Five is Not an Odd Number:

An Exploration of the Benefits of Equal Divisions of the Octave That Are Divisible by 5

By Igliashon Jones

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Contact the author at igliashon.jones@gmail.com

http://www.cityoftheasleep.com

Introduction

For those who assess a metatuning's suitability based on its ability to approximate low-limit Just Intonation intervals, most EDOs that are multiples of 5 (henceforth referred to as "5*n*-EDOs") certainly must seem unsuitable. Not until 35-EDO is the 3/2 "perfect fifth" approximated with less than 20 cents (¢) of error, and even then, it is still 15¢ flat. However, if one is willing to accept a rather wide 3/2 approximation, 5*n*-EDOs offer excellent approximations of many other low-limit JI intervals, especially in the 7-, 11-, and 13-limit; some even offer improvements over 12-EDO in approximating 5-limit intervals! Perhaps more importantly, they have a variety of very *practical* benefits in the ways that they can be applied to musical instruments, benefits which are unmatched by any other family of metatunings.

One of the most popular ways for musicians to get access to alternative metatunings is by refretting guitars (or other fretted stringed instruments). Most alternate divisions of the octave pose unique challenges in how a guitar ought to be tuned, since the standard "4th-4th-4th-Major 3rd-4th" relationship between the strings only works for metatuning systems that closely approximate the 3/2 perfect fifth (*i.e.*, only 12-, 17-, 19-, 22-, 24-, and 26-, and 29-EDO). Most alternate EDOs require the adoption of some rather unfamiliar and more complex open-string tunings, but 5*n*-EDOs are the only ones that allow a *simpler* tuning than the standard. This is because all 5*n*-EDOs have an interval of 480¢, which is exactly 1/5 of 2400¢. What this means is that one can tune every string of the guitar 480¢ apart from its neighbors and the lowest and highest string will come out exactly two octaves apart, for a tuning of 0-480-960-1440*(240)*-1920*(720)*-2400*(0)*¢.

The symmetry and simplicity does not end at guitar tunings, either. All 5*n*-EDOs greater than 10-EDO contain a 10-note scale having 5 large steps (*L*) and 5 small steps (*s*), often referred to as the "Blackwood" scale in honor of Easley Blackwood, the first composer to have recorded music in it. The Blackwood scale has only two modes: Major (*LsLsLsLsLs*) and Minor (*sLsLsLsLsL*). Though it looks like a relative of the familiar diminished or octatonic scale, it behaves quite differently. It can be thought of as two closed cycles of five 720¢ "fifths", offset from each other by a major or minor third[1]. Thus, every note in one cycle will be the tonic for a major triad, and every note in the other cycle will be the tonic of a minor triad. This property is unique to this scale; any other non-chromatic scale of 10 or fewer notes will have at least one break in the cycle of fifths and so will have at least one dissonant triad (or else it will not be an octave-repeating scale). When applied to a 5*n*-EDO guitar in the above-described tuning, this scale is a marvel of simplicity, as it has the same finger positions on each string. This substantially softens the curve when learning to play a xenharmonic guitar, even on one fretted to 30-EDO.

A common reaction among composers who first encounter this scale is to presume it is utterly at odds with tonal music; as it lacks the asymmetry of the diatonic scale, it is presumed that it lacks a feeling of tonal center. However, this 10-note scale actually contains 10 possible 7-note subsets which approximate the structure of the diatonic scale. Consider the example of the 20-EDO Blackwood major scale, 0-180-240-420-

1 See Appendix 2 for visual representations of scales mentioned in this paper.

480-660-720-900-960-1140-1200¢. If one omits the 240¢, 660¢, and 960¢ intervals, what remains is a 7-note scale of 0-180-420-480-720-900-1140-1200¢, which bears close resemblance to the 12-EDO major scale of 0-200-400-500-700-900-1100-1200¢, though the Blackwood version has three step-sizes (60¢, 180¢, and 240¢) rather than two. Such a scale can be constructed on any major degree of the Blackwood scale, and an analogous minor scale can be constructed on any minor degree. So rather than being at odds with tonal music, the Blackwood scale is actually quite *favorable* to it. Perhaps even *more* favorable, since one scale yields 10 possible tonal centers and allows root motion by a (highly tempered) fifth *from* any degree, in either direction—i.e., no matter where you are in the scale, it is always possible to go up or down by a fifth.

