Set 2: Digital Image Fundamentals

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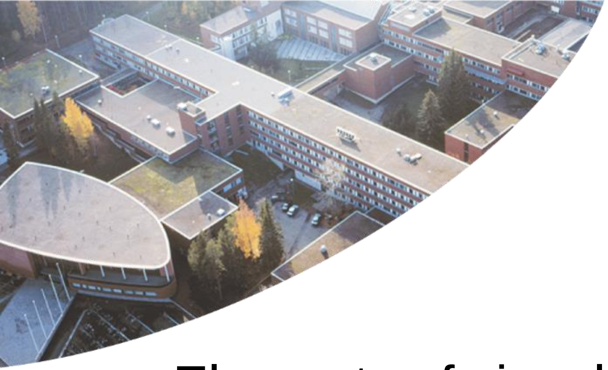


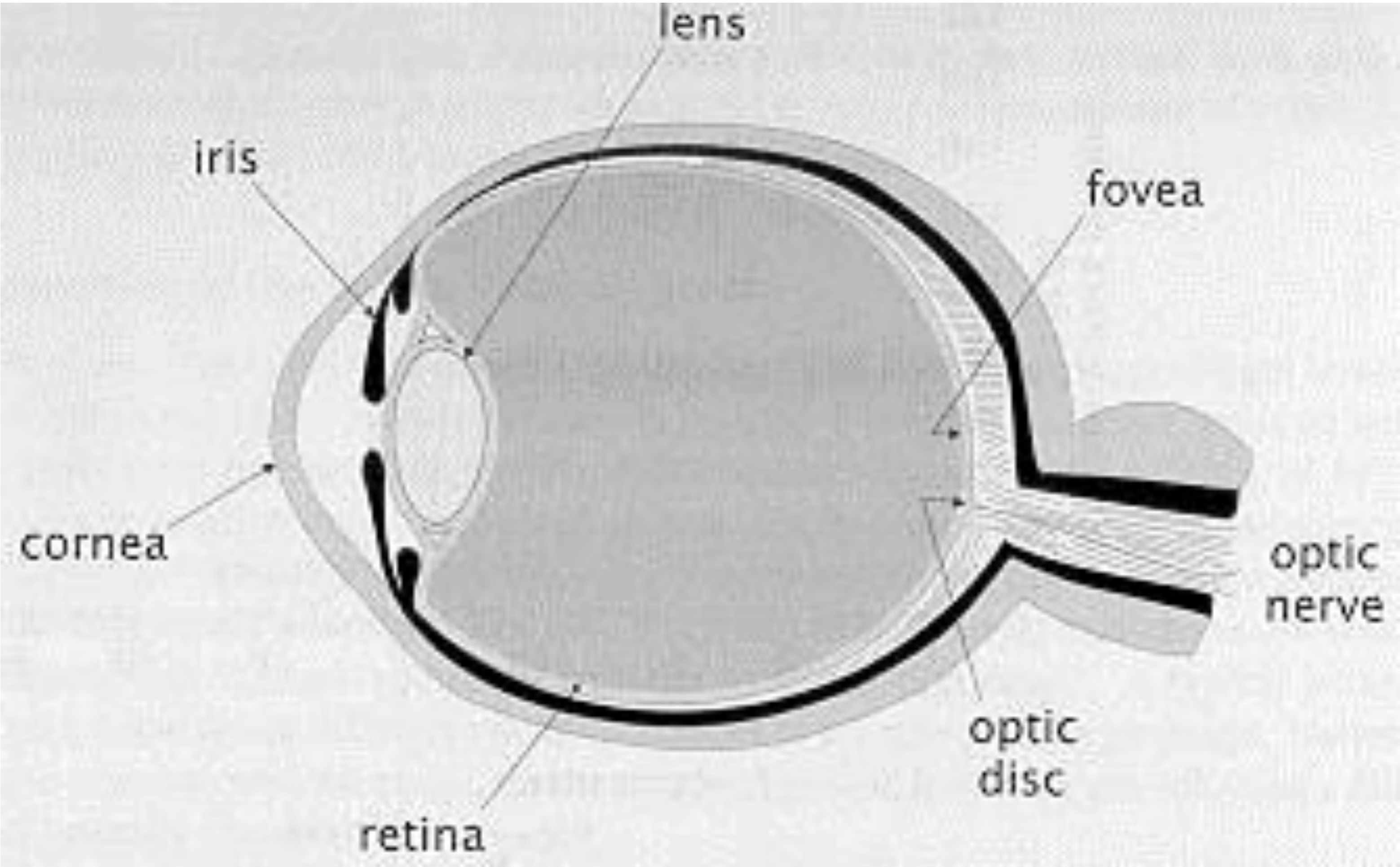
Digital Image Fundamentals

- Elements of visual perception
- A simple image model
- Sampling and quantization
- Basic relationships between pixels
- Imaging geometry

