IE 202 INTRODUCTION TO MODELING AND OPTIMIZATION
MIGROS YANIMDA
STAGE-3 REPORT

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Stage 3

Part-A

Parameters:

p_c= profit of 1 kg cheese.

p_y= profit of 1 kg yoghurt.

p_b= profit of 1 kg butter.

Decision Variables:

x_i= the number of hours machine i works where $i \in \{1, 2, 3\}$.

Model

$$\min(2x_1 + (1.5)x_2 + (1.75)x_3)p_c + (3x_1 + 3x_2 + x_3)p_y + (4x_1 + 2x_2 + (1.5)x_3)p_b$$

s.t.

$$x_1 + x_2 + x_3 \leq 75$$
$$10x_1 + 7.5x_2 + 5x_3 \leq 1000$$
$$x_1 + x_2 + 0.75x_3 \leq 100$$
$$7x_1 + 5x_2 + 2.5x_3 \leq 600$$
$$x_1, x_2, x_3 \geq 0$$

Part B

If $p_c = p_y = p_b = 5$;

Then, sensitivity report is:

Cell	Objective Name	Value
$C$13	"maximize profit"	3375

Cell	Variable Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
$B$3	"x1"	75	0	0	75	3375
$B$4	"x2"	0	0	3375	0	3375
$B$5	"x3"	0	0	3375	0	3375

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	"x1"	75	0	45	1E+30	12,5
$B$4	"x2"	0	-12,5	32,5	12,5	1E+30
$B$5	"x3"	0	-23,75	21,25	23,75	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$10	Waste	75	0	100	1E+30	25
$B$11	yogurt&butter	525	0	600	1E+30	75
$B$8	Time	75	45	75	10,71428571	75
$B$9	Milk supply	750	0	1000	1E+30	250

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Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$3	"x1"	0	75	Contin
$B$4	"x2"	0	0	Contin
$B$5	"x3"	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$10	Waste	75	$B$10<=$E$10	Not Binding	25
$B$11	yogurt&butter	525	$B$11<=$E$11	Not Binding	75
$B$8	Time	75	$B$8<=$E$8	Binding	0
$B$9	Milk supply	750	$B$9<=$E$9	Not Binding	250
$B$3	"x1"	75	$B$3>=0	Not Binding	75
$B$4	"x2"	0	$B$4>=0	Binding	0
$B$5	"x3"	0	$B$5>=0	Binding	0

To maximize the profit, machine 1 should work for 75 hours a week and remaining machines should not work.
In this situation, the optimum value becomes **3375** which is 75*45(final value of x_1 times objective coefficient of x_1).

We can find basic and nonbasic variables from the report.

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \\ \mathrm{x}_4 \\ \mathrm{x}_5 \\ \mathrm{x}_6 \\ \mathrm{x}_7 \end{bmatrix} = \begin{bmatrix} 75 \\ 0 \\ 0 \\ 25 \\ 75 \\ 0 \\ 250 \end{bmatrix}$$

x_4, x_5, x_6 and x_7 are slack variables. x_4 is the slack variable added in the waste constraint. x_5 is added in the yogurt and butter constraint, x_6 is added in time constraint and x_7 is added in milk supply constraint. In here x_1, x_4, x_5 and x_7 are basic variables as their values are different than zero and the remaining ones are non-basic variables. **The final value** of x_1 is 75 and it is 0 for both x_2 and x_3. Also, the slack value for each constraint shows the value of the slack variable that added on that constraint. So, in final we have 4 basic variables and 3 non-basic variables in our optimal basis.

From the variable cells, it is seen that allowable increase for x_1 is infinity;so, if I increase the objective coefficient of x_1 from 45 to infinity, I'm going to have the same set of basic variables. Also, allowable decrease of the objective coefficient of x_1 is 12.5 and it means I can reduce 45 to 32.5.

The allowable decrease of x_2 and x_3 is infinity. And if we increase objective coefficient of both x_2 and x_3 untill 45, we are going to get same optimal basis.

For constraints, rhs of waste constraint can increase until infinity and decrease 25 units. Within these ranges, optimal basis is not gonna change. Rhs is 600 of yogurt and butter constraint and there is no limit for increasing, but the decreasing is limited to 75 units. For time constraint, allowable increase is 10.7 and allowable decrease is 75. The last constraint(which is milk supply) has infinite allowable increase and the allowable decrease is 250. As stated before, if the increases and decreases of right hand sides are in allowable ranges, there will be same optimal basis.

