NANJING UNIVERSITY

图形绘制技术（Rendering）
Chapter 2: Rendering Concepts

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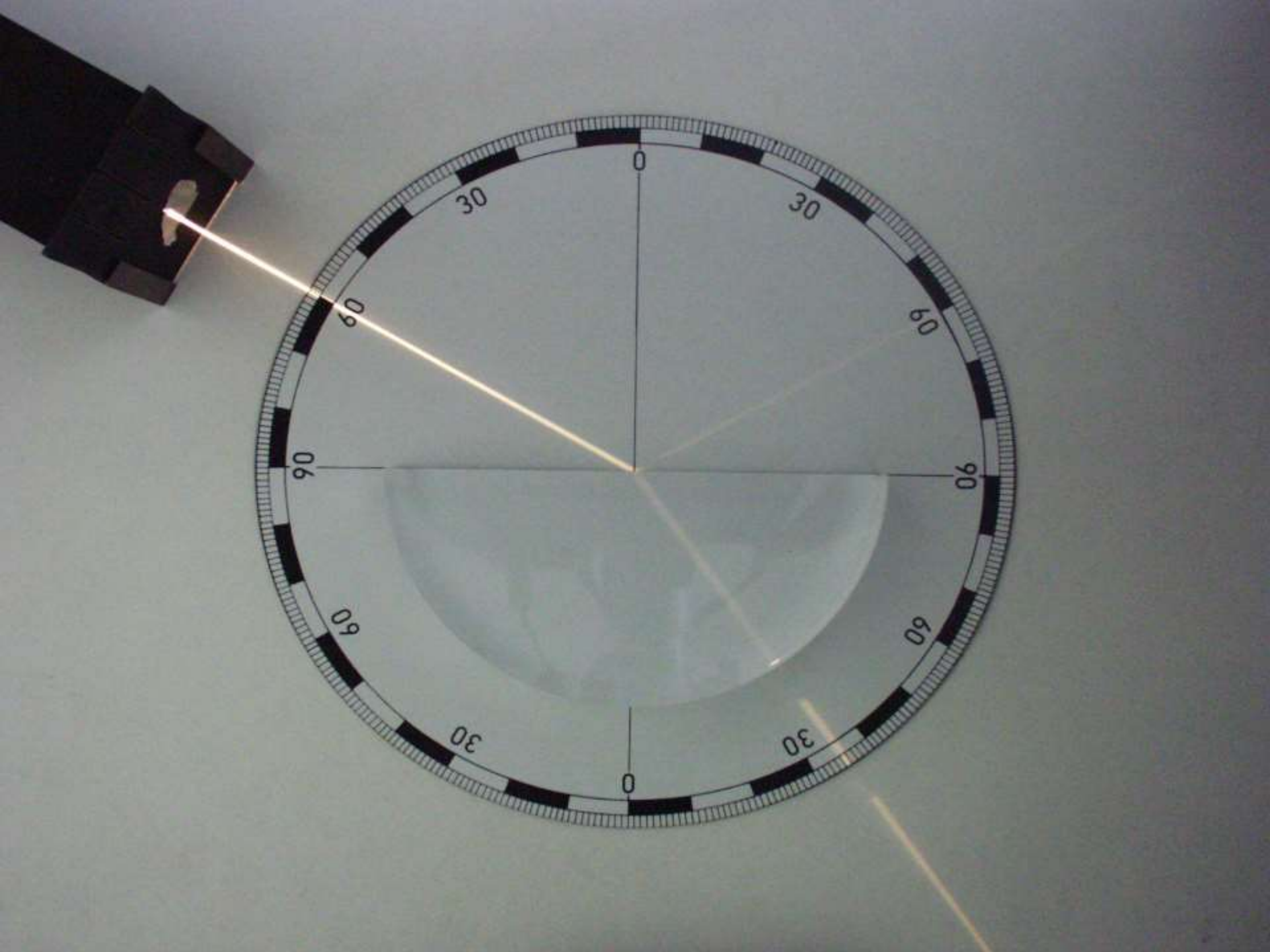
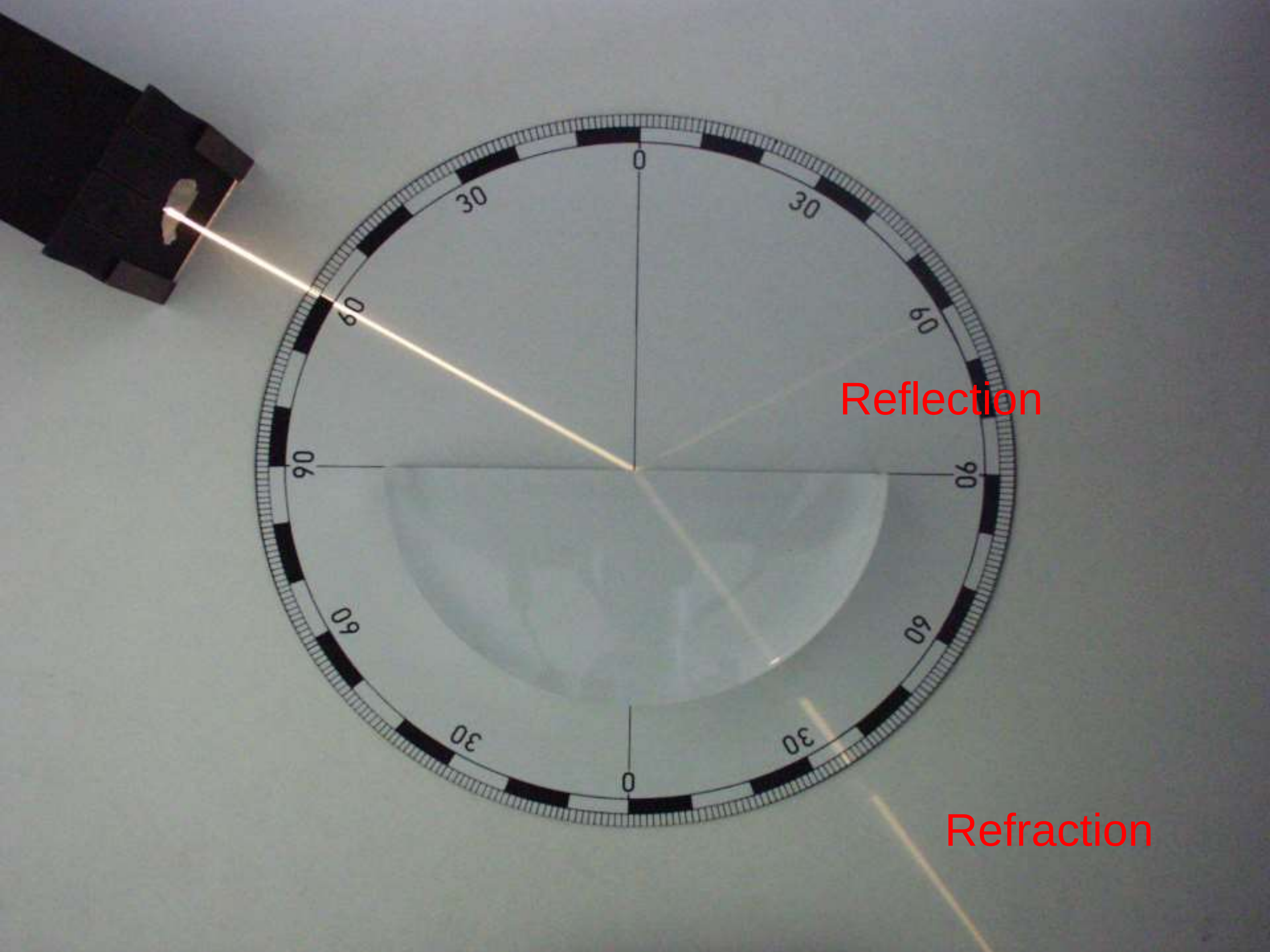
Contents

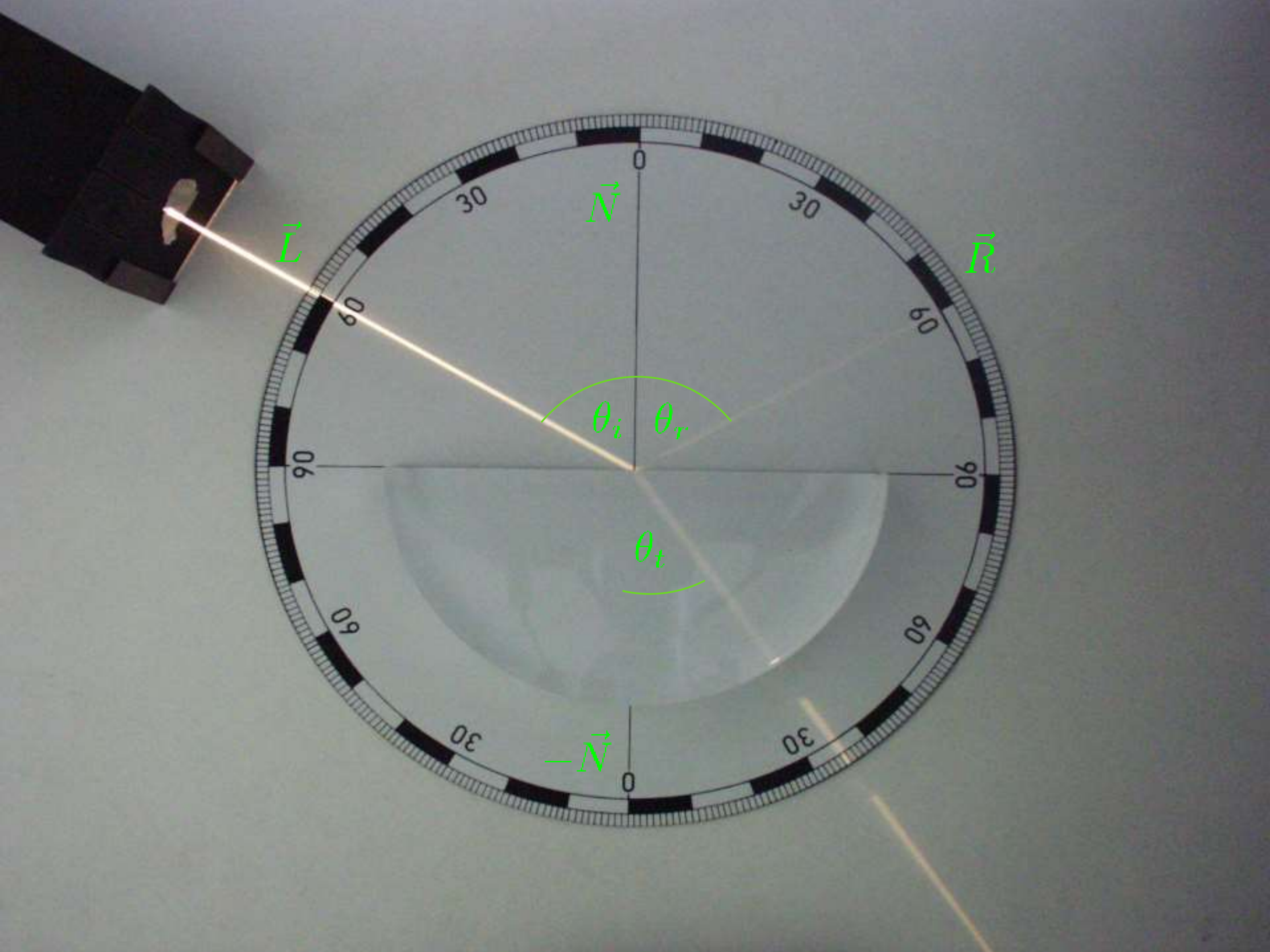
- Geometry Optics（光怎么传播）
- Radiometry and Color（光的能量怎么描述）
- BRDF and Reflection Functions（光和物体的交互）
- The Rendering Equation（统一的框架）

