

Empirical shape function of limit-order books in the Chinese stock market

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Abstract

We have analyzed the statistical probabilities of limit-order book (LOB) shape through building the book using the ultra-high-frequency data from 23 liquid stocks traded on the Shenzhen Stock Exchange in 2003. We find that the averaged LOB shape has a maximum away from the same best price for both buy and sell LOBs. The LOB shape function has nice exponential form in the right tail. The buy LOB is found to be abnormally thicker for the price levels close to the same best although there are much more sell orders on the book. We also find that the LOB shape functions for both buy and sell sides have periodic peaks with a period of five. The 1-min averaged volumes at fixed tick level follow lognormal distributions, except for the left tails which display power-law behaviors, and exhibit long memory. Academic implications of our empirical results are also discussed briefly.

Key words: Econophysics; Stock markets; Continuous double action; Limit-order book shape; Microstructure theory

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1 Introduction

In an order-driven market, limit-order book (LOB) is a queue of orders waiting to be executed and it is the base of continuous double auction mechanism. Orders in the book are sorted according to *price-time priority*. The construction of LOB is a dynamic process. Effective limit orders whose prices do not penetrate the opposite best price are stored in the book, while an effective market order with the price penetrating the opposite best immediately causes a transaction and removes the corresponding orders in the opposite book. In addition, cancelations can also remove the orders in the LOB.

The price levels in the limit-order book are discrete. The difference between two adjacent price levels is the tick size u . It is 0.01 RMB for all stocks in the Chinese market. The price level Δ at any given time t can be defined as follows

$$\Delta = \begin{cases} (p_b - p)/u + 1 & \text{for buy orders} \\ (p - p_a)/u + 1 & \text{for sell orders,} \end{cases} \quad (1)$$

where p is an allowed price in the LOB and p_b and p_a are the best bid and best ask, respectively. According to the definition, $\Delta = 1$ stands for the position at the best bid (ask) in the buy (sell) LOB. Denote $V_b(\Delta, t)$ (respectively $V_s(\Delta, t)$) as the volume at level Δ in the buy (respectively sell) LOB at event time t . $V_b(\Delta, t)$ and $V_s(\Delta, t)$ can be viewed as the instant LOB shape functions on the buy and sell sides, respectively.

The LOB shape function is of crucial importance in the research of market microstructure theory of order-driven markets. A brief discussion is in order. The shape of the LOB affects a trader's strategy and thus influences order aggressiveness [1]. Second, the LOB shape determines the virtual price impact. The price impact $I(\omega)$ of a virtual market order of size ω can be determined as follows [2, 3, 4]

$$I(\omega) = u \times \sup \left\{ n : \sum_{\Delta=1}^n V(\Delta, t) \leq \omega \right\}. \quad (2)$$

It is found that the virtual price impact is much stronger than the actual impact [4] and large price fluctuations are not necessarily caused by large orders but rather the liquidity [5, 6]. It is rational that a large trader prefers to split his large order and submit when the opposite LOB is thick such that the price does not change much. In contrast, an impatient small trader might submit an small order when the opposite LOB is thin for small Δ 's, since usually he does not have ensuing orders. The optimal trading strategy of a large order also depends on the average LOB shape [7, 8], which could be improved if one considers the instant LOB shape function rather than the average.

When we want to investigate the aforementioned topics analytically, the LOB shape function is usually treated as continuous. In the derivation of an optimal execution strategy, many unrealistic LOB shape functions have been proposed [7, 8]. This makes the framework less useful in practice and calls for a realistic shape function. Indeed, the empirical LOB shape function has been investigated in different stock markets. Bouchaud *et al.* found that the LOB shape of individual liquid stocks on the Paris Bourse (February 2001) is symmetrical for buys and sells and has a maximum away from the current bid (ask) [9]. They also found that the distribution of order size at the bid (or ask) can be fitted by a gamma distribution [9]. Potters and Bouchaud investigated three stocks traded on the Nasdaq Stock Market and found that all the LOB shape functions are buy/sell symmetric and only one stock reaches a maximum before relaxation [10]. Similar results on the shape function are also reported using other market data [2, 3, 4, 11].