Reduced cost corresponds to row-0 coefficients of variables in negative value. So, in our final simplex tableau, if I take the row-0 coefficients and take their negative values, I will have the reduced cost. Then, the reduced cost of x_1 is 0 as expected because it is basic variable. The reduced cost of x_2 is -12.5 and it means that if x_2 enters into the basis, it is going to decrease objective by 12.5 per unit.

There are **shadow prices** and these are our dual variable values. So, w_1=0 ,w_2=0 ,w_3=45 ,w_4=0. We can also find these by c_bB^{-1}. So, if the third(time) constraint's right hand side value is increased by one unit, the objective will increase by 45 unit. Because other dual values are zero, they have not any effect on objective.

We have **cell value** of 75 in waste constraint. It means that when we put the final values in this constraint we are going to get 75. The slack is 25 for this constraint because the right hand side is 100. When we subract cell value from right hand side, we get slack value. The time constraint is binding;so, it means that cell value is equal to rhs. We can conclude that in this constraint, we cannot increase our variables' values because they reached their limits for that constraint. We have 2 more binding constraints which state that x_2 and x_3 cannot be less than zero. They are binding because final values of both x_2 and x_3 equal to zero. So, cell value and rhs are equal. The other constraints are not binding;so, we do not have such kind of limitations. For example, the right hand side of yogurt and butter constraint can increase 75 unit as the slack value is 75 and nothing changes in the optimum basis.

Part C

If p_c = 15, p_y= 10, p_b = 5;

Then, sensitivity report is:

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$C$16	"maximize profit"	0	6000

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$3	"x1="	0	75	Contin
$B$4	"x2="	0	0	Contin
$B$5	"x3="	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$10	Waste	75	$B$10<=$E$10	Not Binding	25
$B$11	yogurt&butter	525	$B$11<=$E$11	Not Binding	75
$B$8	Time	75	$B$8<=$E$8	Binding	0
$B$9	Milk supply	750	$B$9<=$E$9	Not Binding	250
$B$3	"x1="	75	$B$3>=0	Not Binding	75
$B$4	"x2="	0	$B$4>=0	Binding	0
$B$5	"x3="	0	$B$5>=0	Binding	0

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	"x1="	75	0	80	1E+30	17,5
$B$4	"x2="	0	-17,5	62,5	17,5	1E+30
$B$5	"x3="	0	-36,25	43,75	36,25	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$10	Waste	75	0	100	1E+30	25
$B$11	yogurt&butter	525	0	600	1E+30	75
$B$8	Time	75	80	75	10,71428571	75
$B$9	Milk supply	750	0	1000	1E+30	250

Our constraints are completely same in this part. The only difference in this part is the price of the products. We see from the sensitivity analisis that the optimal basis is same. The change happens in objective coefficients. The new objective coefficient of x_1 is 80. There is infinite allowable increase of coefficient of x_1, but the allowable decrease is 17.5 . So, within this range the optimal basis stays same.
We have same basic and non-basic variables as:

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \\ \mathrm{x}_4 \\ \mathrm{x}_5 \\ \mathrm{x}_6 \\ \mathrm{x}_7 \end{bmatrix} = \begin{bmatrix} 75 \\ 0 \\ 0 \\ 25 \\ 75 \\ 0 \\ 250 \end{bmatrix}$$

The new objective function is 6000. It is understandable as the only non-zero final value is belongs to x_1 and it is 75 and the objective coefficient is 80. Then $75 * 80 = 6000$. Furthermore, the increase in the profit is expected as the prices are increased.

Reduced costs changed in this part. The reduced cost of x_1 is 0 as expected because it is basic variable. The reduced cost of x_2 is -17.5 and it means that if x_2 enters into the basis, it is going to decrease objective by 17.5 per unit. Same analogy is valid for x_3.

There are **shadow prices** and these are our dual variables' values. So, w_1=0 ,w_2=0 ,w_3=80 ,w_4=0. We can also find these by $c_b B^{-1}$. So, if the third(time) constraint's right hand side value is increased by one unit, the objective will increase by 80 unit.