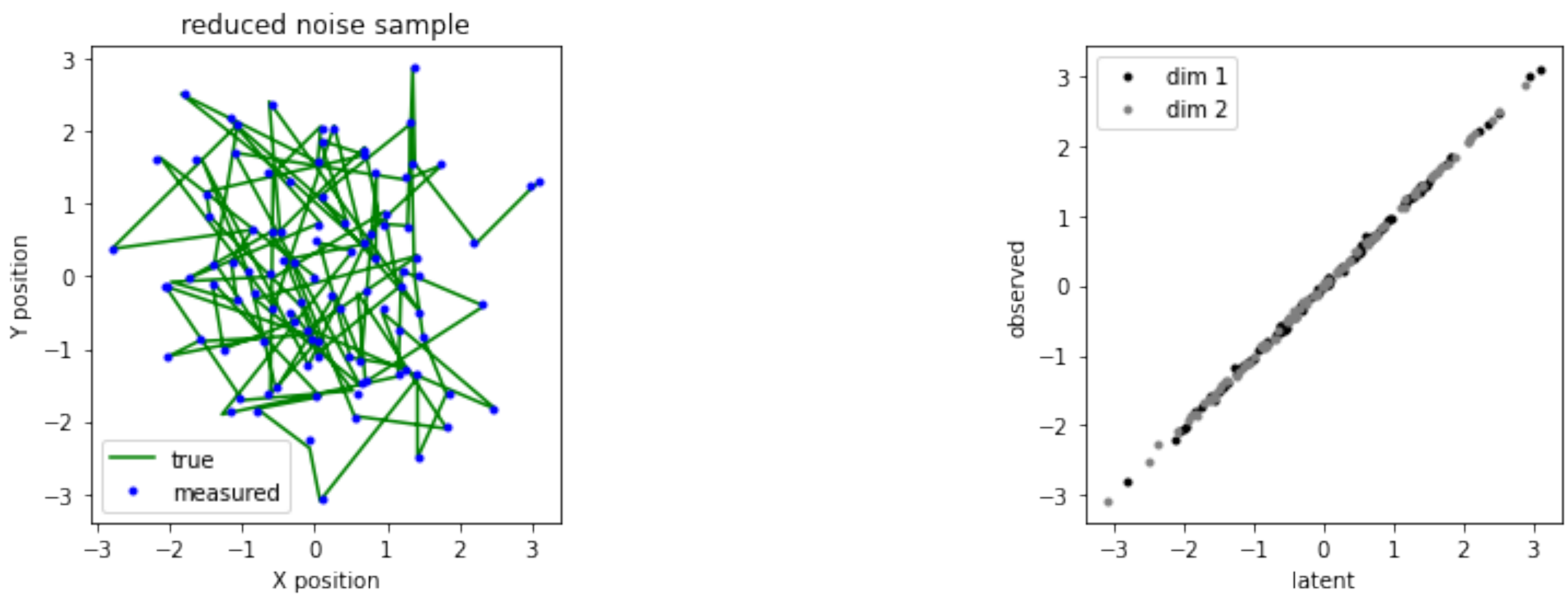
What do you expect should happen?

In [5]:

```python
#### TODO ####
#### reduce observation noise ####
obscov_old = my_kf.observation_covariance.copy()
my_kf.observation_covariance = my_kf.observation_covariance*.01

# plot
for nn in range(3):
    sampled_states_impl, sampled_observations_impl = my_kf.sample(100, initia
    fig = plot_kalman(sampled_states_impl[:,0],sampled_states_impl[:,1],sampl
    plt.axis('square');
```



