MILESTONE 2: LITERATURE REVIEW

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1 Problem Analysis and Its Taxonomy

The aim of the project is try solve the bank queue problem and that is by building a mobile application by which the customer can select which bank to go to and in which time slot. This problem can be considered a variant of Assignment Problem[1] that there is number of Agents(Banks) and number of Tasks(Customers) and any agent can be assigned to perform any task with some cost depending on agent-task assignment. Moreover, our problem can be classified as a type capacitated task allocation problem(CTAP) where the bank branches are our resources and serving each client is a task.

2 Modeling Approaches

Our research led us to find out that a lot of research was done to solve problems similar to ours.

2.1 Integer Quadratic Program

The study[1] tries to solve the general Task Allocation Problem (TAP) where they want to assign tasks to processors in a distributed system. The study stated that most versions of (TAP) can be formulated as an integer quadratic program. Their mathematical model was as follows:

$$min \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} c_{ij}(1 - \sum_{k=1}^{N} x_{ik}x_{jk}) + \sum_{i=1}^{M} \sum_{k=1}^{N} d_{ik}x_{ik} + \sum_{k=1}^{N} s_k y_k \tag{1}$$

$$s.t. \sum_{k=1}^{N} x_{ik} = 1, i = 1, .., M \tag{2}$$

$$\sum_{k=1}^{N} a_i x_{ik} \leq b_k y_k, k = 1, .., N \tag{3}$$

$$x_{ik} \leq y_k, i = 1..M, k = 1, .., N \tag{4}$$

$$x_{ik} \in \{0, 1\}, y_k \in \{0, 1\} \forall i, k \tag{5}$$

[1]Also, Task Allocation Problem

The objective is to minimize the sum of the total communication cost between tasks, the overall execution cost of performing all tasks, and the overall fixed cost of using processors, which are assigned at least one task. The first term of the objective states that the communication cost between two tasks is incurred only if these two tasks are assigned to two different processors. Constraint (2) indicates that each task needs to be assigned to exactly one processor. Constraint (3) states that the total resource usage from the tasks assigned to a processor cannot exceed its capacity. Constraint (4) specifies that any task cannot be assigned to a processor that is not used. Constraint (5) ensures that both x and y are binary variables. Constraint (4) is redundant if a i > 0 for all i because it can be deduced from constraints (2), (3), and (5).

2.2 Linear Physical Program

This study[2] investigates the personnel task assignment problem in central operational departments for the banking sector and it is a recent study done in 2020. This study's main goal is to develop a task assignment methodology that is based on optimization techniques which assigns a set of jobs to a set of employees with different levels of expertise to meet the due dates and satisfy SLAs. In this study, a two-step method is proposed to solve a real life assignment problem. First step prioritizes the jobs coming to the system based on a multi-criteria evaluation. In the second step, a mathematical model is developed to assign jobs to employee groups.

Regarding the mathematical model, it was defined as follows:

Parameters and decision variables of the model arelisted as follows:

- Index
  - j: index for tasks j=1,2,3,...,J
  - i: index for profile groups i=1,2,3,...I
- Parameters:
  - α_{ji}: Competence level for profile group i for task j
  - a_{ji} Ability matrix for profile group i for taskj
  - b_j: Importance level of taskj
  - tp_i: Available time for profile group i
  - k_i: Available employee number for profile group i
  - p: Planning period
  - t_j: Process time of task j
  - c : Minimum capacity usage
- Decision Variables:
  - x_{ji}= 1 if task j is assigned to profile group i, else 0

The proposed Task Assignment model is given as follows:
Objective Function 1:

$$Maximize : (\sum_{j=1}^{J}\sum_{i=1}^{I} x_{ji}\alpha_{ji}) \tag{6}$$

Objective Function 2:

$$Maximize : (\sum_{j=1}^{J}\sum_{i=1}^{I} x_{ji} * b_j) \tag{7}$$

Objective Function 3:

$$Maximize : (c) \tag{8}$$

Subject to:

$$x_{ji} \leq a_{ji} \forall i, j \tag{9}$$

$$\sum_{j=1}^{J} x_{ji} * t_j \leq tp_i \forall i \tag{10}$$

$$\sum_{i=1}^{I} x_{ji} \leq 1 \forall j \tag{11}$$

$$\sum_{i=1}^{I} k_i * p = tp_i \forall j \tag{12}$$

$$\sum_{j=1}^{J} (x_{ji} * t_j)/tp_i \geq c \forall i \tag{13}$$

$$x_{ji} \in \{0,1\}, \forall i, j \tag{14}$$

Eq. (6) tries to maximize the level of task assignments to appropriate profile groups. Eq. (7) tries to maximize assignment level of higher priority tasks. Eq. (8) tries to maximize capacity usage of least occupied profile group. Capacity utilization rates are tried to be balanced.

Eq. (9) tries to ensure that a task can be assigned to a proper profile group. Eq. (10) tries to ensure that available time of profile groups cannot be exceeded. Eq. (11) tries to ensure that each task should be assigned to a profile group. Eq. (12) gives the relation between the total available time and the number of employees in profile groups. Eq. (13) determines the minimum capacity usage of profile groups. Eq. (14) determines the range of variables.

3 Discussion

From the previous literature review we conclude that many studies tend to take the approach of integer programming to solve similar problems in real life. Different integer programming solvers were implemented to solve different problems but they share the same concept. Also the two steps (multi-step) methods were introduced to break down this problem into smaller problems which we will try to achieve in our implementation.

References

[1] Krishnamoorthy Mohan Ernst Andreas, Jiang Houyuan. Exact solutions to task allocation problems. doi:10.1287/mnsc.1060.0578.

[2] Vayvay O. Çetin K., Tuzkaya G. A mathematical model for personnel task assignment problem and an application for banking sector. doi:https://doi.org/10.11121/ijocta.01.2020.00825.