Finally, another benefit of most 5*n*-EDO metatunings is that they are easy to retune to, since (with a few exceptions) their intervals are all multiples of ten cents. Many softsynths and some hosts (such as Apple Logic) allow custom retuning of individual notes within a 12-note set, but only to within one cent of accuracy. This hinders experimentation with 12-note subsets of many larger metatunings because most have intervals that cannot be represented with integer values of cents. In most of the practically-sized 5*n*-EDO metatunings, the intervals all fall on a whole number of cents. Even better, the 10-note Blackwood scale, because of its uniform pattern, maximizes the amount of available keys. Even when limited to a 10-note subset on a retuned softsynth, the composer still has access to 5 distinct Major and 5 distinct Minor keys. This makes experimentation with subsets of these metatunings far less limiting than it is in other metatunings, where the number of notes in the subset will always be fewer than the number of possible tonal centers.

Let us now examine the harmonic and melodic properties of various individual 5*n*-EDO metatunings.

5-EDO

0-240-480-720-960-1200¢

5-EDO is the base scale from which all 5*n*-EDOs are derived. It consists of a slightly sharp 8/7, a very flat (but still functional) 4/3, a very sharp (but still functional) 3/2, and a slightly flat 7/4. It is generally considered to be the smallest EDO that can produce pleasant music, though it is not very versatile. The 240¢ interval seems to function as both a sharp major 2nd and a flat minor 3rd, being almost exactly halfway between 200 and 300¢; yet the 960¢ interval, because of its closeness to the 7th harmonic, feels unambiguously like a flat minor 7th. Overall, the scale has a pleasant, slightly tropical feel, being somewhat similar to the "slendro" scale found in certain species of gamelan music.

10-EDO

0-120-240-360-480-600-720-840-960-1080-1200¢

10-EDO is a fascinating metatuning that is very strong in the 13-limit. It can be thought of two chains of 5-EDO offset by a 360¢ neutral third, which is within half a cent of a Just 16/13, the reciprocal of the 13th Harmonic (also represented in 10-EDO with equal accuracy by the 840¢ interval). The 120¢ and 1080¢

intervals are within 1¢ of 15/14 and 28/15, the septimal minor second and major seventh. Coupled with 5-EDO's excellent approximation of the 7th Harmonic, some very unusual but very consonant harmonies are possible, such as an approximate 7:8:13 triad.

Melodically, 10-EDO contains within it a quasi-diatonic 7-note subset of three 240¢ intervals (*L*) and four 120¢ intervals (*s*), which when arranged as *LssLsLs* produces "consonant" neutral triads on the I, III, IV, V, and VI degrees and diminished triads on the II and VII. Applied to a 10-EDO guitar with strings tuned in 480¢ intervals, this scale provides a very familiar shape, feeling much like a relative of the 12-tone major scale. Because of the neutral intervals, this scale has a mysterious and slightly sinister feel, which makes an interesting counterpoint to the calm simplicity of 5-EDO.

15-EDO

0-80-160-240-320-400-480-560-640-720-800-880-960-1040-1120-1200¢

Harmonically, 15-EDO offers some interesting intervals. It is a strong 11-limit metatuning: the 11th Harmonic and its reciprocal 16/11 are approximated within 9¢ of Just by the 560¢ and 640¢ intervals, and 11/10 and 20/11 are approximated within 5¢ of Just by the 160¢ and 1040¢ intervals (respectively). The 5/4 major third and 8/5 minor sixths are approximated respectively by familiar 400¢ and 800¢ intervals, as in 12-EDO, but the 320¢ minor third and 880¢ major sixth are within 5¢ of a Just 6/5 and 5/3 (respectively)—an improvement over 12-EDO.

