Experiment 15: Inclined plane

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Inhaltsverzeichnis

1 Introduction and goal

The main goal of the experiment is to examine qualitatively the acceleration of three different rolling bodies and to measure it quantitavely for two of them. We want to find an explanation based on the moment of inertia to describe the rolling bodies.
Furthermore, we are going to proof, that the conservation of energy law is fulfilled in our experimental setup.

1.1 Basics

1.1.1 Moment of inertia

According to the equivalence principle for masses, the mass of an object not only determines the gravitational force which acts upon it, but also describes the inertia of it for translational movements, in other words, how difficult it is

to move the object in one direction: $F = m \cdot a$.
The moment of inertia I plays the same role for rotational movements. A larger I implies a larger torque needed for angular acceleration ($M = I \cdot \ddot{\phi}$).
The formal definition of the moment of inertia I is:

$$I = \int r^2 dm \tag{1}$$

Here r is the distance of an infinitesimal mass element dm of the rotational axis. According to equation (1), it does not only depend on the mass, but also on how the mass is distributed on the body.
As an example, the moments of inertia of a homogenous hollow and a solid cylinder will be derived with cylindric coordinates:

$$\begin{aligned}
I_{solid} &= \int r^2 dm = \rho \int r^2 dV && (2)\\
&= \rho \int_0^h \int_0^{2\pi} \int_0^R r^3 dr d\phi && (3)\\
&= \rho 2\pi h \frac{1}{4} R^4 = \frac{1}{2} \rho \pi h R^4 && (4)\\
&= \frac{1}{2} M R^2 && (5)
\end{aligned}$$

Here, ρ is the density of the cylinder, h its height, M its mass and R its radius.
For the hollow cylinder similar:

$$\begin{aligned}
I_{hollow} &= \int r^2 dm = \rho \int r^2 dV && (6)\\
&= \rho \int_0^h \int_0^{2\pi} \int_{r_i}^R r^3 dr d\phi && (7)\\
&= \frac{1}{2} \rho \pi h (R^4 - r_i^4) && (8)
\end{aligned}$$

$$\rho = \frac{M}{\pi h (R^2 - r_i^2)} \tag{9}$$

$$\begin{aligned}
\Rightarrow I &= \frac{1}{2} M \frac{R^4 - r_i^4}{R^2 - r_i^2} && (10)\\
&= \frac{1}{2} M \frac{(R^2 - r_i^2)(R^2 + r_i^2)}{(R^2 - r_i^2)} && (11)\\
&= \frac{1}{2} M (R^2 + r_i^2) && (12)
\end{aligned}$$

Here, r_i stands for the inner radius of the cylinder.

1.1.2 Acceleration on the inclined plane

In this experiment, we want to determine the acceleration of two rolling bodies on the inclined plane.
Therefore, we are looking for a formula for the acceleration a, which only depends on measurable quantities.
The forces acting upon a rolling body on the plane are drawn in Abbildung 1. In

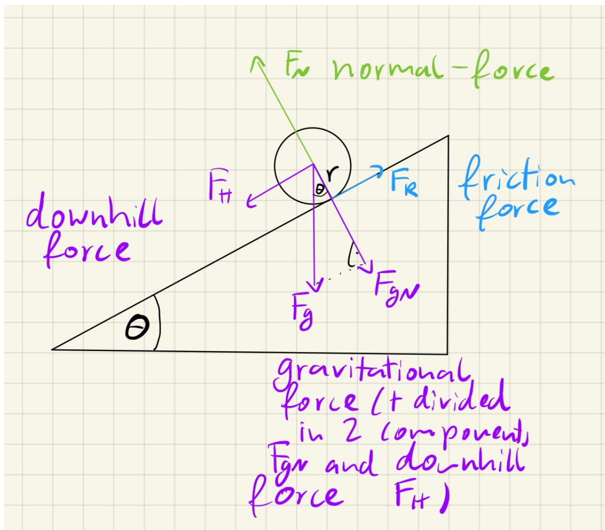

Figure 1: Forces on rolling body

order to get to this equation a force balance is going to be used. The frictional force F_r provokes a torque of:

$$M = F_r \cdot r = I \frac{d\omega}{dt} \tag{13}$$

$$= \frac{I}{r} \frac{dv}{dt} = \frac{I}{r} \cdot a \tag{14}$$

$$\Leftrightarrow F_r = \frac{I}{r^2} \cdot a \tag{15}$$

Here, I used the rolling condition $v = r\omega$, where ω is the angular speed.
The downhill force $F_H = sin\theta F_g = mgsin\theta$(produced by gravity) acts upon the center of mass of the object. The balance of our forces (according to Newtons second law) leads to:

$$ma = mgsin\theta - \frac{I}{r^2} \cdot a \tag{16}$$

$$\Leftrightarrow a = \frac{mgsin\theta}{m + \frac{I}{r^2}} \tag{17}$$

If we plug in the moments of inertia of the hollow and solid cylinder of (12) and (5), we get:

$$a_{hollow} = \frac{2gsin\theta}{3 + \frac{r_i^2}{R^2}} \tag{18}$$

$$a_{solid} = \frac{2}{3}gsin\theta \tag{19}$$

Interestingly we observe, that the radius of the solid cylinder cancels out in the formula and the acceleration is not dependent on it.

