$$\ddot{x} = \frac{u_1 \left(s_\phi s_\psi + c_\phi c_\psi s_\theta\right) - m\left(-\ddot{\beta} s_\beta s_\alpha l - \dot{\beta}^2 c_\beta s_\alpha l - 2\dot{\beta}\dot{\alpha} s_\beta c_\alpha l + \ddot{\alpha} c_\beta c_\alpha l - \dot{\alpha}^2 c_\beta s_\alpha l\right)}{M+m} \tag{1}$$

$$\ddot{y} = \frac{u_1 \left(c_\phi s_\theta s_\psi - c_\psi s_\phi\right) - m\left(\ddot{\beta} c_\beta s_\alpha l - \dot{\beta}^2 s_\beta s_\alpha l + 2\dot{\beta}\dot{\alpha} c_\beta c_\alpha l + \ddot{\alpha} s_\beta c_\alpha l - \dot{\alpha}^2 s_\beta s_\alpha l\right)}{M+m} \tag{2}$$

$$\ddot{z} = \frac{u_1 \left(c_\theta c_\phi\right) - m\left(\ddot{\alpha} s_\alpha l + \dot{\alpha}^2 c_\alpha l\right) - Mg - mg}{M+m} \tag{3}$$

$$\ddot{\alpha} = \left(ml^2 s_\alpha c_\alpha \dot{\beta}^2 - mgl s_\alpha - mlc_\alpha s_\beta \ddot{y} - mlc_\alpha c_\beta \ddot{x} - ml\ddot{z} s_\alpha\right)\left(ml^2 + I_p\right) \tag{4}$$

$$\ddot{\beta} = \frac{mls_\alpha s_\beta \ddot{x} - mls_\alpha c_\beta \ddot{y} - 2ml^2 s_\alpha c_\alpha \dot{\alpha}\dot{\beta}}{ml^2 s_\alpha^2 + I_p} \tag{5}$$

$$\begin{aligned}\ddot{\phi} = \frac{1}{I_x}\Big(&\tau_\phi + \ddot{\psi} I_x s_\theta + \dot{\psi} I_x c_\theta \dot{\theta} + \dot{\psi}^2\left(I_y c_\theta^2 s_\phi c_\phi - I_z c_\theta^2 c_\phi s_\phi\right) + \dot{\theta}^2\left(-I_y c_\phi s_\phi + I_z s_\phi c_\phi\right) \\ &+ \dot{\theta}\dot{\psi}\big(I_y\left(-s_\phi^2 c_\theta^2 + c_\phi^2 c_\theta\right) - I_z\left(c_\phi^2 c_\theta - s_\phi^2 c_\theta\right)\big)\Big)\end{aligned} \tag{6}$$

$$\begin{aligned}\ddot{\theta} = \frac{1}{I_y c_\phi^2 + I_z s_\phi^2}\Big(&\tau_\theta - \dot{\theta}\left(-2\dot{\phi} I_y c_\phi s_\phi + 2\dot{\phi} I_z s_\phi c_\phi\right) \\ &- \ddot{\psi}\left(I_y c_\phi c_\theta s_\phi - I_z s_\phi c_\theta c_\phi\right) \\ &- \dot{\psi}\Big(I_y\left(-\dot{\phi} s_\phi^2 c_\theta - \dot{\theta} c_\phi s_\theta s_\phi + \dot{\phi} c_\phi^2 c_\theta\right) + I_z\left(\dot{\phi} c_\phi^2 c_\theta - \dot{\theta} s_\phi s_\theta c_\phi - \dot{\phi} s_\phi^2 c_\theta\right)\Big) \\ &+ \dot{\psi}^2\left(I_x s_\theta c_\theta - I_y c_\theta s_\theta s_\phi^2 - I_z c_\theta s_\theta c_\phi^2\right) \\ &+ \dot{\theta}\dot{\psi}\left(-I_y c_\phi s_\theta s_\phi + I_z s_\phi s_\theta c_\phi\right) - \dot{\phi}\dot{\psi} I_x c_\theta\Big)\end{aligned} \tag{7}$$

$$\begin{aligned}\ddot{\psi} = \frac{1}{I_x s_\theta^2 + I_y c_\theta^2 s_\phi^2 + I_z c_\theta^2 c_\phi^2}\Big(&\tau_\psi - \dot{\psi}\big(2I_x s_\theta c_\theta \dot{\theta} - 2I_y c_\theta s_\theta \dot{\theta} s_\phi^2 + 2I_y c_\theta^2 s_\phi c_\phi \dot{\phi} \\ &\quad - 2I_z c_\theta s_\theta \dot{\theta} c_\phi^2 - 2I_z c_\theta^2 c_\phi s_\phi \dot{\phi}\big) \\ &- \ddot{\theta}\left(I_y c_\phi c_\theta s_\phi - I_z s_\phi c_\theta c_\phi\right) \\ &- \dot{\theta}\Big(I_y\left(-\dot{\phi} s_\phi^2 c_\theta - \dot{\theta} c_\phi s_\theta s_\phi + \dot{\phi} c_\phi^2 c_\theta\right) \\ &\quad - I_z\left(\dot{\phi} c_\phi^2 c_\theta - \dot{\theta} s_\phi s_\theta c_\phi - \dot{\phi} s_\phi^2 c_\theta\right)\Big) \\ &+ \ddot{\phi} I_x s_\theta + \dot{\phi}\dot{\theta} I_x c_\theta\Big)\end{aligned} \tag{8}$$