Nonlinear Equation System Parameters

The time parameters of the dynamical system are:

$$\tau = 10^{-2}[s]\,; T = 30[s]$$

The covariance matrices are:

$$P_o = \begin{bmatrix} 10^2 & 0 & 0 & 0 & 0 \\ 0 & 15^2 & 0 & 0 & 0 \\ 0 & 0 & 2^2 & 0 & 0 \\ 0 & 0 & 0 & 3^2 & 0 \\ 0 & 0 & 0 & 0 & 0.08^2 \end{bmatrix}; R_0 = \begin{bmatrix} 8^2 & 0 \\ 0 & 4^2 \end{bmatrix}$$

The Initial conditions are:

$$x_0 = 100[m]\,; z_0 = 200[m]\,; \dot{x}_0 = 2\left[\frac{m}{s}\right]\,;\ \dot{z}_0 = 1\left[\frac{m}{s}\right]\,; \theta_0 = 0.01[rad]$$

Using the initial conditions, the term of $\theta(t)$ can be derived from the differential equation:

$$\theta(t) = 0.01t + 0.01$$

By substituting $\theta(t)$ to the accelerations term, we will get:

$$\ddot{x} = 0.4|\sin(t)|\cos(0.01t + 0.01) + 0.2|\cos(t)|\sin(0.01t + 0.01)$$

$$\ddot{z} = 0.2|\cos(t)|\cos(0.01t + 0.01) - 0.4|\sin(t)|\sin(0.01t + 0.01)$$

The measurement model is:

$$\begin{bmatrix} \hat{x} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ \dot{x} \\ \dot{z} \\ \theta \end{bmatrix} + \begin{bmatrix} n_x \\ n_z \end{bmatrix} \ ; \ n_x, n_z \in N(0, \sigma)$$

The continuous non-linear dynamical model is (I solve the model with and without theta, the result was the same):

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \ddot{x} \\ \ddot{z} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ \dot{x} \\ \dot{z} \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.4|\sin(t)|\cos(\theta) + 0.2|\cos(t)|\sin(\theta) \\ 0.2|\cos(t)|\cos(\theta) - 0.4|\sin(t)|\sin(\theta) \\ 0.01 \end{bmatrix}$$

Because of the non-linearity of the system, I solve the differential equation system with ODE45 and with ODE23 (The result of both solvers were the same).

By discretize the continues non-linear dynamical model I'll get the following term:

$$\begin{bmatrix} x \\ z \\ \dot{x} \\ \dot{z} \\ \theta \end{bmatrix}_k = \begin{bmatrix} x \\ z \\ \dot{x} \\ \dot{z} \\ \theta \end{bmatrix}_{k-1} + \Delta t \cdot \begin{bmatrix} \dot{x}_{k-1} \\ \dot{z}_{k-1} \\ 0.4|\sin(t_{k-1})|\cos(\theta_{k-1}) + 0.2|\cos(t_{k-1})|\sin(\theta_{k-1}) \\ 0.2|\cos(t_{k-1})|\cos(\theta_{k-1}) - 0.4|\sin(t_{k-1})|\sin(\theta_{k-1}) \\ 0.01 \end{bmatrix}$$