Geometry Optics

- Law of linear propagation
- Law of reflection
- Law of refraction

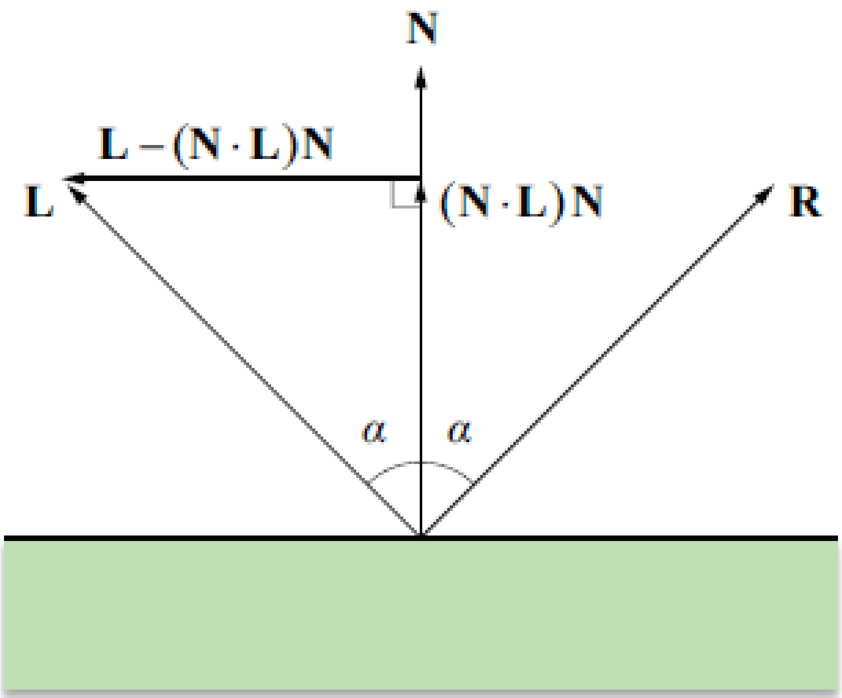

0
30
30
60
60
90
90
60
60
30
30
0


Reflection
Refraction



$\vec{L}$
$\vec{N}$
$\vec{R}$
θ_i
θ_r
θ_t
$-\vec{N}$
0
30
60
90

Reflection Vector Calculation



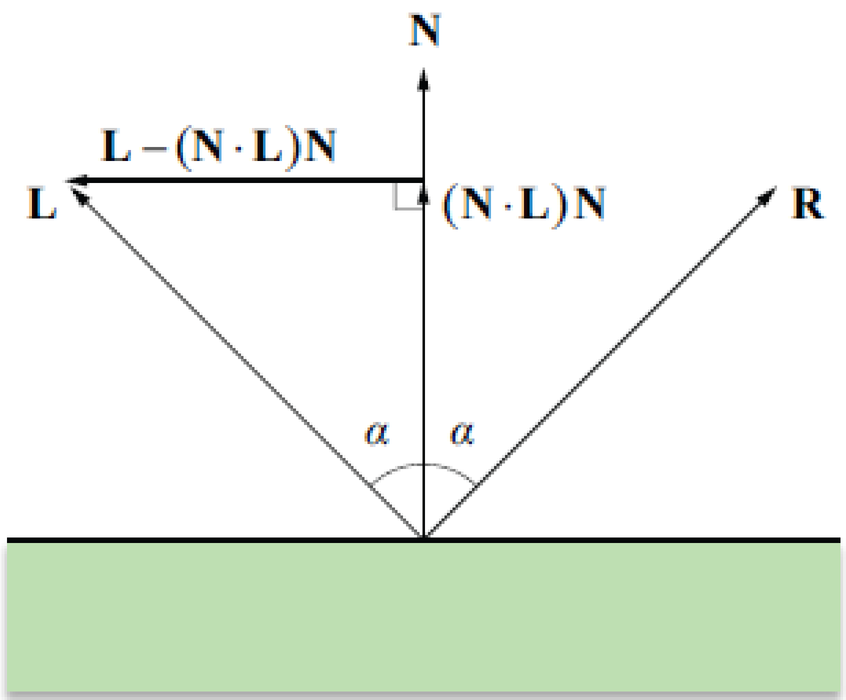
- **N**: surface normal
- **L**: vector pointing towards a light source
- **R**: reflected ray direction

Reflection Vector Calculation

- **N** and **L** have been normalized to unit length

$$\mathrm{perp}_{\mathbf{N}}\mathbf{L} = \mathbf{L} - (\mathbf{N}\cdot\mathbf{L})\mathbf{N}.$$

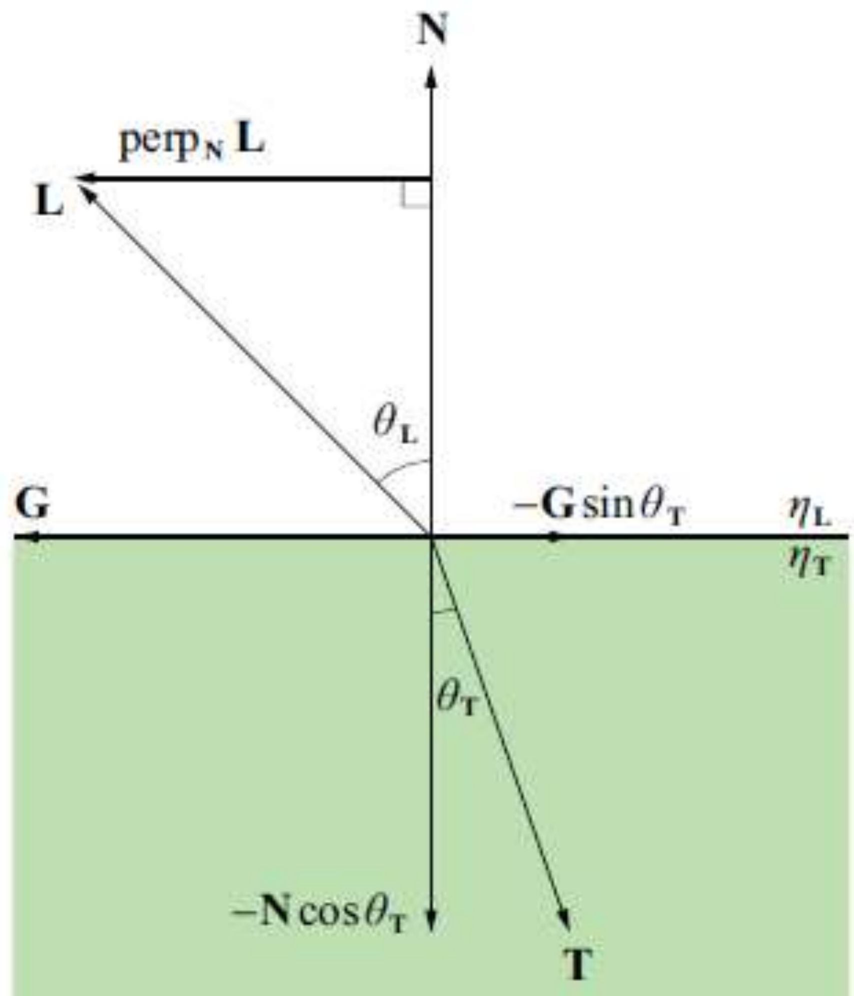
$$\begin{aligned}\mathbf{R} &= \mathbf{L} - 2\,\mathrm{perp}_{\mathbf{N}}\mathbf{L} \\ &= \mathbf{L} - 2[\mathbf{L} - (\mathbf{N}\cdot\mathbf{L})\mathbf{N}] \\ &= 2(\mathbf{N}\cdot\mathbf{L})\mathbf{N} - \mathbf{L}.\end{aligned}$$



Refraction Vector Calculation

- **T**: Refracted vector
- **T** is also normalized
- Snell's law:

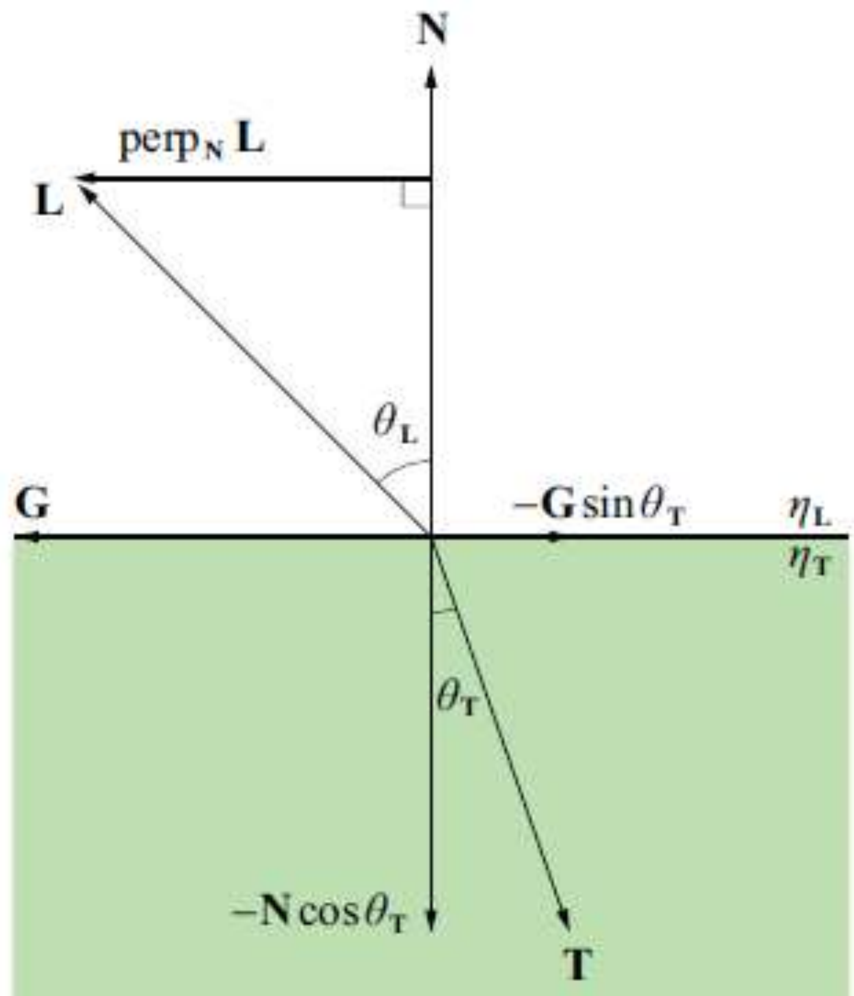
$$\eta_L \sin\theta_L = \eta_T \sin\theta_T;$$



Refraction Vector Calculation

$$\mathbf{G} = \frac{\mathrm{perp}_{\mathbf{N}}\,\mathbf{L}}{\sin\theta_{\mathbf{L}}} = \frac{\mathbf{L} - (\mathbf{N}\cdot\mathbf{L})\mathbf{N}}{\sin\theta_{\mathbf{L}}}$$

$$\begin{aligned}\mathbf{T} &= -\mathbf{N}\cos\theta_{\mathbf{T}} - \mathbf{G}\sin\theta_{\mathbf{T}} \\ &= -\mathbf{N}\cos\theta_{\mathbf{T}} - \frac{\sin\theta_{\mathbf{T}}}{\sin\theta_{\mathbf{L}}}\left[\mathbf{L} - (\mathbf{N}\cdot\mathbf{L})\mathbf{N}\right]\end{aligned}$$



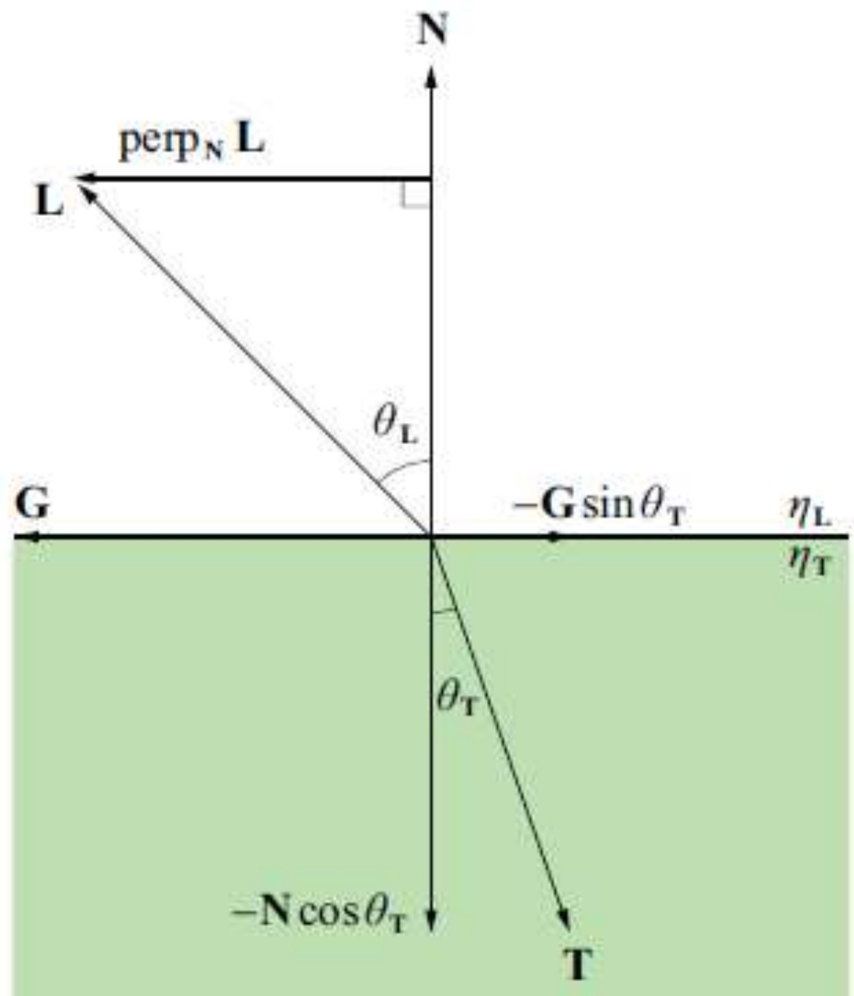
Refraction Vector Calculation

$$\eta_{\mathbf{L}} \sin\theta_{\mathbf{L}} = \eta_{\mathbf{T}} \sin\theta_{\mathbf{T}}.$$

$$\mathbf{T} = -\mathbf{N}\cos\theta_{\mathbf{T}} - \frac{\eta_{\mathbf{L}}}{\eta_{\mathbf{T}}}[\mathbf{L} - (\mathbf{N}\cdot\mathbf{L})\mathbf{N}].$$

