

# IE303 Modeling and Optimization Project 2

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# 1 Question 1

## 1.1 Model

### Description of Model Respectively:

1. The objective is redundant.
2. The A+ donor is next to the B+ donor.
3. Brooke is at one of the ends.
4. The woman wearing a black shirt is somewhere to the left of the 150 lb woman.
5. The actress is next to the chef.
6. Kathleen is 40 years old.
7. The orist is somewhere to the right of the woman wearing the purple shirt.
8. The oldest donor weights 130 lb.
9. Brooke is next to Nichole.
10. The 35 years old woman is exactly to the left of the 30 years old woman.
11. The 120 lb donor is somewhere between the 0- donor and the 150 lb donor, in that order.
12. Kathleen is at one of the ends.
13. The woman wearing the purple shirt is somewhere to the right of the woman wearing the green shirt.
14. The B+ donor weights 140 lb.
15. The youngest woman is next to the 30 years old woman.
16. The woman considered the universal recipient is exactly to the left of the A+ donor.
17. Meghan is somewhere to the right of the woman wearing the purple shirt.
18. The woman wearing the green shirt is somewhere between the actress and the woman wearing the red shirt, in that order.
19. At one of the ends is the 130 lb woman.
20. The universal donor is 35 years old.
21. The orist is somewhere between the actress and the engineer, in that order.
22. The woman wearing a blue shirt is somewhere to the left of the woman wearing a red shirt.
23. The AB+ donor is next to the youngest woman.
24. Every feature needed to be assigned. In other words, summation of every row and column of  $X_{jt}$  is 1 for each  $i$ .

Given table of features:

(i,j)	1	2	3	4	5
1	black	blue	green	purple	red
2	Andrea	Brooke	Kathleen	Meghan	Nichole
3	A+	AB+	B+	B-	0-
4	25	30	35	40	45
5	120	130	140	150	160
6	Actress	Chef	Engineer	Florist	Policewoman

Decision Variables:

$$I = \{1, ..6\}, J = \{1, ..5\}, T = \{1, ..5\}$$

$$X_{ijt} = \begin{cases} 1 & \text{If feature on (i,j) is assigned to (i,t), } i \in \{1, ..6\}, j, t \in \{1, ..5\} \\ 0 & \text{otherwise} \end{cases}$$

Model:

$$\min \sum_{i=1}^6 \sum_{j=1}^5 \sum_{t=1}^5 X_{ijt} \quad (1)$$

subject to

$$X_{31t} \leq X_{33(t+1)} + X_{33(t-1)}, \quad t \in T \quad (2)$$

$$X_{221} + X_{225} = 1 \quad (3)$$

$$X_{11t} \leq X_{54(t+1)} + X_{54(t+2)} + X_{54(t+3)} + X_{54(t+4)}, \quad t \in T \quad (4)$$

$$X_{61t} \leq X_{62(t-1)} + X_{62(t+1)}, \quad t \in T \quad (5)$$

$$X_{23t} = X_{44t}, \quad t \in T \quad (6)$$

$$X_{14t} \leq X_{64(t+1)} + X_{64(t+2)} + X_{64(t+3)} + X_{64(t+4)}, \quad t \in T \quad (7)$$

$$X_{52t} = X_{45t}, \quad t \in T \quad (8)$$

$$X_{22t} \leq X_{25(t-1)} + X_{25(t+1)}, \quad t \in T \quad (9)$$

$$X_{43t} \leq X_{42(t+1)}, \quad t \in T \quad (10)$$

$$X_{511} + X_{515} = 0 \quad (11)$$

$$X_{35t} \leq X_{51(t+1)} + X_{51(t+2)} + X_{51(t+3)}, \quad t \in T$$

$$X_{51t} \leq X_{54(t+1)} + X_{54(t+2)} + x_{54(t+3)}, \quad t \in T$$

$$X_{231} + X_{235} = 1 \quad (12)$$

$$X_{13t} \leq X_{14(t+1)} + X_{14(t+2)} + X_{14(t+3)} + X_{14(t+4)}, \quad t \in T \quad (13)$$

