IE303 Modeling and Optimization Project 1

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1 Question 1

1.1 Model

Description of Model Respectively:

1. Objective function minimizes total daily cost.
2. Demand satisfaction
3. Maximum capacity of machine j restricts the electricity given
4. Working power generators must be able to cope with 1/5 of the demand forecast.
5. If machine does not stop on next period, to be able to not include startup cost new variable Z_{ij} will be 1

Parameters:

Shift number I = {1,..7}, Machine number J = {1,...,25}

D_i: Demand in period i, $i \in I$

CS_j: Startup cost of jth type, $j \in J$

cap_j: max capacity of jth type, $j \in J$

M_j: min output of jth type, $j \in J$

CF_j: Fixed cost of jth type, $j \in J$

CH_j: Hourly cost of jth type, $j \in J$

A_j: Available number of jth type, $j \in J$

T_i: Time interval in period i, $i \in I$

Decision Variables:

$$X_{ij} = \begin{cases} 1 & \text{If jth machine works in ith period,} \quad i \in \{2,3,4,5,6,7\}, \quad j \in J \\ 0 & \text{otherwise} \end{cases}$$

Y_{ij} : Additional amt. of electricity given by jth type of machine on ith period, $i \in I$, $j \in J$

$$Z_{ij} = \begin{cases} 1 & \text{If jth type of machine does not stop in ith period,} \quad i \in \{2,3,4,5,6,7\}, \quad j \in J \\ 0 & \text{otherwise} \end{cases}$$

Model:

$$\min \quad \sum_{i=1}^{7}\sum_{j=1}^{25}(CS_j + CF_jT_i)X_{ij} + Y_{ij}CH_jT_i - CS_jZ_{ij} \tag{1}$$

subject to

$$\sum_{j=1}^{25} M_jX_{ij} + Y_{ij} \geqslant D_i, \quad i \in I \tag{2}$$

$$X_{ij}M_j + Y_{ij} \leqslant cap_j, \quad i \in I, j \in J \tag{3}$$

$$\sum_{j=1}^{25} cap_jX_{ij} \geqslant 1.2D_i \quad i \in I \tag{4}$$

$$X_(i-1)_j + X_{ij} \leqslant 1 + Z_{ij}, \quad i \in \{2,..7\}, j \in J \tag{5}$$

$$X_(i-1)_j + X_{ij} \geqslant 2Z_{ij} \tag{6}$$

1.2 Solution:

```
Run

X(7,18)=0
X(7,19)=0
X(7,20)=0
X(7,21)=0
X(7,22)=0
X(7,23)=1
X(7,24)=1
X(7,25)=1
1319590
End running model

Process exited with code: 0
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The value 1319590 is the objection function value; so, it shows the minimum amount of the total daily cost.

2 Question 2

2.1 Model

Description of Model Respectively:

1. It is a redundant objective function which will give the total number of numbers given initially.
2. If there exists a number t initially corresponding Y variable must be 1 as well.
3. Cells in every column has different numbers.
4. Cells in every row has different numbers.
5. Every cell include a number.
6. Every 9 cell box given by rule include every number differently.

Parameters $\mathrm{P_{tij}}$: 9x9x9 binary matrix that tells initial condition Ex: $\mathrm{P_{611}}$ is 1

Decision Variables:

$$Y_{tij} = \begin{cases} 1 & \text{If the number is located on ith row jth column,} \quad i \in I, j \in J, t \in T \\ 0 & Otw. \end{cases}$$

Model:

$$\min \sum_{t=1}^{9}\sum_{i=1}^{9}\sum_{j=1}^{9} P_{tij}Y_{tij} \tag{1}$$

$$P_{tij} \leqslant Y_{tij}, \quad t \in T, i \in I, j \in J \tag{2}$$

$$\sum_{i=1}^{9} Y_{tij} = 1 \quad t \in T, j \in J \tag{3}$$

$$\sum_{j=1}^{9} Y_{tij} = 1, \quad t \in T, i \in I \tag{4}$$

$$\sum_{t=1}^{9} Y_{tij} = 1 \quad i \in I, j \in J \tag{5}$$

$$Y_{tij} + Y_{t(i+1)j} + Y_{t(i+2)j} + Y_{(ti(j+1)} + Y_{t(i+1)(j+1)} + Y_{t(i+2)(j+1)} + Y_{ti(j+2)} + Y_{t(i+1)(j+2)} + Y_{t(i+2)(j+2)} = 1 \quad t \in T, i \in \{1,4,7\}, j \in \{1,4,7\} \tag{6}$$

2.2 Solution:

Here the solution of 1st puzzle:

$$\begin{bmatrix} 6 & 5 & 9 & 4 & 1 & 3 & 7 & 8 & 2 \\ 3 & 1 & 7 & 2 & 8 & 5 & 4 & 9 & 6 \\ 2 & 4 & 8 & 7 & 9 & 6 & 5 & 3 & 1 \\ 9 & 6 & 5 & 1 & 7 & 2 & 8 & 4 & 3 \\ 8 & 7 & 2 & 9 & 3 & 4 & 1 & 6 & 5 \\ 4 & 3 & 1 & 5 & 6 & 8 & 2 & 7 & 9 \\ 7 & 2 & 6 & 3 & 4 & 1 & 9 & 5 & 8 \\ 5 & 8 & 4 & 6 & 2 & 9 & 3 & 1 & 7 \\ 1 & 9 & 3 & 8 & 5 & 7 & 6 & 2 & 4 \end{bmatrix}$$

6					3	7	8	
	1				5		9	
		8					3	1
			1					
		2		3			6	
					8			9
7								
			6	2	9			
1	9					6		

Since 1st puzzle is correct we can say that our model is true.