Practise session 3: Image transforms, Fourier Transform

1. Fourier Transform and convolution. (1p)
   The discrete Fourier transform for a sequence *x(i)* is defined as
   $$X(k) = \sum_{i=0}^{N-1} x(i) e^{-2\pi j \frac{ik}{N}}$$
   Let *y(i)* be another sequence of length N, and $y_c(i)$ its N-periodic extension $y_c(i) = y(i \bmod N)$. Let us define convolution to be
   $$h(m) = \sum_{i=0}^{N-1} x(i) y_c(m-i)$$
   Prove that the Fourier transform of the convolution of two functions can be calculated by multiplying their Fourier transforms, that is, $H(k) = X(k)Y(k)$ where $H(k)$ is Fourier transform of $h(m)$.

2. $N \log_2 N$ additions and $\frac{1}{2} N \log_2 N$ multiplications are needed to compute the Fast Fourier Transform (FFT) of *N* points. How many additions and multiplications are needed to compute the 2D FFT of and *N x N* image? What if the image is a spectral image of size *N x N x N*? (1p)

3. The following 8 signals *f(x)* are given. Find their Fourier transforms *F(u)* and describe the signals and their corresponding Fourier transforms with words *real, imaginary, complex, even,* and *odd* (using also possible combinations of those words). (1p)

   f(x)
   {1 2 3 4}
   j{1 2 3 4}
   {2 1 1 1}
   {0 -1 0 1}
   j{2 1 1 1}
   j{0 -1 0 1}
   {(4+4 *j*) (3+2*j*) (0+2*j*) (3+2*j*)}
   {(0+0*j*) (1+1*j*) (0+0*j*) (-1-*j*)}

4. Matlab task: Apply Fourier transform to the three images given in Practice session 2. Which functions and options are available in Matlab for FT/DFT/FFT and how the results of them can be displayed/visualized? (1p)