# Homework 2 - IE 453

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# Part A

## Assumptions

1. The solar farm and the demand point are in the same area which makes the transmission distance between them negligible.

2. There is no transmission loss between the grid and the demand point.

3. At the beginning of the week, the amount of water stored in the reservoir is 50% of the reservoir capacity.

4. . At the end of the week, 50% of the reservoir should be filled up.

### Decision Variables

**S**$_t$**:** Amount of water in the reservoir at time t, $t \in \{1, ..., 56\}$.
**Release**$_t$**:** Amount of water released from the reservoir at time $t \in \{1, ..., 56\}$.
**HydroUsed**$_t$**:** Amount of hydro energy (kWh) sent to village from hydropower station at time $t \in \{1, ..., 56\}$.
**Spill**$_t$**:** Amount of water spilled at time $t \in \{1, ..., 56\}$.
**SolarUsed**$_t$**:** Amount of solar energy (kWh) sent to village from solar station at time $t \in \{1, ..., 56\}$.
**SolarCurtailed**$_t$**:** Amount of solar energy (kWh) curtailed at time $t \in \{1, ..., 56\}$.
**Purchase**$_t$**:** Amount of electric energy (kWh) purchased from the grid at time $t \in \{1, ..., 56\}$.

### Parameters - Exogenous Variables

**SolarFarmCapacity:** 80 km$^2$ = $80 \cdot 10^6$ m$^2$
**SolarFarmEfficiency** ($\alpha$) = 0.12
**HydroPowerCapacity** ($\mathbf{S}_{max}$) = 0.1 km$^3$ = $0.1 \cdot 10^9$ m$^3$
**GeneratorEfficency** ($\gamma$) = 0.90
**GeneratorCapacity(**$\mathbf{G}_{max}$**)** = 1.2 GW = $1.2 \cdot 10^6$ kW
**TransmissionLoss** ($l$) = 0.05
**Price**$_t$ = 0.30 cents/kWh at time $t \in \{1, ..., 56\}$.
**Inflow**$_t$**:** Water inflow to the reservoir (m$^3$) at time $t \in \{1, ..., 56\}$.
**Demand**$_t$**:** Electricity demand (kWh) at time $t \in \{1, ..., 56\}$.
**SR**$_t$**:** Solar radiation (kWh/m$^2$) at time $t \in \{1, ..., 56\}$.
**Height (h):** 100 meters
**Density (d):** 1000 kg/m$^3$
**g:** 9.8 m/s$^2$

**Model**

$$\min \sum_{t=1}^{56} Price_t(Purhase_t)$$

$$S_t = S_{t-1} + Inflow_t - Release_t - Spill_t \quad \forall t \in \{2, ..., 56\} \tag{1}$$

$$S_1 = \frac{S_{max}}{2} + Inflow_1 - Release_1 - Spill_1 \tag{2}$$

$$S_{56} = \frac{S_{max}}{2} \tag{3}$$

$$S_t \leq S_{max} \tag{4}$$

$$HydroUsed_t = Release_t \cdot d \cdot g \cdot h \cdot \gamma/[(3600) \cdot 10^3)] \quad \forall t \in \{1, ..., 56\} \tag{5}$$

$$SR_t \cdot SolarFarmCap \cdot \alpha = SolarCurtailed_t + SolarUsed_t \quad \forall t \in \{1, ..., 56\} \tag{6}$$

$$Demand_t = SolarUsed_t + HydroUsed_t(1 - l) + Purchase_t \quad \forall t \in \{1, ..., 56\} \tag{7}$$

$$HydroUsed_t \leq G_{max} \cdot 3 \quad \forall t \in \{1, ..., 56\} \tag{8}$$

$$\text{all } dv's \geq 0 \tag{9}$$

# Part B

## Result

We implemented our model on Python's PuLp package. The model results in 139,385,246.64 cents.

# Part C-1

### Decision Variables

The decision variables are the same as Part A, except $S_t$ is removed as there is not a reservoir.

### Parameters - Exogenous Variables

The parameters are the same as in Part A.

