Sliding Mode Control

Yagut Badalova

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State-Space Model

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$

$$\begin{bmatrix} \dot{p} \\ \dot{v} \\ \dot{\alpha} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{f_w}{m_w+m_p} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{f_w m_p l_{sp}}{J_a(m_w+m_p)} & \frac{m_p l_{sp} g}{J_a} & -\frac{f_p}{J_a} \end{bmatrix} \begin{bmatrix} p \\ v \\ \alpha \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_w+m_p} \\ 0 \\ -\frac{m_p l_{sp}}{J_a(m_w+m_p)} \end{bmatrix} \tau$$

$$\alpha_d = 0, \quad \tilde{\alpha} = \alpha_d - \alpha = -\alpha$$
$$p_d = 0, \quad \tilde{p} = p_d - p = -p$$

Sliding Surface Definition

The sliding surface s is defined as:

$$s = \left(\frac{d}{dt} + \lambda\right)\tilde{\alpha} + \left(\frac{d}{dt} + \beta\right)\tilde{p}$$

Expanding:

$$s = \dot{\tilde{\alpha}} + \lambda\tilde{\alpha} + \dot{\tilde{p}} + \beta\tilde{p} = \dot{\alpha} - \lambda\alpha + \dot{p} - \beta p = 0$$

Differentiating s:

$$\dot{s} = -\dot{\omega} - \lambda\omega - \ddot{p} - \beta v = 0$$

where - $\tilde{\alpha} = \alpha_d - \alpha$ - $\tilde{p} = p_d - p$

$$\dot{s} = -\frac{f_w m_p l_{sp}}{J_a(m_w+m_p)}v - \frac{m_p l_{sp} g}{J_a}\alpha + \frac{m_p l_{sp}}{J_a(m_w+m_p)}\tau + \left(\frac{f_p}{J_a} - \lambda\right)\omega + \frac{f_w}{m_w+m_p}v - \frac{1}{m_w+m_p}\tau - \beta v =$$
$$= -\frac{f_w m_p l_{sp}}{J_a(m_w+m_p)}v - \frac{m_p l_{sp} g}{J_a}\alpha + \left(\frac{f_p}{J_a} - \lambda\right)\omega + \left(\frac{f_w}{m_w+m_p} - \beta\right)v + \frac{m_p l_{sp} - J_a}{J_a(m_w+m_p)}\tau$$

Control Law

Considering the system parameters below

m_p	$= 0.329$	Pendulum mass [kg]
m_w	$= 3.2$	Cart mass [kg]
l_{sp}	$= 0.44$	Pendulum length [m]
f_w	$= 6.2$	Cart friction
f_p	$= 0.009$	Pendulum friction
g	$= 9.81$	Gravity [m/s^2]
J_a	$= 0.072$	Pendulum inertia

The Control law will be as follows

$$\tau = \hat{\tau} - k\,\mathrm{sgn}(s)$$

Where

$$\hat{\tau} = \frac{J_a(m_w + m_p)}{m_p l_{sp} - J_a}\left(\frac{f_w m_p l_{sp}}{J_a(m_w + m_p)}v + \frac{m_p l_{sp} g}{J_a}\alpha - \left(\frac{f_p}{J_a} - \lambda\right)\omega - \left(\frac{f_w}{m_w + m_p} - \beta\right)v\right)$$

$$= 12.335\,v + 68.877\,\alpha - 3.492\big(0.125 - \lambda\big)\,\omega - 3.492\big(1.757 - \beta\big)\,v$$

and k is the switching factor that should be tuned.