Basics.dfw -- M. Edward Borasky, Borasky Research, 16 June 2001
Basic calculations from Partch and Sethares

References: Partch, Harry (1974), *Genesis of a Music*
Sethares, William (1999), *Tuning, Timbre, Spectrum, Scale*
Xenakis, Iannis (1992), *Formalized Music*

The Partch system, called *Monophony*, has several elements. It is based on so-called *11-limit just intonation*, where the *frequencies* of the musical tones are related to a central pitch and to each other by *ratios* of *positive integers*. A ratio, called a *rational number* by mathematicians, has a *numerator* and a *denominator* that are integers, and the term *11-limit* refers to the fact that none of the ratios used has a numerator or denominator with a factor greater than 11. Thus, 2*3*5*7*11/7*3*5 is within the 11-limit, but 13/11 is not.

List of identities (Odentities or Udentities)

```
#1:   Identities := [1, 9, 5, 11, 3, 7]
```

Function to rescale a ratio between 1 and 2

```
      Rescale(ratio) :=
        If ratio > 2
           Rescale(ratio/2)
#2:        If ratio < 1
              Rescale(ratio·2)
              ratio
```

Tonality diamond (rows are Otonalities, columns are Utonalities)

#3: $$\text{TonalityDiamond} := \text{VECTOR}\left(\text{VECTOR}\left(\text{Rescale}\left(\frac{\text{Identities}_j}{\text{Identities}_i}\right), j, 1, \text{DIMENSION(Identities)}\right), i, 1, \text{DIMENSION(Identities)}\right)$$

Cents from a ratio

```
#4:   Cents(ratio) := 1200·LOG(ratio, 2)
```

Ratio from cents

#5: $\text{Ratio(cents)} := 2^{\text{cents}/1200}$

Sethares dissonance functions: From *Tuning, Timbre, Spectrum, Scale* Appendix E. The exact form of these equations follows the Matlab version. Note that the scaling by 5 isn't really necessary since we normalize the plotted points to a peak value of 1 later, but is done here so the formulas match the code in the book.

Frequency scaling function

#6: $$\text{Fscale(f)} := \frac{0.24}{0.0207 \cdot f + 18.96}$$

Dissonance function for two sines with f1 < f2

```
#7:   Diss1(f1, f2, a1, a2) := a1·a2·(5·EXP(- 3.51·Fscale(f1)·(f2 - f1)) -

        5·EXP(- 5.75·Fscale(f1)·(f2 - f1)))
```

Symmetric dissonance function for two sines

```
      Dissonance(f1, f2, a1, a2) :=
        If f1 > f2
#8:        Diss1(f2, f1, a2, a1)
           Diss1(f1, f2, a1, a2)
```

Intrinsic dissonance of a sound

```
#9:   fv :ε Vector

#10:  av :ε Vector
```

#11: $$\text{IntrinsicDissonance(fv, av)} := \frac{1}{2} \cdot \sum_{i=1}^{\text{DIMENSION(fv)}} \sum_{j=1}^{\text{DIMENSION(fv)}} \text{Dissonance}(fv_i, fv_j, av_i, av_j)$$

Test data: 1/1 = G 392

```
#12:  Npartials := 11

#13:  Gfreq := VECTOR(392·i, i, 1, Npartials)

#14:  Gunitamp := VECTOR(1, i, 1, Npartials)
```

#15: $\text{Gscaledamp} := \text{VECTOR}\left(\frac{1}{i}, i, 1, \text{Npartials}\right)$

Dissonance of an interval

#16: $\text{IntervalDissonance}(fv, av, ratio) := \text{IntrinsicDissonance}(fv, av) + \text{IntrinsicDissonance}(ratio\cdot fv, av) + \sum_{i=1}^{\text{DIMENSION}(fv)} \sum_{j=1}^{\text{DIMENSION}(fv)} \text{Dissonance}(fv_i, ratio\cdot fv_j, av_i, av_j)$

Function to normalize a plot matrix to peak=1

#17: plotmat :ε Vector

#18: $\text{Scaleplot}(plotmat, factor) := plotmat\cdot\begin{bmatrix} 1 & 0 \\ 0 & factor \end{bmatrix}$

