Homework 1 - IE 453

Simge Çınar 21901465
Ayşe Beyza Çanakçı 21901849
Sena Ceren Göktaş 21704202
Yağız Yaman 21702889

12 March 2023

Part A

In this part of the question, we have followed two different approaches.

In one approach, we assumed the number of wind turbines as a decision variable and the storage system capacity as a parameter (1,000,000 Wh). By doing so, we intended to observe if 10 wind turbines are enough to meet the demand of the village.

In the other approach, we assumed the storage system's capacity as a decision variable and the number of wind turbines as a parameter (10). Then, we observed the required storage capacity when the number of wind turbines is 10. If it is bigger than the current storage capacity, it means the current system is not feasible.

In the following part, you may view the models for both approaches and the interpretations of results.

Model for the Approach 1

Decision Variables

WindGenerator: The number of wind turbines needs to be installed.
WindUsed$_t$**:** Amount of wind energy (Wh) sent to village from wind turbines at time
$t \in \{1, ..., 168\}$.
WindStored$_t$**:** Amount of wind energy (Wh) sent to storage system from wind turbines at time
$t \in \{1, ..., 168\}$.
StorageDischarged$_t$**:** Amount of wind energy (Wh) discharged to village from the storage system at time
$t \in \{1, ..., 168\}$.
WindCurtailed$_t$**:** Amount of wind energy (Wh) curtailed at time $t \in \{1, ..., 168\}$.
StorageLevel$_t$**:** Stored wind energy level (Wh) at the beginning of time $t \in \{1, ..., 169\}$.

Parameters

TurbineCost = \$2,000,000
gamma = 0.85
StorageCapacity = 1,000,000 Wh
Interest = 0.005
LifetimeWind = -52* 25
Demand(Wh)$_t$ = Demand of consumer at time $t \in \{1, ..., 168\}$.
GeneratedEnergy(Wh)$_t$ = Energy generated at time $t \in \{1, ..., 168\}$.

Discount rates for Wind Turbines and the Storage System are as follows, respectively, throughout the question (except Part D):

$$\frac{\text{Interest}}{1-(1+\text{Interest})^{\text{LifetimeWind}}} = 0.0050020897 \tag{1}$$

$$\frac{\text{Interest}}{1-(1+\text{Interest})^{\text{LifetimeStorage}}} = 0.005028099568 \tag{2}$$

Model

$$\begin{aligned} \min \quad & \frac{\text{Interest}}{1-(1+\text{Interest})^{\text{LifetimeWind}}} * \text{TurbineCost} * \text{WindGenerator} \\ & + \frac{\text{Interest}}{1-(1+\text{Interest})^{\text{LifetimeStorage}}} * \text{StorageCapacity} * \text{StorageCost} \end{aligned}$$

$$Demand(Wh)_t = WindUsed_t + StorageDischarged_t \tag{3}$$

$$GeneratedEnergy(Wh)_t * WindGenerator = WindUsed_t + WindStored_t + WindCurtailed_t \tag{4}$$

$$StorageLevel_1 = 0 \tag{5}$$

$$StorageLevel_{t+1} = StorageLevel_t + gamma * WindStored_t - \frac{1}{gamma} * StorageDischarged_t \tag{6}$$

$$StorageLevel_t \leq StorageCapacity_t \qquad \forall t \in \{1, .., 168\} \tag{7}$$

$$\text{all } dv's \geq 0 \tag{8}$$

Model for the Approach 2

Decision Variables

StorageCapacity: The storage capacity (Wh) of the storage system.
WindUsed$_t$: Amount of wind energy (Wh) sent to village from wind turbines at time $t \in \{1, ..., 168\}$.
WindStored$_t$: Amount of wind energy (Wh) sent to storage system from wind turbines at time $t \in \{1, ..., 168\}$.
StorageDischarged$_t$: Amount of wind energy (Wh) discharged to village from the storage system at time
$t \in \{1, ..., 168\}$.
WindCurtailed$_t$: Amount of wind energy (Wh) curtailed at time $t \in \{1, ..., 168\}$.
StorageLevel$_t$: Stored wind energy level (Wh) at the beginning of time $t \in \{1, ..., 169\}$.