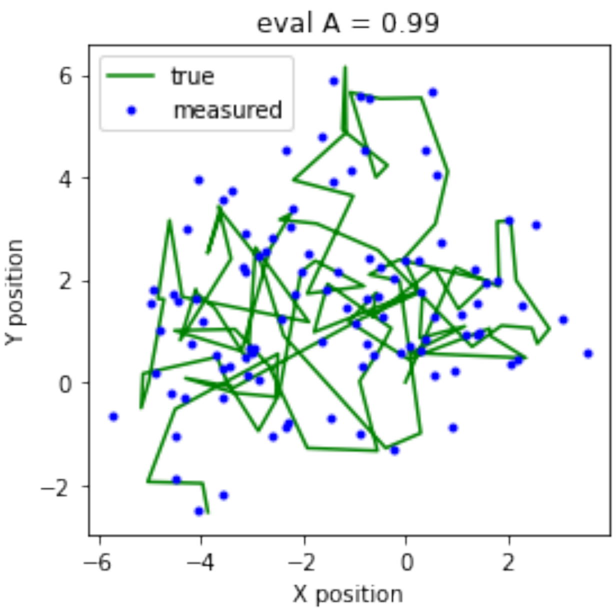


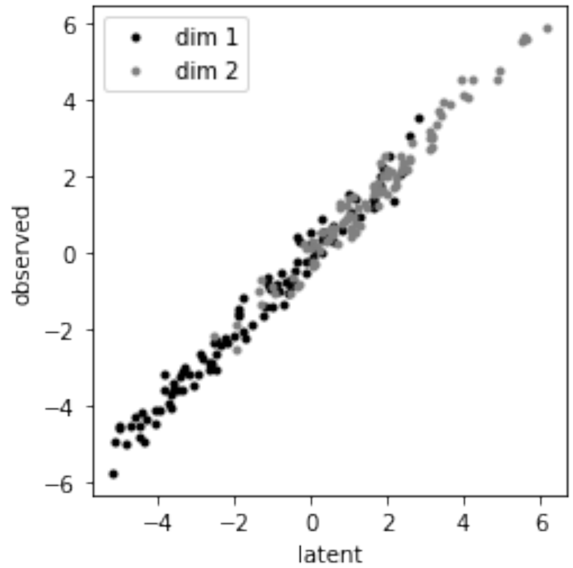


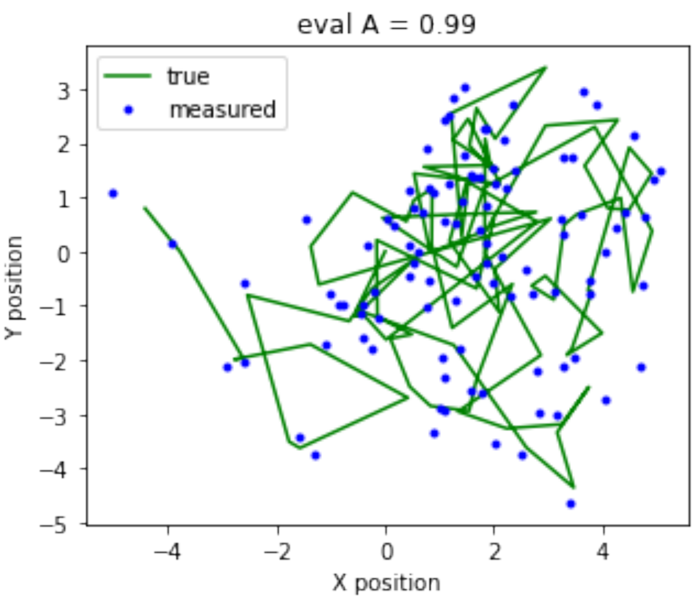
increase the respective temporal dyamics

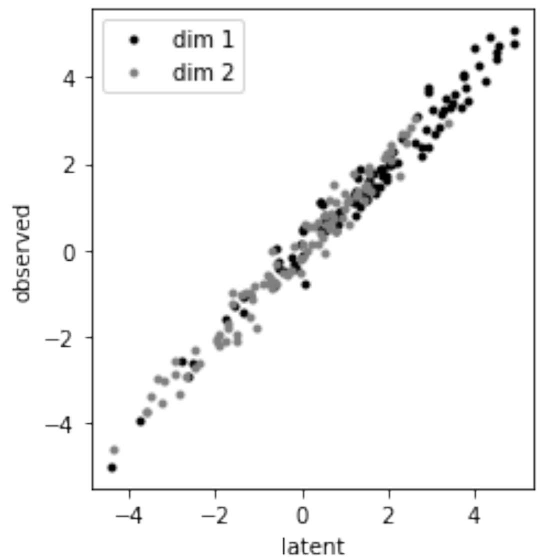
What do you expect should happen?

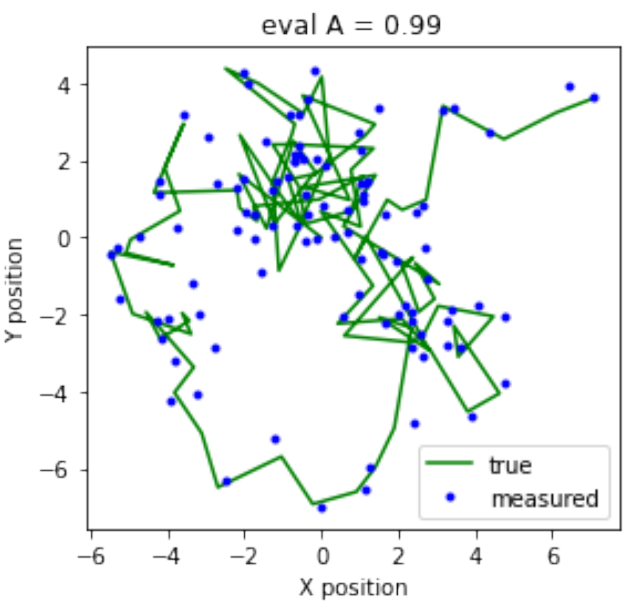
```
In [6]: #### TODO ####
        #### increase latent temporal dependency ####
        my_kf.observation_covariance = obscov_old.copy()
        my_kf.transition_matrices = np.eye(n_dim_state)*.9

        # plot
        for nn in range(3):
            sampled_states_impl, sampled_observations_impl = my_kf.sample(100, initia
            fig = plot_kalman(sampled_states_impl[:,0],sampled_states_impl[:,1],
                              sampled_observations_impl[:,0],sampled_observations_imp
```