1.1.3 Law of conservation of energy

When a body rolls down the inclined plane under an energetic perspective, the initial potential height energy is converted into kinetic rotation and translation energy. So the total energy can be expressed as:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh \tag{20}$$

Here v is the translation velocity, I the moment of inertia and w the angular velocity.

2 Lab report

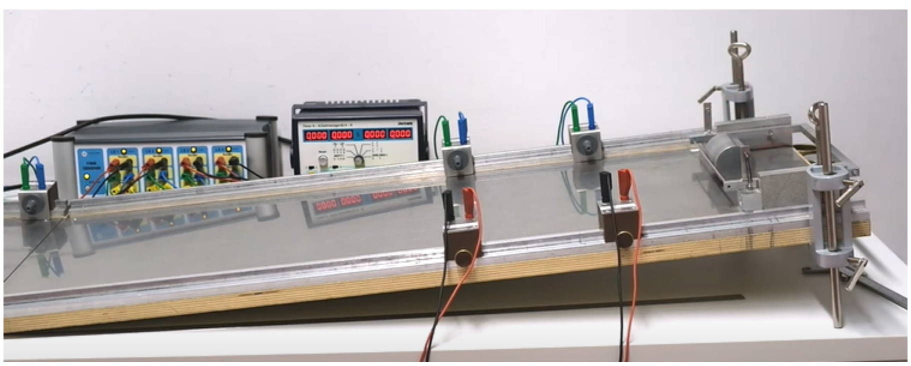

Figure 2: Inclined plane with light barriers

On todays experiment, we are going to use an inclined plane with light barriers connected to a control unit. Each light barrier corresponds to one stopwatch on the control unit. When the body starts rolling, a sensor automatically activates the stopwatches. The light barriers consist of a transmitter, an infrared LED, which sends light signals to a receiver, a photodiode, on the other side of the plane. If the body passes this barriers and therefore interrupts the light signals being send, the stopwatches are triggered and stop counting the time.

In the first step of the experiment, we measure all the relevant quantities which we will need later on in the evaluation: diameters of solid and hollow (inner and outer) cylinders with a capillar. With a spirit level we are going to check if the plane has the same inclination at all points and readjust the plane if necessary.
In the second step, we want to check, that the bodies roll uniformly accelerated on the plane and observe qualitatively the acceleration of the three different cylinders.
In the third step, the acceleration for the hollow and solid cylinder are going to be measured. Therefore, the distance of the light barriers from the starting point must be measured. Here, we measure 5 times the times for each cylinder.
In the fourth and last step, the translational speed of the bodies on the horizontal section (end of inclined plane) are going to be measured 5 times each with only 2 light barriers. Also, the height of the inclined plane is measured.

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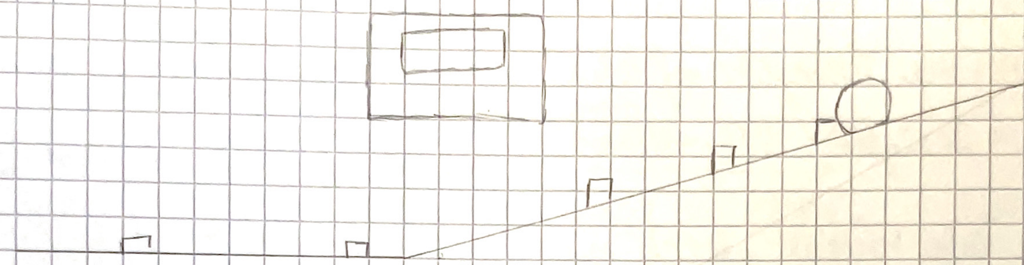
Measurement Protocol: Inclined Plane



Sketch 1: Inclined Plane

Devices used

- Height-adjustable runway
- Light barrier with control unit
- Spirit level
- Ruler
- Solid cylinder (∅ aluminium, $\rho = 2{,}70 g/cm^3$)
- Hollow cylinder (brass, $\rho = 8{,}44 g/cm^3$)
- Composite cylinder: sheath of aluminium, core of brass
- callipers
- balance

2.) We firstly check that the plain shows the same slopeness for the whole ramp with the spirit level.

We also measure the main characteristics of both cylinders used with the callipers.

Hollow: $d_i = 4{,}060 cm$ $d_a = 5{,}000 cm$ $m = 442{,}9 g$
Solid: $d = 5{,}000 cm$ $m = 444{,}2 g$

We also measure ~~the plane's length and height~~ the important values to calculate the plane's angle.