Parameters

WindGenerator = 10
TurbineCost = $2000000
StorageCost = 0.6 cent/Wh (assumed for this approach, even though not given yet.)
gamma = 0.85
Interest = 0.005
LifetimeWind = -52* 25
LifetimeStorage = -52* 20
$\mathbf{Demand(Wh)}_t$ = Demand of consumer at time $t \in \{1, ..., 168\}$.
$\mathbf{GeneratedEnergy(Wh)}_t$ = Energy generated at time $t \in \{1, ..., 168\}$.

Model

$$\min \quad \frac{\text{Interest}}{1-(1+\text{Interest})^{\text{LifetimeWind}}} * \text{TurbineCost} * \text{WindGenerator} + \frac{\text{Interest}}{1-(1+\text{Interest})^{\text{LifetimeStorage}}} * \text{StorageCapacity} * \text{StorageCost}$$

$$Demand(Wh)_t = WindUsed_t + StorageDischarged_t \tag{1}$$

$$GeneratedEnergy(Wh)_t * WindGenerator = WindUsed_t + WindStored_t + WindCurtailed_t \tag{2}$$

$$StorageLevel_1 = 0 \tag{3}$$

$$StorageLevel_{t+1} = StorageLevel_t + gamma * WindStored_t - \frac{1}{gamma} * StorageDischarged_t \tag{4}$$

$$StorageLevel_t \leq StorageCapacity_t \qquad \forall t \in \{1, .., 168\} \tag{5}$$

$$\text{all } dv's \geq 0 \tag{6}$$

Results

In the first approach, the model suggests installing 24 wind turbines which signal that 10 wind turbines are insufficient to supply the decentralized electrification system, with a fixed storage capacity of 1MWh.

In the second approach, the model suggests installing a storage system of approximately 3MWh which is three times greater than the already existing capacity (1MWh). Hence, we can conclude that with 10 wind turbines, in order to supply the system sufficiently, we need to have a storage capacity of 3MWh rather than 1MWh.

Alternative ways to satisfy the unmet demand are:

1. To install 13 more wind turbines, as the model suggests, with a fixed storage capacity of 1MWh.
2. To upgrade the storage system capacity to 3MWh, with a fixed number of wind turbines, which is 10.
3. Create a hybrid system. Adopt a dispatchable and preferably fast energy source (e.g., diesel) to prevent infeasibility in the case where the wind is not available and the storage system is not sufficiently charged, or charged at all.
4. Utilize the long-distance transmission lines / connect the system to the grid.
5. Get the benefit of the demand side management to reduce the high energy load at times of low wind speed.
6. Keep the number of wind turbines (10) and storage system capacity (1 MWh) fixed, but install wind turbines with higher blade sizes to increase the energy generated by the available wind. Please see the graphs in Part D.

Note: It sounds rational to suggest installing the storage system, as fully or partially charged at the beginning. However, when we adopt this to the model, by removing the constraint that represents the initial emptiness, and making the initial state of the storage system as fully charged ($1,000,000$Wh), we again get infeasibility. This suggests that with the given number of wind turbines and the storage system capacity, the system is not self-sustainable, independent of the initial state of the storage size.

Part B

We tackled this part of the question similar to the first approach in Part A, where the number of wind turbines is a decision variable.

Decision Variables

WindGenerator: The number of wind turbines needs to be installed.
WindUsed$_t$: Amount of wind energy (Wh) sent to village from wind turbines at time $t \in \{1, ..., 168\}$.
WindStored$_t$: Amount of wind energy (Wh) sent to storage system from wind turbines at time $t \in \{1, ..., 168\}$.
StorageDischarged$_t$: Amount of wind energy (Wh) discharged to the village from the storage system at time
$t \in \{1, ..., 168\}$.
WindCurtailed$_t$: Amount of wind energy (Wh) curtailed at time $t \in \{1, ..., 168\}$.
StorageLevel$_t$: Stored wind energy level (Wh) at the beginning of time $t \in \{1, ..., 169\}$.