$$X_{33t} = X_{53t}, \quad t \in T \quad (14)$$

$$X_{41t} \leq X_{42(t-1)} + X_{42(t+1)}, \quad t \in T \quad (15)$$

$$X_{32t} \leq X_{31(t+1)} \quad t \in T \quad (16)$$

$$X_{14t} \leq X_{24(t+1)} + X_{24(t+2)} + X_{24(t+3)} + X_{24(t+4)}, \quad t \in T \quad (17)$$

$$X_{131} + X_{135} = 0 \quad (18)$$

$$X_{61t} \leq X_{13(t+1)} + X_{13(t+2)} + X_{13(t+3)}, \quad t \in T$$

$$X_{13t} \leq X_{15(t+1)} + X_{15(t+2)} + X_{15(t+3)}, \quad t \in T$$

$$X_{521} + X_{525} = 1 \quad (19)$$

$$X_{35t} = X_{43t}, \quad t \in T \quad (20)$$

$$X_{641} + X_{645} = 0 \quad (21)$$

$$X_{61t} \leq X_{64(t+1)} + X_{64(t+2)} + X_{64(t+3)}, \quad t \in T$$

$$X_{64t} \leq X_{63(t+1)} + X_{63(t+2)} + X_{63(t+3)}, \quad t \in T$$

$$X_{12t} = X_{15(t+1)} + X_{15(t+2)} + X_{15(t+3)} + X_{15(t+4)}, \quad t \in T \quad (22)$$

$$X_{32t} \leq X_{41(t+1)} + X_{41(t-1)}, \quad t \in T \quad (23)$$

$$\sum_{j=1}^5 X_{ijt} = 1, \quad i \in I, \quad t \in T \quad (24)$$

$$\sum_{t=1}^5 X_{ijt} = 1, \quad i \in I, \quad j \in J$$

## 1.2 Result of the model:

Since we have one variable the result is very easy to be interpreted the solution of the problem. Because of the consecutive indexed constraints we have defined t index as 1 to 10 to not having out of index problem. So our focus is the indexes between 2 to 6 in the below picture.

```
x = [[0
      1 0 0 0 0 0 0 0 0]
      [0 0 0 0 1 0 0 0 0]
      [0 0 1 0 0 0 0 0 0]
      [0 0 0 1 0 0 0 0 0]
      [0 0 0 0 0 1 0 0 0]]
[[0 0 0 1 0 0 0 0 0]
 [0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 1 0 0 0]
 [0 0 0 0 1 0 0 0 0]
 [0 0 1 0 0 0 0 0 0]]
[[0 0 0 0 1 0 0 0 0]
 [0 0 0 1 0 0 0 0 0]
 [0 0 0 0 0 1 0 0 0]
 [0 1 0 0 0 0 0 0 0]
 [0 0 1 0 0 0 0 0 0]]
[[0 0 0 1 0 0 0 0 0]
 [0 0 0 1 0 0 0 0 0]
 [0 0 1 0 0 0 0 0 0]
 [0 1 0 0 0 0 0 0 0]
 [0 0 1 0 0 0 0 0 0]]
[[0 0 0 1 0 0 0 0 0]
 [0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 1 0 0 0]
 [0 0 0 0 1 0 0 0 0]
 [0 0 1 0 0 0 0 0 0]]
[[0 1 0 0 0 0 0 0 0]
 [0 0 1 0 0 0 0 0 0]
 [0 0 0 0 0 1 0 0 0]
 [0 0 0 0 1 0 0 0 0]
 [0 0 0 1 0 0 0 0 0]]];
```

As it is seen we have 6 2x2 matrices for each feature of people respectively to the above scheme (Color of shirt, Name,...). There are 30 ones in the matrix which tells the positions of the features. For instance, black is assigned to its first place which is understood from 1st one of the matrix. Remaining work is putting the features accordingly and we have reached the following result.

Donors	Donor 1	Donor 2	Donor 3	Donor 4	Donor 5
Shirt	black	green	purple	blue	red
Name	Brooke	Nichole	Andrea	Meghan	Kathleen
Blood	B-	O-	AB+	A+	B+
Age	45	35	30	25	40
Weight	130	160	120	150	140
Job	Actress	Chef	Policewoman	Florist	Engineer

## 2 Question 2

In this part of the question, we're going to search for finding the solution by using elimination strategies and we will check whether the resulting table will match with the result we found in the first part by using modelling.