#19: $\text{Normalize}(plotmat) := \text{Scaleplot}\left(plotmat, \frac{1}{\text{MAX}(plotmat\ \text{COL}\ 2)}\right)$

Plot a dissonance curve starting at G392

Number of points to plot

#20: Npoints := 300

Points for unit-amplitude partials

#21: $\text{UnitDissonancePoints} := \text{Normalize}\left(\text{VECTOR}\left([x, \text{APPROX}(\text{IntervalDissonance}(\text{Gfreq}, \text{Gunitamp}, x))], x, 1, 2, \frac{1}{\text{Npoints}}\right)\right)$

Points for 1/n scaled-amplitude plots

#22: $\text{ScaledDissonancePoints} := \text{Normalize}\left(\text{VECTOR}\left([x, \text{APPROX}(\text{IntervalDissonance}(\text{Gfreq}, \text{Gscaledamp}, x))], x, 1, 2, \frac{1}{\text{Npoints}}\right)\right)$

The Partch scale

#23: $\text{Pscale} := \left[1, \frac{81}{80}, \frac{33}{32}, \frac{21}{20}, \frac{16}{15}, \frac{12}{11}, \frac{11}{10}, \frac{10}{9}, \frac{9}{8}, \frac{8}{7}, \frac{7}{6}, \frac{32}{27}, \frac{6}{5}, \frac{11}{9}, \frac{5}{4}, \frac{14}{11}, \frac{9}{7}, \frac{21}{16}, \frac{4}{3}, \frac{27}{20}, \frac{11}{8}, \frac{7}{5}, \frac{10}{7}, \frac{16}{11}, \frac{40}{27}, \frac{3}{2}, \frac{32}{21}, \frac{14}{9}, \frac{11}{7}, \frac{8}{5}, \frac{18}{11}, \frac{5}{3}, \frac{27}{16}, \frac{12}{7}, \frac{7}{4}, \frac{16}{9}, \frac{9}{5}, \frac{20}{11}, \frac{11}{6}, \frac{15}{8}, \frac{40}{21}, \frac{64}{33}, \frac{160}{81}, \frac{2}{1}\right]$

Points for unit-amplitude spectrum at the Partch scale degrees

#24: $\text{UnitPartchPoints} := \text{Normalize}(\text{VECTOR}([\text{Pscale}_i, \text{APPROX}(\text{IntervalDissonance}(\text{Gfreq}, \text{Gunitamp}, \text{Pscale}_i))], i, 1, \text{DIMENSION}(\text{Pscale})))$

Points for scaled-amplitude sprectrum at the Partch scale degrees

#25: $\text{ScaledPartchPoints} := \text{Normalize}(\text{VECTOR}([\text{Pscale}_i, \text{APPROX}(\text{IntervalDissonance}(\text{Gfreq}, \text{Gscaledamp}, \text{Pscale}_i))], i, 1, \text{DIMENSION}(\text{Pscale})))$

MIDI calculations: MIDI note numbers range from 0 to 127, where middle C (C below A440) = note 60, and thus A440 = 69

Frequency of a note number

#26: $$\mathrm{MIDIFreq}(\mathrm{notenumber}) := 440\cdot 2^{(\mathrm{notenumber}-69)/12}$$

Note number of a frequency: ***this will not necessarily be an integer***. For anything but 12-TET frequencies, it will be an integer plus a fractional part. The note can be played on many synthesizers by sending the integer note number and a pitch bend, which can be computed from the fractional part. The pitch bend value ranges from 0 to 16383, with a bend value of 8192 representing no pitch bend and a semitone having a bend value of 4096. Since there's a note number every semitone, we only need to use positive bend values in the range 8192 - 12288. Note that these functions have ***not*** been tested.

#27: $$\mathrm{MIDInotenumber}(\mathrm{freq}) := \frac{12\cdot \mathrm{LN}\left(\frac{\mathrm{freq}}{55}\right)}{\mathrm{LN}(2)} + 33$$