

Here is a plot of remainder $R = (xq - p)/(p + q)$ against $x = \text{interval}$.

$p = \text{Round}[x \cdot k / (x - 1)]$, $q = \text{Round}[k / (x - 1)]$ where $k = 1, 2, 3, \dots$ the k 'th wave convergent. R represents the difference between our original frequencies, x and 1, and the rational values $p \cdot g$ and $q \cdot g$, where $g = (x + 1)/(p + q)$, the GCD between p and q . Since $p \cdot g \sim x$, $q \cdot g \sim 1$, then g also represents the *Approximate GCD* (\sim GCD) between our original frequencies. Observe also that $k = (p - q)$ which is the difference frequency for the rational counterparts.

The zeros of the graph occur at $(xq - p) = 0$, that is, at rational intervals $x = p/q$. R is negative for the lhs side of these zeros, positive for the rhs. We can clearly see the distance ('closeness') of x from each wave convergent k along the x axis. For e.g., $81/64 = 1.265\dots$ is close to $5/4 = 1.25$ in both the 1st and 2nd WC's i.e. first two graphs.

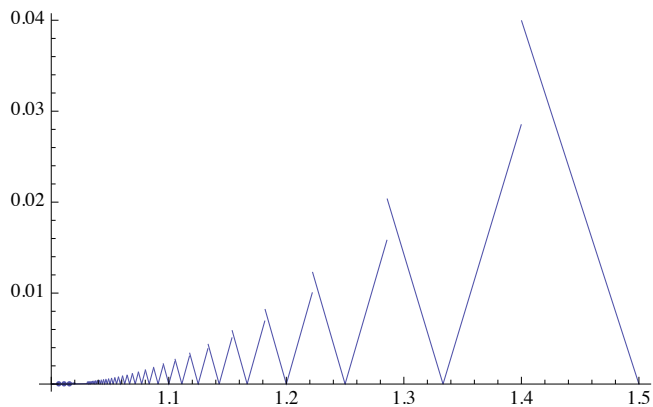
Observe that there are jump discontinuities at all of the zeros of the *other* convergents. This is useful for separating intervals. Observe also that the slope of each line is $\pm q/(p + q)$.

Finally, notice also that intervals b/w higher harmonics appear closer to unison toward the left. The triangles appear smaller because there is less manoeuvring room b/w intervals. The largeness of the simpler intervals (to the right) show that there is much more room for error. OTOH, the larger convergents give tighter triangles.

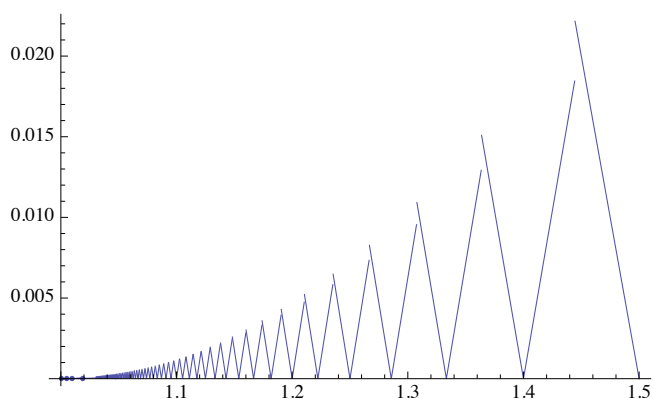
The formula for the max value of x for the $(p - q)$ 'th convergent is $x(\text{max}) = [(p - q)/(q \pm 1/2)] + 1$. (Whether this is inclusive or the cut off point (discontinuity) I don't know).

Here are 1, 2, 3 and 7 WC's:

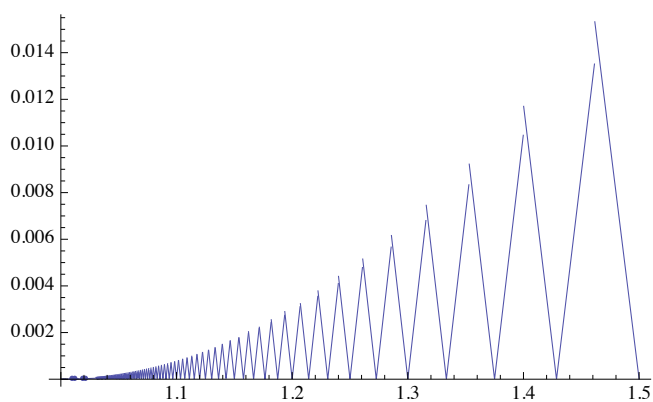
Plot[Abs[(x * Round[1 / (x - 1)] - (Round[x * 1 / (x - 1)])] / (Round[x * 1 / (x - 1)] + Round[1 / (x - 1)]), {x, 1, 1.5}]



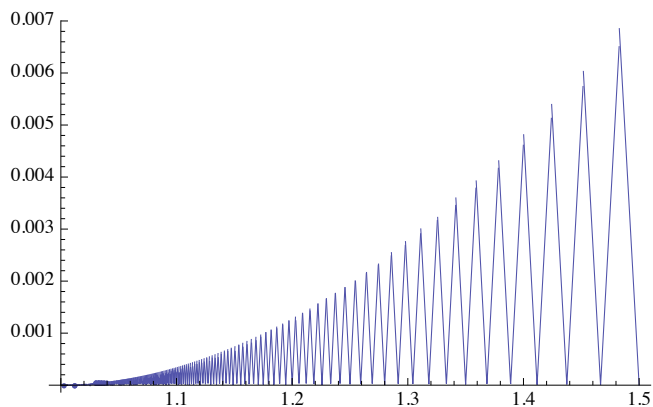
```
Plot[Abs[(x * Round[2 / (x - 1)] - (Round[x * 2 / (x - 1)])) /  
(Round[x * 2 / (x - 1)] + Round[2 / (x - 1)]), {x, 1, 1.5}]
```



```
Plot[Abs[(x * Round[3 / (x - 1)] - (Round[x * 3 / (x - 1)])) /  
(Round[x * 3 / (x - 1)] + Round[3 / (x - 1)]), {x, 1, 1.5}]
```