### Model

$$\min \sum_{t=1}^{56} Price_t(Purhase_t)$$

$$Inflow_t = Release_t + Spill_t \quad \forall t \in \{1, ..., 56\} \tag{1}$$

$$HydroUsed_t = Release_t \cdot d \cdot g \cdot h \cdot \gamma/[(3600) \cdot 10^3)] \quad \forall t \in \{1, ..., 56\} \tag{2}$$

$$SR_t \cdot SolarFarmCap \cdot \alpha = SolarCurtailed_t + SolarUsed_t \quad \forall t \in \{1, ..., 56\} \tag{3}$$

$$Demand_t = SolarUsed_t + HydroUsed_t(1 - l) + Purchase_t \quad \forall t \in \{1, ..., 56\} \tag{4}$$

$$HydroUsed_t \leq G_{max} \cdot 3 \quad \forall t \in \{1, ..., 56\} \tag{5}$$

$$\text{all } dv's \geq 0 \tag{6}$$

## Result

We implemented our model on Python's PuLp package. The model results in 139,781,757.12 cents.

# Part C-2

### Decision Variables

The decision variables are the same as Part A.

### Parameters - Exogenous Variables

The parameters are the same as Part A, except for the following parameter:
**GeneratorCapacity($\mathbf{G}_{max}$)** = 15000 kW

### Model

The model is the same as Part A.

### Result

We implemented our model on Python's PuLp package. The model results in 141,477,705 cents.

# Part D

## Capacity Factor

We will find capacity factors based on the model proposed in Part A. We will adopt the following formula as we progress in this part:

$$\text{Capacity Factor } = \frac{\text{Actual Output}}{\text{Potential Output}} \tag{1}$$

## 1- Solar Farm

In order to find the **potential** output of the solar farm, we assumed the solar radiation for one week to take on its maximum value which is 2.367 kWh/m$^2$. Considering the solar farm efficiency as $\alpha = 0.12$, and the capacity as $80 \cdot 1000000$ m$^2$, we found the potential output as follows:

$$80 \cdot 1000000 \cdot 2.367 \cdot 0.12 \cdot 56 = 1.2724992 \cdot 10^9 \text{ kWh} \tag{2}$$

In order to find the **actual** output of the solar farm, we summed the energy generated ($SolarCurtailed$ + $SolarUsed$) over the one-week period which turned out to be:

Hence, the capacity factor for the solar farm is $\frac{285\ 974\ 400}{1\ 272\ 499\ 200} = 0.224$

### 2- Hydro-Power Station

In order to find the **potential** output of the hydro-power station, we considered the generator efficiency as $\gamma = 0.90$, and the capacity as $1.2 \cdot 1000000$ kW, and we found the potential output as follows:

$$1\ 200\ 000 \cdot 3 \cdot 56 \cdot 0.90 = 181\ 440\ 000 \text{ kWh} \tag{1}$$

In order to find the **actual** output of the hydro-power station, we summed the energy generated ($HydroUsed$) over the one-week period which turned out to be: 9 456 959.2 kWh

Hence, the capacity factor for the hydro-power station is $\frac{9\ 456\ 959.2}{181\ 440\ 000} = 0.052$

# Part E

### Assumptions

1. The solar farm and the demand point are in the same area which makes the transmission distance between them negligible.
2. There is no transmission loss between the grid and the demand point.
3. At the beginning of the week, the amount of water stored in the upper reservoir is 50% of the reservoir capacity.
4. At the beginning of the week, the lower reservoir is empty.

### Decision Variables

$\mathbf{S}_{t,U}$**:** Amount of water (m$^3$) in the upper reservoir at time $t \in \{1, ..., 56\}$.
$\mathbf{S}_{t,L}$**:** Amount of water in the lower reservoir at time t, $t \in \{1, ..., 56\}$.
**Release**$_t$**:** Amount of water (m$^3$) released from the reservoir at time $t \in \{1, ..., 56\}$.
**HydroUsed**$_t$**:** Amount of hydro energy (kWh) sent to village from hydropower station at time $t \in \{1, ..., 56\}$.
**Spill**$_{t,U}$**:** Amount of water (m$^3$) spilled from the upper reservoir at time $t \in \{1, ..., 56\}$.
**Spill**$_{t,L}$**:** Amount of water spilled from the lower reservoir at time $t \in \{1, ..., 56\}$.
**SolarUsed**$_t$**:** Amount of solar energy (kWh) sent to village from solar station at time $t \in \{1, ..., 56\}$.
**SolarCurtailed**$_t$**:** Amount of solar energy (kWh) curtailed at time $t \in \{1, ..., 56\}$.
**Purchase**$_t$**:** Amount of electric energy (kWh) purchased from the grid at time $t \in \{1, ..., 56\}$.
**Pump**$_t$**:** Amount of water (m$^3$) pumped from lower reservoir to upper reservior at time $t \in \{1, ..., 56\}$.
**SolarPump**$_t$**:** Amount of energy (kWh) used from solar to pump the water from the lower reservoir to the upper reservoir at time $t \in \{1, ..., 56\}$.