In 15-EDO, we find the first appearance of the 10-note Blackwood scale, with step sizes of *L*=160¢ and *s*=80¢. Melodically, the 160¢ "neutral second" is a fairly weak interval, as it is too narrow to act as a whole tone but too wide to act as a semitone. Compared to the higher 5*n*-EDOs, melodies in the 15-EDO Blackwood scale seem to have an almost "goofy" feel because of the instability of the 160¢ interval. For those who season their music with a dash of comedy, 15-EDO can be great fun because of this. For dramatic or "serious" compositions in 15-EDO, percussive and inharmonic timbres seem to work best, at least in the author's experience. Some sources[2] suggest a scale of *LLLLLLLs* as a tonal basis for 15-EDO, where *L*=160¢ and *s*=80¢. In this scale, six degrees act as tonics to consonant triads, and one triad has both major and minor thirds. However, because the 160¢ interval seems so melodically weak, it is the author's opinion that the Blackwood scale is a better tonal basis for 15-EDO.

20-EDO

0-60-120-180-240-300-360-420-480-540-600-660-720-780-840-900-960-1020-1080-1140-1200¢

20-EDO can be thought of as a combination of 10-EDO and 15-EDO, since 20 is twice 10 and so contains it, but also has an unequal 10-note scale like that of 15. Harmonically, 20-EDO approximates the 11th Harmonic and its reciprocal almost as closely as 15, though it is around 11¢ flat rather than 9¢ sharp. Since 20

2 "The Pentadecaphonic System". http://www.inteas.com/Penta02.htm. Accessed 3/11/10.

contains 5-EDO and 10-EDO, it contains their excellent approximations of the 7th and 13th Harmonics, allowing for some excellent Harmonic Series-based chords (like an approximate 7:8:11:13 tetrad). It also offers approximations of the 10/9 minor whole tone and 9/5 minor seventh (within less than 3¢ of Just, at 180¢ and 1020¢ intervals, respectively). The major third of 20-EDO is very sharp, being about 2.5¢ from a 14/11 and not approximating 5/4, so its minor sixth is flat and closer to 11/7 than 8/5. Also, since 20 is a multiple of 4, 20-EDO contains 4-EDO, which is a diminished chord of 0-300-600-900-1200¢. Thus, the minor third and major sixth are the same as in 12-EDO.

Melodically, 20-EDO performs admirably. The Blackwood scale is generated by an *L* of 180¢ and an *s* of 60¢, so the semitone is one-third of a whole-tone. This is considered an ideal ratio by some theorists[3], and is one of the reasons a metatuning like 17-EDO is considered to be very strong from a melodic standpoint. Interestingly, the 180¢ major second, consonant in and of itself, is actually dissonant in this metatuning when played over a full triad, because it is 540¢ from the 720¢ fifth; this makes for some interesting compositional challenges when composing a melody over a chord progression. Beyond the Blackwood scale, 20-EDO has a few other scales that may be of interest: since it contains 10-EDO, the 7-note "neutral diatonic" is available, and there is also a 7-note "quasi-equal" scale composed of six 180¢ steps and one 120¢ step. The octatonic scale is also present, in two flavors: one in which *L*=180¢ and *s*=120¢, and another in which *L*=240¢ and *s*=60¢

25-EDO

0-48-96-144-192-240-288-336-384-432-480-528-576-624-672-720-768-816-864-912-960-1008-1056-1104-1152-1200¢

