

# 1. PID algorithm theory

## 1.1 Introduction to the algorithm

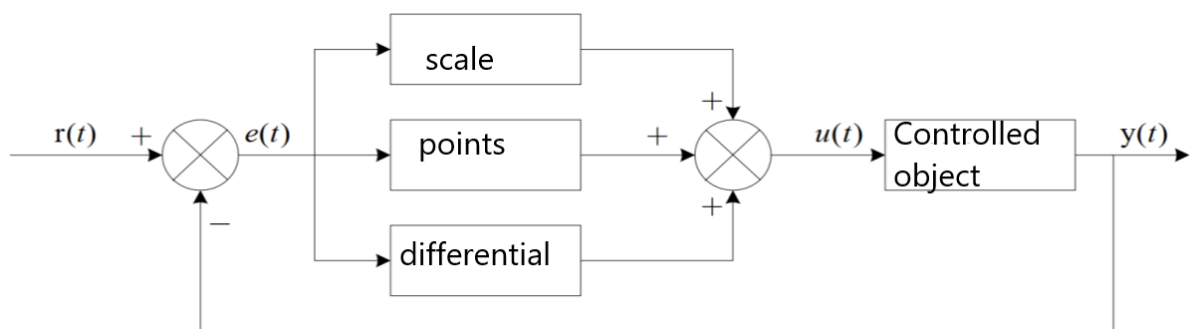
PID, is the proportional integral differential operation on the input deviation, the superposition result of the operation to control the actuator. The formula is as follows :

$$u(t) = K_p[e(t) + \frac{1}{T_i} \int_0^t e(t) dt + T_d \frac{de(t)}{dt}]$$

It consists of three parts:

- P is the proportion, which is the input deviation multiplied by a factor;
- I is the integral, which is the integration of the input deviation;
- D is the differential, which differentiates the input deviation.

A basic PID controller is shown below:



### 1.1.1, Proportional part

The proportional part of the mathematical formula is expressed as:  $K_p * e(t)$

In the analog PID controller, the role of the proportional part is to react to the deviation instantly. Once the deviation is generated, the controller immediately produces a control effect, so that the control amount to reduce the deviation of the direction of change. The strength of the control role depends on the proportion coefficient, the larger the proportion coefficient, the stronger the control role, the faster the transition process, the static deviation of the control process is also smaller; but the larger, but also more prone to oscillations, destroying the stability of the system. Therefore, the example coefficient must be selected appropriately, in order to transition time is less, the static deviation is small and the effect of stability.

Advantage : Adjusting the open-loop proportionality coefficient of the system improves the steady state accuracy of the system, reduces the inertia of the system, and accelerates the response speed.

Disadvantage : With P controller only, too large open-loop proportionality coefficient will not only increase the overshoot of the system, but also make the system stability margin become small or even unstable.

### 1.1.2 Integral Part

The mathematical equation of the integral part is expressed as:  $\frac{K_p}{T_i} \int_0^t e(t) dt$   
The larger the integration constant is, the weaker the accumulation effect of the integral is, when the system will not oscillate in the transition; however, increasing the integration constant will slow down the elimination process of the static error, and the time required for eliminating the deviation is longer, but it can reduce the amount of overshooting and improve the stability of the

system. When  $T_i$  is small, the role of the integral is stronger, then the system may generate oscillation in the transition time, but the time required to eliminate the deviation is shorter. Therefore,  $T_i$  must be determined according to the specific requirements of the actual control.

Advantage: Elimination of steady state error.

Disadvantage: the addition of integral controller will affect the stability of the system, so that the stability margin of the system is reduced.

### 1.1.3, Differential Part

The mathematical equation of the differential part is expressed as:  $K_p T_d \frac{de(t)}{dt}$

The function of the differential part makes to stop the change of deviation. It is controlled according to the tendency (speed of change) of the deviation. The faster the deviation changes, the greater the output of the differential controller will be and will be able to correct the deviation value before it becomes large. The introduction of differential action will help to reduce the amount of overshoot, overcome oscillations, and stabilize the system, which is especially beneficial for high order systems, and it speeds up the tracking speed of the system. However, the role of differentiation is very sensitive to the noise of the input signal, for those systems with large noise, generally do not use differentiation, or filter the input signal before the role of differentiation. The role of the differential part is determined by the differential time constant  $T_d$ . The larger  $T_d$  is, the stronger it suppresses the deviation change; the smaller  $T_d$  is, the weaker it resists the deviation change. The differential component obviously plays a significant role in the stabilization of the system. By choosing the differential constant  $T_d$  appropriately, the differential effect can be optimized.