**Parameters - Exogenous Variables**

**SolarFarmCapacity:** 80 km$^2$ = $80 \cdot 10^6$ m$^2$
**SolarFarmEfficiency** ($\alpha$) = 0.12
**HydroPowerCapacity** ($\mathbf{S}_{max,U}$) = 0.1 km$^3$ = $0.1 \cdot 10^9$ m$^3$
**HydroPowerCapacity** ($\mathbf{S}_{max,L}$) = 0.05 km$^3$ = $0.05 \cdot 10^9$ m$^3$
**GeneratorEfficency** ($\gamma$) = 0.90
**GeneratorCapacity**($\mathbf{G}_{max}$) = 1.2 GW = $1.2 \cdot 10^6$ kW
**TransmissionLoss** ($l$) = 0.05
**Price**$_t$ = 0.30 cents/kWh at time $t \in \{1, ..., 56\}$.
**Inflow**$_t$**:** Water inflow to the upper reservoir (m$^3$) at time $t \in \{1, ..., 56\}$.
**Demand**$_t$**:** Electricity demand (kWh) at time $t \in \{1, ..., 56\}$.
**SR**$_t$**:** Solar radiation (kWh/m$^2$) at time $t \in \{1, ..., 56\}$.
**Height (h):** 100 meters
**Density (d):** 1000 kg/m$^3$
**g:** 9.8 m/s$^2$

**Model**

$$\min \sum_{t=1}^{56} Price_t(Purhase_t)$$

$$S_{t,U} = S_{t-1,U} + Inflow_t - Release_t + Pump_t - Spill_{t,U} \quad \forall t \in \{2, ..., 56\} \tag{1}$$

$$S_{1,U} = Smax/2 + Inflow_1 - Release_1 + Pump_1 - Spill_{1,U} \tag{2}$$

$$S_{t,L} = S_{t-1,L} + Release_t - Pump_t - Spill_{t,L} \quad \forall t \in \{2, ..., 56\} \tag{3}$$

$$S_{0,L} = 0 \tag{4}$$

$$S_{1,L} = S_{0,L} + Release_1 - Pump_1 - Spill_{1,L} \tag{5}$$

$$SR_t \cdot SolarFarmCapacity \cdot \alpha = SolarPumped_t + SolarCurtailed_t + SolarUsed_t \quad \forall t \in \{1, ..., 56\} \tag{6}$$

$$HydroUsed_t = Release_t \cdot d \cdot g \cdot h \cdot \gamma \cdot 1/[(3600) \cdot 10^3)] \quad \forall t \in \{1, ..., 56\} \tag{7}$$

$$HydroUsed_t \leq G_{max} \cdot 3 \quad \forall t \in \{1, ..., 56\} \tag{8}$$

$$Pump_t \cdot d \cdot g \cdot h \cdot (1/\gamma) \cdot 1/[(3600) \cdot 10^3)] = SolarPumped_t \quad \forall t \in \{1, ..., 56\} \tag{9}$$

$$Purchase_t + HydroUsed_t \cdot (1 - l) + SolarUsed_t \geq Demand_t \quad \forall t \in \{1, ..., 56\} \tag{10}$$

$$S_{t,U} \leq S_{max,U} \quad \forall t \in \{1, ..., 56\} \tag{11}$$

$$S_{t,L} \leq S_{max,L} \quad \forall t \in \{1, ..., 56\} \tag{12}$$

$$all\ dv's \geq 0 \quad \forall t \in \{1, ..., 56\} \tag{13}$$

**Results**

We implemented our model on Python's PuLp package. The model results in 123,818,143.23 cents.