The length of the inclined part: 99,5cm

height: ~~19,4cm~~ ~~15,3cm~~ 17,3cm   height at centre of mass ~~17,5cm~~ 15,3cm

We know check qualitatively that the ~~bodys~~ bodies roll with constant acceleration, to do so, we choose quadratic distances between the light sensors:
5cm, 20cm, 45cm, 80cm, and calculate the time difference between each one, our expectations would be for every Δt to be the same and that is also what we observe, (with maximal deviations of 2ms).

Quantitative measurements:

We know put each light sensor at a distance from each other of 15cm:
(We measure the time 5 times for each)

hollow cylinder	
solid cylinder	

	t_1 [s]	t_2 [s]	t_3 [s]	t_4 [s]
hollow cylinder	0,522	0,792	0,982	1,142
	0,528	0,800	0,988	1,148
	0,529	0,797	0,988	1,147
	0,530	0,797	0,988	1,147
	0,524	0,794	0,982	1,141
solid cylinder	0,471	0,710	0,884	1,024
	0,471	0,711	0,885	1,025
	0,480	0,715	0,888	1,028
	0,477	0,713	0,887	1,026
	0,480	0,718	0,891	1,031

Table 1: acceleration measurements

We ~~know we~~ have measured the velocity of the bodies after falling when they reach the not inclined plane:

	t_1 [s]	t_2 [s]
hollow cylinder	1,522	1,647
	1,518	1,643
	1,527	1,662
	1,526	1,651
	1,521	1,647
solid cylinder	1,369	1,482
	1,363	1,477
	1,371	1,485
	1,368	1,482
	1,368	1,482

Table 2: ~~to~~ end velocity measurements

error calculation:

- Callipor: 0,005cm (scaling)
- ruler: 0,05cm for the scaling, for
  the calculation of height and length: 0,2cm
  extra for the difficulty of perfectly positioning
  the ruler:

$$\sqrt{0{,}05^2+0{,}2^2}\ \text{cm} = 0{,}21\ \text{cm}$$

- $\Delta m = 0{,}1\ g$
- time calculations: statistic and systematic
  errors.

3 Evaluation

3.1 Errors assumed in evaluation

Before I start with the evaluation of the experiment, I want to quickly explain the errors which we assumed during the experiment.

To begin with, the caliper used to measure the diameters of the cylinders, had a scaling of $0,01cm$ and therefore we are going to assume an error of $0,005cm$. The ruler, used to measure the the horizontal length and height of the plane, has a scaling error of $0,05cm$. Additionally, we want to consider an extra error of $0,2cm$ because of the difficulty of positioning the ruler exactly parallel to the lenghts of interest. Quadratic addition leads to a total error of: $\sqrt{0,05^2 + 0,2^2}cm = 0,21cm$.
However, for the distances of the light barriers on the plane, for which we also used the ruler, I am not going to assume the extra error of $0,2cm$, because on the plane was drawn the thin scale lines by pencil and therefore we were able to place the ruler in its exact position.
We also measured the masses of the cylinders with a digital scale with an error of $0,1g$.

3.2 Qualitative analysis of the rolling bodies on inclined plane

To begin with, as in later calculations we are always going to assume, the type of movement of the rolling bodies is going to be uniformly accelerated, we want to test this fact before.
If the acceleration would be constant, the distance must be proportional to the square of the time:

$$s \propto t^2 \Leftrightarrow t \propto \sqrt{s} \tag{21}$$

This would mean, if we multiply the distance by 4, the time must double, if me multiply it by 9 it must triple and so on. Therefore, we placed the light barriers in distances of $5; 20; 45; 80cm$. If the times measured in the experiment show a constant difference from each other, we can assume, that the acceleration is constant.
We made two measurements to check this condition and in both cases, the deviation of the measured time differences between two light barriers was maximal $2ms$ - which is a non significant deviation - and therefore we can assume uniformly accelerated movement for the rest of the experiment.

Furthermore, we want to qualitatively check, which of the three available cylinders (solid, hollow and composite) arrives first at the ending of the inclined part under the condition that all start with zero velocity from the same starting point.
In Abbildung 3 we can observe the different accelerations of all cylinders. The