In this paper, we shall study in detail the LOB shape of 23 liquid stocks traded on the Shenzhen Stock Exchange (SZSE) in China. The rest of the paper is organized as follows. In Section 2, we describe briefly the database we adopt. Section 3 introduces the average shape of buy and sell LOBs. We then discuss in Section 4 the probability distributions and time dependency of volumes at the first three best. The last section concludes.

2 Data sets

The Chinese stock market is a pure order-driven market where orders are matched resulting in transactions. Our data contain ultra-high-frequency data of 23 liquid stocks listed on the Shenzhen Stock Exchange in 2003 [12]. We find that the results for different stocks are qualitatively similar. Hence we will present the results for a very liquid stock. In 2003, only limit orders were allowed to submit and the market constituted opening call auction, cooling period and continuous double auction. We focus on the LOB in continuous double auction.

As an example, our presentation is based on the order flow data for a stock named Shenzhen Development Bank Co., LTD (code 000001), whose time stamps are accurate to 0.01 second including details of every event, with the information containing date, order size, limit price, time, best bid, best ask, transaction volume, and aggressiveness identifier (which identifies whether a record is a buy order, a sell order, or a cancelation). The database totally records 3,925,832 events, including 1,718,156 buy orders, 1,595,961 sell orders, 598,750 cancelations and 12,965 invalid orders. Using this nice database, we rebuild the LOB according to the trading rules [13] and study the statistical probabilities of LOB shape.

3 Averaged shape

In the continuous double auction mechanism, order placement adds volume to the book, while order cancelation or transaction removes volume from the book. It is clear that these three types of events (order placement, order cancelation and transaction) can change the shape of the LOB. In what follows we use event time, not clock time. In this way, the event time t advances by 1 when an event occurs. At every time t , we have an instant LOB shape $V_{b,s}(\Delta, t)$ on each side (buy or sell). The averaged shape of the buy (sell) LOB can be calculated as follows

$$V_{b,s}(\Delta) = \frac{1}{M} \sum_{t=1}^M V_{b,s}(\Delta, t), \quad (3)$$

where M is the number of total events in 2003 for the stock we analyzed.

It is known that traders tend to place their orders on the same best price [9, 10, 14, 15, 16]. On the other hand, the orders placed near the same best have a higher execution probability, and impatient traders are likely to make a cancelation when these orders are not executed immediately. It is thus not clear what is the LOB shape under these opposite forces. Fig. 1 shows the shapes of buy and sell LOBs.

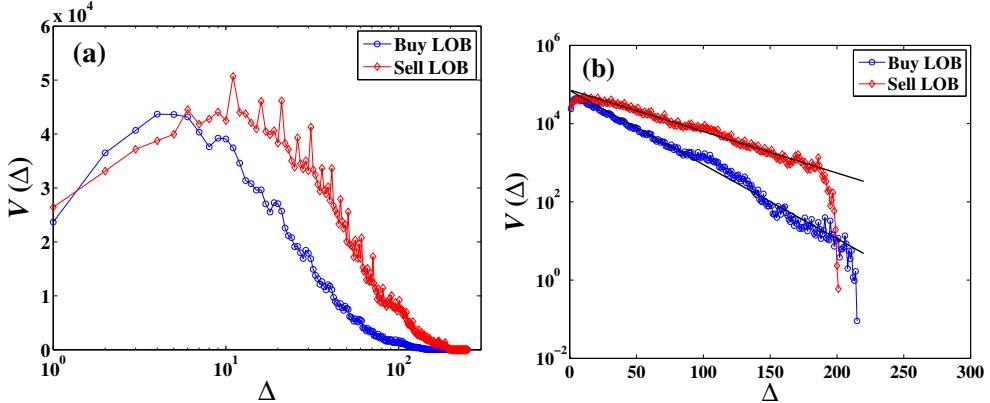


Fig. 1. LOB shape $V(\Delta)$ as a function of relative distance Δ for buy and sell limit-order books in log-linear coordinates (a) and linear-log coordinates (b).