### 2.1 Finding Who Are at The Ends

Firstly, we want to find who are at the corner points in our table.

	1	2	3	4	5
Shirt-colors	black	blue	green	purple	red
Names	Andrea	Brooke	Kathleen	Meghan	Nichole
Blood-types	A+	AB+	B+	B-	O-
Ages	25	30	35	40	45
Weights	120	130	140	150	160
Jobs	Actress	Chef	Engineer	Florist	Policewoman

Now, let's write the constraints that strictly help while trying to find end ones.

2) Brooke is at one of the ends.

11) Kathleen is at one of the ends.

3) The woman wearing a black shirt is somewhere to the left of the 150 lb woman.

→ From (3) We can conclude that the woman wearing black shirt can not be at the right corner woman who is 150 lb can not be at the left corner.

6) The florist is somewhere to the right of the woman wearing the purple shirt.

→ From (6) We can conclude that the florist can not be at the left corner and woman wearing purple shirt cannot be at the right corner.

9) The 35 years old woman is exactly to the left of the 30 years old woman.

→ From (9) we can conclude that 35 years old cannot be at the right corner and 30 years old cannot be at the left corner.

10) The 120 lb donor is somewhere between the 0- donor and the 150 lb donor, in that order.

→ From (10) we can conclude that woman who is 120 lb cannot be at any corner.

0- donor cannot be at the right corner.

150-lb donor cannot be at the left corner.

12) The woman wearing the purple shirt is somewhere to the right of the woman wearing the green shirt.

→ From (12) we can conclude that woman with purple shirt cannot be at the left corner. Also, in (6), we found that woman with purple shirt cannot be at the right corner. So, woman with purple shirt cannot be at any corner. Also, it is seen that woman with green shirt cannot

be at the right corner. So, the only places for woman with purple shirt are 3 4. Also, the only places for woman with green shirt are 2 3.

15) The woman considered the universal recipient is exactly to the left of the A+ donor.  
→ From (12) we can conclude that AB+ donor cannot be at the right corner and A+ donor cannot be at the left corner.

16) Meghan is somewhere to the right of the woman wearing the purple shirt.  
→ From this constraint, it is seen that purple shirt cannot be at the right corner, but we deduced this fact from (12). So, this part is useless.  
The other inference of this constraint is Meghan cannot be at the right corner, but this one is also useless because from (2) (11), we know that Brooke and Kathleen are at the ends. So, we cannot conclude anything about the corner ones from that constraint.  
However, the interesting part is we can see that the woman wearing the purple shirt can be at position 2 3. It cannot be at position 4 because Meghan is at the right of it and we know the Brooke and Kathleen are at the ends. Then, the positions for Meghan are either 3 or 4. Now, We know from (12) that the only places for woman with purple shirt are 3 4. And now, we found that the woman wearing the purple shirt can be at position 2 3. **So, woman wearing purple shirt is at position 3.** What is more is that in (16) it is given that Meghan is somewhere to the of the woman wearing the purple shirt and we know that it cannot be at the position-5. **So, Meghan is at position 4.** Furthermore, in (12), we found that the only places for woman with green shirt are 2 3 and because the woman wearing purple shirt is at position 3, **we see that woman wearing green shirt is at position 2.**

17) The woman wearing the green shirt is somewhere between the actress and the woman wearing the red shirt, in that order.  
→ We know that the woman wearing the green shirt is at position 2. So, because the actress must be at somewhere to the left of the woman wearing the green shirt, **then the actress is at position 1.** Also, the red shirt cannot be at the left corner.

18) At one of the ends is the 130 lb woman.

20) The florist is somewhere between the actress and the engineer, in that order.  
→ It means that florist cannot be at any corner. Actress cannot be at the right corner and engineer cannot be at the left corner.

21) The woman wearing a blue shirt is somewhere to the left of the woman wearing a red shirt.  
→ We can deduce that woman with blue shirt cannot be at the right corner and woman with red shirt cannot be at the left corner.