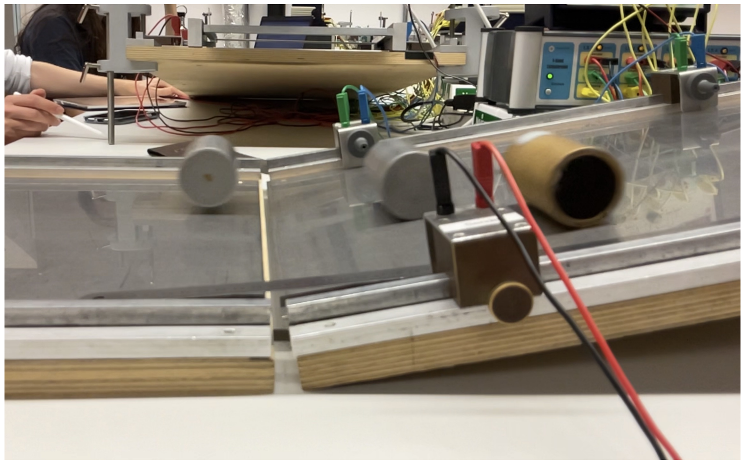


Figure 3: Race of the three cylinders

first arriving cylinder is the composite one, the solid cylinder comes next and the hollow one last.
We have also created a slow-motion video of the race of all three cylinders which can be seen under the following link: CLICK HERE.
The explanation for the order of the arriving cylinders goes as follows: The solid and hollow cylinder have very similar masses (see second section of measurement protocol) and also very similar diameters, so those factors could not have an influence on the acceleration. The main physical quantity which differs between them is the moment of inertia I. In the introduction we derived the formulas for a hollow and solid cylinder: (12) and (5). These equations imply for equal masses and outer diameters:

$$I_{solid} < I_{hollow} \tag{22}$$

Formula (17) states, that a larger moment of inertia implies a smaller acceleration. We can see the acceleration relation in formulas (18) and (19), where:

$$a_{solid} > \frac{2}{3} \cdot \frac{gsin\theta}{1 + \frac{r_i^2}{3R^2}} = a_{hollow} \tag{23}$$

because $\frac{r_i^2}{R^2}$ is always positive. Therefore, the hollow cylinder, with the larger moment of inertia is accelerated less, while the solid one is accelerated more.

3.3 Acceleration: $s - t^2$-diagram

In this section, the acceleration of the hollow- and solid cylinder will be determined using two different methods: one is a graphical approach and the other a numerical.

For the graphical approach, we want to use our observation from before, that the acceleration of the bodies is constant on the plane. If we integrate over time two times (and assume that starting velocity is zero, as in the experiment) we get

$$a = const. \tag{24}$$

$$v(t) = \int adt = at \tag{25}$$

$$s(t) = \int vdt = \int atdt = \frac{a}{2}t^2 + s_0 \tag{26}$$

for the distance of the object at a certain time and with a starting distance of s_0. We see that $s \propto t^2$. Therefore, if we plot the distance s versus t^2 we get a straight line, whith a slope of $\frac{a}{2}$ - according to equation (26). By this, we can therefore calculate the acceleration.
We made 5 measurements for all the times. For each time, the mean and the standard error of the mean is computed. Then, all the mean times are squared and for their errors according to Gauss

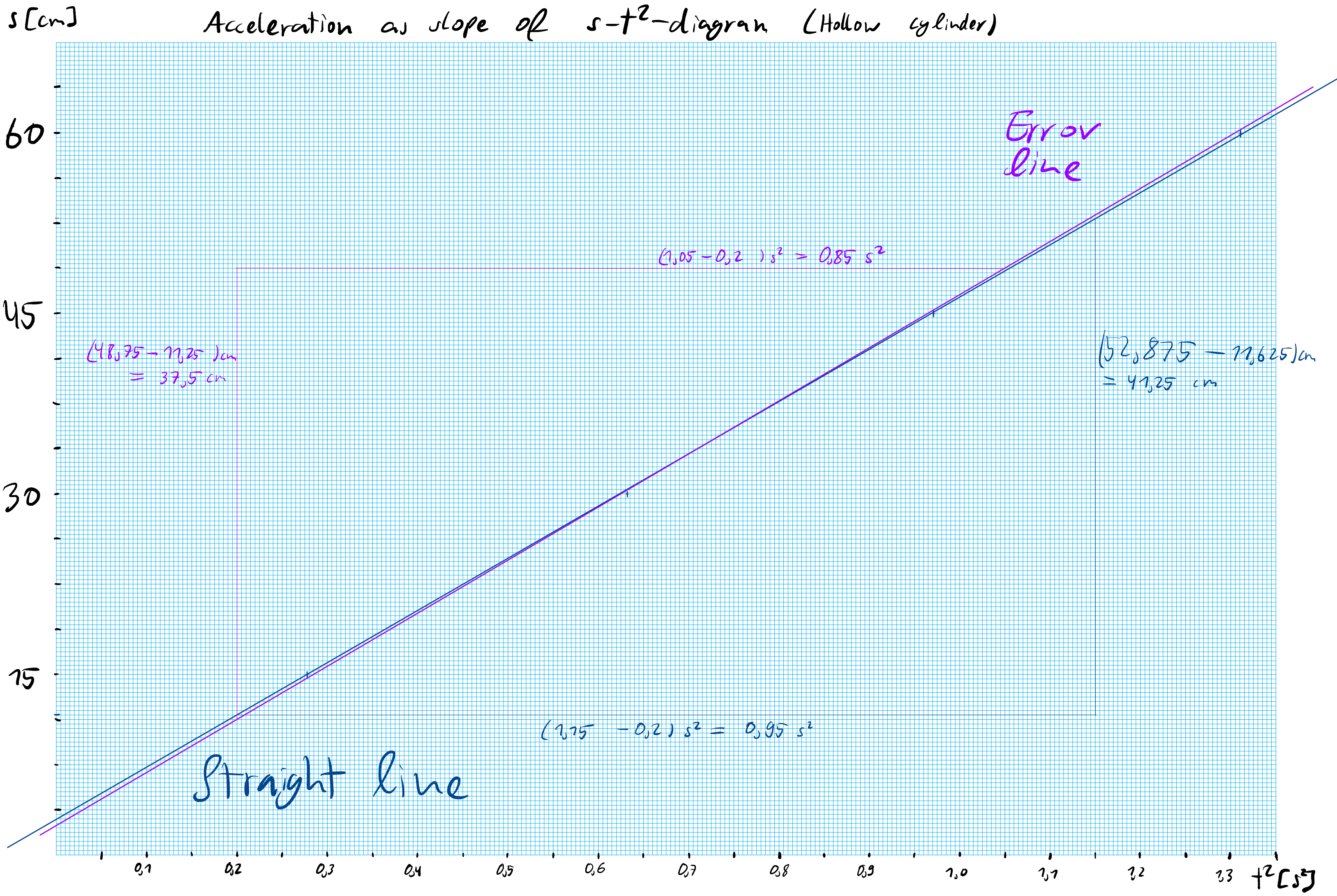
$$\triangle(t^2) = 2t\triangle t \tag{27}$$

