MILESTONE 3: MATHEMATICAL MODEL

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1 Problem's Input

1.1 b

Number of bank branches.

1.2 $\mathbf{s}_j$

Number of slots per working day for branch j.

1.3 r

Number of requests per day.

1.4 m

Number of available services across all branches.

1.5 $\mathbf{cap}_j$

Capacity of branch j per slot.
$1 \leq j \leq b$

1.6 $\mathbf{rs}_i$

The service required by request i.
$1 \leq rs_i \leq m \qquad 1 \leq i \leq r$

1.7 $\mathbf{slots}_{zk}$

Number of slots required to complete service z at branch j.
$1 \leq z \leq m$
$1 \leq j \leq b$

1.8 $\mathbf{dist}_{ij}$

The distance between the location of request i and the location of branch j.
$1 \leq i \leq r$
$1 \leq j \leq b$

1.9 $\mathbf{p}_i$

The priority of request i.
$1 \leq i \leq r$

1.10 $\mathbf{serves}_{jwz}$

$serves_{jwz} = 1$, if counter w at branch j provides service z.
$serves_{jwz} = 0$, otherwise.
$1 \leq j \leq b, 1 \leq z \leq m, 1 \leq w \leq cap_j$

1.11 d

The maximum accepted distance between a branch and a request.

Both this constant and $dist_{ij}$ are of the same numerical value.

2 Decision Variables

2.1 x_{ijkw}

$X_{ijkw} = 1$ if request i is handled in bank j at slot k at counter w.
$X_{ijkw} = 0$ otherwise.

$1 \leq i \leq r \quad 1 \leq j \leq b \quad 1 \leq k \leq s_j \quad 1 \leq w \leq cap_j$

3 Objective Function

The aim is to maximize:

$$\alpha F0 - \beta F1 + \sigma F2$$

where α, β, σ are non-negative tunable parameters.

3.1 F0: The number of matchings

$$\sum_{i=1}^{r} \sum_{j=1}^{b} \sum_{k=1}^{s_j} \sum_{w=1}^{cap_j} x_{ijkw}$$

3.2 F1: The Distance between the customer and the branch of the matched requests.

$$\sum_{i=1}^{r} \sum_{j=1}^{b} \sum_{k=1}^{s_j} \sum_{w=1}^{cap_j} x_{ijkw} * dist_ij$$

3.3 F2: The priority of the matched requests.

$$\sum_{i=1}^{r}\sum_{j=1}^{b}\sum_{k=1}^{s_j}\sum_{w=1}^{cap_j} x_{ijkw} * p_i$$

4 Constraints

4.1 Domain Constraint

$$x_{ijkw} \in [0,1] \qquad \text{if } serves_{jwrs_i} = 1 \text{ and } (slots_{rs_i} + k - 1) \leq s_j \text{ and } dist_{ij} \leq d$$
$$x_{ijkw} = 0 \qquad \text{otherwise.}$$

$$\forall i \in [1, r]$$

$$\forall j \in [1, b]$$

$$\forall k \in [1, s_j]$$

$$\forall w \in [1, cap_j]$$

4.2 Each Customer is handled at most once

$$\sum_{j=1}^{b}\sum_{k=1}^{s_j}\sum_{w=1}^{cap_j} x_{ijkw} \leq 1$$

$$\forall i \in [1, r]$$

4.3 No two requests are being handled at the same counter twice

$$x_{ijkw} \leq \mu * (1 - x_{i_2jk_2w})$$

where μ is a very large number (∞)

$$\forall i \in [1, r]$$

$$\forall i_2 \in [1, r] - \{i\}$$

$$\forall j \in [1, b]$$

$$\forall k \in [1, s_j]$$

$$\forall k_2 \in [k, k + slots_{rs_i} - 1]$$

$$\forall w \in [1, cap_j]$$