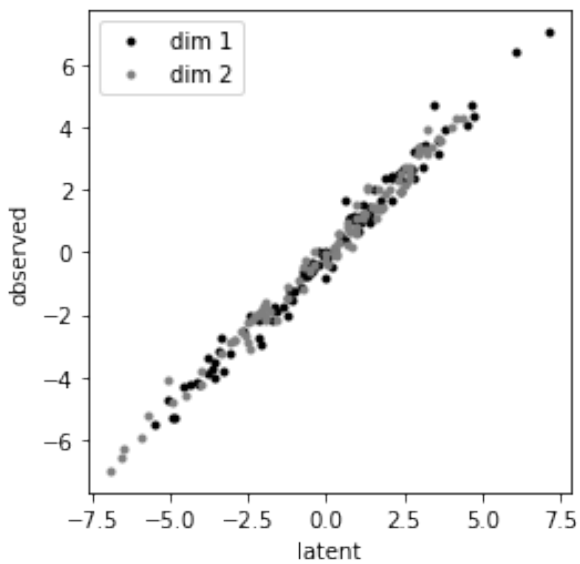










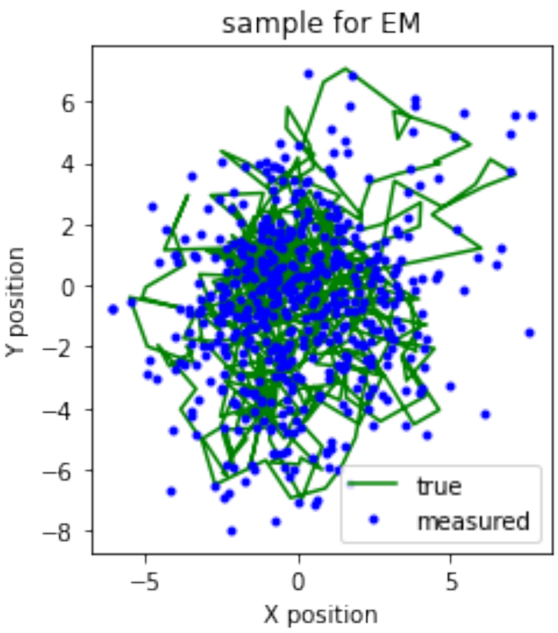


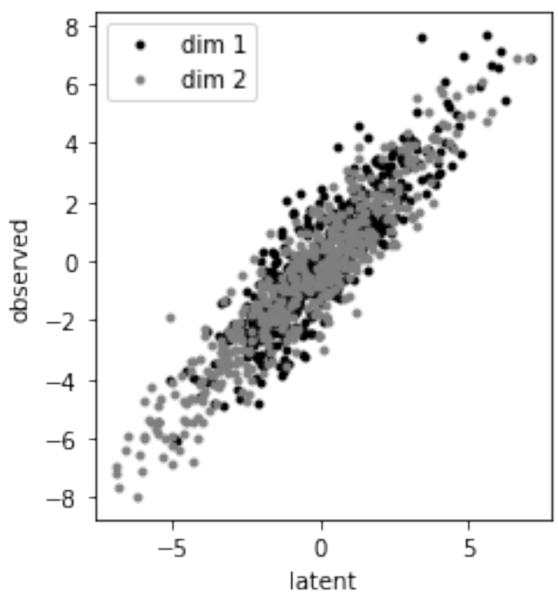
EM

data to use

In [7]:
```
kf_GT = KalmanFilter(n_dim_state=n_dim_state, n_dim_obs=n_dim_obs)
# set paramters
kf_GT.transition_matrices = np.eye(n_dim_state)*.9
kf_GT.transition_covariance = np.eye(n_dim_obs)
kf_GT.observation_matrices = np.eye(n_dim_state)
kf_GT.observation_covariance = np.eye(n_dim_obs)
kf_GT.initial_state_mean = np.zeros(n_dim_state)
kf_GT.initial_state_covariance = np.eye(n_dim_state)*.1
# import to your own kalman object
my_kf_GT = MyKalmanFilter(n_dim_state=n_dim_state, n_dim_obs=n_dim_obs)
my_kf_GT.import_param(kf_GT)
# print the parameters
print_parameters(my_kf_GT, evals=True)


# sample
latent, data = kf_GT.sample(500, initial_state=kf_GT.initial_state_mean, rand
_, _ = kf_GT.filter(data)
estlat,_ = kf_GT.smooth(data)
fig = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1], title='sample
```

```
transition_matrices = [0.9 0.9], shape = (2,)

transition_covariance = [1. 1.], shape = (2,)

observation_covariance = [1. 1.], shape = (2,)

initial_state_covariance = [0.1 0.1], shape = (2,)

observation_matrices = [[1. 0.]
 [0. 1.]], shape = (2, 2)

initial_state_mean = [0. 0.], shape = (2,)
```