Very close to the quarter-tone system of 24-EDO, 25-EDO manifests a rather different harmonic palette. 25 approximates a variety of low-limit Just intervals very closely, but still misses the 3/2 by the same amount as its smaller siblings. Of particular note is the 5/4 major third (5th Harmonic) and its complement the 8/5 minor sixth, approximated within 2¢ of Just by the 384¢ intervals. The 432¢ and 768¢ intervals are within 3¢ of the 9/7 septimal major third 14/9 septimal minor sixth, and the 13/11 tridecimal minor third and 22/13 tridecimal major sixth are both represented within 2¢ of Just by the 288¢ and 912¢ intervals. 25-EDO offers 2 tritones, at 576¢ and 624¢, which represent 7/5 and 10/7 within 7¢ of Just. The 192¢ interval is essentially a "meantone"[4] major second halfway between 9/8 and 10/9. With the excellent representations of 5th and 7th Harmonics, and the passable representation of the 9th, near-Just 4:5:7:9 tetrads are possible. However, the 11th and 13th Harmonics are *not* well-represented at all, so 25-EDO is much worse than its siblings at producing more xenharmonic harmonies, with the exception of the 144¢ and 1056¢ intervals which miss 12/11 and 11/6 by about 6¢.

Melodically, 25 offers a variety of possibilities. It offers two versions of the Blackwood 10-note scale, one where the *L*=192 and *s*=48 (referred to here as "Unfair" because the *L* is so much larger than the *s*), and one

3 Secor, George. "The 17-tone Puzzle — And the Neo-medieval Key That Unlocks It". http://xenharmony.wikispaces.com/space/showimage/17puzzle.pdf. Accessed 3/11/10.

4 **Meantone** refers to an interval approximately halfway between a Just 9/8 (~203¢) and 10/9 (~182¢).

where *L*=144 and *s*=96 (referred to here as "Fair" because of the closeness in size of the *L* and *s* intervals). In the Unfair scale, triads will have the 432¢ major third and the 288¢ minor third; in the Fair scale, they will have 384¢ major thirds and 336¢ minor third. Though the Fair and Unfair Blackwood scales are similar in structure, they sound *very* different. The Fair scale is exceedingly *consonant* harmonically, yet very *dissonant* melodically, which nearly obviates the harmonic consonance. On the other hand, the Unfair scale is quite dissonant harmonically (owing to the near-9/7 major third), but is fairly consonant melodically (the sub-quarter-tone that acts as a semitone is not very effective, but at least the whole-tone is a true whole-tone).

There are also some more xentonal possibilities in 25-EDO, such as the 7-note “antimajor” scale of *ssLsssL*, where *s*=144 and *L*=240. This scale, which is found also in 16-EDO with slightly different interval sizes, has a VERY flat fifth of 672¢, with major thirds of 384¢ and minor thirds of 288¢. This scale is sort of a bizarro mirror-image of the common-practice major scale of *LLsLLLs,* such that on every degree that the major scale forms a major triad, the antimajor forms a minor (and vice-versa; likewise, the VII degree is augmented in the antimajor, rather than diminished as in the major). However, the lack of a true semitone makes melodies very difficult, and the 672¢ fifth, though only 2¢ further from a Just 3/2 than the 720¢ fifth, is considerably less tolerable.

30-EDO

0-40-80-120-160-200-240-280-320-360-400-440-480-520-560-600-
640-680-720-760-800-840-880-920-960-1000-1040-1080-1120-1160-1200¢

Being twice 15 and three times 10, 30-EDO of course has all the properties of both. This means it approximates the 7th, 11th, and 13th Harmonics and their reciprocals, as well as 11/10, 6/5, 5/3 and 20/11, very well. The 9/7 septimal major third and 14/9 septimal minor sixth are approximated within 5¢ of Just by the 440¢ and 760¢ intervals. Being divisible by 6, 30-EDO contains the Whole Tone scale of 0-200-400-800-1000-1200¢. This grants an excellent representation of the 9th Harmonic as well, allowing for a good representation of the 7:9:11:13 tetrad. Though it does not do so as well as 25-EDO, 30-EDO also represents the 13/11 tridecimal minor third and 22/13 tridecimal major sixth, within about 10¢ of Just, at 280¢ and 920¢.