**Possible Extension of the Model**

In the case of having alternating electricity prices, rather than a fixed one as in our case (30 cent/kWh), we could purchase electricity for two reasons: to store in the PHES when electricity prices are low and to transmit it to the demand point. In this case, we would have two decision variables as:
**Purchase$_{1,t}$:** Electric energy purchased from the grid and sent to the PHES at time $t \in \{1, ..., 56\}$.
**Purchase$_{2,t}$:** Electric energy purchased from the grid and sent directly to the demand point at time $t \in \{1, ..., 56\}$.
In this case, we would change the objective in Part E as follows:

$$\min \sum_{t=1}^{56} Price_t (Purhase_{1,t} + Purhase_{2,t})$$

and change the constraints (9) and (10) of the model in Part E as follows:

$$Pump_t \cdot d \cdot g \cdot h \cdot 1/[(3600) \cdot 10^3)] = SolarPumped_t \cdot \gamma + Purchase_{1,t} \cdot \gamma \quad \forall t \in \{1, ..., 56\} \tag{1}$$

$$Demand_t \leq Purchase_{2,t} + HydroUsed_t (1 - l) + SolarUsed_t \quad \forall t \in \{1, ..., 56\} \tag{2}$$

## Part F

Proposed demand-side management strategy for this section is load shifting. Therefore, the given model in Part E will be modified according to this strategy.

### Decision Variables

The decision variables are same as Part E. Only additional decision variable is:
**Shift$_t$:** Amount of demand (kWh) shifted from time t to t+1, $\forall t \in \{1, ..., 56\}$.

### Parameters - Exogenous Variables

The parameters are same as Part E.

### Model:

$$\min \sum_{t=1}^{56} Price_t(Purhase_t)$$

$$S_{t,U} = S_{t-1,U} + Inflow_t - Release_t + Pump_t - Spill_{t,U} \quad \forall t \in \{2, ..., 56\} \tag{1}$$

$$S_{1,U} = Smax/2 + Inflow_1 - Release_1 + Pump_1 - Spill_{1,U} \tag{2}$$

$$S_{t,L} = S_{t-1,L} + Release_t - Pump_t - Spill_{t,L} \quad \forall t \in \{2, ..., 56\} \tag{3}$$

$$S_{0,L} = 0 \tag{4}$$

$$S_{1,L} = S_{0,L} + Release_1 - Pump_1 - Spill_{1,L} \tag{5}$$

$$SR_t \cdot SolarFarmCapacity \cdot \alpha = SolarPumped_t + SolarCurtailed_t + SolarUsed_t \quad \forall t \in \{1, ..., 56\} \tag{6}$$

$$HydroUsed_t = Release_t \cdot d \cdot g \cdot h \cdot \gamma \cdot 1/[(3600) \cdot 10^3)] \quad \forall t \in \{1, ..., 56\} \tag{7}$$

$$HydroUsed_t \leq G_{max} \cdot 3 \quad \forall t \in \{1, ..., 56\} \tag{8}$$

$$Pump_t \cdot d \cdot g \cdot h \cdot (1/\gamma) \cdot 1/[(3600) \cdot 10^3)] = SolarPumped_t \quad \forall t \in \{1, ..., 56\} \tag{9}$$

$$Purchase_1 + HydroUsed_1 \cdot (1 - l) + SolarUsed_1 \geq Demand_1 - Shift_1 \tag{10}$$

$$Purchase_t + HydroUsed_t \cdot (1 - l) + SolarUsed_t \geq Demand_t - Shift_t + Shift_{t-1} \quad \forall t \in \{1, ..., 56\} \tag{11}$$

$$Shift_t \leq Demand_t \quad \forall t \in \{1, ..., 56\} \tag{12}$$

$$Shift_{56} = 0 \tag{13}$$

$$S_{t,U} \leq S_{max,U} \quad \forall t \in \{1, ..., 56\} \tag{14}$$

$$S_{t,L} \leq S_{max,L} \quad \forall t \in \{1, ..., 56\} \tag{15}$$

$$all\ dv's \geq 0 \quad \forall t \in \{1, ..., 56\} \tag{16}$$

The new constraints (10), (11), (12) and (13) can be explained as follows:

The constraint (10) written for the first period where the shift from a previous period is not possible. Therefore, it can only reduce the demand by passing it to the next periods where $t \geq 1$. Constraint (11) is the balance equation for the other periods. Constraint (12) limits the shifted amount by the amount of demand to prevent the constant shift of demand. Lastly, constraint (13) prevents the shift in the last period by equalizing it to the zero since the planning horizon ends.

**Results**

We implemented our model on Python's PuLp package. The model results in 119,500,624.53 cents.

**How this strategy affects cost of electricity:**

The objective function is improved by 3.48% compared to the Part E. The reason behind this is with a load shifting demand side management, the peak demand is reduced and shifted to other periods if necessary. Therefore, assuming periods where demand is relatively high and where demand is low (which means there are extra supply of renewable energy since it is not fully used), some portion of the demand shifted to the second case period. Therefore, purchase from the grid has been reduced.