Classification of musical scales

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We intent to provide a simple, yet systematic classification for musical scales. Several classications of musical scales have been provided more or less explictly. For example Ellis [2] introduced two classes: cyclic, which equates equal tempered (ET), or linear which would mean all the non-ET scales.

Mark Lindley & Ronald Turner-Smith [3] make a much more explicit attempt to classify all the scales. However, I personally found their description a bit confusing, specially their definition of coherent systems ('in which all the pitch classes make one chain of 5ths') and non-coherent systems ('which have proven musically so awkward that no well-known composer has ever written music for such a system').

Here we attempt to provide a simple classification which pretends to create clear and mutually exclusive categories. This note uses comments (which I thank much) provided in the tuning-math group, but any mistakes or innacuracies found here are just my own.

The first division we propose is derived from the ratio used to create the pitch classes. As we will see later, the classification will then go to consider the number of generators used to complete the scale. In this sense, the equivalence ratio (octave of otherwise) could be consider as a generator itself, however because of the current preeminence of the octave equivalent we have opted to put it up explicitly upfront.

As a consequence the first branch is:

- Octave equivalent scales
- Other ratio equivalent scales (e.g. Bohlen-Pierce)

Within each of these categories we propose that the basic division should be that of the number of generators needed to complete the scale. Hence the second branching of our classication would be

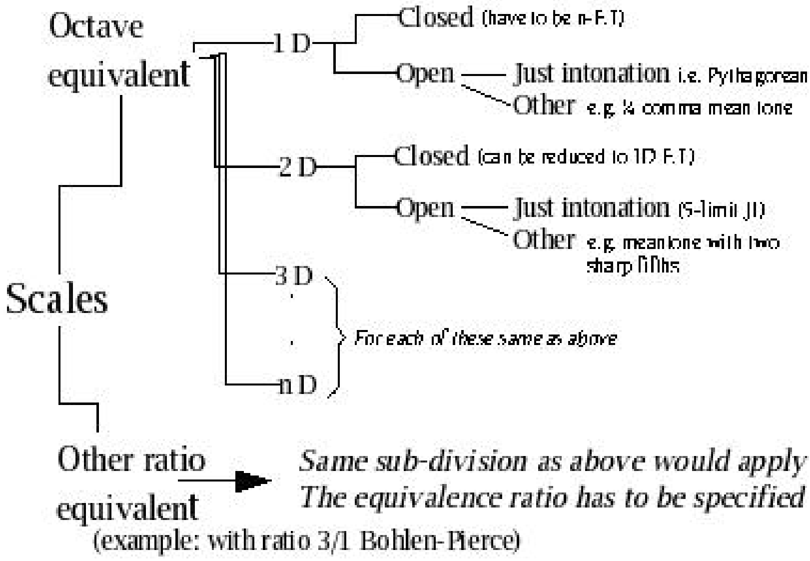

Figure 1: Schematic representation of classification of musical scales

- 1 dimensional scales (one generator, e.g. Pitagorean tunning, 1/4 comma meantone scale)
- 2 dimensional scales (two generators, e.g. 5-limit just scales, two sharp fifths meantone [1])
- 3 dimensional scales (three generators, e.g. 7-limit just scales, some others meantone scales)
- and so on N dimensional scales

For each of these categories we would then propose a new branch into 'open' and 'closed' scales. Closed scales allways end up beign ET scales. In 1 dimension this is obvious. In 2 or more dimensiones (and this the discussion we had in the group about the Cartesian product of two scales) they can allways be reduce to a 1 dimensional case with a generator $lcm(all-the-generators)^{-1}$. The 'open' scales can be dividen in two other categories 'just intonation' in which all the ratios has to be of the form $2^p3^q5^r7^s11^t\ldots$ or 'other', in which the ratios as derived based on other considerations, but normally an approximation to the abover, this last would include the traditiona meantone.

Figure 1 intends to provide additional clarification.

References

[1] J. Murray Barbour. *Tunning and Temperament: A Historical Survey*. Michigan State College Press, East Lansing, 2nd ed. (1961) edition.

[2] Alexander J. Ellis. *On temperament*. Dover Publications, 1885. Section A Annex XX of Hermann Helmholtz's book.

[3] Mark Lindley & Ronald Turner-Smith. An algebraic approach to mathematical models of scales. Music Theory Online (3), June 1993.