IE 202 Project Solution

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PART A

Parameters

- districts = {1..10}
- types = {1=jet, 2=2M, 3=3M, 4=5M}
- c : cost of travel
- OC : Overcapacity cost
- K : Maximum difference between 3M and Jet
- D_i : Demand for district i
- $Dist_{ij}$: Distance between districts i and j
- f_t : Fixed building cost of store type t
- cap_t : Capacity of store type t
- r_t : range of store type t
- M : A very large value to be used in big-M linearizations
- $n_{ij} = \begin{cases} 1 & \text{if the districts i and j are neighbours} \\ 0 & \text{if not} \end{cases}$
- $Servability_{tij} = \begin{cases} 1 & \text{if the store type t opened in district i can serve to district j} \\ 0 & \text{if not} \end{cases}$

Decision Variables

- x_{ijt} : The amount of demand that is satisfied for district j from the shop in district i which has type t
- $y_{it} = \begin{cases} 1 & \text{if store that has type t is opened in district i} \\ 0 & \text{otherwise} \end{cases}$

- $O_{it} = \begin{cases} 1 & \text{if store type t in district i decides to use overcapacity} \\ 0 & \text{otherwise} \end{cases}$

Objective Function

$$\textbf{minimize} \qquad \sum_{t=1}^{4}\sum_{i=1}^{10} f_t * y_{it} + \sum_{i=1}^{10}\sum_{j=1}^{10}\sum_{t=1}^{4} x_{ijt} * dist_{ij} * c + \sum_{i=1}^{10}\sum_{t=1}^{4} OC * O_{it}$$

s.t

1. Demand for every district should be met

$$\sum_{i=1}^{10}\sum_{t=1}^{4} x_{ijt} \geq D_j \qquad \text{forall j in districts}$$

2. Constraint for 5M Migros

$$\sum_{i=1}^{k} n_{ij} * y_{i4} = 0 \iff y_{j4} = 1 \qquad \text{forall j in districts}$$

leads to

$$\sum_{i=1}^{10} n_{ij} * y_{i4} \leq M * (1 - y_{j4})$$

$$1 - y_{j4} \leq \sum_{i=1}^{10} n_{ij} * y_{i4}$$

3. Difference between 3M and Jet

$$| \sum_{i=1}^{10} y_{i1} - \sum_{i=1}^{10} y_{i3} | \leq K$$

leads to

$$\sum_{i=1}^{10} y_{i1} - \sum_{i=1}^{10} y_{i3} \leq K$$

$$\sum_{i=1}^{10} y_{i3} - \sum_{i=1}^{10} y_{i1} \leq K$$

4. After observing demand values Migros concludes that district 4 and 8 have similar demand behaviours.

$$(y_{4t} - y_{8t}) = 0 \qquad \text{forall t in types}$$

5. Range Constraint

$$(1 - Servability_{tij}) * x_{ijt} \leq 0 \qquad \text{forall i,j in districts, forall t in types}$$

6. Capacity Constraint

$$\sum_{j=1}^{10} x_{ijt} \leq y_{it} * cap_t + 0.1 * O_{it} * cap_t \qquad \text{forall i in districts, forall t in types}$$

7. Relating the variables

$$x_{ijt} \leq y_{it} * M \qquad \text{forall i,j in districts, forall t in types}$$

8. Relating the variables

$$y_{it} \leq O_{it} \qquad \text{forall i in districts, forall t in types}$$

Thus the model in compact form becomes:

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{4}\sum_{i=1}^{10} f_t * y_{it} + \sum_{i=1}^{10}\sum_{j=1}^{10}\sum_{t=1}^{4} x_{ijt} * dist_{ij} * c + \sum_{i=1}^{10}\sum_{t=1}^{4} OC * O_{it} \\
\text{s.t.} \quad & \sum_{i=1}^{10}\sum_{t=1}^{4} x_{ijt} \geq D_j && \forall j \in \text{districts} \\
& \sum_{i=1}^{10} n_{ij} * y_{i4} \leq M * (1 - y_{j4}) && \forall j \in \text{districts} \\
& 1 - y_{j4} \leq \sum_{i=1}^{10} n_{ij} * y_{i4} && \forall j \in \text{districts} \\
& \sum_{i=1}^{10} y_{i1} - \sum_{i=1}^{10} y_{i3} \leq K \\
& \sum_{i=1}^{10} y_{i3} - \sum_{i=1}^{10} y_{i1} \leq K \\
& y_{4t} = y_{8t} && \forall t \in \text{types} \\
& (1 - Servability_{tij}) * x_{ijt} \leq 0 && \forall i, j \in \text{districts}\ , \forall t \in \text{types} \\
& \sum_{j=1}^{10} x_{ijt} \leq y_{it} * cap_t + 0.1 * O_{it} * cap_t && \forall i \in \text{districts}\ , \forall t \in \text{types} \\
& x_{ijt} \leq y_{it} * M && \forall i, j \in \text{districts}\ , \forall t \in \text{types} \\
& y_{it} \leq O_{it} && \forall i \in \text{districts}\ , \forall t \in \text{types} \\
& x_{ijt} \geq 0 && \forall i, j \in \text{districts}\ , \forall t \in \text{types} \\
& y_{it} \in \{0, 1\} && \forall i \in \text{districts}\ , \forall t \in \text{types} \\
& O_{it} \in \{0, 1\} && \forall i \in \text{districts}\ , \forall t \in \text{types}
\end{aligned}
$$

PART B

Parameters

- districts = $\{1..10\}$
- $Served_i = \begin{cases} 1 & \text{if district } i \text{ has a store according to the results of part(a)} \\ 0 & \text{if not} \end{cases}$
- $Dist_{ij}$: Distance between district i and warehouse j

Decision Variables

- t : The maximum distance a truck has to travel in a single journey
- $x_{ij} = \begin{cases} 1 & \text{if district j is being served by warehouse in district i} \\ 0 & \text{otherwise} \end{cases}$
- $y_i = \begin{cases} 1 & \text{if a warehouse is opened in district i} \\ 0 & \text{otherwise} \end{cases}$

Objective Function

$$\textbf{minimise} \qquad t$$

s.t

1. min of max constraint

$$t \geq dist_{ij} * x_{ij} \qquad \text{forall i,j in districts}$$

2. 4 warehouses should be opened

$$\sum_{i=1}^{10} y_i = 4$$

3. Districts with stores should be served with a single warehouse

$$\sum_{i=1}^{10} x_{ij} = Served_j \qquad \text{forall j in districts}$$

4. Relating variables

$$y_i \geq x_{ij} \qquad \text{forall i,j in districts}$$

Thus, the model in compact form becomes:

$$
\begin{array}{lll}
\min & t & \\
\text{s.t.} & t \geq dist_{ij} * x_{ij} & \forall i, j \in \text{districts} \\
& \displaystyle\sum_{i=1}^{10} y_i = 4 & \\
& \displaystyle\sum_{i=1}^{10} x_{ij} = Served_j & \forall j \in \text{districts} \\
& y_i \geq x_{ij} & \forall i, j \in \text{districts} \\
& x_{ij} \in \{0, 1\} & \forall i, j \in \text{districts} \\
& y_i \in \{0, 1\} & \forall i \in \text{districts}
\end{array}
$$

PART C

Parameter Additions to Part A

- ST : Given threshold

Decision Variable Additions to Part A

- $z_{ijt} = \begin{cases} 1 & \text{if store that has type t is opened in district i serves j} \\ 0 & \text{otherwise} \end{cases}$

Constraint Additions to Part A

1. Relating variables

$$x_{ijt} \leq M * z_{ijt} \qquad \text{forall i,j in districts, forall t in types}$$

2. The served districts cannot exceed treshhold

$$\sum_{i=1}^{10} z_{ijt} \leq ST \qquad \text{forall j in districts, forall t in types}$$

3. Binary variable

$$z_{ijt} \in \{0, 1\} \qquad \text{forall i,j in districts, forall t in types}$$

Thus, the model in compact form becomes:

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{4}\sum_{i=1}^{10} f_t * y_{it} + \sum_{i=1}^{10}\sum_{j=1}^{10}\sum_{t=1}^{4} x_{ijt} * dist_{ij} * c + \sum_{i=1}^{10}\sum_{t=1}^{4} OC * O_{it} & \\
\text{s.t.} \quad & \sum_{i=1}^{10}\sum_{t=1}^{4} x_{ijt} \geq D_j & \forall j \in \text{districts} \\
& \sum_{i=1}^{10} n_{ij} * y_{i4} \leq M * (1 - y_{j4}) & \forall j \in \text{districts} \\
& 1 - y_{j4} \leq \sum_{i=1}^{10} n_{ij} * y_{i4} & \forall j \in \text{districts} \\
& \sum_{i=1}^{10} y_{i1} - \sum_{i=1}^{10} y_{i3} \leq K & \\
& \sum_{i=1}^{10} y_{i3} - \sum_{i=1}^{10} y_{i1} \leq K & \\
& y_{4t} = y_{8t} & \forall t \in \text{types} \\
& (1 - Servability_{tij}) * x_{ijt} \leq 0 & \forall i,j \in \text{districts}, \forall t \in \text{types} \\
& \sum_{j=1}^{10} x_{ijt} \leq y_{it} * cap_t + 0.1 * O_{it} * cap_t & \forall i \in \text{districts}, \forall t \in \text{types} \\
& x_{ijt} \leq y_{it} * M & \forall i,j \in \text{districts}, \forall t \in \text{types} \\
& y_{it} \leq O_{it} & \forall i \in \text{districts}, \forall t \in \text{types} \\
& x_{ijt} \leq M * z_{ijt} & \forall i,j \in \text{districts}, \forall t \in \text{types} \\
& \sum_{i=1}^{10} z_{ijt} \leq ST & \forall j \in \text{districts}, \forall t \in \text{types} \\
& x_{ijt} \geq 0 & \forall i,j \in \text{districts}, \forall t \in \text{types} \\
& y_{it} \in \{0,1\} & \forall i \in \text{districts}, \forall t \in \text{types} \\
& O_{it} \in \{0,1\} & \forall i \in \text{districts}, \forall t \in \text{types} \\
& z_{ijt} \in \{0,1\} & \forall i,j \in \text{districts}, \forall t \in \text{types}
\end{aligned}
$$

DATA

- If districts are closer than 140 they are considered neighbours
- Travel cost = 0.1
- Overcapacity cost = 60
- Max difference between 3M and Jet = 2
- The service threshold for part(c) = 1
- Demand :

$$\begin{bmatrix} 483 & 426 & 736 & 790 & 894 & 524 & 386 & 784 & 302 & 662 \end{bmatrix}$$

- Distances :

$$\begin{bmatrix} 0 & 102 & 934 & 948 & 136 & 757 & 522 & 596 & 79 & 849 \\ 102 & 0 & 427 & 36 & 144 & 483 & 80 & 803 & 740 & 718 \\ 934 & 427 & 0 & 874 & 439 & 176 & 70 & 540 & 925 & 22 \\ 948 & 36 & 874 & 0 & 901 & 475 & 246 & 807 & 327 & 901 \\ 136 & 144 & 439 & 901 & 0 & 830 & 985 & 812 & 976 & 561 \\ 757 & 483 & 176 & 475 & 830 & 0 & 144 & 945 & 567 & 96 \\ 522 & 80 & 70 & 246 & 985 & 144 & 0 & 847 & 76 & 757 \\ 596 & 803 & 540 & 807 & 812 & 945 & 847 & 0 & 807 & 729 \\ 79 & 740 & 925 & 327 & 976 & 567 & 76 & 807 & 0 & 684 \\ 849 & 718 & 22 & 901 & 561 & 96 & 757 & 729 & 684 & 0 \end{bmatrix}$$

- Fixed Cost :

$$\begin{bmatrix} 8000 & 16000 & 45000 & 94000 \end{bmatrix}$$

- Capacity :

$$\begin{bmatrix} 300 & 400 & 500 & 600 \end{bmatrix}$$

- Range :

$$\begin{bmatrix} 150 & 250 & 400 & 600 \end{bmatrix}$$