must be used. This values are shown in Tabelle 1. The error of the ruler is

Barrier	Mean time $\bar{t}$ [s]	Error of the mean [s]	t^2 [s^2]	$\triangle(\bar{t}^2)$ [s^2]
First: s=15cm	0,53	0,00154	0,277	0,0016
Second: s=30cm	0,80	0,00138	0,63	0,0022
Third: s=45cm	0,99	0,00147	0,97	0,0029
Fourth: s=60cm	1,15	0,00145	1,31	0,003

Table 1: Hollow Cylinder values for graphic

$\triangle s = 0,05cm$ as discussed before. A straight line and a error line is interpoled between the values:



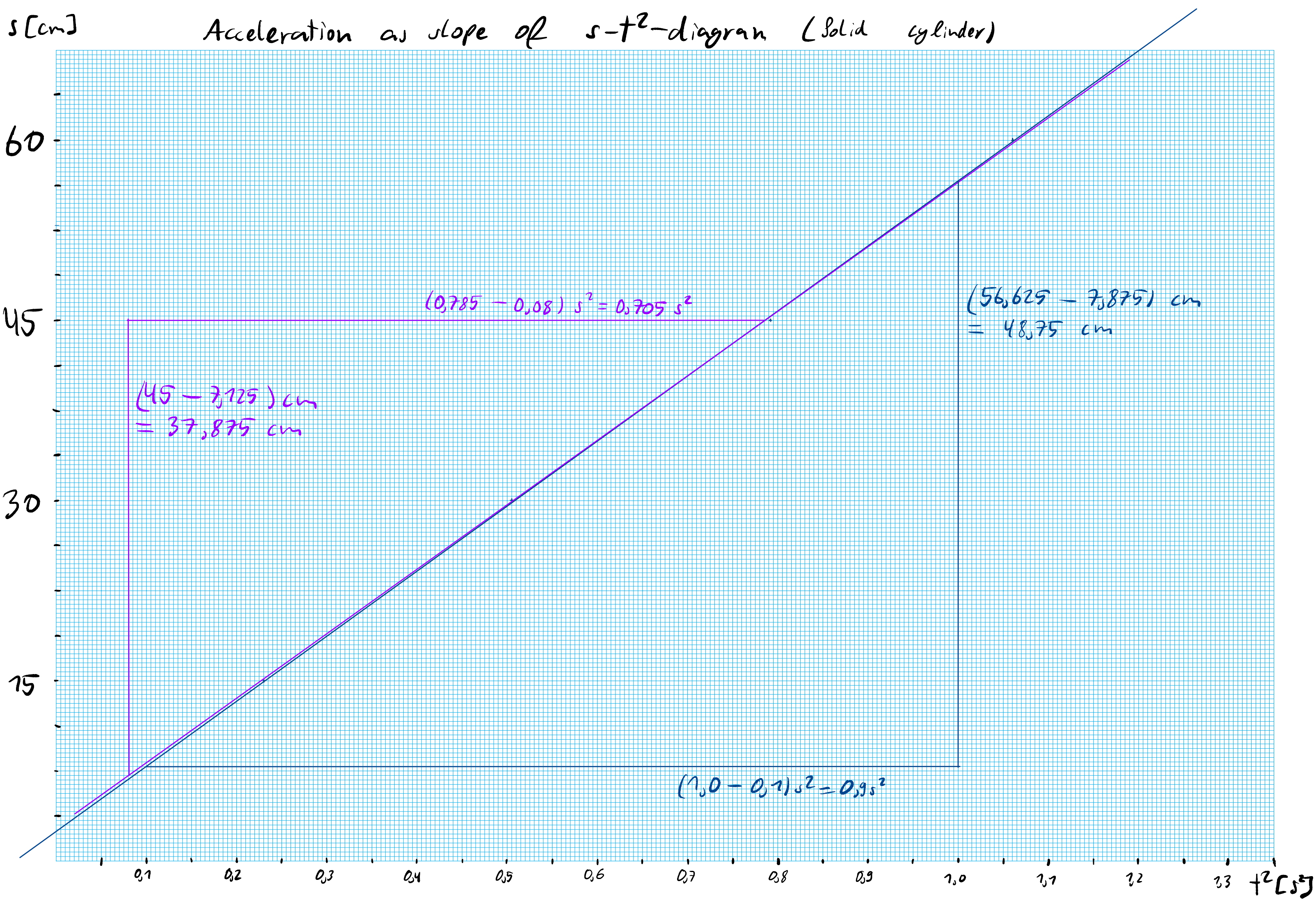
s [cm]
Acceleration as slope of s-t²-diagram (Hollow cylinder)
60
45
30
15
Error line
(1,05 − 0,2) s² = 0,85 s²
(48,75 − 11,25) cm = 37,5 cm
(52,875 − 11,625) cm = 41,25 cm
(1,15 − 0,2) s² = 0,95 s²
Straight line
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1,0
1,1
1,2
1,3
t² [s²]