Let's trace back the constraints;

We can see that in (6) it is given that the florist is somewhere to the right of the woman wearing the purple shirt. Later, we found that the woman wearing the purple shirt is at position 3. So, the only places for the florist are position 4 or 5.  
In (20), it is written that the florist is somewhere between the actress and the engineer, in

that order. So, the florist cannot be at the 5th position. Then, we can conclude that the florist is at position 4. Then, it is seen that the engineer is at position 5. Now, we know a lot about who cannot be at the ends in terms of shirt color. Below table shows which ones cannot be at which corner.

Left	Right
Purple	Black
Red	Purple
Green	Green
	Blue

What we found until here are;  
The only choice for the right end is woman with red shirt and only 2 choices for the left end are woman with blue/black colors.  
We know that the woman wearing the green shirt is at position 2 and the woman wearing the purple shirt is at position 3.  
We know that Meghan is at position 4.  
We found that the actress is at position 1, the florist is at position 4 and the engineer is at position 5. From (18), we know that at one of the ends is the 130 lb woman.  
Also we know that Brooke Kathleen are at ones of the ends.

So, for now, the table is;

	1	2	3	4	5
Shirt-colors	black/blue	green	purple	—	red
Names	Brooke/Kathleen			Meghan	Brooke/Kathleen
Blood-types					
Ages					
Weights	130/				130/
Jobs	actress			florist	engineer

## 2.2 Matching Constraints

Now, we're going to check the matching constraints.

5) Kathleen is 40 years old.

7) The oldest donor weights 130 lb.

→ From (18), we know that at one of the ends is the 130 lb woman. So, oldest donor is also at the end. More importantly, we know from (5) that Kathleen is 40 years old and it is not the oldest age. We know that the other woman at the corner is Brooke; so, Brooke is the oldest donor and he weights 130 lb.



13) The B+ donor weighs 140 lb.

19) The universal donor(0-) is 35 years old.

→ We know from (10) that 0- donor cannot be at the right corner; so, 35 years old cannot be at the right corner. We know the ages of the end ones differ from 35 years old. So, it can be at position 2,3 or 4.

Now, let's create the table again with new information;

	1	2	3	4	5
Shirt-colors	black/blue	green	purple		red
Names	Brooke/Kathleen			Meghan	Brooke/Kathleen
Blood-types					
Ages	45/40				45/40
Weights	130/				130/
Jobs	actress			florist	engineer

### 2.3 Next-to Constraints

Now, let's check the constraints who give information about who is next-to whom.

1) The A+ donor is next to the B+ donor.

4) The actress is next to the chef.

→ It means that the chef is at position 2. The remaining job is policewoman;so, policewoman is at position 3.

8) Brooke is next to Nichole.

→ We know that Megan is at position 4. So, Nichole must be in position 2 and it means that Brooke is at position 1 and Kathleen is at position 5. The remaining name is Andrea; so, Andrea is at position 3.

14) The youngest woman is next to the 30 years old woman.

22) The AB+ donor is next to the youngest woman.

→ From (14) (22), we can conclude that the youngest woman is not AB+.

Now, the table becomes;

	1	2	3	4	5
Shirt-colors	black/blue	green	purple		red
Names	Brooke	Nichole	Andrea	Meghan	Kathleen
Blood-types					
Ages	45				40
Weights	130				
Jobs	actress	chef	policewoman	florist	engineer

## 2.4 Trace-backing the Constraints

Now, we will trace-back the constraints to find the relevant parts.

Write down the constraints that are helpful to find the remaining places;

- 1) The  $A+$  donor is next to the  $B+$  donor.
- 3) The woman wearing a black shirt is somewhere to the left of the 150 lb woman.
- 9) The 35 years old woman is exactly to the left of the 30 years old woman.
- 10) The 120 lb donor is somewhere between the 0- donor and the 150 lb donor, in that order.
- 13) The  $B+$  donor weights 140 lb.
- 14) The youngest woman is next to the 30 years old woman.
- 15) The woman considered the universal recipient is exactly to the left of the  $A+$  donor.
- 19) The universal donor is 35 years old.
- 22) The  $AB+$  donor is next to the youngest woman.

Now, let's make some inferences.