Replacing $\cos\theta_{\mathbf{T}}$ with $\sqrt{1-\sin^2\theta_{\mathbf{T}}}$

$$\mathbf{T} = -\mathbf{N}\sqrt{1 - \frac{\eta_{\mathbf{L}}^2}{\eta_{\mathbf{T}}^2}\sin^2\theta_{\mathbf{L}}} - \frac{\eta_{\mathbf{L}}}{\eta_{\mathbf{T}}}[\mathbf{L} - (\mathbf{N}\cdot\mathbf{L})\mathbf{N}].$$



Refraction Vector Calculation

Replacing $\sin^2\theta_{\mathbf{L}}$ with $1-\cos^2\theta_{\mathbf{L}} = 1-(\mathbf{N}\cdot\mathbf{L})^2$ finally yields

$$\boxed{\mathbf{T} = \left(\frac{\eta_{\mathbf{L}}}{\eta_{\mathbf{T}}}\mathbf{N}\cdot\mathbf{L} - \sqrt{1-\frac{\eta_{\mathbf{L}}^2}{\eta_{\mathbf{T}}^2}\left[1-(\mathbf{N}\cdot\mathbf{L})^2\right]}\right)\mathbf{N} - \frac{\eta_{\mathbf{L}}}{\eta_{\mathbf{T}}}\mathbf{L}.}$$

Total Internal Reflection: $\sin\theta_{\mathbf{L}} \le \eta_{\mathbf{T}}/\eta_{\mathbf{L}}$

Basic Assumptions about Light

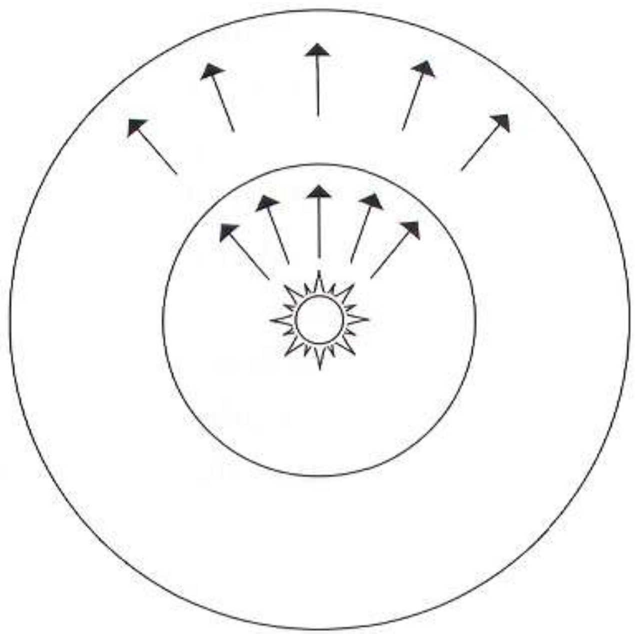
- **Linearity**: the combined effect of two inputs is equal to the sum of effects
- **Energy conservation**: scattering event can't produce more energy than they started with
- **Steady state**: light is assumed to have reached equilibrium, so its radiance distribution isn't changing over time.
- **No polarization**: we only care the frequency of light but not other properties (such as phases)
- **No fluorescence or phosphorescence**: behavior of light at a wavelength or time doesn't affect the behavior of light at other wavelengths or time

Radiometry

- Radiometry is the science of measuring radiant energy transfers.
- The study of the propagation of electromagnetic radiation in an environment
- Radiometric Quantities
  - Energy
  - Radiant power (total flux) Φ, P
  - Irradiance (flux density) E
  - Radiosity (flux density) B
  - Intensity I
  - **Radiance** L

Radiant Power (Flux)

- Total amount of energy passing through a surface per unit of time (J/s,W)



Irradiance & Radiosity

- Irradiance E is the total radiant power per unit area (flux density) *incident onto* a surface with a fixed orientation.

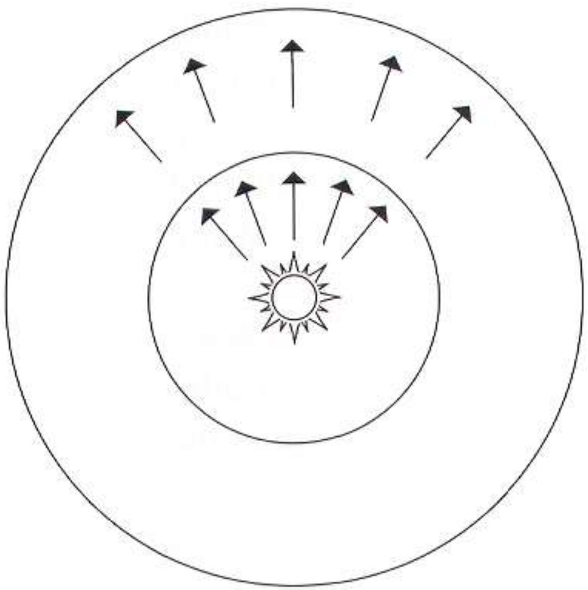
- Radiosity B is defined as the total radiant power per unit area (flux density) *leaving* a surface.

Irradiance

- Area density of flux (W/m^2) $$E = \frac{d\Phi}{dA}$$

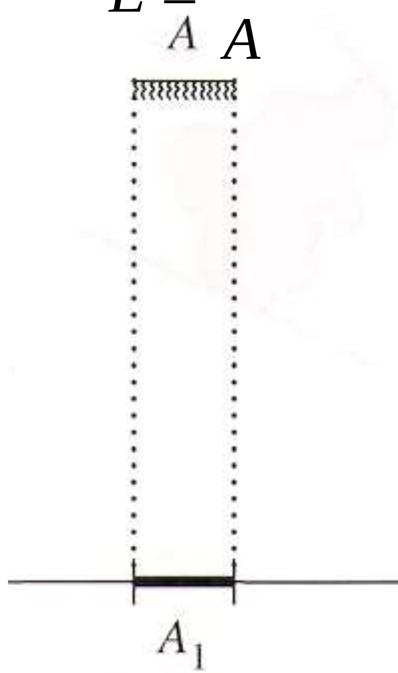
Inverse square law

$$E = \frac{\Phi}{4\pi r^2}$$

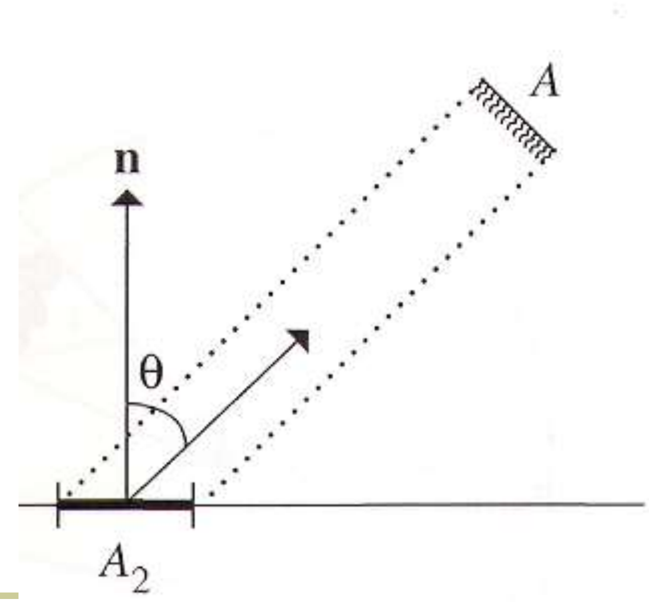


Lambert's law

$$E = \frac{\Phi}{A}$$

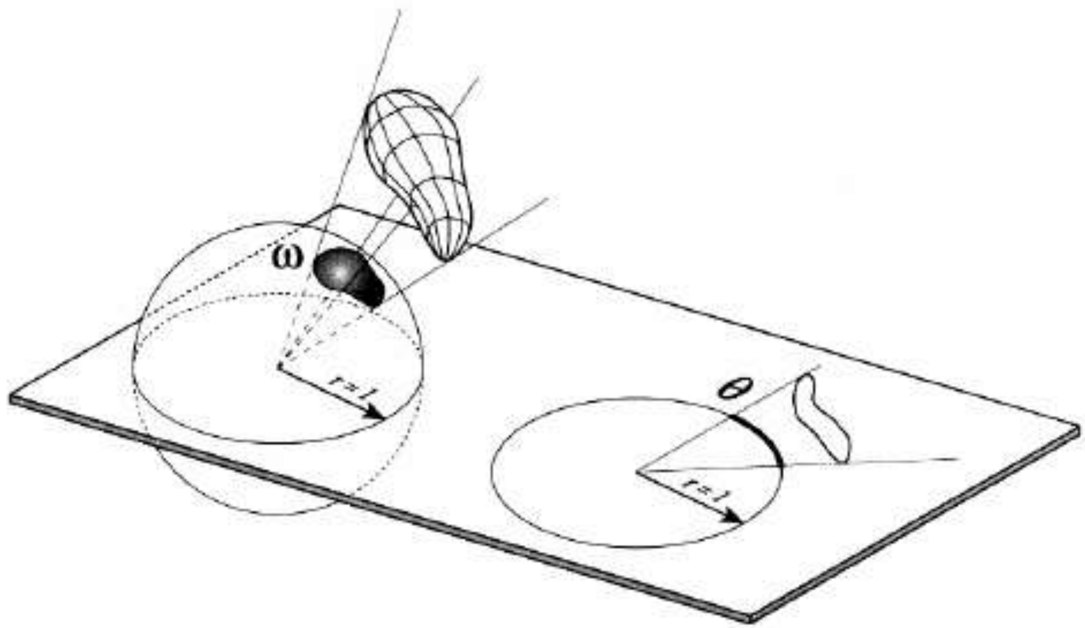


$$E = \frac{\Phi \cos\theta}{A}$$



Angles and Solid Angles

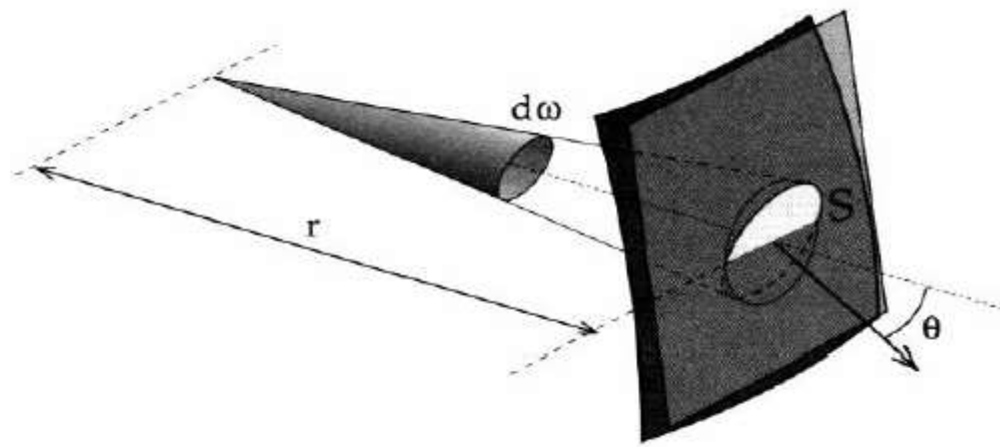
- θ: the angle subtended by a curve in the plane is the length of the projected arc on the unit circle.
- ω: the solid angle subtended by an object is the surface area of its projection onto the unit sphere (steradians [sr]).