In Fig. 1(a), we in general find that the LOB shape function has a maximum away from the same best ($\Delta = 1$) and is roughly symmetrical to the maximum, which consists with the result of Bouchaud *et al.* [9]. The LOB shapes are asymmetric between buy orders and sell orders. The LOB shape $V(\Delta)$ increases when $\Delta \leq \Delta_{\max}$ and decreases afterwards, where $\Delta_{\max} = 4$ for buys and $\Delta_{\max} = 11$ for sells. We note that only two (000088 and 000539) of the 23 stocks do not have clear maxima and the values of Δ_{\max} vary from stock to stock. In addition, the total volume of sell orders is greater than that of buy orders, which is especially visible for large Δ . This phenomenon is also observed for other stocks except that two stocks (000088 and 000089) have comparable buy and sell volumes, which is consistent with the

fact that the Chinese stock market in 2003 was in the middle of a long-lasting bearish antibubble from 2001 to 2005 [17] and more market participants tended to sell their shares.

There are two more features arise in the empirical LOB shape function. Although there are more sell limit orders in the book, the buy LOB is still thicker than sell LOB for small Δ in Fig. 1(a). In 2003, only the information on the first three visible levels ($\Delta = 1, n_2$, and n_3 such that the instant LOB shape function $V(n_1) \neq 0$, $V(n_2) \neq 0$ and $V(\Delta) = 0$ for other relative distances less than n_3) were disposed to traders. We find that, 10 stocks have thicker sell books, 10 stocks have thicker buy books, and the other three have comparable book thickness. This observation is very interesting since the traders faced a very strong illusionary signal that there were more buy orders while the market was bearish. Another interesting feature is the presence of periodic peaks at $\Delta = 5n + 1$ for $n = 0, 1, 2, \dots$, which are observed in all 23 stocks. The periodic peaks are higher for sell orders than buy orders. The underlying mechanism of this universal behavior is unclear, which might be related to the trading strategy of larger traders or people's irrational preference of some numbers like 5, 10 or their multiples [18]. These two features call for further investigation, which is however beyond the scope of this work.

In Fig. 1(b), we show the shape functions in linear-log coordinates to study the functional form for large Δ . The volumes in both buy and sell LOBs decrease exponentially,

$$V_{b,s}(\Delta) \sim e^{-\beta_{b,s}\Delta}. \quad (4)$$

Using least-squares fitting method, we obtain that $\beta_b = 0.044 \pm 0.0004$ for buy LOB and $\beta_s = 0.025 \pm 0.0002$ for sell LOB. The decreasing speed of buy LOB is faster than that of sell LOB, which means that there is a larger proportion of more aggressive orders in the buy LOB than in the sell LOB. It seems that buyers pay more attention to the execution probability, while sellers consider the return of their investigation more important. We notice that most of other stocks have similar exponentially decreasing shapes. In contrast, Bouchaud *et al.* have found that the LOB shape tails have power-law behaviors for the three liquid stocks traded on the Paris Bourse [9]. In addition, $V_{b,s}(\Delta)$ abruptly plummet to zero at the tail ends, which is caused by the 10% price fluctuation limitation compared to the close price on the previous trading day.

We have studied the event-time averaged volume placed at each tick levels in the LOB. However, the volume may have large fluctuations and greatly deviate from the mean. It is necessary to analyze the fluctuations of volumes at each tick levels. Here, we study the standard deviation σ as a function of the relative distance Δ , that is,

$$\sigma_{b,s}(\Delta) = \sqrt{\langle V_{b,s}(\Delta)^2 \rangle - \langle V_{b,s}(\Delta) \rangle^2}. \quad (5)$$

The standard deviations for buy and sell LOBs are presented in Fig. 2. We find that the functional form of $\sigma(\Delta)$ is very similar to that of the shape for both buy and

sell LOBs. The standard deviation $\sigma(\Delta)$ increases with Δ at the first few levels and then decreases exponentially. When comparing the buy and LOBs, the sell LOB is found to be thicker with larger fluctuations.

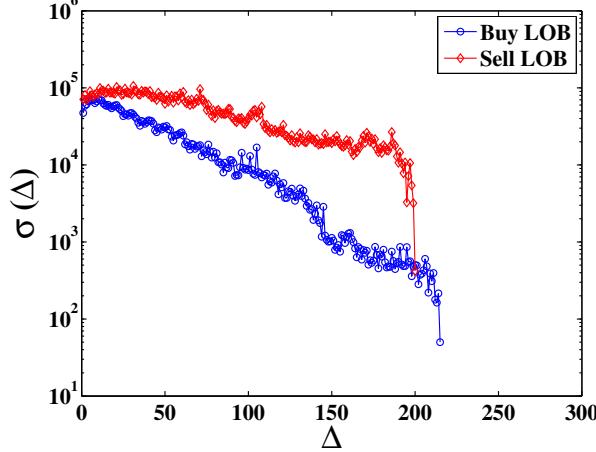


Fig. 2. Plot of the standard deviations $\sigma(\Delta)$ as a function of the relative distance Δ for buy and sell LOBs.