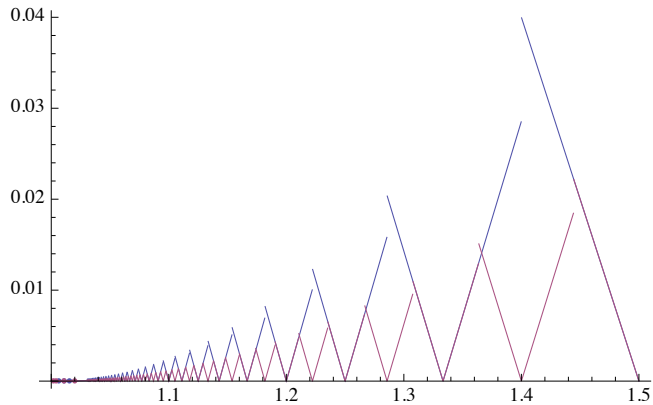


```
Plot[Abs[(x * Round[7 / (x - 1)] - (Round[x * 7 / (x - 1)])) /  
(Round[x * 7 / (x - 1)] + Round[7 / (x - 1)]), {x, 1, 1.5}]
```



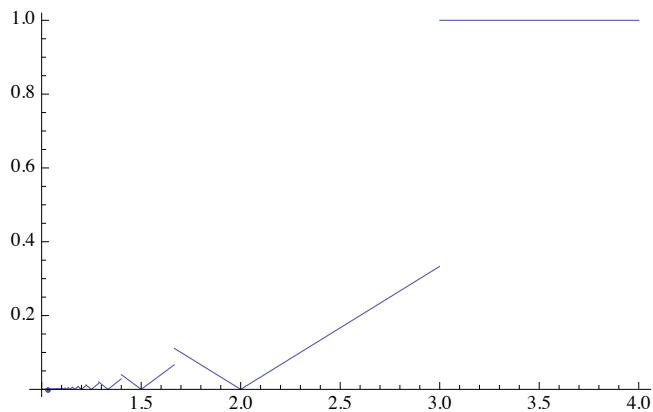
The 1st and 2nd convergents together (blue and red respectively).

```
Plot[{Abs[
  (x * Round[1 / (x - 1)] - (Round[x * 1 / (x - 1)])) / (Round[x * 1 / (x - 1)] + Round[1 / (x - 1)]],
  Abs[(x * Round[2 / (x - 1)] - (Round[x * 2 / (x - 1)])) /
    (Round[x * 2 / (x - 1)] + Round[2 / (x - 1)])]], {x, 1, 1.5}]
```

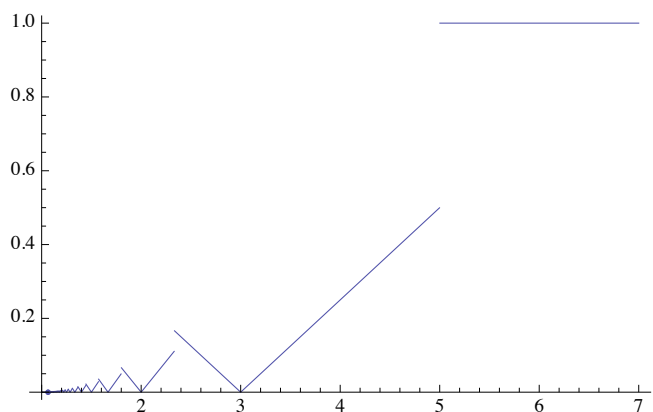


The 1st convergents 'flatline' at $x = 5$. The 2nd at $x = 5$. In general the $(p - q)$ 'th convergent terminates at $(2(p - q) + 1)$. Its value from hereon in will always be unity. This appears to put a limiting range upon the intervals.

```
Plot[Abs[(x * Round[1 / (x - 1)] - (Round[x * 1 / (x - 1)])) /
  (Round[x * 1 / (x - 1)] + Round[1 / (x - 1)])], {x, 1, 4}]
```



```
Plot[Abs[(x * Round[2 / (x - 1)] - (Round[x * 2 / (x - 1)])) /
  (Round[x * 2 / (x - 1)] + Round[2 / (x - 1)])], {x, 1, 7}]
```



Example: Listing the WC's of the tempered major seventh $2^{(11/12)}$ =
 1 . 8 8 7 7 . . . g i v e s :

```
List[
  Round[(2^(11/12)) * Range[10] / ((2^(11/12)) - 1)] / Round[Range[10] / ((2^(11/12)) - 1)]
  {2, 2, 2, 2, 9/5, 11/6, 13/7, 15/8, 17/9, 19/10, 21/11}]
```

Here are the 5th, 6th and 7th WC's:

```
Plot[{Abs[(x * Round[5 / (x - 1)] - (Round[x * 5 / (x - 1)])) /
  (Round[x * 5 / (x - 1)] + Round[5 / (x - 1)])], Abs[
  (x * Round[6 / (x - 1)] - (Round[x * 6 / (x - 1)])) / (Round[x * 6 / (x - 1)] + Round[6 / (x - 1)]),
  Abs[(x * Round[7 / (x - 1)] - (Round[x * 7 / (x - 1)])) /
  (Round[x * 7 / (x - 1)] + Round[7 / (x - 1)])]], {x, 1.5, 2}]
```