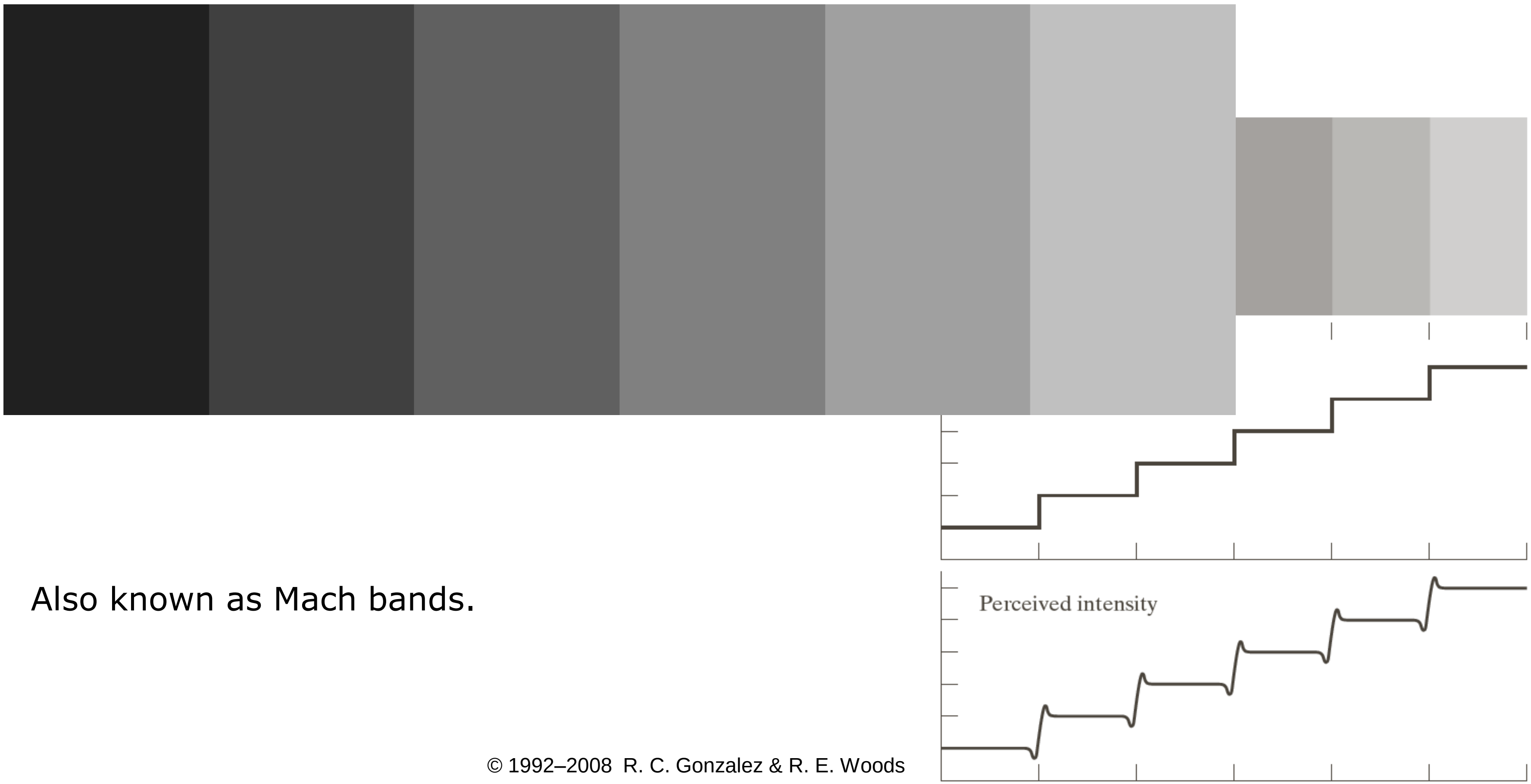
Elements of Visual Perception

- Structure of the human eye
  - Simplified diagram of a cross section of the human eye
  - Distribution of rods and cones in the retina
- Image formation in the eye
- Brightness adaptation and discrimination
  - Mach band pattern and other “optical illusions”
  - Examples showing that perceived brightness is not a simple function of intensity
  - Example of simultaneous contrast
- Different sizes of variation are perceived differently by the eye, which needs to be taken into consideration in evaluation print quality (e.g. mottling)

Structure of the Human Eye



Optical illusion on gray-scale values



Also known as Mach bands.

Optical illusion on geometry

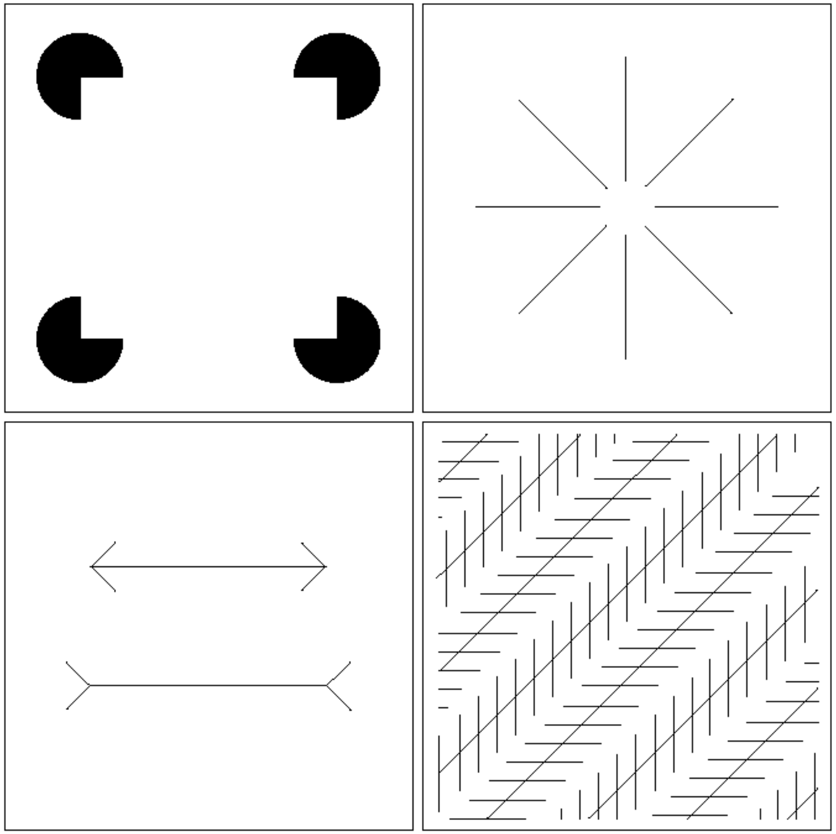
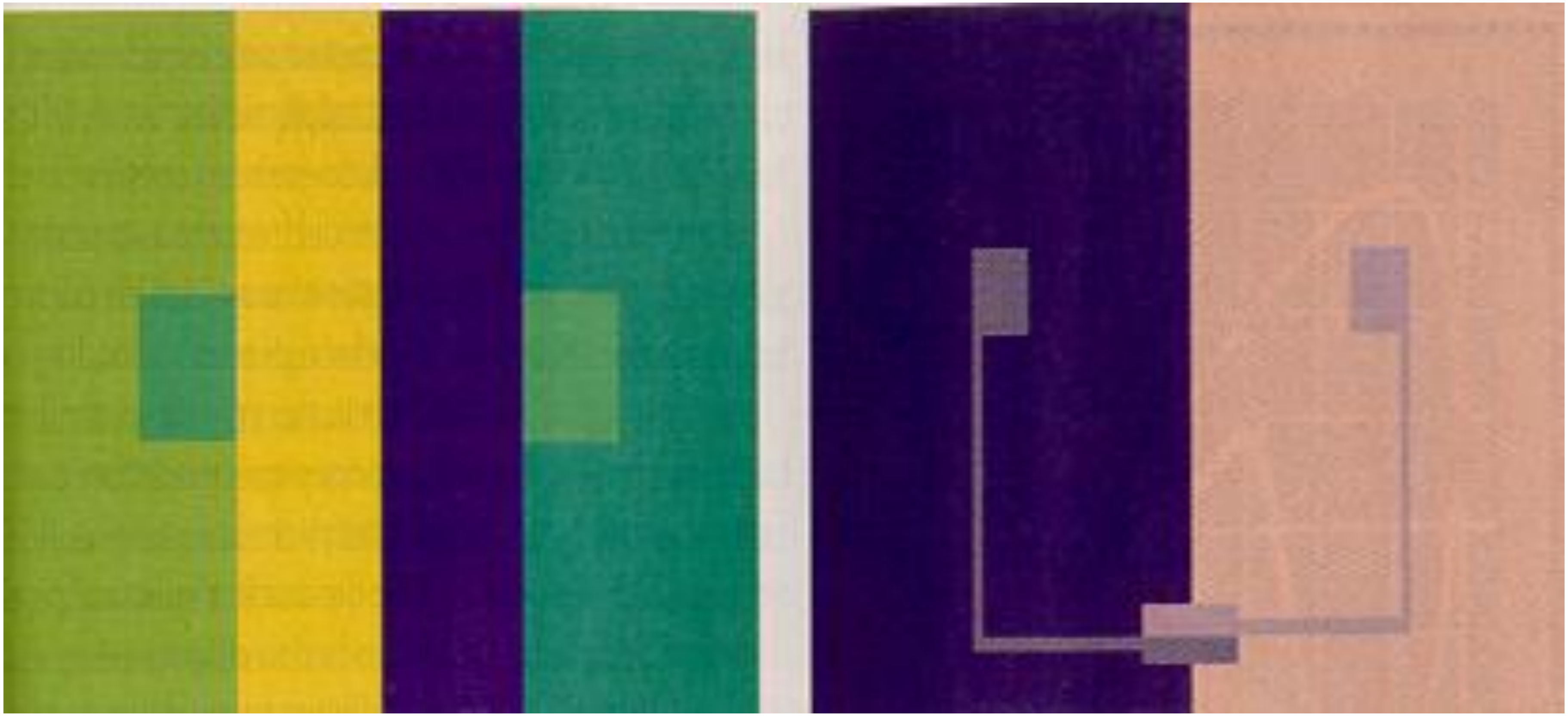