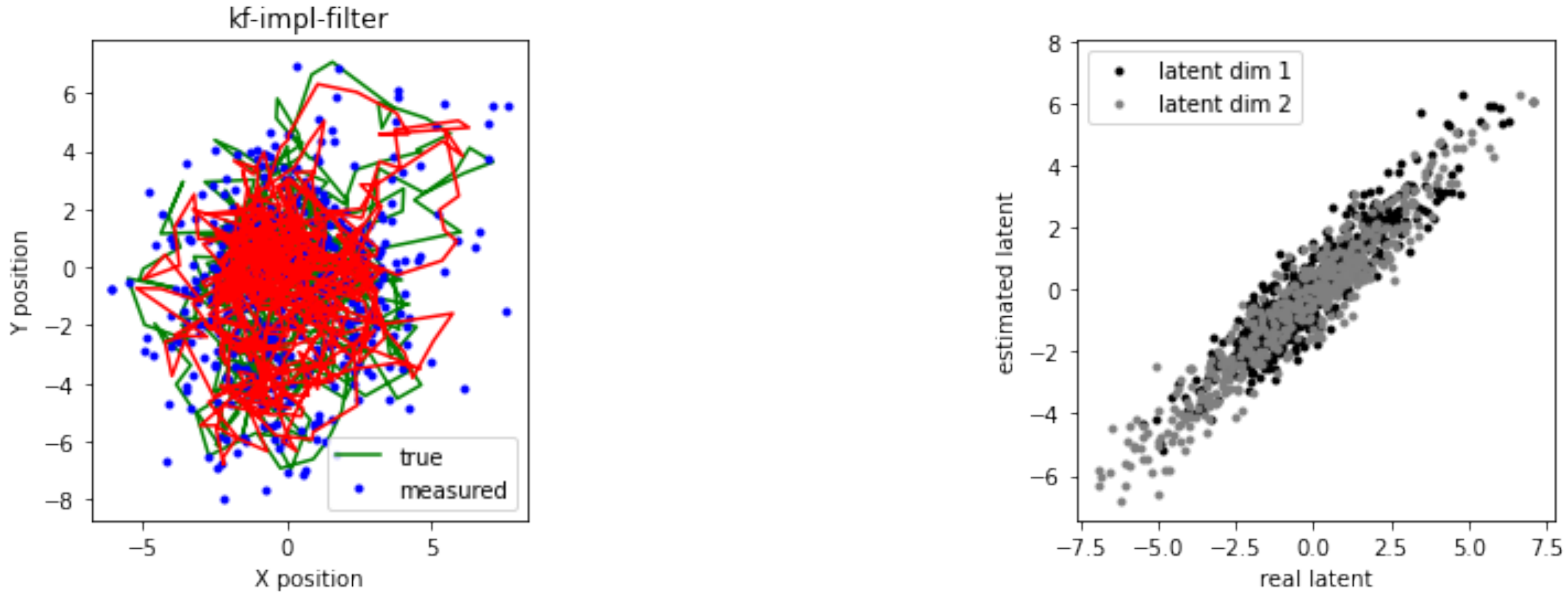




Filtering

with known parameters

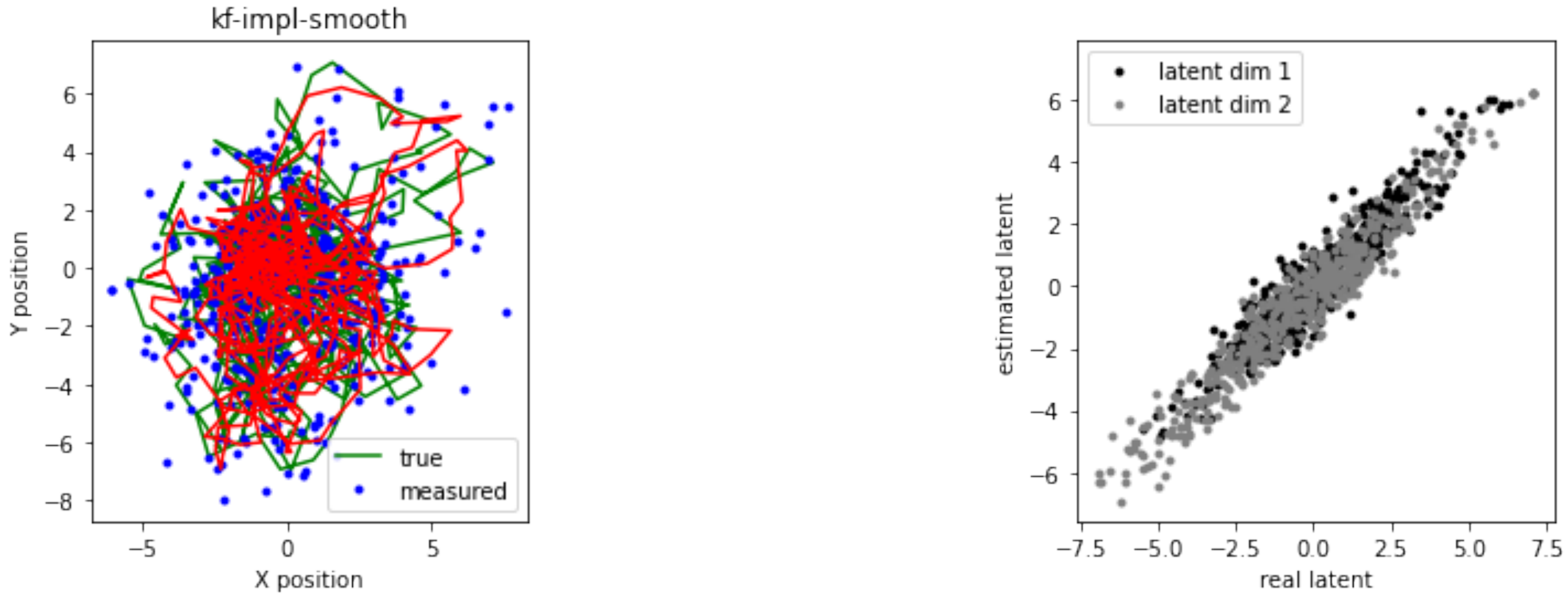
In [8]:
```
filtered_state_means_impl, filtered_state_covariances_impl = my_kf_GT.filter(
fig = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1], filtered_state_
plt.axis('square');
```



Smoothing

with known parameters