Solid Angle for a Small Area

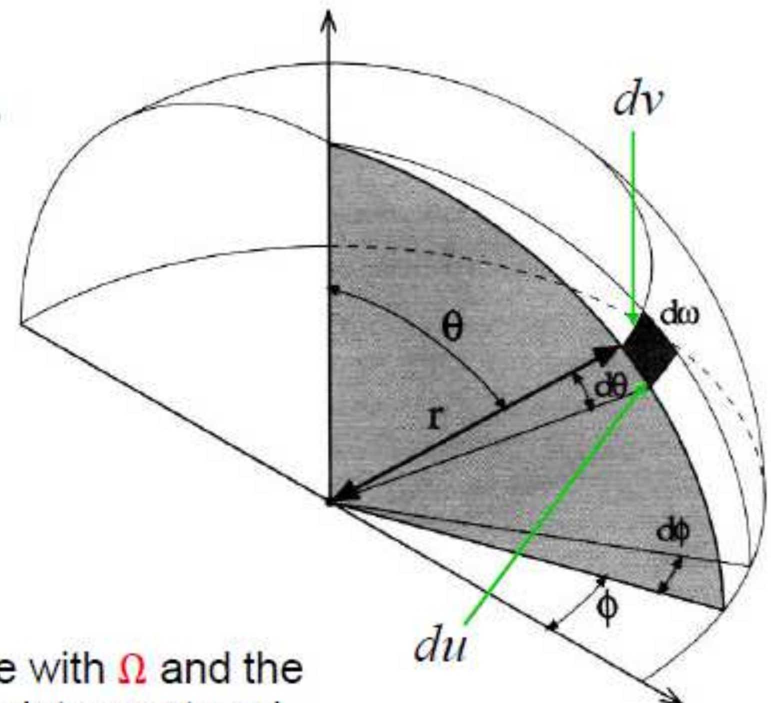
- The solid angle subtended by an (infinitely) small surface patch S with area dA is obtained by dividing the projected area dAcosθ by the square of the distance to the origin:

$$d\omega = \frac{dA \cos\theta}{r^2}$$



Solid Angle in Spherical Coordinates

- **Infinitesimally small solid angle**
  - $du = r\, d\theta$
  - $dv = r \sin\theta\, d\phi$
  - $dA = du\, dv = r^2 \sin\theta\, d\theta\, d\phi$
  - $\Rightarrow d\omega = \frac{dA}{r^2} = \sin\theta\, d\theta\, d\phi$

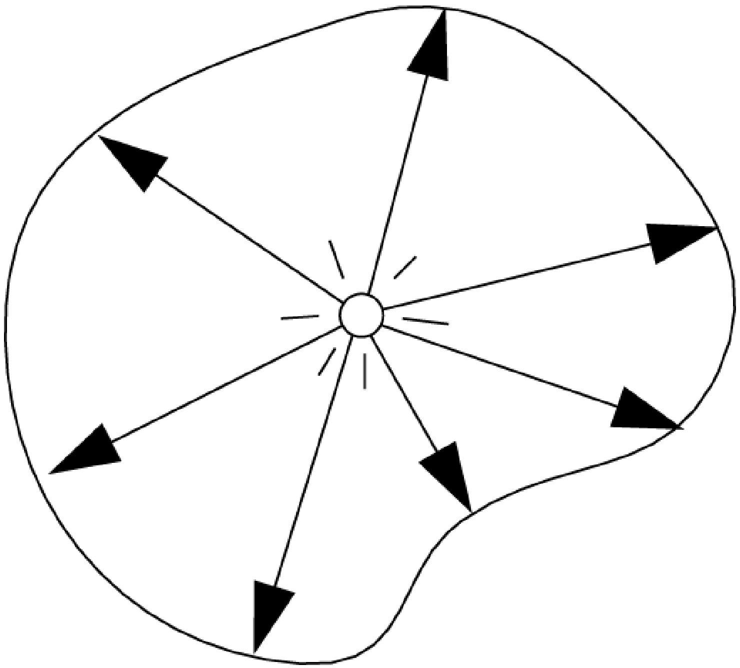


- **Finite solid angle of an surface S**
  - $\omega = \int_S \sin\theta\, d\theta\, d\phi$
- **Definition:**
  - We denote the entire Sphere with Ω and the (positive) hemisphere at a point x centered around its normal vector with Ω_+

Intensity

- Flux density per solid angle $I = \frac{d\Phi}{d\omega}$
- Intensity describes the directional distribution of light
- Isotropic point source

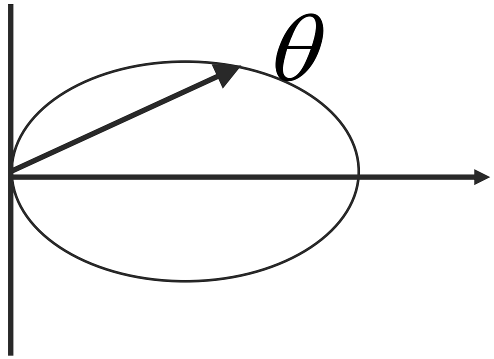
$$
\begin{aligned}
\Phi_e &= \int_{\text{Sphere}} I_e(\omega)\, d\omega \\
&= I \int_{\text{Sphere}} d\omega \\
&= I \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\varphi \, d\theta \\
&= 4\pi I
\end{aligned}
$$



Intensity

- Warn's spotlight

If the total flux is Φ, what is the intensity?

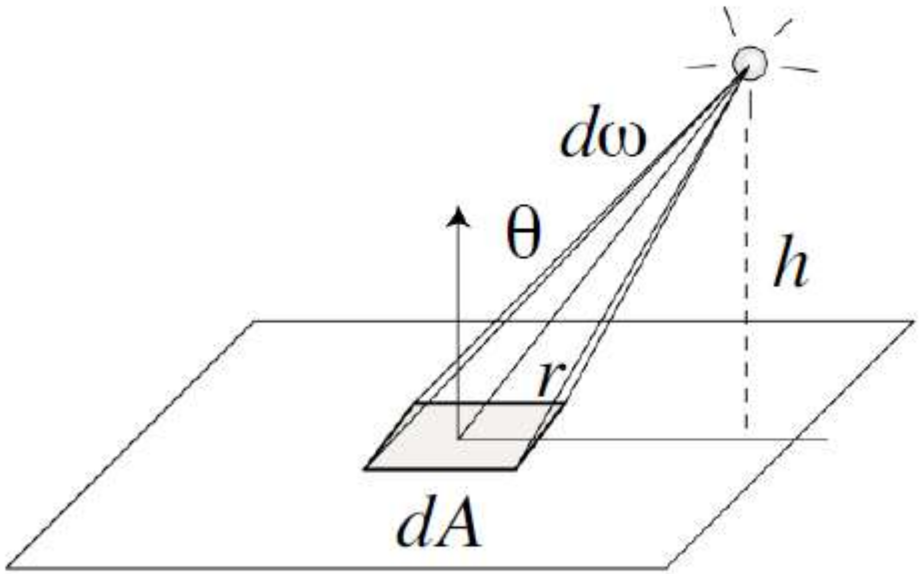


$$I(\omega)=\begin{cases} c\cos^{S}\theta & \theta<\pi/2 \\ 0 & otherwise \end{cases}$$

$$\Phi = c\int_0^{2\pi}\int_0^1 \cos^S\theta d\cos\theta d\phi = 2\pi c\int_0^1 \cos^S\theta d\cos\theta$$

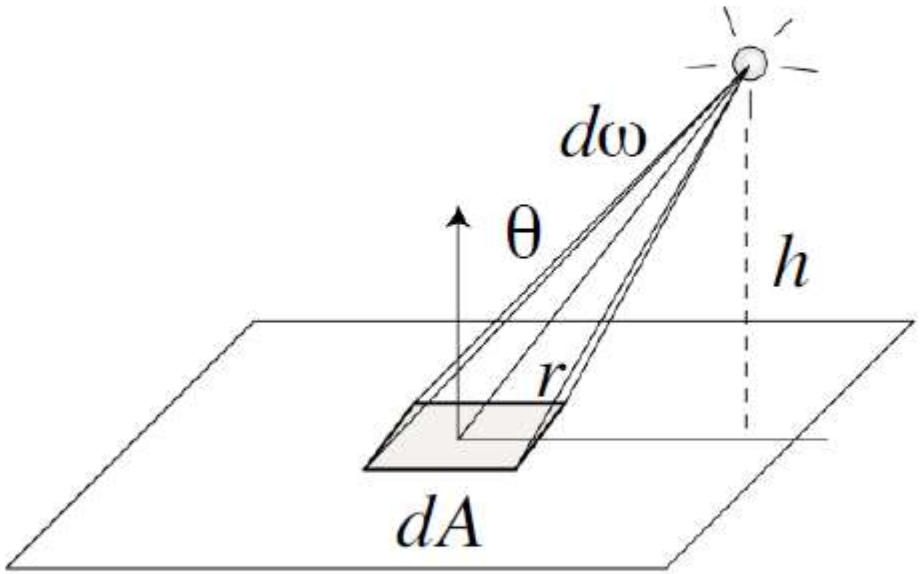
$$= 2\pi c \left.\frac{y^{S+1}}{S+1}\right|_{y=0}^{y=1} = \frac{2\pi c}{S+1} \longrightarrow c=\frac{S+1}{2\pi}\Phi$$

Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

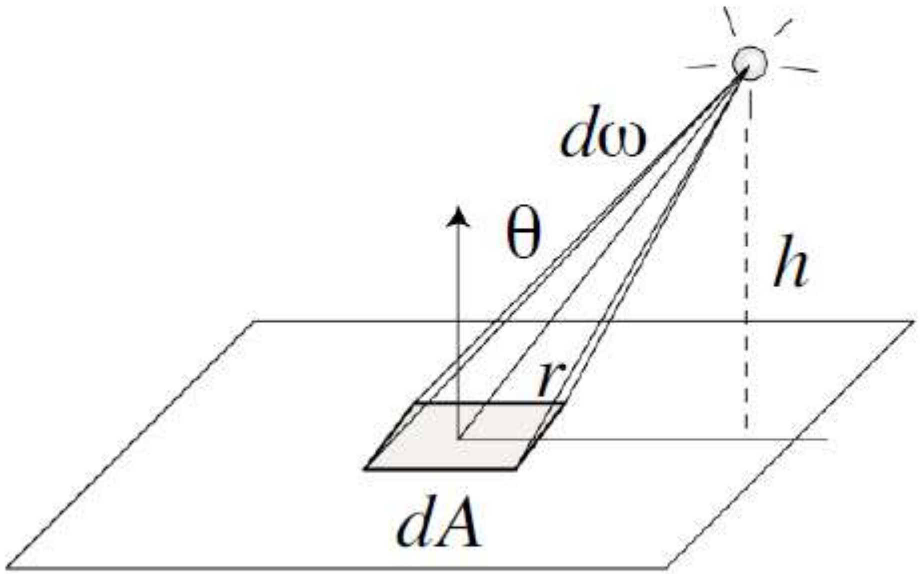
Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

$$d\Phi = I\, d\omega$$

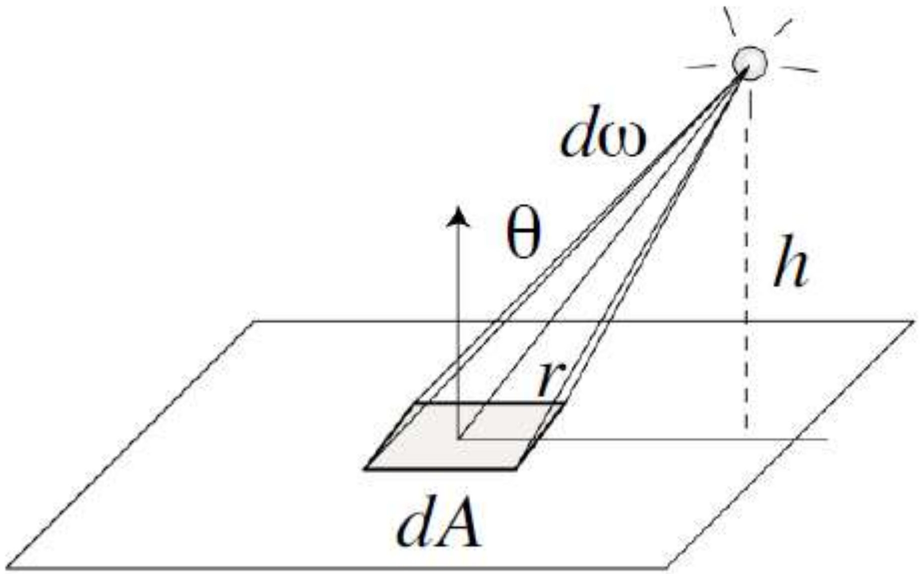
Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

$$d\omega = \frac{\cos\theta}{r^2} dA$$

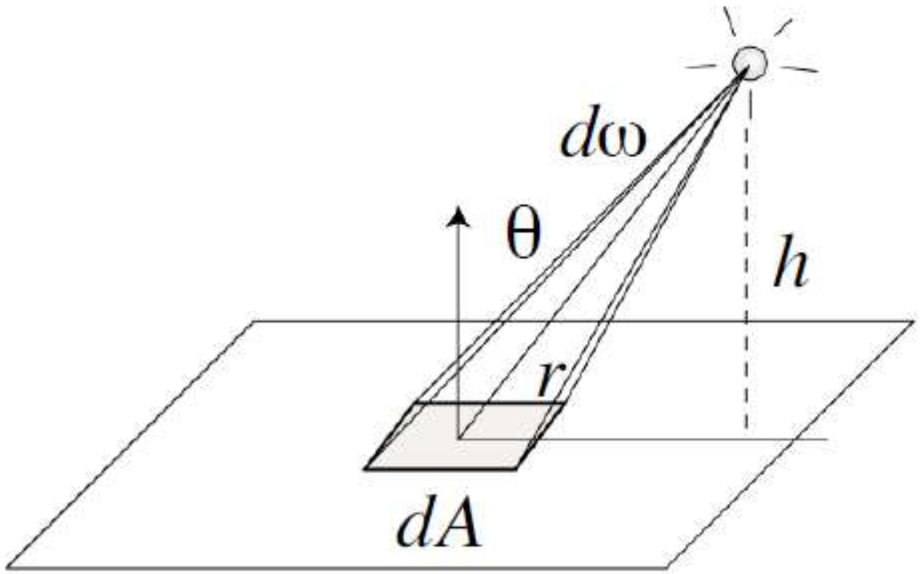
Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

$$I\,d\omega = \frac{\Phi}{4\pi}\frac{\cos\theta}{r^2}dA$$

Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

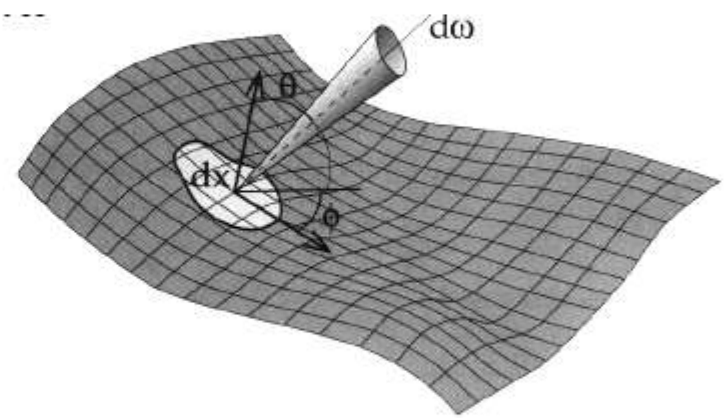
$$I\,d\omega = \frac{\Phi}{4\pi}\frac{\cos\theta}{r^2}dA = E\,dA \quad \Rightarrow \quad E = \frac{\Phi}{4\pi}\frac{\cos\theta}{r^2}$$