FIGURE 2.9 Some well-known optical illusions.

Elements of Visual Perception

- Left: Simultaneous contrast to make the same colors look different.
- Right: Simultaneous contrast can make different colors look the same.



A Simple Image Model

- Image *f(x,y)*: (intensity) value *f*, spatial coordinates *x* and *y*, resolution in pixels
- $0 < f(x,y) < \infty$
- $f(x,y) = i(x,y)\, r(x,y)$
    - *i(x,y)* illumination, $0<i(x,y)<\infty$
        - Characterized by the illumination source
    - *r(x,y)* reflectance, $0<r(x,y)<1$
        - Characterized by the imaged object
    - What if imaging is based on transmission *t(x,y)*, like X-ray?

Sampling and Quantization

- Resolution $N * M$
  - Effects of reducing spatial resolution.
- Number of gray-levels $G = 2^m$
  - Effects of reducing number of gray-levels.
- Number of bits

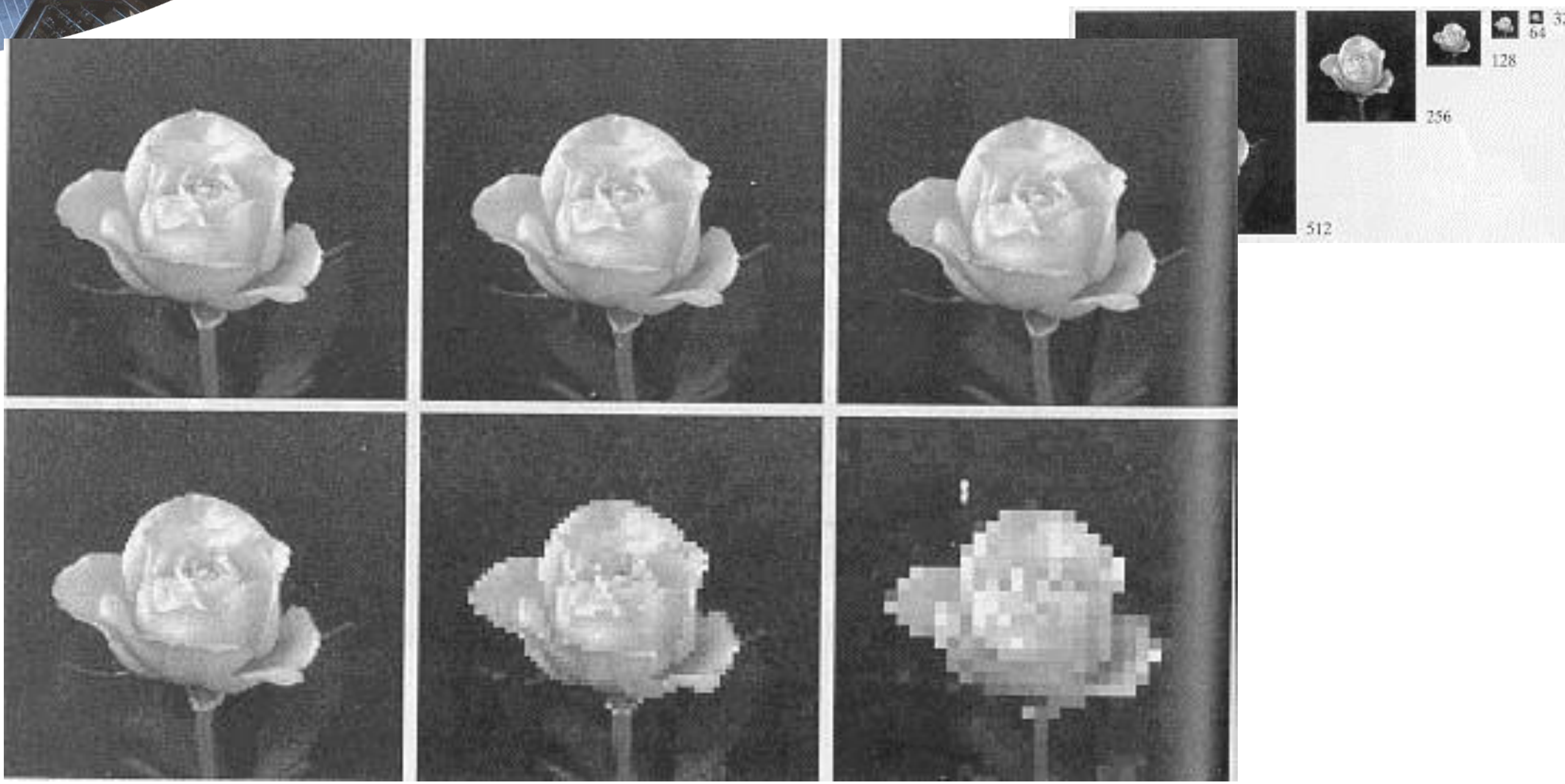
$$b = N * M * m$$

- Note: color images typically contain 3 times as many bits (R, G and B channel each have their own intensity value)
- Spectral images contain

$$b = N * M * B * m$$

where *B* is the number of bands ($B = a * 10^1 \ldots 10^3$)