4 Statistical properties of volumes at individual tick levels

4.1 Probability distribution

We have analyzed the averaged volume above. Here we focus on the time averaged volume over a fixed clock time interval δt at individual levels

$$v_{b,s}(\Delta, t) = \frac{1}{N} \sum_{t=1}^N V_{b,s}(\Delta, t_i), \quad (6)$$

where t_i is the time moments of the N events occur in the interval $(t - \delta t, t]$ and N is a function of t and δt . We use $\delta t = 1$ min to calculate the time-averaged volume at each price level.

Fig. 3 shows the probability density functions (PDFs) for $\Delta = 1, 2$, and 3 . In Fig. 3 (a), we find that $\ln v$ in general is normally distributed

$$f(\ln v) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[-\frac{(\ln v - \mu)^2}{2\pi\sigma^2} \right], \quad (7)$$

that is, v is log-normally distributed with the PDF being¹

$$p(v) = f(\ln v)/v \quad (8)$$

This is also different from the Paris Bourse stocks where the volumes on the best are distributed according to a Gamma distribution [9]. With the increase of the relative distance Δ , the mean of $\ln v$, μ , increases, which is line with the result in Fig. 1. We can also project that μ decreases for large Δ . More generally, we find that the 1-min volumes at other tick levels for different stocks are basically lognormally distributed.

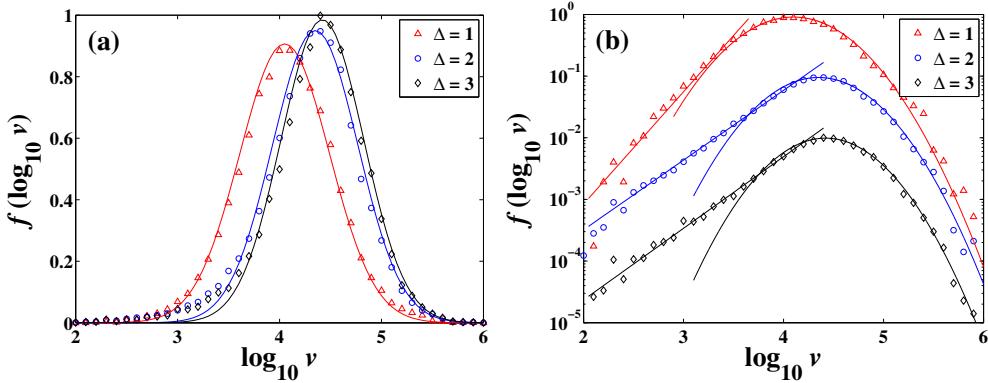


Fig. 3. Probability density functions $f(\ln v)$ of 1-min averaged logarithmic volumes at the first three tick levels on the buy LOB in a linear-linear scale (a) and linear-log scale (b). The curves corresponding to $\Delta = 2$ and $\Delta = 3$ in (b) have been vertically translated downward for clarity. The results are similar on the sell side.

When v is small, we find that the empirical curves deviate from the lognormal distribution $f(\ln v)$. We plot the probability density functions $f(\ln v)$ of $\ln v$ in a linear-log scale, which is presented in Fig. 3 (b). It is clear that the small volumes v deviate from the corresponding lognormal distributions and exhibit power-law behaviors

$$f(\ln v) \sim v^{\beta_\Delta} \text{ or } p(v) \sim v^{\beta_\Delta - 1}. \quad (9)$$

Using least-squares fitting, we obtain that $\beta_1 = 4.19 \pm 0.09$ ($2.2 < \log_{10} v < 3.5$) for $\Delta = 1$, $\beta_2 = 2.61 \pm 0.03$ ($2.1 < \log_{10} v < 4.2$) for $\Delta = 2$, and $\beta_3 = 2.67 \pm 0.05$ ($2.1 < \log_{10} v < 4.2$) for $\Delta = 3$.