Radiance

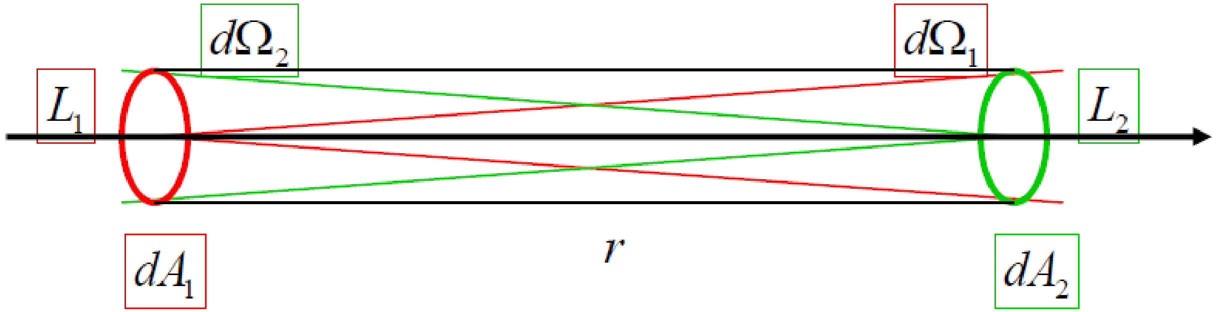
- Radiance L is defined as the total flux (radiant power) traveling at some point x in a specified direction ω, per unit area perpendicular to the direction of travel, per unit solid angle.
- The differential flux $d^2\Phi$ radiated through the differential solid angle $d\omega$, from the projected differential area $dA\cos\theta$ is:

$$d^2\Phi = L(x,\omega)dA\ \cos\theta\ d\omega$$

$$L(x,\omega) = \frac{d^2\Phi}{dA\ \cos\theta\ d\omega}$$



Radiance in Space



Flux leaving surface 1 must be equal to flux arriving on surface 2

$$L_1 \cdot d\Omega_1 \cdot dA_1 = L_2 \cdot d\Omega_2 \cdot dA_2$$

From geometry follows $\quad d\Omega_1 = \frac{dA_2}{r^2} \qquad d\Omega_2 = \frac{dA_1}{r^2}$

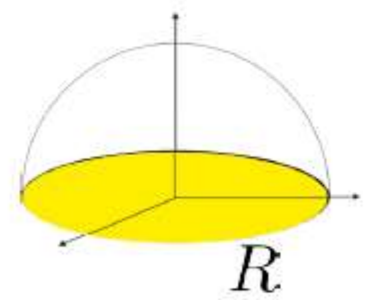
The radiance in the direction of a light ray remains constant as it propagates along the ray.

Sensors response is proportional to radiance (human eye, camera)

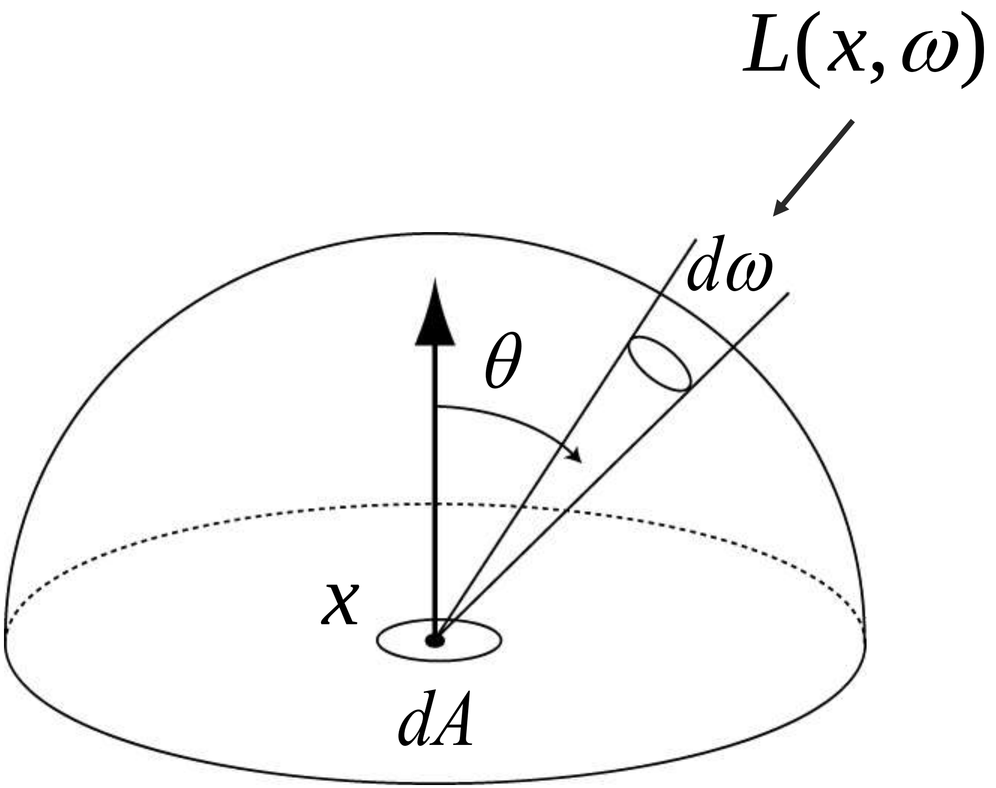
Radiance

- Uniform diffuse area source (with radius *R*)

$$
\begin{aligned}
\Phi_e &= \int_{Area} \int_{\text{Hemisphere}} L_e(x,\omega) \cos\theta \, d\omega \, dA \\
&= L \int_{Area} \int_{\text{Hemisphere}} \cos\theta \, d\omega \, dA \\
&= L\pi \int_{Area} dA \\
&= L\pi^2 R^2
\end{aligned}
$$



Calculate Irradiance From Radiance



$$E(x) = \frac{d\Phi}{dA} = \int_{\Omega} L(x,\omega)\cos\theta d\omega$$

Spectral Quantities

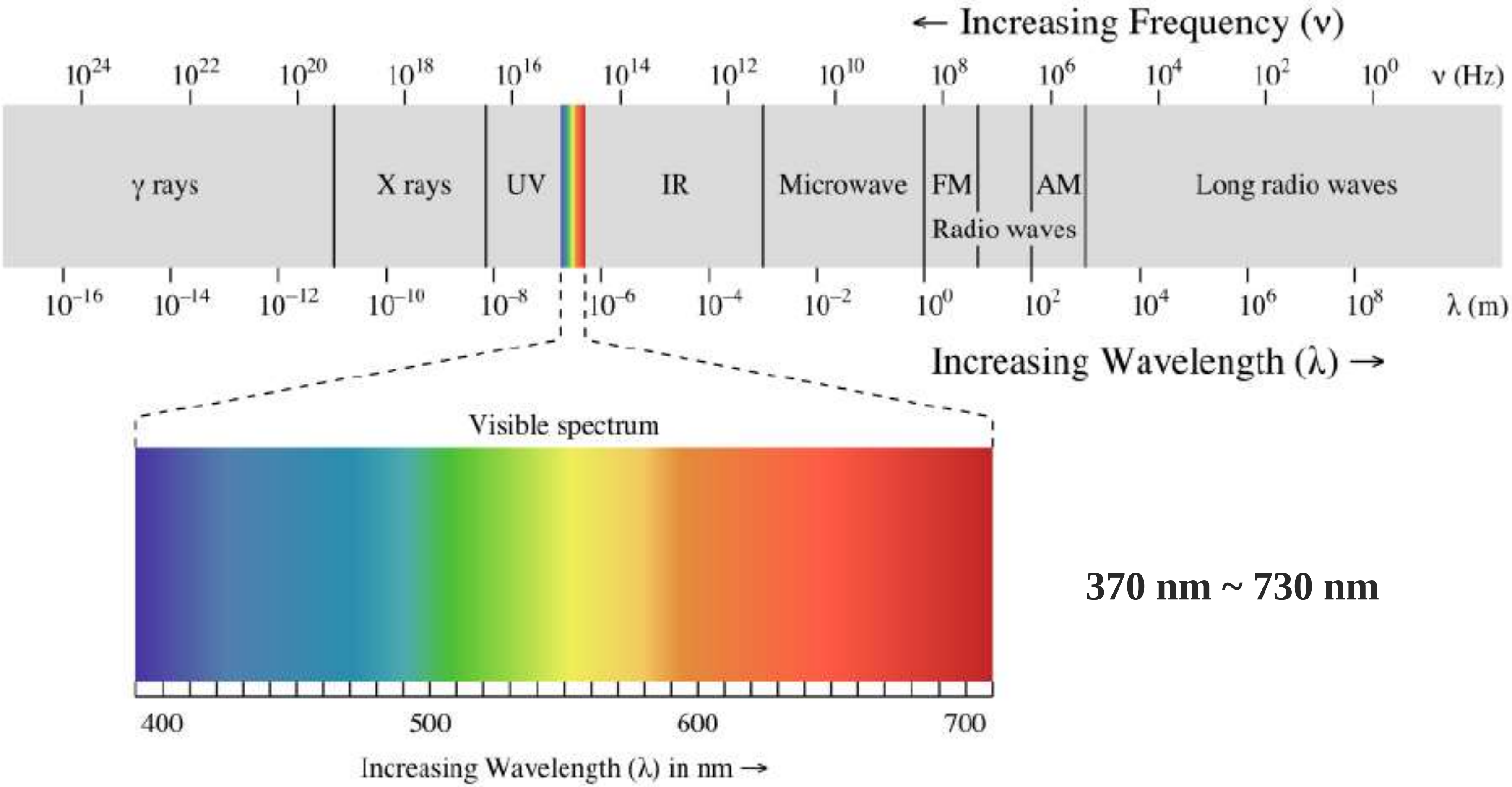
- Radiometric quantity per wavelength

e.g. spectral radiance

$$L_{e,\lambda}(x,\omega)\ [\mathrm{W sr^{-1} m^{-2} nm^{-1}}]$$

$$L_{e,\lambda}(x,\omega) = \frac{d^2\Phi_e}{d\omega\, dA \cos\theta\, d\lambda}$$

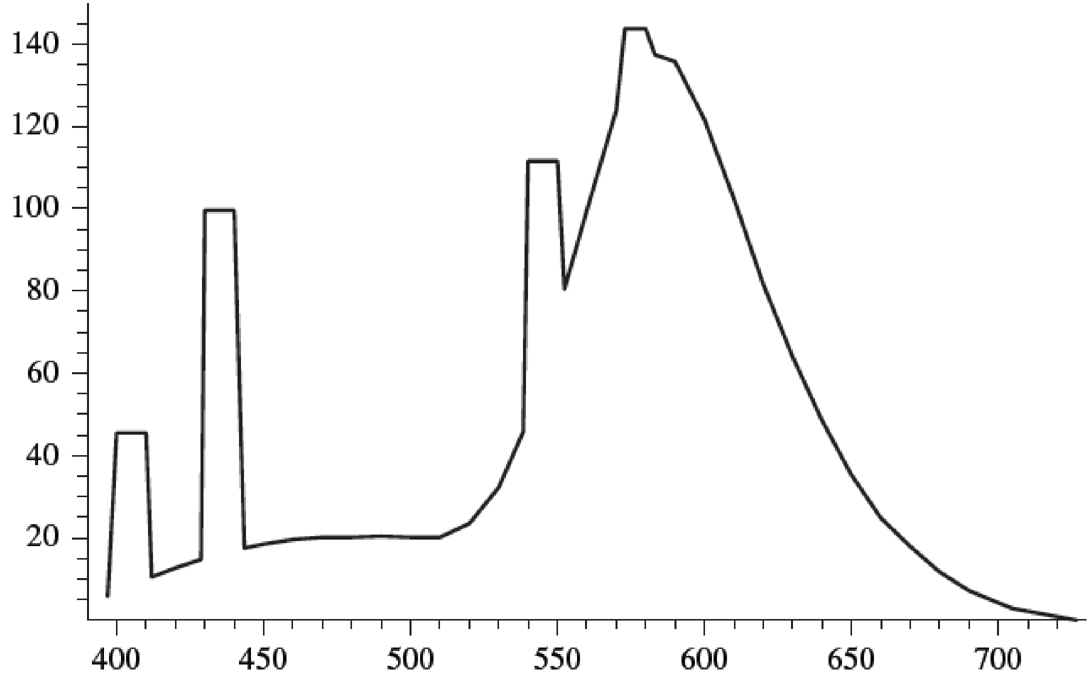
Spectrum



370 nm ~ 730 nm

Spectral Power Distribution (SPD)