In [9]:
```
smoothed_state_means_impl, smoothed_state_covariances_impl = my_kf_GT.smooth(
fig = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1],
                  smoothed_state_means_impl[:,0], smoothed_state_means_impl[:
plt.axis('square');
```



run EM

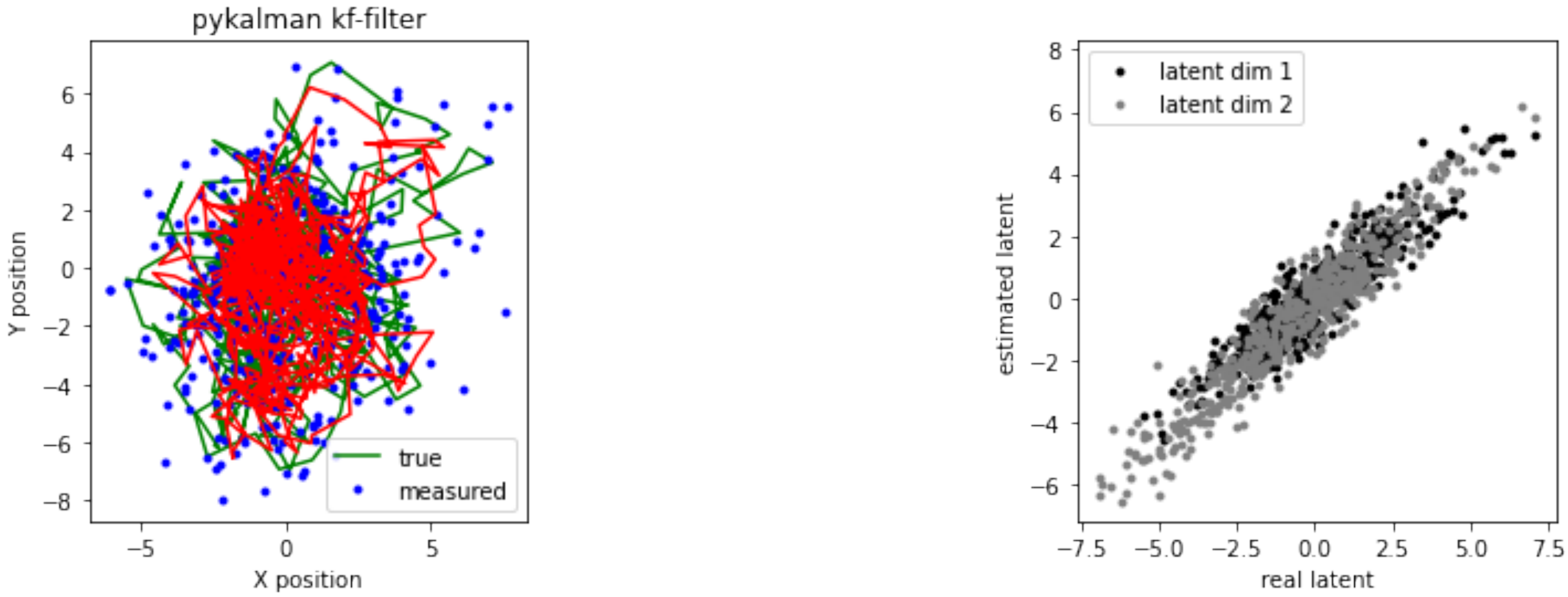
to learn parameters (M-step)