4.2 Long memory

Temporal dependency can be quantitatively assessed by the autocorrelation function $C(\ell)$, which describes the average correlation between two points with time lag ℓ . Many processes have the autocorrelation function decaying exponentially

¹ Denote $g(y)$ and $h(x)$ the PDFs of y and x , respectively. If y is a function of x , we have $g(y)dy = h(x)dx$. It follows immediately that $h(x) = g(y)dy/dx = g(\ln x)/x$.

$(C(\ell) \sim e^{-\ell/\ell_0}$ for $\ell \rightarrow \infty$), which means these processes exhibit short memory with a characteristic timescale ℓ_0 . On the other hand, when the autocorrelation function is not integrable, for example, $C(\ell)$ decaying as a power-law behavior ($C(\ell) \sim \ell^{-\gamma}$), the process has long memory without any characteristic timescale, which means that the values in the past have potential predictive power for the future.

The property of temporal dependency is equivalently characterized by the Hurst index H , and the relationship between the autocorrelation exponent γ (assuming $C(\ell) \sim \ell^{-\gamma}$) and the Hurst index H can be expressed by $\gamma = 2 - 2H$ [19, 20]. Detrended fluctuation analysis (DFA) is a popular method to estimate the Hurst index [19, 21, 22]. We perform DFA on the 1-min averaged volumes at the first three tick levels on the buy LOB. The detrended fluctuation functions $F(\ell)$ are presented in Fig. 4. Sound power-law relations are observed in the three curves and the Hurst indexes are $H_1 = 0.76 \pm 0.01$ for $\Delta = 1$, $H_2 = 0.83 \pm 0.01$ for $\Delta = 2$, and $H_3 = 0.81 \pm 0.01$ for $\Delta = 3$, respectively. With the Hurst indexes H significantly larger than 0.5, we argue that the 1-min averaged volumes at the first three tick levels exhibit long memory. Quantitatively similar results are observed for the sell LOB and for other stocks. This agrees well with the fact that order signs have long memory [23, 24].

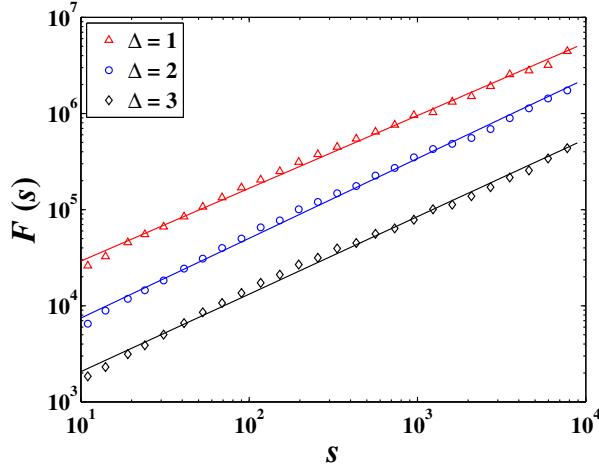


Fig. 4. Plot of the detrended fluctuation functions $F(\ell)$ of 1-min averaged volumes at the first three tick levels on the buy limit-order book. The results corresponding to $\Delta = 2$ and $\Delta = 3$ have been vertically translated downwards for clarity.

5 Conclusion

We have investigated the limit-order book shapes of 23 stocks traded on the Shenzhen Stock Exchange in the whole year 2003. For brevity, we presented the results of a very liquid stock (Shenzhen Development Bank Co., LTD, 000001). For most of the stocks, the averaged shape has a maximum away from the same best and

the volumes in the LOBs decrease exponentially. The LOB shapes are asymmetric between buy and sell orders and the sell LOB shape relaxes much slower. The probability density functions of 1-min averaged volumes at the first three tick levels follow lognormal distributions with a power-law behavior for small volumes in the left tails. Using detrended fluctuation analysis, we confirmed that the 1-min averaged volumes at a fixed tick level on the LOB exhibit long memory. When compared with the Paris Bourse stocks [9], we find that the LOB shapes are qualitative similar but quantitatively different.

Several problems arise that need to be addressed: why the buy LOB is abnormally thicker for the price levels close to the same best and why there are relatively large volume on the tick levels of $\Delta = 5n + 1$? It is also noteworthy that our results on the empirical LOB shape functions can be used to develop more realistic optimal trading strategy for large traders.

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