By checking (14), we see that the 25 years old woman is next to the 30 years old woman. Then, we can say that 35 years old woman cannot be at the position 3. So, the only available places for the 35 years old woman are position 2 4. Also, by from (9), we see that 35 years old woman is exactly to the left of the 30 years old woman. Then, **the 35 years old woman must be in position 2. We also know that 35 years old woman is 0- from (19).**

Now, in (22) it written that the  $AB+$  donor is next to the youngest woman. So, the  $AB+$  donor cannot be at the position 1. From (13) we know that the  $B+$  donor weights 140 lb, then it also cannot be at position 1. In (1), it is given that the  $A+$  donor is next to the  $B+$  donor. So,  $A+$  cannot be at position 1 because we know that 0- is in position 2. **Because the remaining blood type is  $B-$ , it is seen that  $B-$  donor is at place 1.** Now, the table becomes,

	1	2	3	4	5
Shirt-colors	black/blue	green	purple		red
Names	Brooke	Nichole	Andrea	Meghan	Kathleen
Blood-types	$B-$	0-			
Ages	45	35			40
Weights	130				
Jobs	actress	chef	policewoman	florist	engineer

From (1), we know that the  $A+$  donor is next to the  $B+$  donor. So,  $AB+$  donor cannot be at the place 4. In (22), it is given that the  $AB+$  donor is next to the youngest woman. So, the youngest woman(25 years-old) must be at the position 2 or 4. It cannot not be at the position 2 because there is the 35 years-old donor there, **then the youngest donor is at the position 4.** Now, the remaining position for the age is position 3 and the remaining age is 30. **So, the 30 years-old donor is at the position 3.**

	1	2	3	4	5
Shirt-colors	black/blue	green	purple		red
Names	Brooke	Nichole	Andrea	Meghan	Kathleen
Blood-types	$B-$	$O-$			
Ages	45	35	30	25	40
Weights	130				
Jobs	actress	chef	policewoman	florist	engineer

Now, from (22) we know that the  $AB+$  donor is next to the youngest woman. So, it is either in the position 3 or 4. Also by checking (1), we can see that the  $A+$  donor is next to the  $B+$  donor. Then  $AB+$  donor cannot be at the position 4. **Then  $AB+$  donor is at position 3 for sure.**

By checking (10), we see that the 120 lb donor is somewhere between the  $O-$  donor and the 150 lb donor, in that order. So, 2nd donor cannot be either 120 lb or 150 lb. Also in (13) it is written that the  $B+$  donor weights 140 lb. **So, 160 lb is at the position 2.**

The table becomes;

	1	2	3	4	5
Shirt-colors	black/blue	green	purple		red
Names	Brooke	Nichole	Andrea	Meghan	Kathleen
Blood-types	$B-$	$O-$	$AB+$		
Ages	45	35	30	25	40
Weights	130	160			
Jobs	actress	chef	policewoman	florist	engineer

In (13), it is written that the  $B+$  donor weights 140 lb. So, it is either position 4 or 5. Also, in (10) it is given that the 120 lb donor is somewhere between the  $O-$  donor and the 150 lb donor, in that order. So, 120 lb donor is either at the position 3 or 4 and the 150 lb donor is at the position 4 or 5. Then, it is seen that **the 120 lb donor is at position 3.**

In (15), it is written that the woman considered the universal recipient is exactly to the left of the  $A+$  donor. Then, it is seen that **the  $A+$  donor is at the position 4. Then,  $B+$  donor is at position 5.** In (13), it is given that the  $B+$  donor weights 140 lb. So, **140 lb donor is at the position 5 and 150 lb donor is at the position 4.**

Finally, as we know from (3) that the woman wearing a black shirt is somewhere to the left of the 150 lb woman, **woman wearing black shirt is at position 1.**

The final table is;

	1	2	3	4	5
Shirt-colors	black	green	purple	blue	red
Names	Brooke	Nichole	Andrea	Meghan	Kathleen
Blood-types	$B-$	$O-$	$AB+$	$A+$	$B+$
Ages	45	35	30	25	40
Weights	130	160	120	150	140
Jobs	actress	chef	policewoman	florist	engineer

This table is exactly same with the one we found in part (a).