Effects of Reducing Spatial Resolution

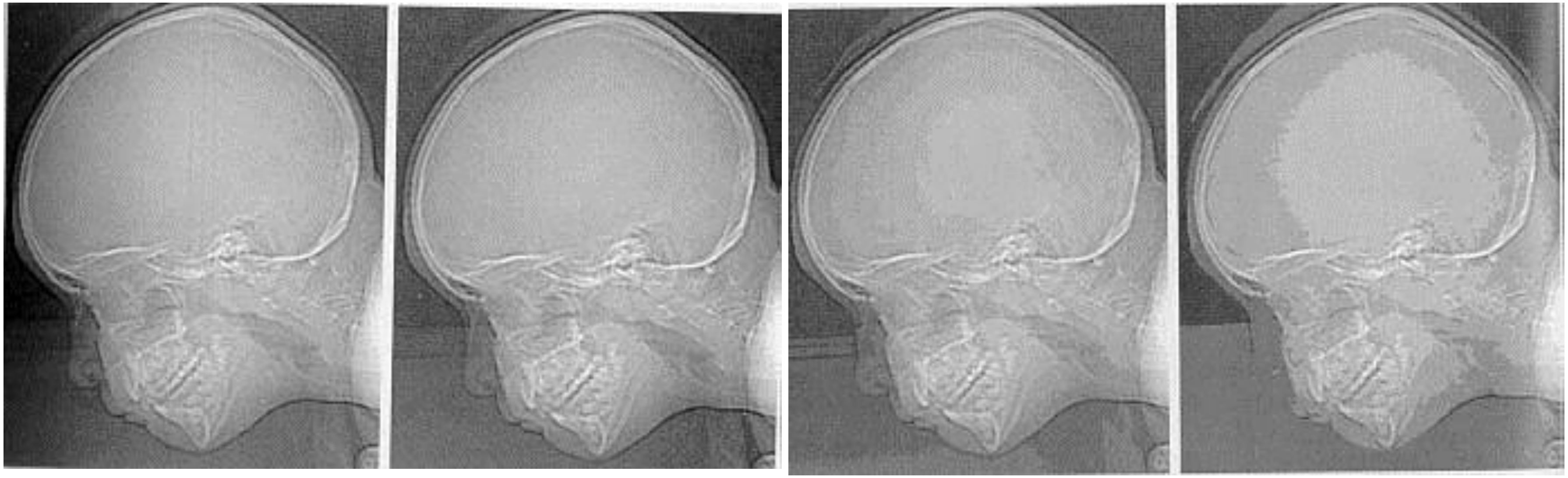


Effects of Sampling and Quantization

- Left: The reduced spatial resolution
- Right: The reduced number of gray-levels per pixel

Effects of Reducing Number of Gray-levels

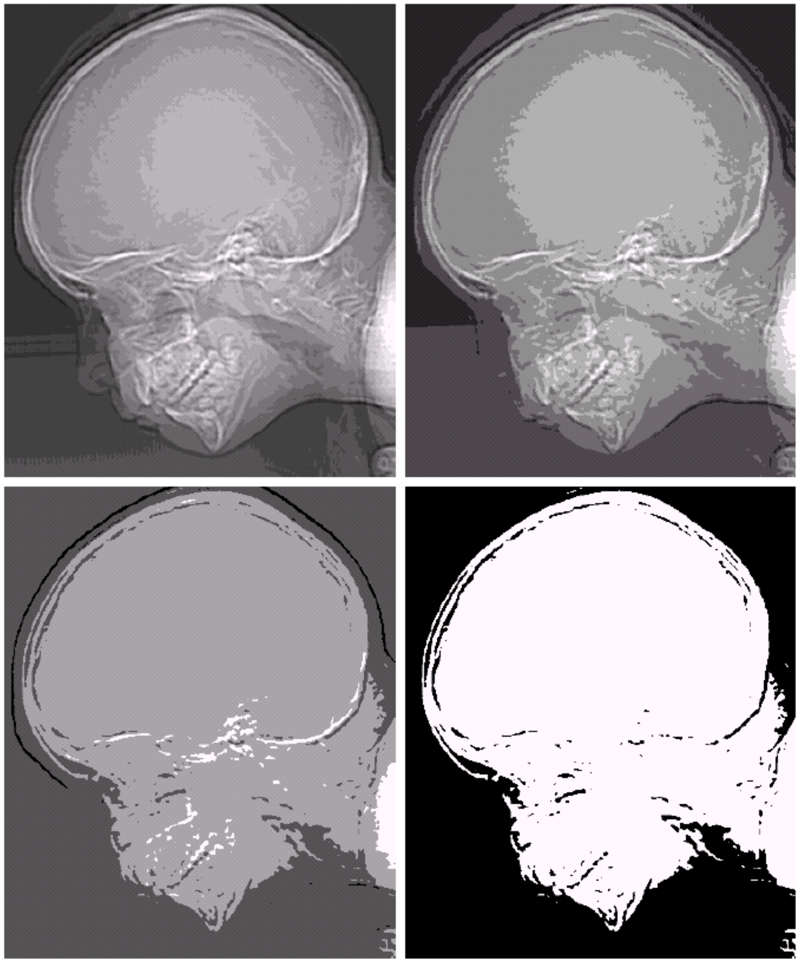
- 128, 64, 16, 8-level images from left to right.
- False contouring in two rightmost images.