- A distribution function of wavelength that describes the amount of light at each wavelength



spectral distribution of emission from a fluorescent light

Color

- Need a compact, efficient and accurate way to represent functions like these
- Find proper basis functions to map the infinite-dimensional space of all possible SPD functions to a low-dimensional space of coefficients
- For example, $B(\lambda)=1$ is a trivial but bad approximation

XYZ Color

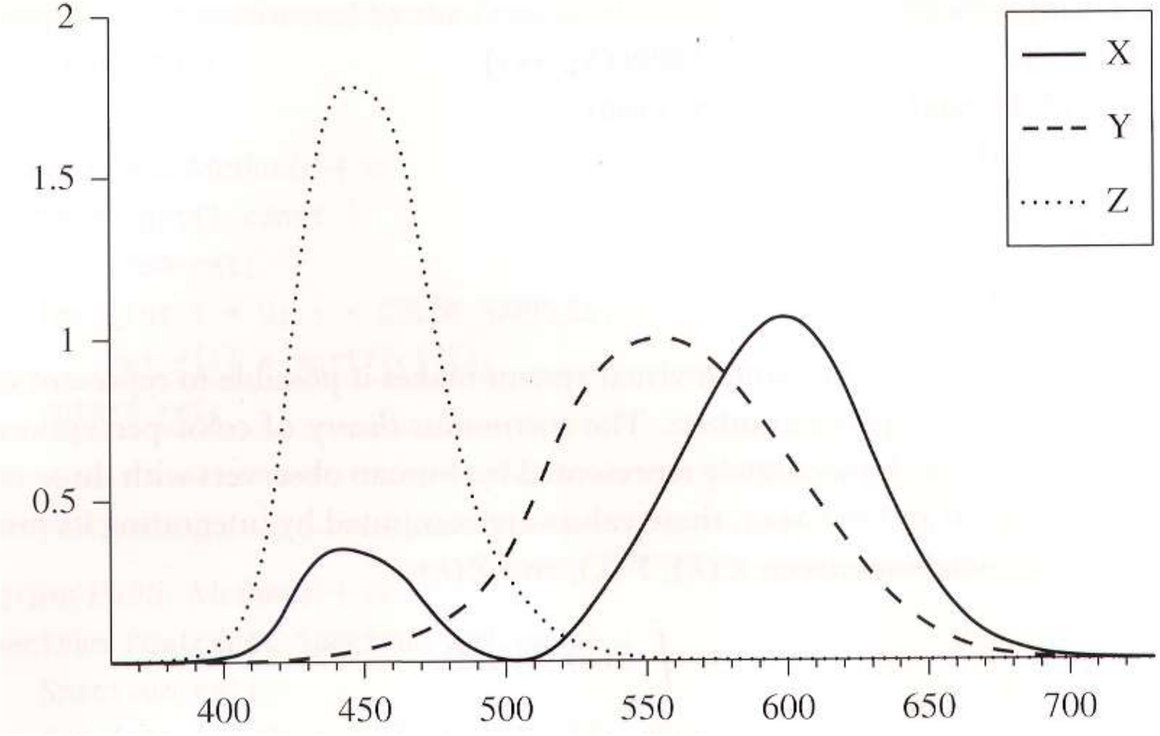
- Tristimulus theory: all visible SPDs S can be accurately represented for human observers with three values, x_λ, y_λ and z_λ.
- The basis are the *spectral matching curves*, X(λ), Y(λ) and Z(λ) determined by CIE.

$$x_\lambda = \frac{1}{\int Y(\lambda) d\lambda} \int_\lambda S(\lambda)\, X(\lambda)\, d\lambda$$

$$y_\lambda = \frac{1}{\int Y(\lambda) d\lambda} \int_\lambda S(\lambda)\, Y(\lambda)\, d\lambda$$

$$z_\lambda = \frac{1}{\int Y(\lambda) d\lambda} \int_\lambda S(\lambda)\, Z(\lambda)\, d\lambda.$$

XYZ Color



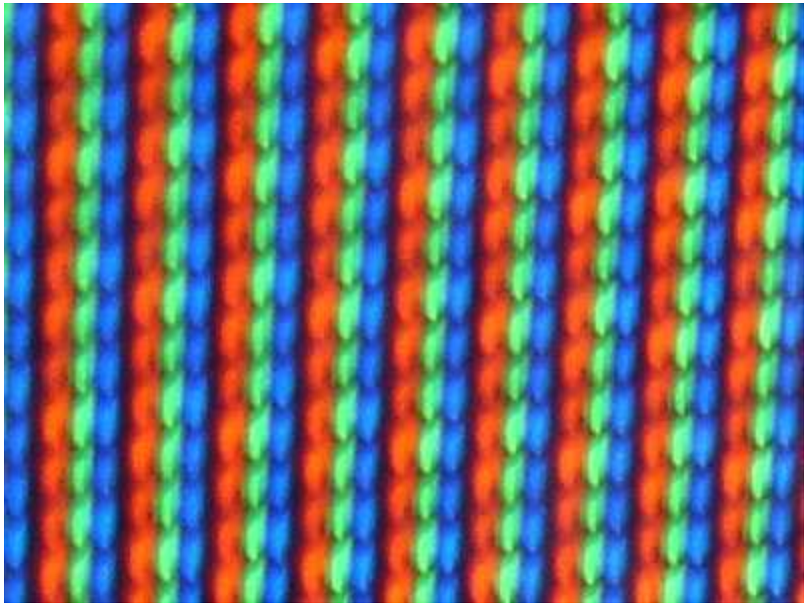
three matching curves

XYZ Color

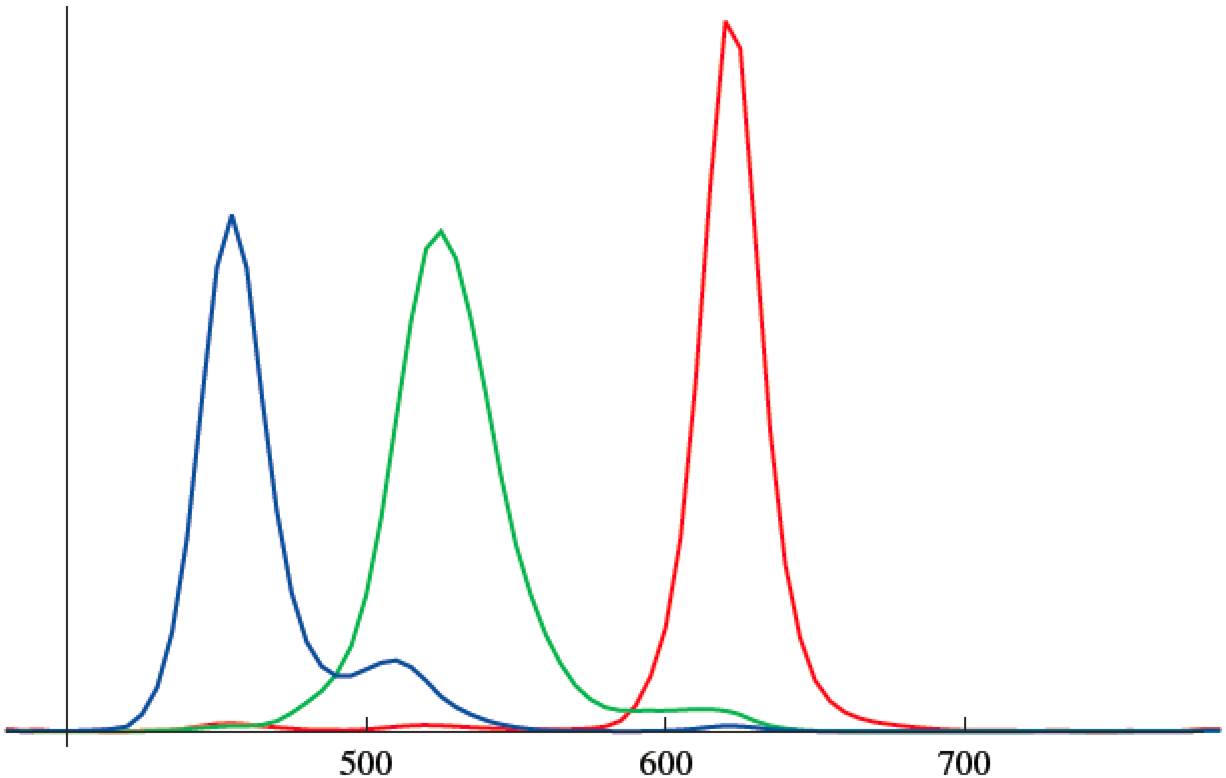
- 国际照明委员会(CIE)1931年制定，又称为“CIE1931标准色度系统”
- XYZ color is device-independent.
- The *y* coordinate of XYZ color is closely related to *luminance*.

RGB Color

- 与屏幕显示相关，最常用的颜色空间模型。



RGB Color



Red, Green, and Blue Emission Curves for an LCD Display

Conversion between XYZ and RGB

- Given an $(x_\lambda, y_\lambda, z_\lambda)$ representation of an SPD, we can convert it to corresponding RGB coefficients:

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{pmatrix} \int R(\lambda)X(\lambda)\mathrm{d}\lambda & \int R(\lambda)Y(\lambda)\mathrm{d}\lambda & \int R(\lambda)Z(\lambda)\mathrm{d}\lambda \\ \int G(\lambda)X(\lambda)\mathrm{d}\lambda & \int G(\lambda)Y(\lambda)\mathrm{d}\lambda & \int G(\lambda)Z(\lambda)\mathrm{d}\lambda \\ \int B(\lambda)X(\lambda)\mathrm{d}\lambda & \int B(\lambda)Y(\lambda)\mathrm{d}\lambda & \int B(\lambda)Z(\lambda)\mathrm{d}\lambda \end{pmatrix} \begin{bmatrix} x_\lambda \\ y_\lambda \\ z_\lambda \end{bmatrix}$$

Conversion between XYZ and RGB

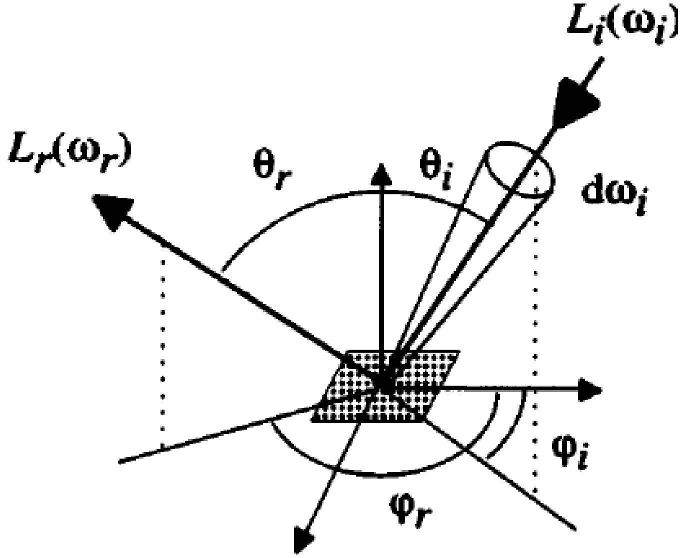
```
inline void XYZToRGB(const float xyz[3], float rgb[3]) {
    rgb[0] =  3.240479f*xyz[0] - 1.537150f*xyz[1] - 0.498535f*xyz[2];
    rgb[1] = -0.969256f*xyz[0] + 1.875991f*xyz[1] + 0.041556f*xyz[2];
    rgb[2] =  0.055648f*xyz[0] - 0.204043f*xyz[1] + 1.057311f*xyz[2];
}
```

```
inline void RGBToXYZ(const float rgb[3], float xyz[3]) {
    xyz[0] = 0.412453f*rgb[0] + 0.357580f*rgb[1] + 0.180423f*rgb[2];
    xyz[1] = 0.212671f*rgb[0] + 0.715160f*rgb[1] + 0.072169f*rgb[2];
    xyz[2] = 0.019334f*rgb[0] + 0.119193f*rgb[1] + 0.950227f*rgb[2];
}
```