The same process is repeated for the solid cylinder, for which we get the following values:

Barrier	Mean time $\bar{t}$ [s]	Error of the mean [s]	t^2 $[s^2]$	$\triangle(\bar{t}^2)$ $[s^2]$
First: s=15 cm	0,48	0,002	0,2304	0,0019
Second: s=30 cm	0,71	0,0014	0,5041	0,0020
Third: s=45 cm	0,89	0,0012	0,7921	0,0022
Fourth: s=60 cm	1,03	0,0012	1,0609	0,0026

Table 2: Solid cylinder values for graphic

Acceleration as slope of s-t^2-diagram (Solid cylinder)



For the slopes m_x of the respective straight lines, we get:

$$m_{hollow} = (43,4 \pm 0,7)\frac{cm}{s^2} \tag{28}$$

$$m_{solid} = (54,2 \pm 0,4)\frac{cm}{s^2} \tag{29}$$

For the acceleration, the slopes have to be multiplied by factor 2 according to (26).

$$\boxed{a_{hollow} = (86,8 \pm 1,4)\frac{cm}{s^2}} \tag{30}$$

$$\boxed{a_{solid} = (108,4 \pm 0,8)\frac{cm}{s^2}} \tag{31}$$

The solid cylinder has in fact, a larger acceleration than the hollow one, as discussed before.

3.4 Acceleration: Numerical method

Now, we want to use a numerical method for getting the acceleration.
In the basics we have derived the following formulas (18) and (19) for the acceleration of the hollow and the solid:

$$a_h = \frac{2gsin\theta}{3 + \frac{r_i^2}{R^2}} \tag{32}$$

$$a_s = \frac{2}{3}gsin\theta \tag{33}$$

The respective total differentials and errors - according to Gaussian error propagation - are:

$$da_h = \frac{a_h}{g}dg + \frac{a_h}{tan\theta}d\theta - \frac{2a_h r_i}{R^2(3+\frac{r_i^2}{R^2})}dr_i + \frac{2a_h r_i^2}{R^3(3+\frac{r_i^2}{R^2})}dR \tag{34}$$

$$\triangle a_h = \sqrt{\left(\frac{a_h}{g}\triangle g\right)^2 + \left(\frac{a_h}{tan\theta}\triangle\theta\right)^2 + \left(\frac{2a_h r_i}{R^2(3+\frac{r_i^2}{R^2})}\triangle r_i\right)^2 + \left(\frac{2a_h r_i^2}{R^3(3+\frac{r_i^2}{R^2})}\triangle R\right)^2} \tag{35}$$

$$da_s = \frac{a_s}{g}dg + \frac{a_s}{tan\theta}d\theta \tag{36}$$

$$\triangle a_s = \sqrt{\left(\frac{a_s}{g}\triangle g\right)^2 + \left(\frac{a_s}{tan\theta}\triangle\theta\right)^2} \tag{37}$$

In the experiment, we did not measure the angle θ, but we measured the length, $99,5cm$, and height, $17,3cm$, of the inclined plane part. So we can determine the angle with:

$$\boxed{\theta = arctan\left(\frac{17,3cm}{99,5cm}\right) = 0,1721rad = 9,86^\circ} \tag{38}$$