Effects of Reducing Number of Gray-levels

e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)



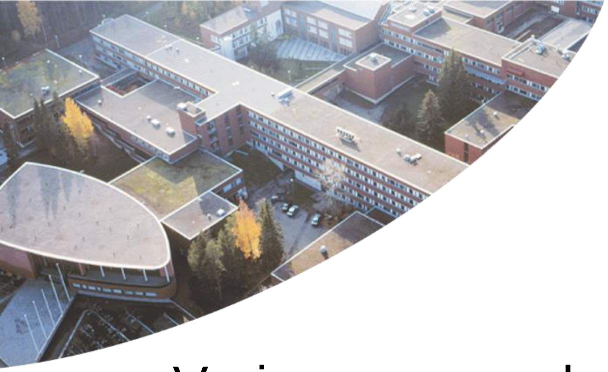


Image Interpolation

- Various approaches for resampling
  - Applying exiting data to estimate values in unknown locations

  - Nearest neighbor interpolation
    - assigns a value of the nearest neighbour

  - Bilinear interpolation

$$v(x, y) = ax + by + cxy + d$$

  - Bicubic interpolation

$$v(x, y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$$

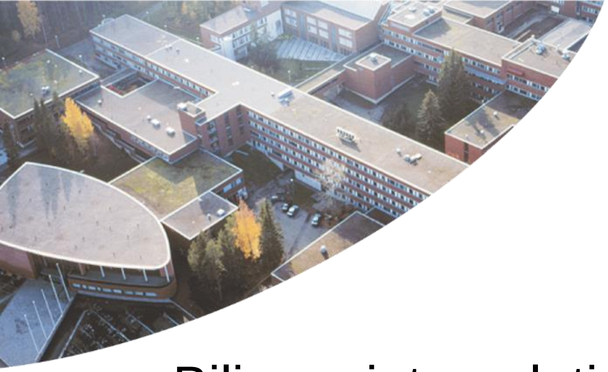
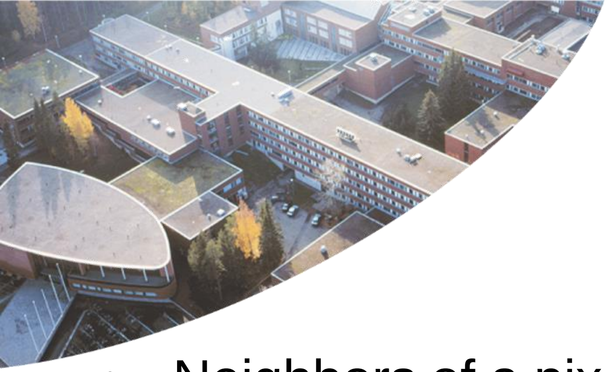


Image Interpolation

- Bilinear interpolation
- Bi-cubic interpolation

- Top row: original image 72 dpi
- Bottom row: original image 150 dpi



Basic Relationships Between Pixels

- Neighbors of a pixel $p(x, y)$
  - 4-neighbors
    $N_4(p)$: $(x+1, y), (x-1, y), (x, y+1), (x, y-1)$
  - Diagonal neighbors
    $N_D(p)$: $(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$
  - 8-neighbors $N_8(p)$: $N_4(p)$ *and* $N_D(p)$
  - Adjacency between pixels p and q: adjacent if they are neighbors

- Connectivity of pixels
  - 4-connectivity, 8-connectivity
  - m-connectivity
  - A path from $p(x, y)$ to $q(s, t)$ is defined based on connectivity

Pixel connectivity, m-Connectivy

m-Connectivity

$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$	$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$
4-connected	8-connected	m-connected

V – set of intensity values used to define connectivity. In binary images, $V = \{1\}$ – refers to connectivity of pixels with value 1.

Two pixels, p and q, with values from *V* are *m-connected* if:

(i) q is in $N_4(p)$ *or*

(ii) q is in $N_D(p)$ and $N_4(p) \cap N_4(q)$ is empty.

Basic Relationships Between Pixels

- Distance measures between pixels *p(x,y)* and *q(s,t)*
  - Euclidean distance

    $$D_e(p,q): D_e(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$$

  - D_4 distance $D_4(p,q)$ (city-block distance)

    $$D_4(p,q) = |x-s| + |y-t|$$

  - D_8 distance $D_8(p,q)$ (chessboard distance)

    $$D_8(p,q) = \max(|x-s|, |y-t|)$$

  - D_m distance is minimum distance between two pixels
    - consider an example, distance between *p0=1* and *p4=1*

$$\begin{array}{ccc} & p_3 & p_4 \\ p_1 & p_2 & \\ p_0 & & \end{array}$$

p1=0, p3=0; m-distance = ?

p1=1, p3=0: m-distance = ?

p1=0, p3=1: m-distance = ?

p1=1, p3=1: m-distance = ?

Mathematical tools in DIP

- Linear vs. nonlinear operations
  - $H\left[a_i f_i(x,y) + a_j f_j(x,y)\right] = a_i H[f_i(x,y)] + a_j H\left[f_j(x,y)\right]$
  - Additivity (+) and homogenity (*H(af)* vs. *aH(f))*
  - e.g. max-operator? An example may show that it is non-linear

- Arithmetic/Logic operations for pixels
  - Addition, subtraction, multiplication, and division
  - AND, OR, COMPLEMENT
  - Example of filtering
- Distance Transforms