BRDF

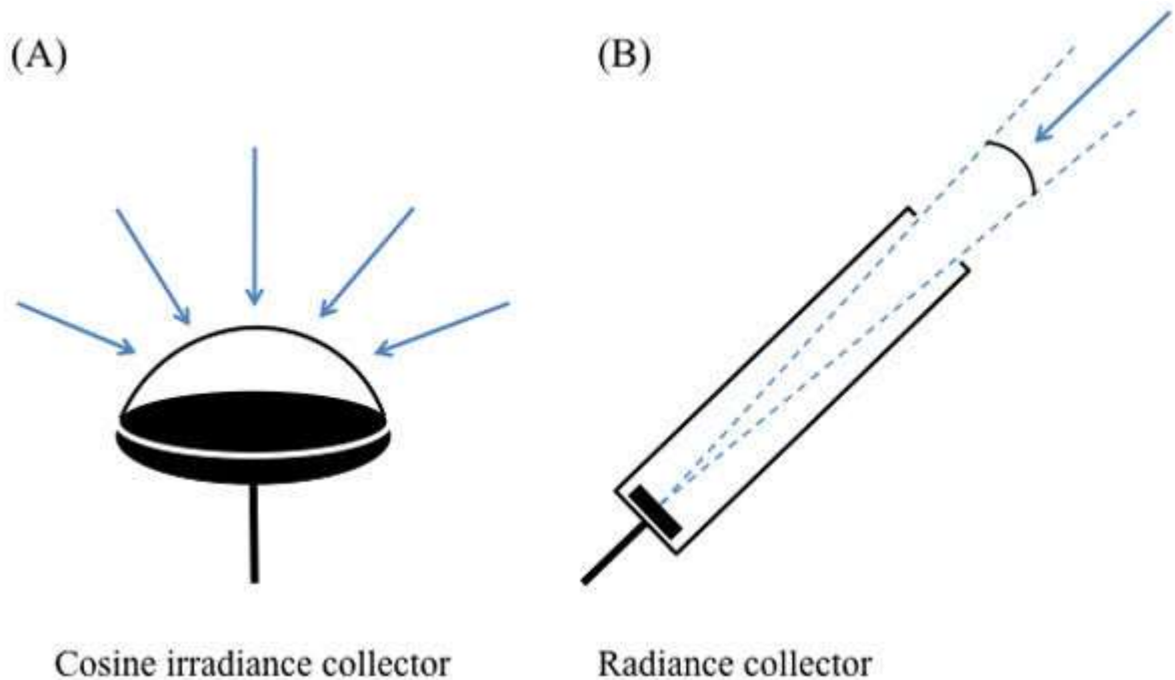
- Bidirectional Reflectance Distribution Function
- BRDF f_r describes surface reflection at a point x for light incident from direction $\omega_i = (\theta_i, \varphi_i)$ reflected into direction $\omega_r = (\theta_r, \varphi_r)$

$$f_r(\vec{\omega}_i \rightarrow \vec{\omega}_r) \equiv \frac{L_r(\vec{\omega}_r)}{L_i(\vec{\omega}_i) \cos \theta_i \, d\omega_i}$$



BRDF

- 为什么BRDF要定义成辐射率（L）和辐照度（E）的比值，而不是直接定义为辐射率和辐射率比值？



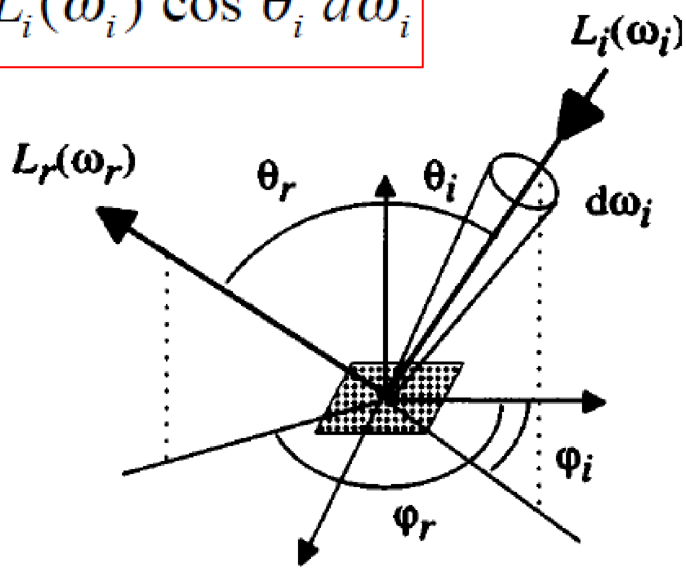
BRDF

- **为什么BRDF要定义成辐射率（L）和辐照度（E）的比值，而不是直接定义为辐射率和辐射率比值？**
- 测平面上一点在某一个方向的出射辐射率很简单，只需要用仪器（B）从该方向对准该点就可以了。而测平面一点入射的辐射率则没有那么简单，必须保证光源正好覆盖测量仪开口立体角，大了该点会接受到比测量值更多的光照，导致测量值比实际值小，小了则与仪器的设计立体角不一致，可在实际中是基本做不到光源大小正好覆盖测量仪开口立体角的。而测表面的辐照度则简单得多，只要保证光源很小，而且没有来自其他方向的光干扰，这时候测到的辐照度就是平面上来自光源方向的微分辐照度。

The Reflection Equation

- The reflected radiance is due to the radiance arriving from all directions weighted by the BRDF relating the incoming:

$$\boxed{L_r(\vec{\omega}_r) = \int_{\Omega_i} f_r(\vec{\omega}_i \to \vec{\omega}_r)\, L_i(\vec{\omega}_i) \cos\theta_i \, d\omega_i}$$



The Reflection Equation

$$L_r(\vec{\omega}_r) = \int_{\Omega_i} f_r(\vec{\omega}_i \to \vec{\omega}_r)\, L_i(\vec{\omega}_i) \cos\theta_i \, d\omega_i$$

- Given the incident light distribution and the BRDF of the material, we can solve this equation.
- It is often called **a local (direct) illumination** model.
- The easiest case is one with no occlusion and direct illumination from a point light.

Direct Illumination

The Reflection Equation

- How to add shadows?
- How to extend to a **global illumination** model?

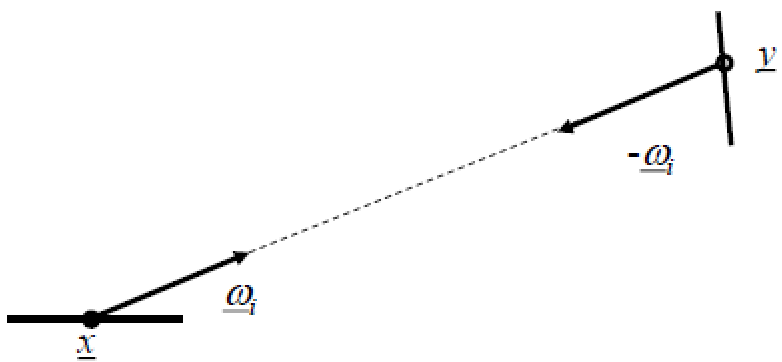
The Rendering Equation

- Surface balance equation

$$[\text{outgoing}] = [\text{emitted}] + [\text{reflected}]$$

The Rendering Equation

- **Light exiting at some point**
  - Given by emitted light plus reflected incoming light at x
  - $L_o(x,\omega_o) = L_e(x,\omega_o) + L_r(x,\omega_o)$
    $= L_e(x,\omega_o) + \int_{\omega_i \in \Omega_+} f_r(\omega_i, x, \omega_o)\, L_i(x,\omega_i) \cos\theta_i \, d\omega_i$
- **Coupling output back to input**
  - Light incident at x is the light exiting at some other point y
    - $L_i(x,\omega_i) = L_o(y,-\omega_i) = L_o(RT(x,w_i),-\omega_i)$
  - With the visibility or ray-tracing operator RT
    - $y = RT(x,\omega_i)$



The Rendering Equation

- **Rendering Equation**
  - Parameterized by direction

$$L_o(x,\omega_o) = L_e(x,\omega_o) + \int_{\omega_i \in \Omega_+} f_r(\omega_i, x, \omega_o)\, L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

  - Parameterized by position over all surfaces S

$$L_o(x,\omega_o)$$
$$= L_e + \int_{y \in S} f_r(\omega_i, x, \omega_o)\, L_o\left(y, \frac{x-y}{\|x-y\|}\right) V(x,y) G(x,y) dA_y$$

    - with $V(x,y)$ giving visibility between x und y,
    - and the Geometric Term G given by
      - $d\omega_i = dA_y \frac{\cos\theta_y}{\|x-y\|^2}$
      - $\mathrm{G}(x,y) = \frac{\cos\theta_x \cos\theta_y}{\|x-y\|^2}$

The Rendering Equation

$$L_o(x,\omega_o) = L_e(x,\omega_o) + \int_{y\in S} f_r(\omega_i, x, \omega_o)\, L_o(y(x,\omega_i), -\omega_i) V(x,y) G(x,y) dA_y$$

- **Properties**
  - Mathematical: Fredholm equation of the 2-nd kind
  - Global coupling of illumination
    - Each point potentially influences each other point
    - Often still a sparse operator due to occlusion
  - Linear transport operator **T**
    - Solution can be computed separately for each light source
      - And accumulated
      - Dimmed lights result in dimmed solutions
  - Volume effects are not considered !!

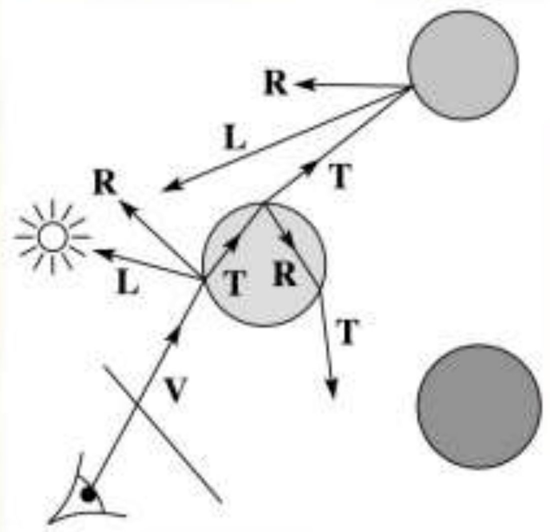
► Lighting Simulation == Solving the Rendering Equation

Solving The Rendering Equation

- **Monte Carlo methods**
  - **Ray tracing**
  - Path tracing (distributed ray tracing)
  - Bidirectional path tracing
  - Path guiding
  - Photon mapping
  - And more…
- **Finite element methods**
  - Classic radiosity

Whitted ray-tracing algorithm

- In 1980, Turner Whitted introduced ray tracing to the graphics community.
  - Combines eye ray tracing + rays to light
  - Recursively traces rays

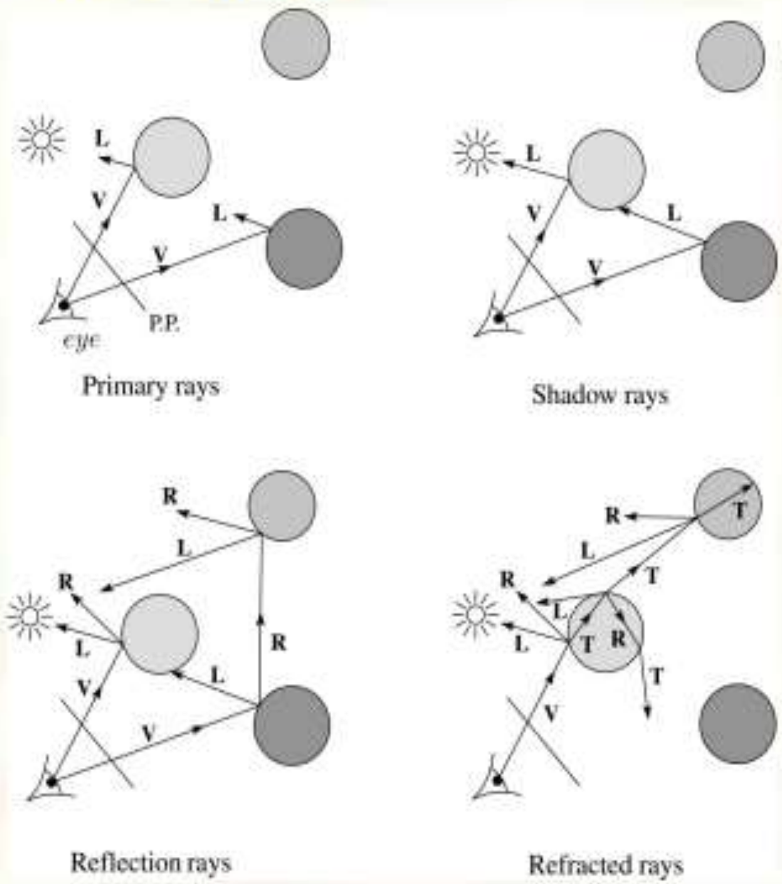


- <u>Algorithm:</u>

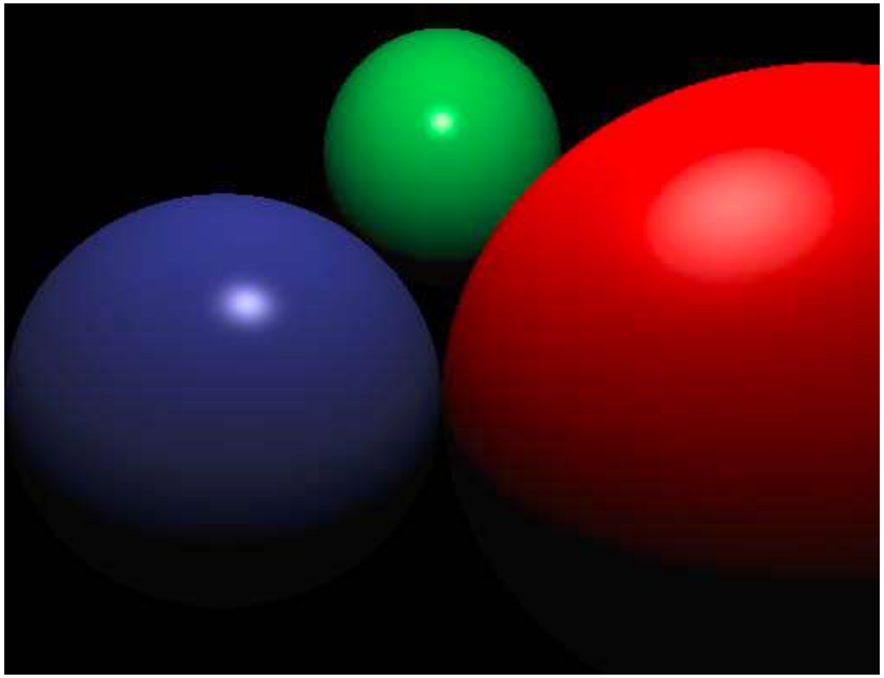
1. For each pixel, trace a **primary ray** in direction **V** to the first visible surface.
2. For each intersection, trace **secondary rays**:
   - **Shadow rays** in directions $\mathbf{L}_i$ to light sources
   - **Reflected ray** in direction **R**.
   - **Refracted ray** or **transmitted ray** in direction **T**.