Parameters

TurbineCost = \$2000000
gamma = 0.85
StorageCapacity = $1.5 * 1,000,000$Wh
Interest = 0.005
LifetimeWind = -52*25
Demand(Wh)$_t$ = Demand of consumer at time $t \in \{1, ..., 168\}$.
GeneratedEnergy(Wh)$_t$ = Energy generated at time $t \in \ 1, ..., 168\}$.

Model

$$\min \quad \frac{\text{Interest}}{1-(1+\text{Interest})^{\text{LifetimeWind}}} * \text{TurbineCost} * \text{WindGenerator} + \frac{\text{Interest}}{1-(1+\text{Interest})^{\text{LifetimeStorage}}} * \text{StorageCapacity} * \text{StorageCost}$$

$$Demand(Wh)_t = WindUsed_t + StorageDischarged_t \tag{1}$$

$$GeneratedEnergy(Wh)_t * WindGenerator = WindUsed_t + WindStored_t + WindCurtailed_t \tag{2}$$

$$StorageLevel_1 = 0 \tag{3}$$

$$StorageLevel_{t+1} = StorageLevel_t + gamma * WindStored_t - \frac{1}{gamma} * StorageDischarged_t \tag{4}$$

$$StorageLevel_t \leq StorageCapacity_t \qquad \forall t \in \{1, .., 168\} \tag{5}$$

$$\text{all } dv's \geq 0 \tag{6}$$

Results

The model suggests installing 21 wind turbines.
Formula Used to Find the Capacity Factor:

$$\frac{\text{Actual Output}}{\text{Potential Output}} \tag{1}$$

Actual output is the sum of energy generated for a week. This information is extracted from the one-week power curve provided to us.
Potential output is calculated using the storage system's maximum capacity, $1.5 * 1,000,000$ Wh. Since the actual output is in weekly units, we multiplied the maximum capacity by 168 hours.

Hence,

$$\frac{164086000.0}{1.5*1,000,000} \approx 0.65 \tag{2}$$

Part C

We tackled this part of the question similarly to the second approach in part B, where the storage capacity is a decision variable.

Decision Variables

StorageCapacity: The storage capacity (Wh) of the storage system.
WindUsed$_t$: Amount of wind energy (Wh) sent to village from wind turbines at time $t \in \{1, ..., 168\}$.
WindStored$_t$: Amount of wind energy (Wh) sent to storage system from wind turbines at time $t \in \{1, ..., 168\}$.
StorageDischarged$_t$: Amount of wind energy (Wh) discharged to village from the storage system at time
$t \in \{1, ..., 168\}$.
WindCurtailed$_t$: Amount of wind energy (Wh) curtailed at time $t \in \{1, ..., 168\}$.
StorageLevel$_t$: Stored wind energy level (Wh) at the beginning of time $t \in \{1, ..., 169\}$.

Parameters

WindGenerator = 18
TurbineCost = \$2000000
StorageCost = 0.6 cent/Wh
gamma = 0.85
Interest = 0.005
LifetimeWind = -52* 25
LifetimeStorage = -52* 20
Demand(Wh) $_t$ = Demand of consumer at time $t \in \{1, ..., 168\}$.
GeneratedEnergy(Wh) $_t$ = Energy generated at time $t \in \{1, ..., 168\}$.