With an error of

$$d\theta = \frac{1}{x(1+\frac{h^2}{x^2})}dh - \frac{h}{x^2(1+\frac{h^2}{x^2})}dx \tag{39}$$

$$\triangle\theta = \sqrt{\left(\frac{\triangle h}{x(1+\frac{h^2}{x^2})}\right)^2 + \left(\frac{\triangle xh}{x^2(1+\frac{h^2}{x^2})}\right)^2} \tag{40}$$

$$= 0,0021rad = 0,12^\circ \tag{41}$$

$$\Rightarrow \theta = (0,1721 \pm 0,0021)rad = (9,86 \pm 0,12)^\circ \tag{42}$$

For the gravitational acceleration we use the value given in the internship script at page 45 for Heidelberg: $g = (9,80984 \pm 0,00002)\frac{m}{s^2}$.
The inner and outer radius of the hollow cylinder are:

$$r_i = \frac{4,060cm}{2} = (2,0300 \pm 0,0025)cm \tag{43}$$

$$R = \frac{5cm}{2} = (2,5000 \pm 0,0025)cm \tag{44}$$

The error for both comes from the error of the caliper divided by two.
Now I have calculated and listed all the necessary values for formulas (19) and (18). For the acceleration we get:

$$\boxed{a_h = (91,8 \pm 1,1)\frac{cm}{s^2}} \tag{45}$$

$$\boxed{a_s = (112,0 \pm 1,4)\frac{cm}{s^2}} \tag{46}$$

These values will be compared with the ones from the first method in the discussion.

3.5 Energy conservation

In the last step of the experiment we measured five times the times on the horizontal plane with 2 light barriers in order to get the velocity and to be able to determine the kinetic energy of the rolling bodies. We did this in order to compare it to the total potential energy of the bodies at the peak of the inclined plane at the beginning and therefore check the law of conservation of energy.

According to equations (20), (12) and (5) the kinetic energy can be written as:

$$E_{kin} = \frac{m}{2}v^2 + \frac{I}{2}w^2 = \frac{m}{2}v^2 + \frac{I}{2}\left(\frac{v}{R}\right)^2 \tag{47}$$

$$E_{kinhollow} = \frac{m}{2}v^2 + \frac{\frac{1}{2}m(R^2 + r_i^2)}{2}\left(\frac{v}{R}\right)^2 \tag{48}$$

$$= \frac{mv^2}{2} \cdot \left(\frac{3}{2} + \frac{r_i^2}{2R^2}\right) \tag{49}$$

$$E_{kinsolid} = \frac{m}{2}v^2 + \frac{\frac{1}{2}mR^2}{2}\left(\frac{v}{R}\right)^2 \tag{50}$$

$$= \frac{3}{4}mv^2 \tag{51}$$

They have errors of:

$$\triangle E_{kh} = \sqrt{\left(\frac{E_{kh}}{m}\triangle m\right)^2 + \left(\frac{2E_{kh}}{v}\triangle v\right)^2 + \left(\frac{mv^2 r_i}{2R^2}\triangle r_i\right)^2 + \left(-\frac{mv^2 r_i^2}{4R^3}\triangle R\right)^2} \tag{52}$$

$$\triangle E_{ks} = E_{ks}\sqrt{\left(\frac{1}{m}\triangle m\right)^2 + \left(\frac{2}{v}\triangle v\right)^2} \tag{53}$$

The needed values for the kinetic energy are listed in Tabelle 3: For the calcu-

	Hollow	Solid
Inner radius [cm]	2,03	-
Error [cm]	0,0025	-
Outer radius [cm]	2,5	2,5
Error [cm]	0,0025	0,0025
Mass [g]	442,9	444,2
Error [g]	0,1	0,1
$\overline{t_1}$ [s]	1,5228	1,3678
Error [s]	0,0017	0,0013
$\overline{t_2}$ [s]	1,648	1,4816
Error [s]	0,0016	0,0013
Velocity [m/s]	1,198	1,318
Error [m/s]	0,022	0,021

Table 3: All values for kinetic energy

lation of the (constant) velocity of the rolling bodies, I determined the mean of the measured times, considered the error of the mean and used:

$$v = \frac{0,15m}{\overline{t_2} - \overline{t_1}} \tag{54}$$

$$\triangle v = \frac{v}{\overline{t_2} - \overline{t_1}}\sqrt{\triangle\overline{t_1}^2 + \triangle\overline{t_2}^2} \tag{55}$$

We get:

$$\boxed{\begin{aligned} E_{kinhollow} &= (0,582 \pm 0,021)J \quad (56)\\ E_{kinsolid} &= (0,579 \pm 0,019)J \quad (57)\end{aligned}}$$

The potential energy can be easily computed with:

$$E_{pot} = mgh \tag{58}$$

$$\triangle E_{pot} = E_{pot} \cdot \sqrt{\left(\frac{\triangle m}{m}\right)^2 + \left(\frac{\triangle g}{g}\right)^2 + \left(\frac{\triangle h}{h}\right)^2} \tag{59}$$

$$\boxed{\begin{aligned} \Rightarrow E_{poth} &= (0,665 \pm 0,009)J \quad (60)\\ E_{pots} &= (0,667 \pm 0,009)J \quad (61)\end{aligned}}$$

Here h was the height of the center of mass, as the gravitational force acts upon this point.