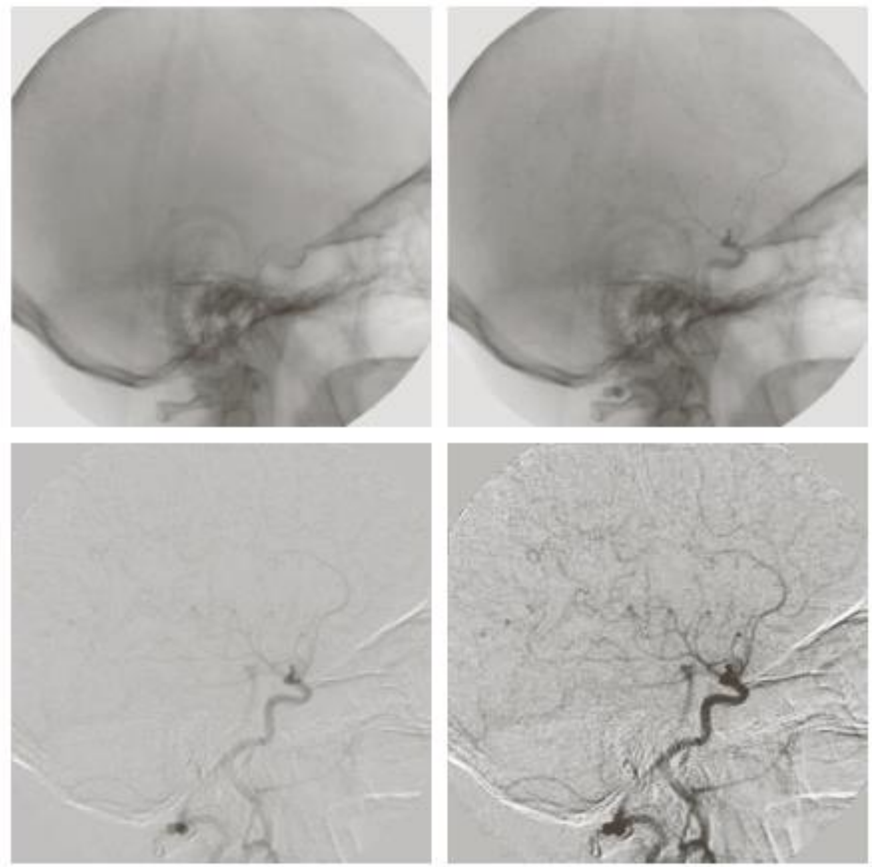
Mathematical tools in DIP

- Example on X-ray imaging
- Original image,
- injection applied

  Difference image,

  contrast enhanced

  difference image

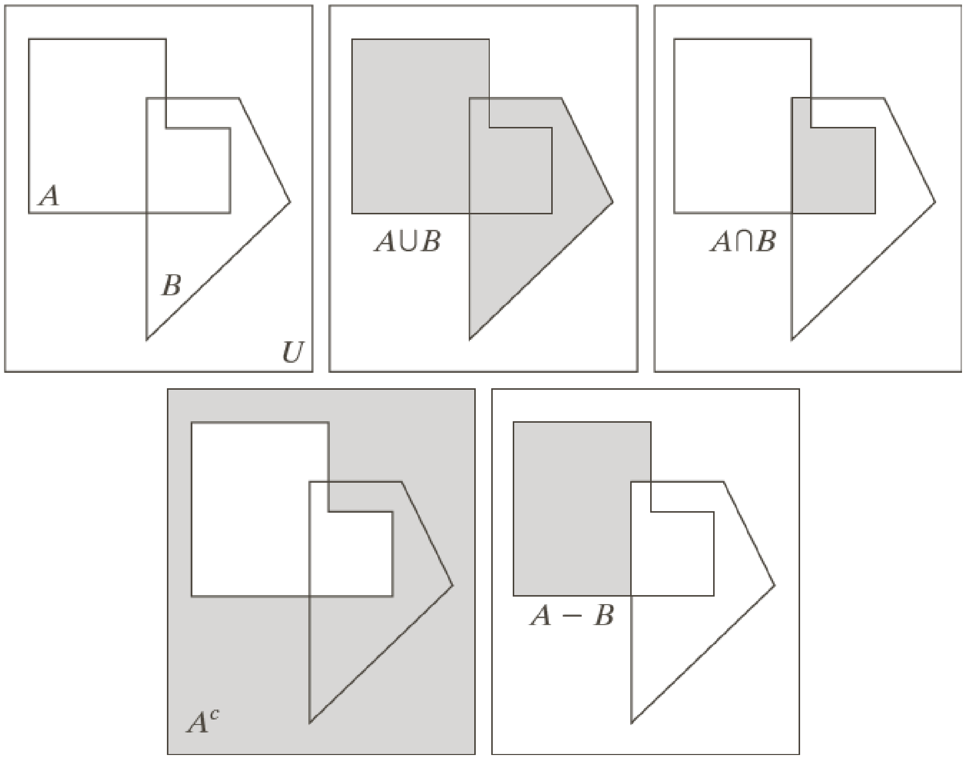


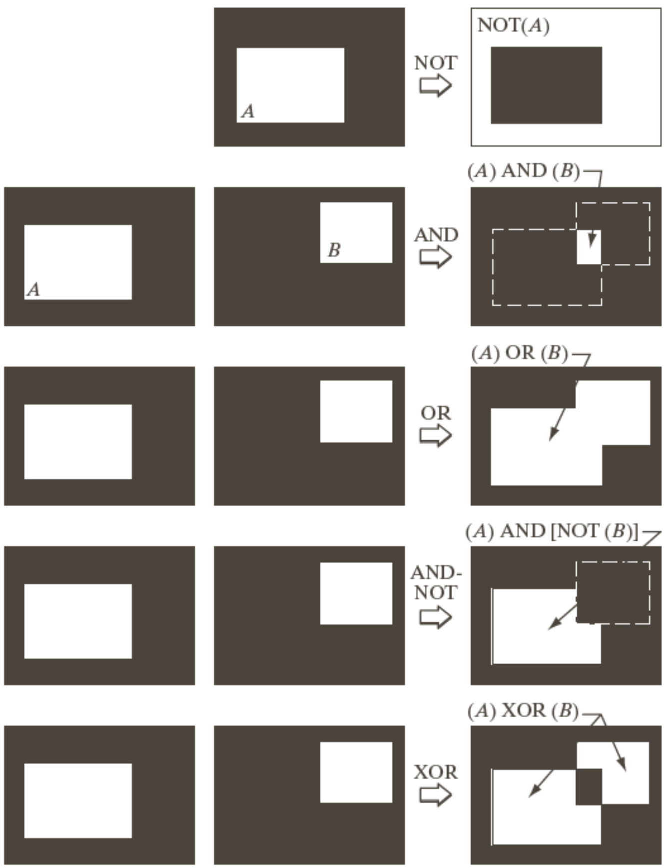
Mathematical tools in DIP

- Shading correction with $g(x,y) = f(x,y) * h(x,y)$
  - $g(x,y)$ known
  - $f(x,y)$ unknown
  - $h(x,y)$?