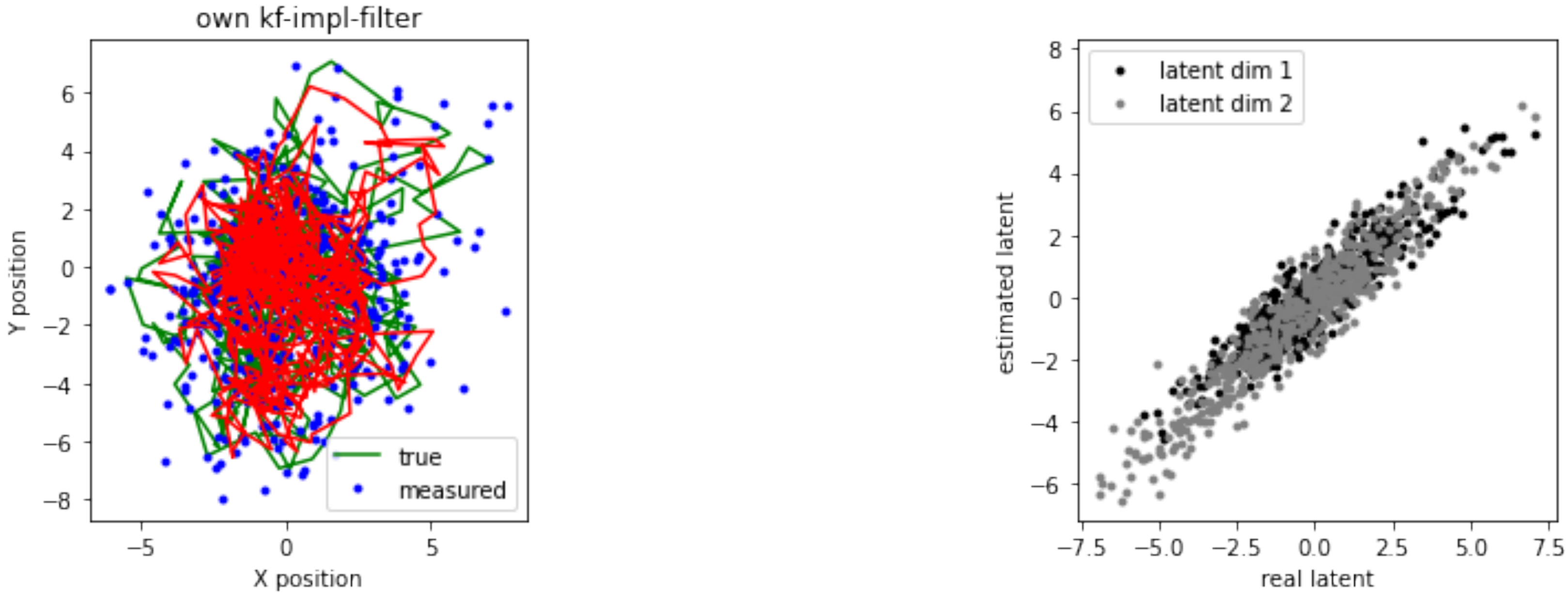
In [10]:

```
np.random.seed(0)

iters = 10
# perturb starting parameters
kf = KalmanFilter(n_dim_state=data.shape[1], n_dim_obs=data.shape[1],
                  transition_matrices= np.eye(data.shape[1]) * 0.95,
                  observation_matrices= np.eye(data.shape[1])+np.random.randn(
                  transition_covariance= np.eye(data.shape[1]),
                  observation_covariance = np.eye(data.shape[1]),
                  initial_state_mean=np.random.randn(data.shape[1]),
                   initial_state_covariance = np.eye(data.shape[1]),
                  em_vars = ['transition_matrices', 'observation_matrices','tr
                           'initial_state_mean', 'initial_state_covariance'])

my_kf = MyKalmanFilter(n_dim_state=data.shape[1], n_dim_obs=data.shape[1])
my_kf.import_param(kf)

kf.em(data, n_iter=iters)
my_kf.em(data, max_iter=iters)

print('           pykalman EM:')
print(' ')
print_parameters(kf, evals=True)
print('           own implementation EM:')
print(' ')
print_parameters(my_kf, evals=True)
```