Advantages: faster response, reduced overshooting, reduced oscillations, and a "predictive" effect on the dynamic process.

## 1.2 Selection of PID algorithm

Digital PID control algorithms can be divided into positional PID and incremental PID control algorithms. Then we should understand the principle of the PID algorithm before deciding to use it:

### 1.2.1, Positional PID algorithm

$$u(k) = K_p e(k) + K_I \sum_{i=0} e(i) + K_D [e(k) - e(k-1)]$$

$e(k)$ : user-set value (target value) - current state value of the control object

Proportion P :  $e(k)$

Integral I : Accumulation of  $\sum e(i)$  error

Differential D :  $e(k) - e(k-1)$  This error - Last error

That is, positional PID is the deviation of the actual position of the current system from the desired position you want to achieve for PID control

Because there is an error integral  $\sum e(i)$ , which is always cumulative, that is, the current output  $u(k)$  is related to all the past states, and the cumulative value of the error is used; (the error  $e$  will have the error cumulative), the output  $u(k)$  corresponds to the actual position of the actuator, and once the control output is wrong (the problem of the current state value of the control object), the drastic change of  $u(k)$  causes the system to change drastically and the Positional PID can be integrated into the control system in the same way. And positional PID in the integral term reaches saturation, the error will still continue to accumulate under the integral effect, once the

error begins to change in the reverse direction, the system needs a certain amount of time to exit from the saturation zone, so in the  $u(k)$  to reach the maximum and minimum, to stop the integral effect, and to have the integral limit and the output limit, so in the use of the positional PID, generally we directly use the PD control.

Positional PID is suitable for objects where the actuator does not have an integral part, such as servos and balancing trolleys for upright and temperature control systems.

Advantage: Positional PID is a non-recursive algorithm, which can directly control the actuator (e.g., balancing trolley), the value of  $u(k)$  and the actual position of the actuator (e.g., current angle of the trolley) correspond to each other, so it can be used for objects where the actuator is not equipped with an integrating part.

Disadvantage: Each output is related to a past state, and the calculation requires the accumulation of  $e(k)$ , which makes the calculation laborious.

### 1.2.2 Incremental PID Algorithm

$$\Delta u(k) = u(k) - u(k-1) = K_P[e(k) - e(k-1)] + K_I e(k) + K_D[e(k) - 2e(k-1) + e(k-2)]$$

Proportion P :  $e(k) - e(k-1)$  this error - last error

Integral I :  $e(k)$  error

Differential D :  $e(k) - 2e(k-1) + e(k-2)$  this time error - 2\*last time error + last time error

Incremental PID According to the formula, once  $K_P$ ,  $T_I$  and  $T_D$  are determined, the deviation of the three measurements before and after can be used to derive the control increment from the formula, and the resulting control amount  $\Delta u(k)$  corresponds to the increment of the position error of the last few times, not to the deviation from the actual position. There is no accumulation of errors, that is to say, there is no need to accumulate in the incremental PID. The control increment  $\Delta u(k)$  is only related to the last three sampling values, it is easy to get a better control effect through weighting, and in the event of a system problem, incremental type will not seriously affect the system's work.

Summarize: incremental PID, is the position type PID to take the increment, this time the controller output is the difference between the two adjacent sampling moments of the calculated position value, the result obtained is incremental, that is, in the previous control amount of the basis needs to be increased (negative value means decrease) control amount.

Advantages:

- ① The impact is small in the case of malfunction, and the error data can be removed by the method of logical judgment if necessary.
- ② Small impact when switching manually/automatically, making it easy to realize non-disturbance switching. When the computer fails, the original value can still be maintained.
- ③ Accumulation is not required in the arithmetic equation. The determination of the control increment  $\Delta u(k)$  is only related to the last 3 sampling values.

Disadvantages:

- ① Large integral truncation effect with steady state error;
- ② The effect of overflow is large. It is not good to use incremental type for some controlled objects;

### 1.2.3 Difference between incremental type and positional type

(1) The incremental algorithm does not need to do the accumulation, and the determination of the control quantity increment is only related to the last several deviation sampling values, so the influence of the calculation error on the control quantity calculation is small. The positional algorithm has to use the accumulated value of the past deviation, which is easy to produce a large accumulation error.