- Masking with ROI

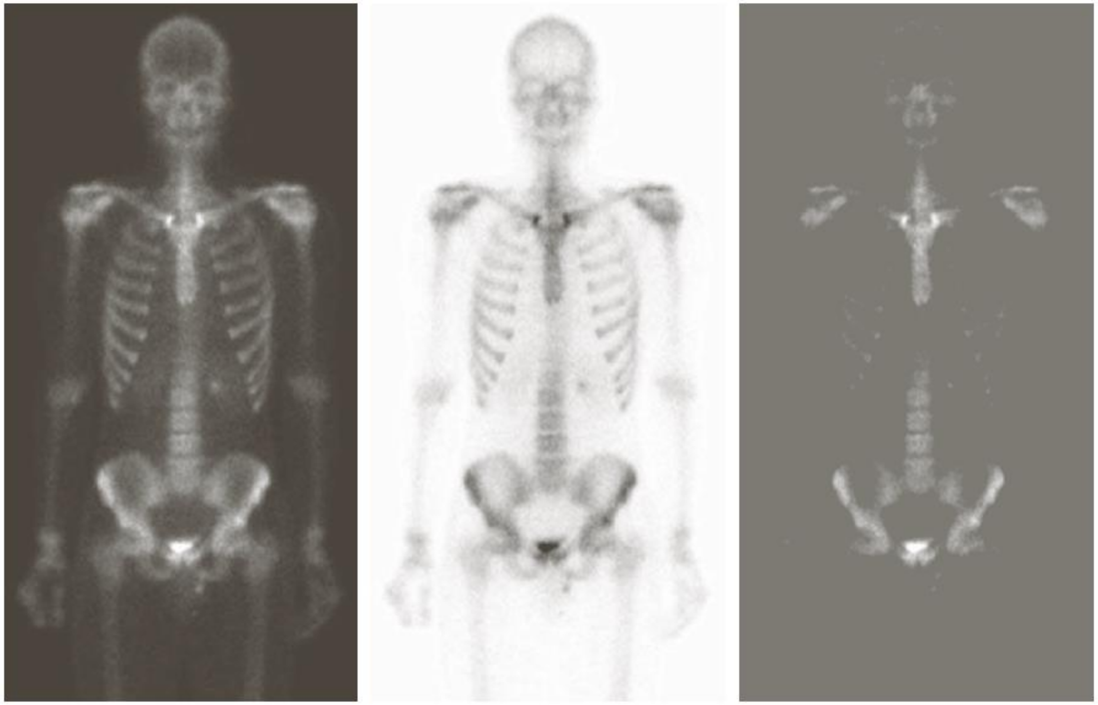
Mathematical tools in DIP





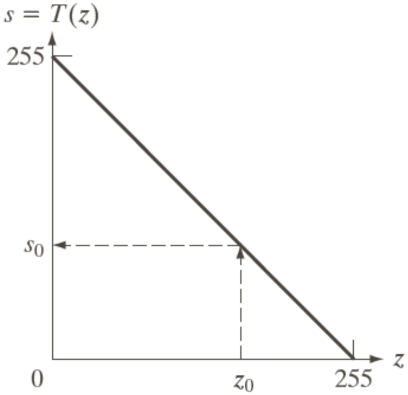
Mathematical tools in DIP

- - Negative image



a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)



Imaging Geometry

- Basic transformations
  - Translation, rotation, scaling, shearing

- Transformation in general is
- $(x, y) = T\{(v, w)\}$

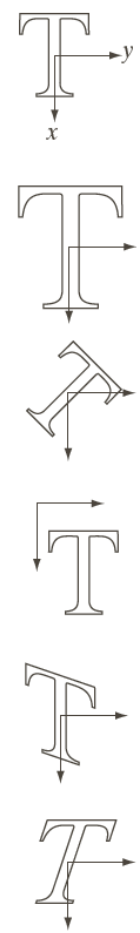
- In the following the computations are

- $[x\ y\ 1] = [v\ w\ 1]T = [v\ w\ 1]\begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$

Imaging Geometry

- Affine transformations

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	



Imaging Geometry

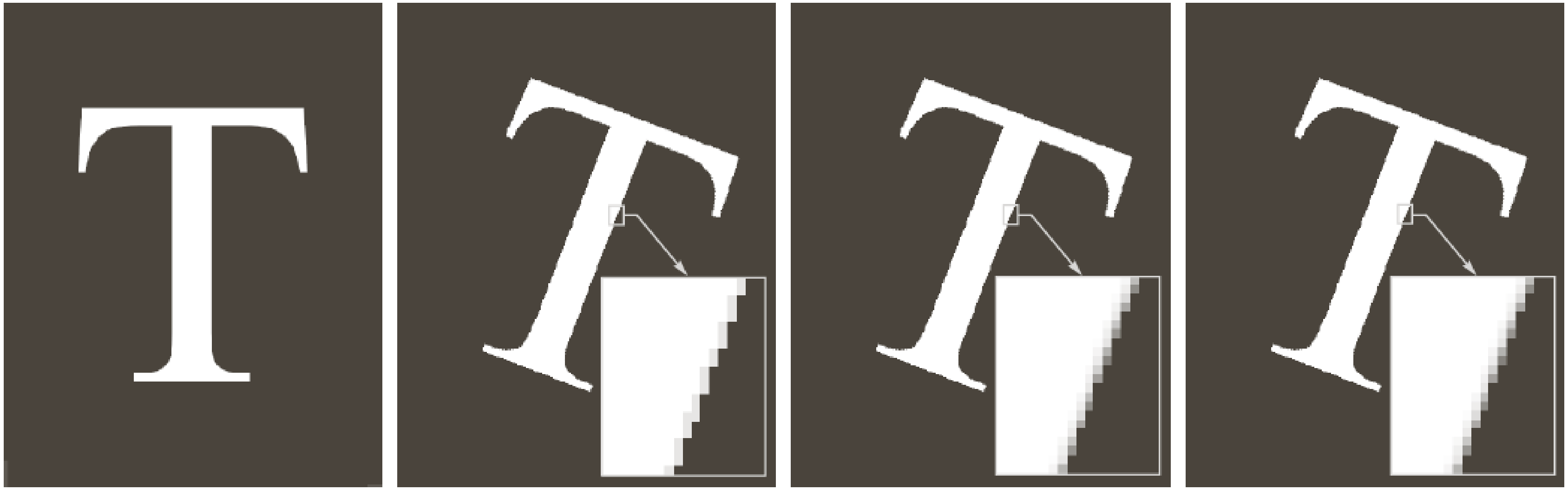
- Two approaches in the computations
  - *Forward mapping*
- $[x\ y\ 1] = [v\ w\ 1]T$
    - Straight-forward approach to find new location and intensity for each pixel
    - Problem: multiple original values may map to one pixel only?
  - *Inverse mapping*

$$[v\ w\ 1] = T^{-1}[x\ y\ 1]$$

    - Interpolation needed in defining the intensities for the pixels

Imaging Geometry

- Example on inverse mapping



a b c d

FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

Imaging Geometry

- In previous mappings the transformation T was known
- *Image registration* for aligning two (or more) images
  - Two images:
    - *Input image* which we want to transform and
    - *Reference image* against which we want to register
  - Differences may come from viewing angle, distance, orientation, sensor resolution, object position, etc.