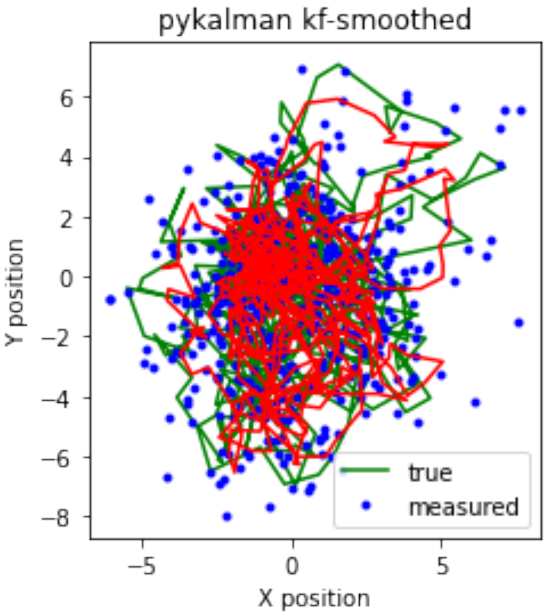
```
           pykalman EM:

transition_matrices = [0.87764566 0.92906913], shape = (2,)

transition_covariance = [0.68854391 0.94375978], shape = (2,)

observation_covariance = [0.97845363 0.88969914], shape = (2,)

initial_state_covariance = [0.04314256 0.06181459], shape = (2,)

observation_matrices = [[1.12314164 0.02460226]
 [0.1431459  1.01994597]], shape = (2, 2)

initial_state_mean = [-1.20813372  0.51062271], shape = (2,)

           own implementation EM:

transition_matrices = [0.87703758 0.92961749], shape = (2,)

transition_covariance = [0.68882951 0.9447692 ], shape = (2,)

observation_covariance = [0.97828376 0.8894537 ], shape = (2,)

initial_state_covariance = [0.0431518  0.06180991], shape = (2,)

observation_matrices = [[1.12300893 0.02462427]
 [0.14317923 1.01988379]], shape = (2, 2)

initial_state_mean = [-1.20845467  0.5091462 ], shape = (2,)
```

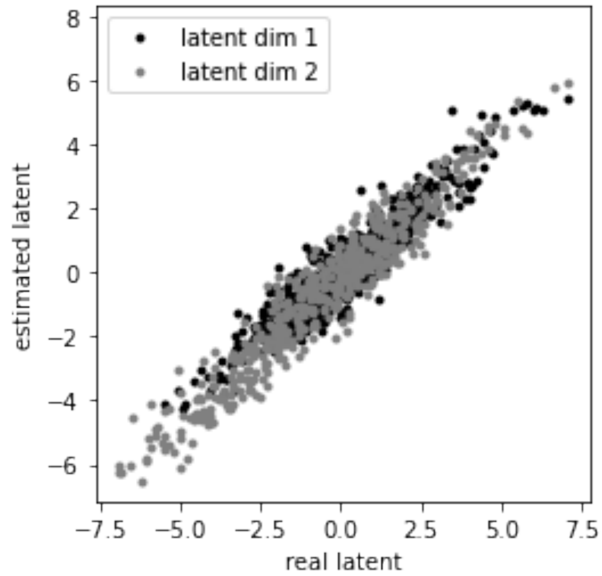
In [11]:
```python
# compare the filter results
filtered_state_means, filtered_state_covariances = kf.filter(data)
filtered_state_means_impl, filtered_state_covariances_impl = my_kf.filter(dat
_ = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1],
                filtered_state_means[:,0], filtered_state_means[:,1], "r-", t
plt.axis('square');
_ = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1],
                filtered_state_means_impl[:,0], filtered_state_means_impl[:,1
plt.axis('square');
```