Model

$$\begin{aligned} \min \quad & \frac{\text{Interest}}{1-(1+\text{Interest})^{\text{LifetimeWind}}} * \text{TurbineCost} * \text{WindGenerator} \\ & + \frac{\text{Interest}}{1-(1+\text{Interest})^{\text{LifetimeStorage}}} * \text{StorageCapacity} * \text{StorageCost} \end{aligned}$$

$$Demand(Wh)_t = WindUsed_t + StorageDischarged_t \tag{1}$$

$$GeneratedEnergy(Wh)_t * WindGenerator = WindUsed_t + WindStored_t + WindCurtailed_t \tag{2}$$

$$StorageLevel_1 = 0 \tag{3}$$

$$StorageLevel_{t+1} = StorageLevel_t + gamma * WindStored_t - \frac{1}{gamma} * StorageDischarged_t \tag{4}$$

$$StorageLevel_t \leq StorageCapacity_t \qquad \forall t \in \{1, .., 168\} \tag{5}$$

$$\text{all } dv's \geq 0 \tag{6}$$

Results

The model suggests adopting the storage system of capacity 1818824Wh which is approximately 1.8 times the suggested capacity.

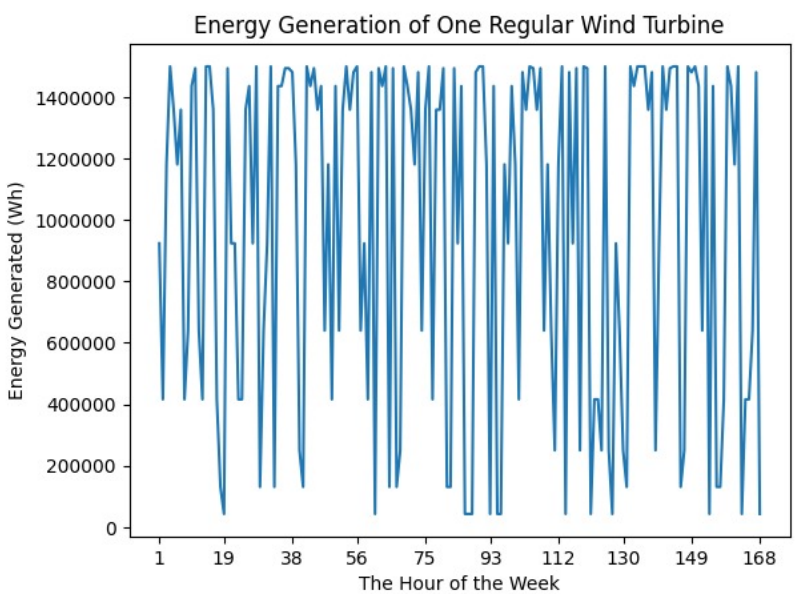
Objective Function Value: $185762.5954

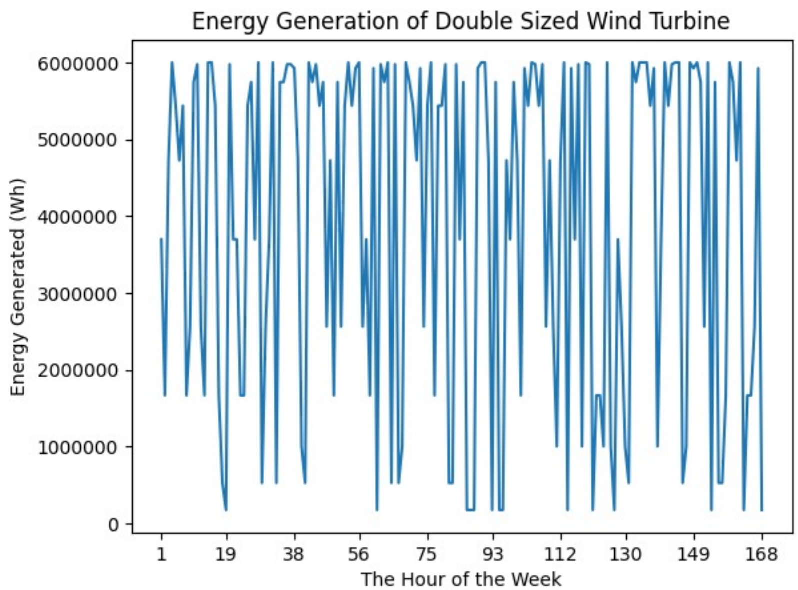
Part D

This part includes the new version of the wind turbines which is more eco-friendly. The blade length in this new version is twice the old one. Given the wind power formula:

$$W = 0.63Av^3 \tag{1}$$

it can be understood that the output of wind turbines will increase by 4. Therefore, the distribution of the energy generated for each hour t (168 hours total in a week) can be viewed below.