(2) The incremental algorithm is the increment of the control quantity, for example, in the valve control, only the part of the output of the valve opening change, the impact of false operation is small, if necessary, can also be restricted or prohibited by logical judgment of the output, will not seriously affect the work of the system. On the other hand, the output of the positional type corresponds directly to the output of the object, so it has a greater impact on the system.

(3) The incremental PID control output is the increment of the control quantity, and there is no integral effect, so the method is applicable to the object of the actuator with integral parts, such as stepping motors, etc., and the positional PID is applicable to the object of the actuator without integral parts, such as electro-hydraulic servo valves.

(4) For PID control, positional PID requires integral limit and output limit, while incremental PID requires only output limit.

Positional PID and incremental PID is only a digital PID control algorithm of the two forms of realization, the essence is exactly the same. The main difference is that the integral term is stored in different ways, positional PID integral term alone, incremental PID integral term as part of the output storage, online players also have their own unique insights and views on the use of positional and incremental, or to look at our specific application scenarios which algorithm is suitable for.

## 1.3 Debugging of PID parameters

There are many ways to select the parameters of PID controller, such as trial and error method, critical proportionality method, expanding the critical proportionality method and so on. However, for PID control, parameter selection is always a very complicated work, need to be constantly adjusted to get a more satisfactory control effect. Based on experience, the general PID parameters are determined as follows:

### 1.3.1. Determine the proportionality coefficient $K_p$

Determine the proportionality coefficient  $K_p$ , first of all, remove the integral term and differential term of PID, you can make  $T_i = 0$ ,  $T_d = 0$ , so that it becomes purely proportional regulation. The input is set to 60% to 70% of the maximum allowable output value of the system, and the proportionality coefficient  $K_p$  is gradually increased from 0 until the system oscillates; and then in turn, the proportionality coefficient  $K_p$  is gradually reduced from this point until the disappearance of the system oscillation. Record the proportionality coefficient  $K_p$  at this time, and set the proportionality coefficient  $K_p$  of PID to 60% to 70% of the current value.

### 1.3.2. Determine the integration time constant $T_i$

After the proportionality coefficient  $K_p$  is determined, set a larger integral time constant  $T_i$ , and then gradually reduce  $T_i$  until the system oscillation, and then in turn, gradually increase  $T_i$  until the system oscillation disappears. Record the  $T_i$  at this time, and set the integral time constant  $T_i$  of PID to 150% to 180% of the current value.

### 1.3.3. Determine the differential time constant Td

Differential time constant Td generally do not need to set, 0 can be, at this time the PID regulation is converted to PI regulation. If you need to set, with the same method to determine the Kp, take the value of 30% of the non-oscillation.

### 1.3.4. System no-load, with load intermodulation

Fine-tune the PID parameters until they meet the performance requirements.

Of course, this is only my personal debugging method, not necessarily suitable for everyone and every environment, only to provide you with a reference; however, the Internet has also circulated the debugging PID of the classic try to put together the mnemonic, I also posted for your reference:

Parameter setting to find the best, from small to large order to check.

First is the proportion after the integral, and then finally add the differential.

Curve oscillation is very frequent, the proportion dial should be enlarged.

The curve is drifting around a big bend, the proportional dial should be adjusted to a smaller size.

The curve is slow to recover from deviation, the integration time goes down.

The curve fluctuates for a long period, so the integration time is increased.

The curve oscillates quickly, lower the differential first.

The differential time should be lengthened.

Ideal curve has two waves, high in front and low in the back, four to one.

If you look at the curve and analyze it, the quality of adjustment will not be low.

PID is a proportional (P), integral (I), differential (D) control algorithm, not necessarily at the same time with these three algorithms, but also can be PD, PI, or even only P algorithm control. I used to have a closed-loop control of one of the most simple idea of only P control, the current results back, and then subtracted from the target, if positive, then decelerate, if negative, then accelerate, of course, this is only the simplest closed-loop control algorithm, but also still back to the previous section of the positional and incremental summary, specific reference to our current control environment, because of the differences in the control system that will allow us to achieve the most stable results of the system parameters are certainly not the same. The most stable effect of the parameters is certainly OK.