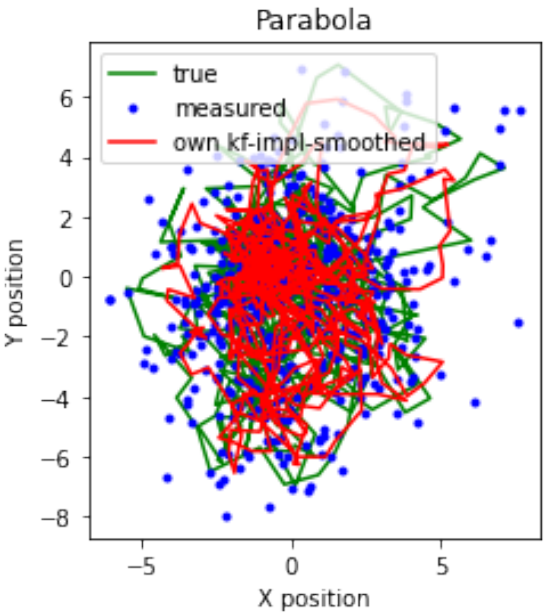


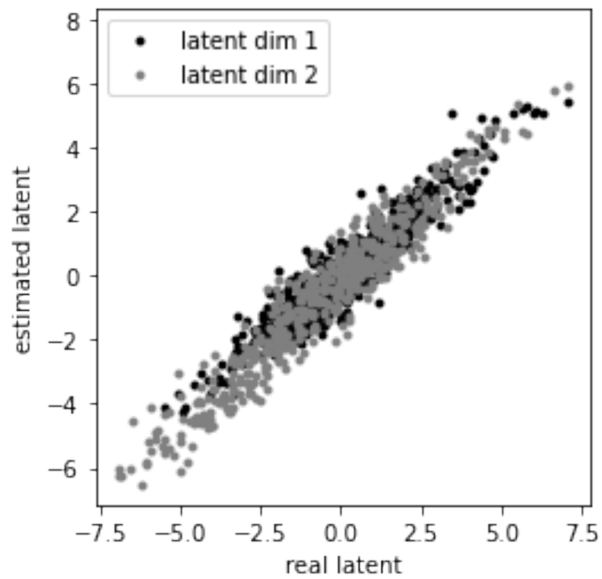


In [12]:
```python
# compare the smooth results
smoothed_state_means, smoothed_state_covariances = kf.smooth(data)
smoothed_state_means_impl, smoothed_state_covariances_impl = my_kf.smooth(dat
_ = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1],
                smoothed_state_means[:,0], smoothed_state_means[:,1], "r-", t
plt.axis('square');
_ = plot_kalman(latent[:,0],latent[:,1],data[:,0],data[:,1],
                smoothed_state_means_impl[:,0], smoothed_state_means_impl[:,1
plt.axis('square');
```





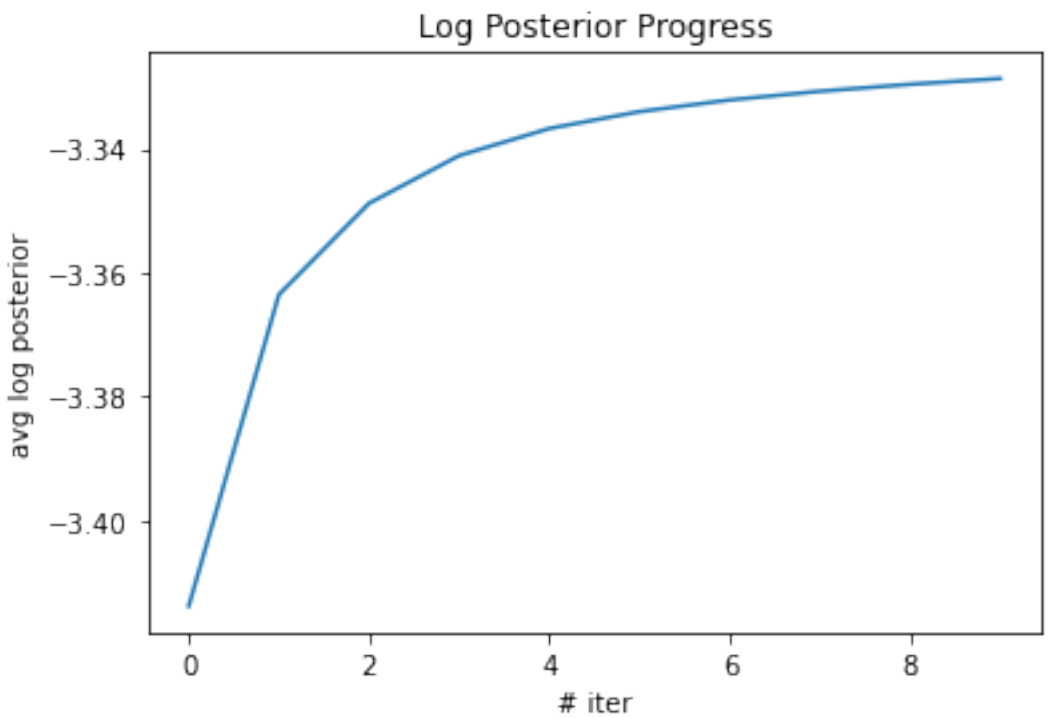




In [13]:

```
# visualize the change of avg log posterior
print(kf.__dict__.keys())
visualize_line_plot(my_kf.avg_em_log_posterior, "# iter", "avg log posterior"
```

```
dict_keys(['transition_matrices', 'observation_matrices', 'transition_covarian
ce', 'observation_covariance', 'transition_offsets', 'observation_offsets', 'i
nitial_state_mean', 'initial_state_covariance', 'random_state', 'em_vars', 'n_
dim_state', 'n_dim_obs'])
```



In [ ]:

In [ ]:

In [ ]:

Please turn in the code as a notebook AND as a pdf before 10/14/2019 11:55 pm. Please name your notebook netid.ipynb.

Your work will be evaluated based on the code and plots. You don't need to write down your answers to these questions in the text blocks.

In [ ]:

In [ ]:

Processing math: 100%