These new turbines' lifetime is 30 years with a cost of \$2.5 million. Also, it is known that the storage system's cost is 60 cents/Wh with a lifetime of 20 years. The assumption that the initial storage capacity is zero from the previous question also holds.

Therefore, the formulation of the model to optimize both the number of turbines and the storage size while satisfying demand is as follows:

Decision Variables

WindGenerator: The number of wind turbines needs to be installed.
StorageCapacity: The storage capacity (Wh) of the storage system.
WindUsed$_t$: Amount of wind energy (Wh) sent to village from wind turbines at time $t \in \{1, ..., 168\}$.
WindStored$_t$: Amount of wind energy (Wh) sent to the storage system from wind turbines at time $t \in \{1, ..., 168\}$.
StorageDischarged$_t$: Amount of wind energy (Wh) discharged to the village from the storage system at time $t \in \{1, ..., 168\}$.
WindCurtailed$_t$: Amount of wind energy (Wh) curtailed at time $t \in \{1, ..., 168\}$.
StorageLevel$_t$: Stored wind energy level (Wh) at the beginning of time $t \in \{1, ..., 169\}$.

Parameters

TurbineCost = \$2500000
StorageCost = 0.6 cent/Wh
gamma = 0.85
Interest = 0.005
LifetimeWind = -52* 30

LifetimeStorage = -52* 20
Demand(Wh)$_t$ = Demand of consumer at time $t \in \{1, ..., 168\}$.
GeneratedEnergy(Wh)$_t$ = Energy generated at time $t \in \{1, ..., 168\}$.

Discount rates for Wind Turbines and the Storage System are as follows, respectively:

$$\frac{\text{Interest}}{1 - (1 + \text{Interest})^{\text{LifetimeWind}}} = 0.00500208975 \tag{2}$$

$$\frac{\text{Interest}}{1 - (1 + \text{Interest})^{\text{LifetimeStorage}}} = 0.005028099567 \tag{3}$$

Model

$$\begin{aligned} \min \quad & \frac{\text{Interest}}{1 - (1 + \text{Interest})^{\text{LifetimeWind}}} * \text{TurbineCost} * \text{WindGenerator} \\ & + \frac{\text{Interest}}{1 - (1 + \text{Interest})^{\text{LifetimeStorage}}} * \text{StorageCapacity} * \text{StorageCost} \end{aligned}$$

$$Demand(Wh)_t = WindUsed_t + StorageDischarged_t \tag{1}$$

$$GeneratedEnergy(Wh)_t * WindGenerator = WindUsed_t + WindStored_t + WindCurtailed_t \tag{2}$$

$$StorageLevel_1 = 0 \tag{3}$$

$$StorageLevel_{t+1} = StorageLevel_t + gamma * WindStored_t - \frac{1}{gamma} * StorageDischarged_t \tag{4}$$

$$StorageLevel_t \leq StorageCapacity_t \qquad \forall t \in \{1, .., 168\} \tag{5}$$

$$\text{all } dv's \geq 0 \tag{6}$$

Results

The model suggests installing 1 wind turbine and a storage system with a capacity of 3943530Wh.

Objective Function Value: $24,403.7

Note 1: We have intentionally written all our objectives in cost terms.
Note 2: Throughout the assignment, we have used Wh as the energy unit.
Note 3: All models are implemented in Python's PuLP library.
Note 4: All codes of our models are in the Python codes provided in the .zip file.