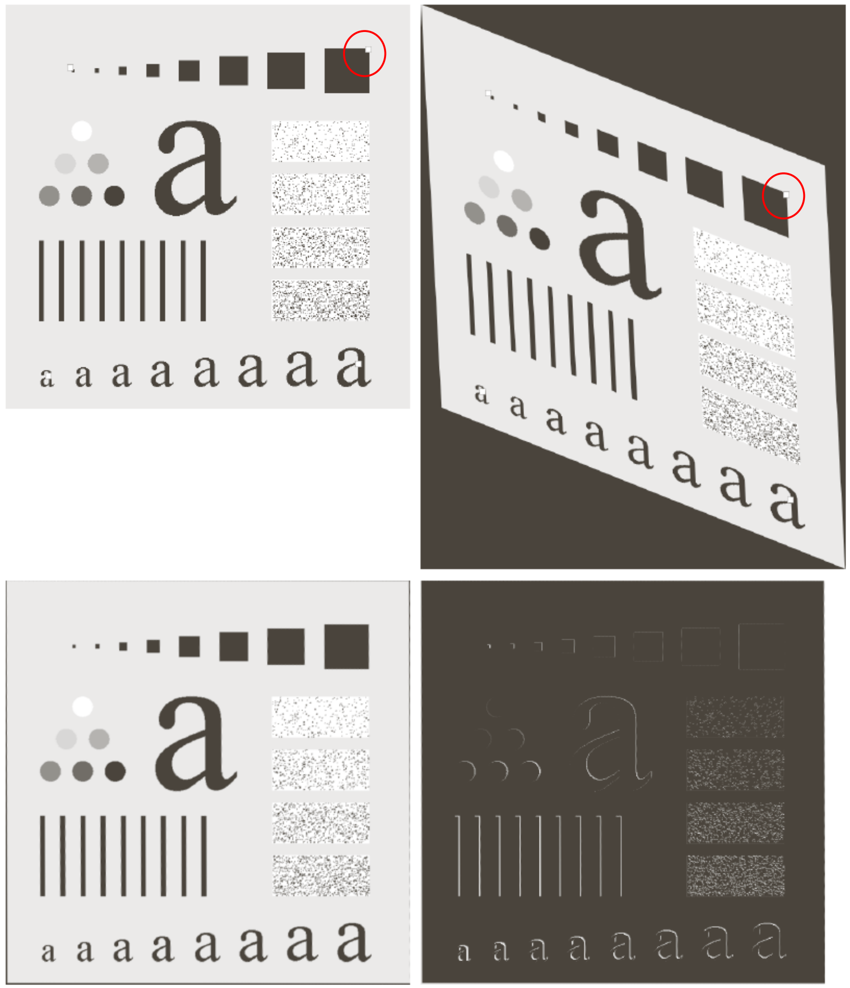
Imaging Geometry

- Basic steps in registration
  1. Define control points in the two images (How?)
  2. Find the transformation matrix
  3. Apply the transformation on input image (location, intensity)
  4. Apply desired operations on the two images
- For example, in step 2

$$x = c_1 v + c_2 w + c_3 vw + c_4$$
$$y = c_5 v + c_6 w + c_7 vw + c_8$$

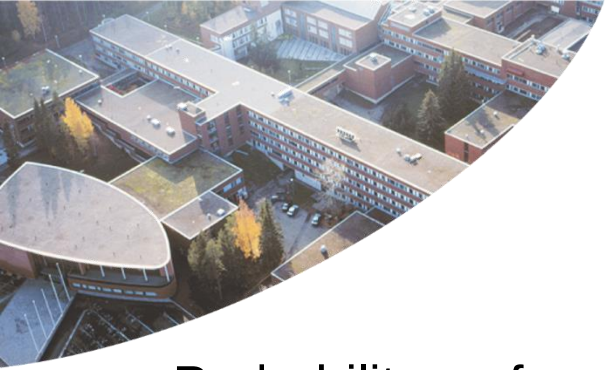
- For higher quality use more corresponding pixels
  - Quadrilaterals as subimages
  - Polynomials as more complex models (LSQ algorithms)
- Intensity interpolation for finding the intensities for the pixels

Imaging Geometry



a b
c d

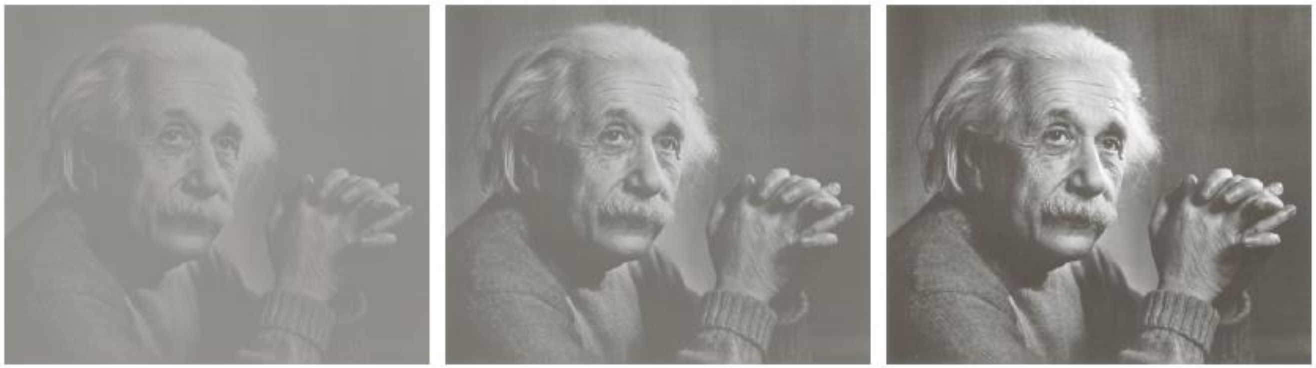
FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the borders). (d) Difference between (a) and (c), showing more registration errors.



Probabilistic models

- Probability p of an intensity level z_k in an image of size MN
- $p(z_k) = \frac{n_k}{MN}$
- Mean m and variance σ^2 of the intensities
  - also other moments (bias, kurtosis(heaviness of the tail))

$$\mu_n(z) = \sum_{k=0}^{L-1} (z_k - m)^n p(z_k), n = 0,1,2,3,4$$



a b c

FIGURE 2.41 Images exhibiting (a) low contrast, (b) medium contrast, and (c) high contrast.

Imaging Geometry

- Basic transformations

- Perspective transformation
  - The camera coordinate system *(x,y,z)* is aligned to the world coordinate system *(X,Y,Z)*
- Camera model
  - Imaging geometry with two coordinate systems.
  - Camera viewing a 3-D scene
- Camera calibration
- Stereo imaging
- Motion, tracking