Whitted ray-tracing algorithm

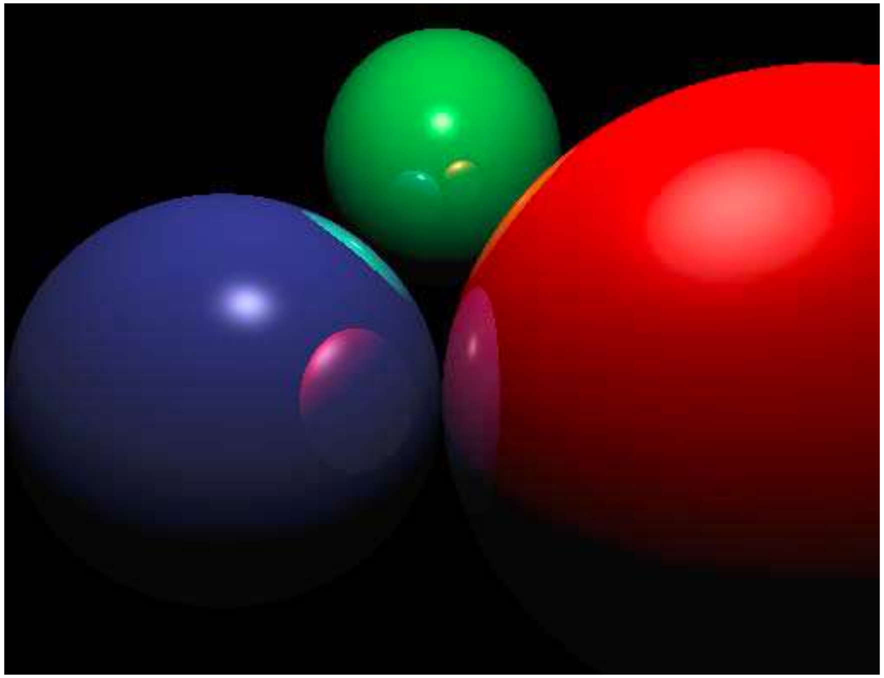
Let's look at this in stages:



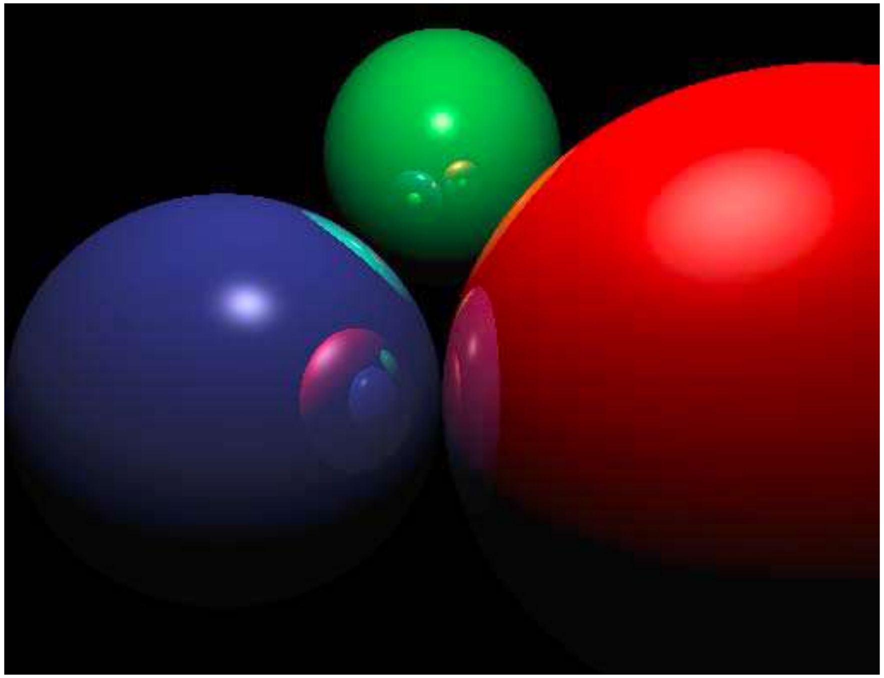
Example: Reflections at depth = 0



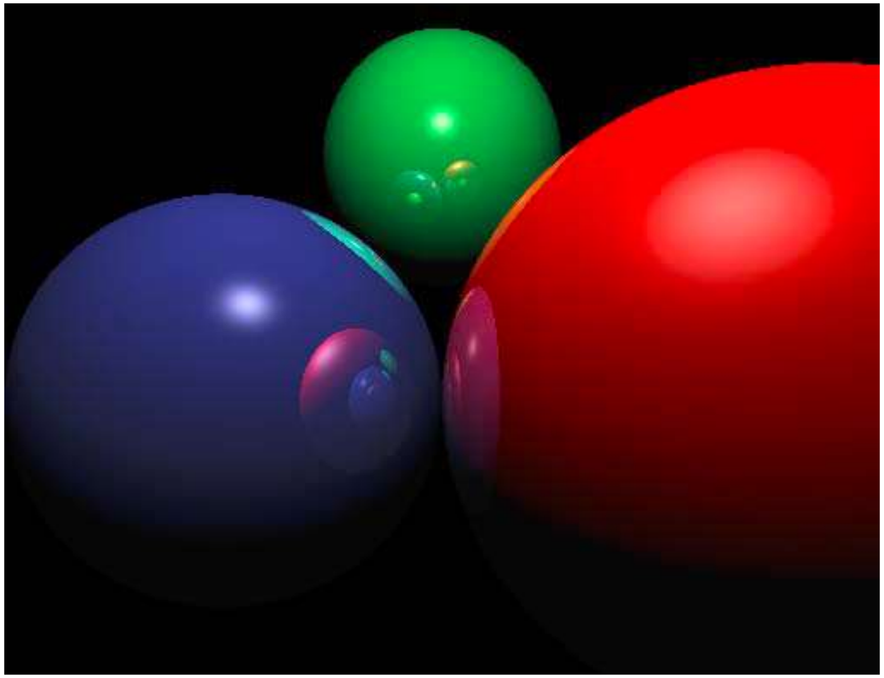
Example: Reflections at depth = 1



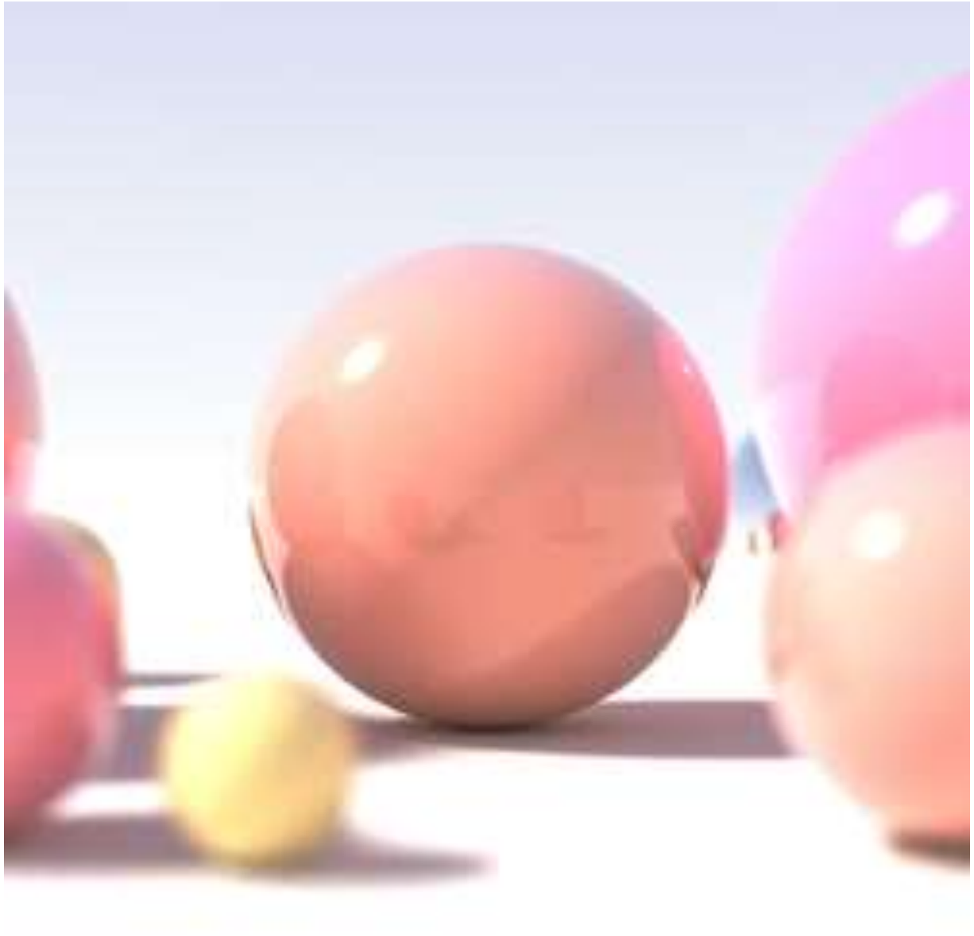
Example: Reflections at depth = 2



Example: Reflections at depth = 3



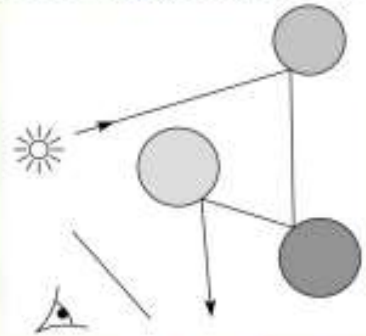
Ray tracing example



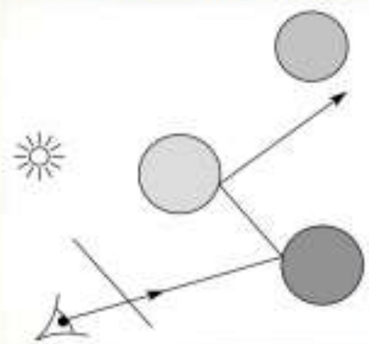
Ray tracing example

Eye vs. light ray tracing

- Where does light begin?
- At the light: light ray tracing (a.k.a., forward ray tracing or photon tracing)



- At the eye: eye ray tracing (a.k.a., backward ray tracing)



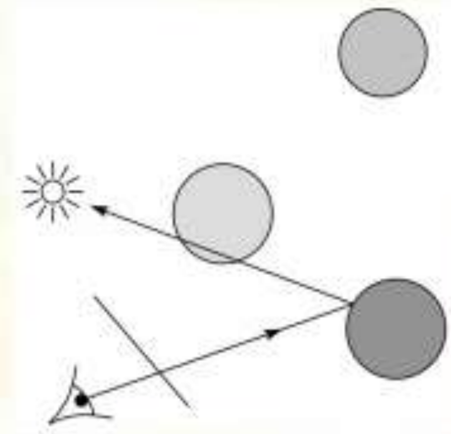
- We will generally follow rays from the eye into the scene.

Ray Casting v.s. Ray Tracing

- Direct illumination
- Indirect illumination
- Global illumination

Ray Casting v.s. Ray Tracing

- Local illumination
  - Cast one eye ray,
    then shade according to light



- Appel (1968)
  - Cast one eye ray + one ray to light