The deviation of the computed kinetic and potential energy is going to be commented in the discussion.

4 Summary and discussion

In a nutshell, in this experiment we wanted to analyse the movement of different rolling cylinders on the inclined plane.

First, we did an examination of the acceleration type of the rolling bodies on the inclined plane. As our hypothesis was an uniformly accelerated movement we adjusted the light barriers used to measure the time at the inclined plane at quadratic distances. Measuring the time stamps of one rolling cylinder at all these barriers, we observed, that the difference in time between two light barriers was constant and therefore our hypothesis was right - the rolling bodies fullfill an uniformly accelerated type of movement.
Then, we qualitatively compared the acceleration of a solid, hollow and composite cylinder and got to the result, that the composite is accelerated the most, then the solid cylinder and last the hollow one. In our evaluation we gave an explanation for this behaviour considering the difference in the moments of inertia of the bodys.
In addition, we determined the acceleration over two different methods: On one hand we used a graphical approach, plotting the distance versus the time squared and measuring the slope of the straight line. This method delivered an acceleration of $a_{hollow} = (86,8 \pm 1,4)\frac{cm}{s^2}$ and $a_{solid} = (108,4 \pm 0,8)\frac{cm}{s^2}$. On the other hand, with a force equilibrium we derived a formula for the acceleration which was only dependent on easily measurable quantities, so we measured those in order to gain a second numerical solution: $a_h = (91,8 \pm 1,1)\frac{cm}{s^2}$ and $a_s = (112,0 \pm 1,4)\frac{cm}{s^2}$.

In the last step of the experiment, we determined the potential energy over a height measurment and the kinetic translational and rotational energy over the measurement of the constant velocity on the horizontal part after the inclination part of the plane. This lead us to $E_{kinhollow} = (0,582 \pm 0,021)J$, $E_{kinsolid} = (0,579 \pm 0,019)J$, $E_{poth} = (0,665 \pm 0,009)J$ and $E_{pots} = (0,667 \pm 0,009)J$.

In the following I am going to calculate the deviation between some of the results, discuss its meaning and suggest some changes to the experiment in order to reduce them and to make the results more precise. For the sigma deviations I am going to use:

$$\frac{|value1 - value2|}{\sqrt{error1^2 + error2^2}} \tag{62}$$

Comparing the acceleration determined with the graphical method with the acceleration of the numerical solution we find a $2,8\sigma$ deviation between both results for the hallow cylinder and a deviation of $2,2\sigma$ for the solid cylinder which is therefore under the significance threshold of 3σ. Probably the Sigma-Deviation is due to a too small consideration of the error for the numerical method, especially the angle, as it was really difficult to position the ruler correctly for the length and the height. The first graphical method however seems preciser. The reason for this is, that in the graphical solution we only had two error sources: one was the measurement of the lengths at which the light barriers were and the other one were the times measured with the light barriers. Especially the second one was very low. On the other side, for the numerical approach, we had to also consider the error of the gravitational constant, of the inclination angle and the radiuses of the objects.
In order to minimize the error of the second method we therefore would need to improve all of our measurement devices. For the graphic method, the length measurement could be further improved by painting a really precise scala (perhaps with a machine) on the inclined plane.

Regarding the energic balance, we find that there is an absolute error (difference of kinetic and potential energy) of $-0,088J$ for the solid cylinder and of $-0,083J$ for the hollow cylinder - which corresponds to a 4σ deviation for both cylinders. This big sigma-deviation probably was enhanced by the fact that the assumed errors for the energies are really low. But as it is that big and interestingly the difference between kinetic and potential energy is negative in both cases - we should take a closer look at this and try to find a systematic error.
In an ideal experimental setup, the potential energy should equal the kinetic energy because of the law of energy conservation. In this experiment however the potential energy is not only transformed into kinetic energy, but also into heat because of friction - which we have not considered in our evaluation. This was probably especially relevant at the intersection point of the horizontal and the inclined part of the plane, because as they were two separate pieces, we could not avoid a little gap between both. Especially when the cylinders hit

this gap, energy is also transformed into heat energy. Therefore the energetic balance turns out to be negative in our calculations.
In order to reduce friction, we could, in example, use an air track and let objects with less weight slight down the inclined plane.

Overall I found the experiment very interesting - especially the qualitative part where we had to determine which of the rolling bodies would arive sooner. A fundamental understanding of the moment of inertia is crucial in any modern physics applications.