Like 25-EDO, 30-EDO also has two versions of the 10-note Blackwood scale, one where *L*=200 and *s*=40, and one identical to that of 15-EDO. Melodically, 30-EDO's proprietary Blackwood scale, with an *L:s* ratio of 5:1, is not very useful, as 40¢ too narrow to function effectively as a semitone. Like 25, it also contains a version of the antimajor scale, having step sizes *L*=200, *s*=160. In 30-EDO, this scale is slightly more consonant, as the fifth is now a 680¢ interval which is about as flat from a Just 3/2 as the 720¢ fifth is sharp. However, the major and minor thirds in this scale are very close, at 360 and 320¢, respectively; as such, this scale can almost be considered a “quasi-equal heptatonic”, as opposed to a true antimajor.

35-EDO and Above

At 35-EDO, we lose the 5*n*-EDO property of having every interval be expressible in whole numbers of cents. Since this loss is not accompanied by any harmonic or melodic improvements, it hardly seems worth devoting space in this paper to such a large and complex tuning. Beyond 35, the number of notes per octave becomes quite unmanageable, and neither harmonic nor melodic properties are significantly improved (from a Blackwood standpoint). In fact, 5*n*-EDOs from 40 on seem to work better for meantone-type music than for Blackwood. However, for reference purposes, a table of interval sizes of 5*n*-EDOs from 35 to 50 is listed in Appendix 4.

Comparisons

Each of these metatunings has its own unique strengths and weaknesses, though they have much in common. Here is a brief overview of what the author feels to be the stand-out aspects of each metatuning in comparison to its siblings:

5-EDO: Simplicity is really the only thing this metatuning has over the others, but this should not be taken lightly. It is well-suited to percussive instruments and would be excellent if applied to instruments for non-musicians (like children, the mentally-handicapped, or those who lack the patience to learn more developed musical systems). It is also a good first scale for those new to instrument-building.

10-EDO: Only marginally more complex than 5, 10 offers many more resources. The 7-note neutral scale can function tonally, and a near-Just 13th Harmonic enters the intervallic vocabulary. It has a dark and alien mood, unlike 5's whimsical brightness or 15's goofiness. 10 does not represent the 5-limit *at all*, so it is much more xenharmonic than 15-EDO. It is a good choice for musicians seeking unusual new sounds but who don't want to deal with the complexity of more than 12 notes in an octave.

15-EDO: Compared to 10-EDO, 15 sounds very normal, as it represents 5-limit thirds better than even 12-EDO. Compared to 20, 15 represents the 5th and 11th Harmonics much better but does not represent the 13th at all; it is possibly the least xenharmonic of all the 5*n*-EDOs. However, it *is* one of the most "xenmelodic", as it misses a traditional whole-tone by 40¢ in either direction. Using an 11/10 neutral second or and 8/7 augmented second over 5-limit harmonies sounds very strange indeed, and in this regard, 15 is the most unusual of the bunch. Its mood, in the author's opinion, seems somewhat goofy, especially when it is pushed in a direction contrary to its tendencies (i.e. when one attempts common-practice music with it). It is well-suited to the guitar and keyboard (even a standard keyboard can play in 15 with little trouble). It is excellent to facilitate transition into the world of xentonality, for while it has a very simple structure and familiar harmonies, it is so at odds with the common practice that it forces the player out of familiar patterns.

20-EDO: In the author's humble opinion, 20 is one of the most overlooked tunings among the community of microtonalists, as it is often overshadowed by its neighbors 19- and 22-EDO. 19 and 22 both offer *excellent* 5-limit and 7-limit harmonies and very acceptable 3/2's, whereas 20 barely approximates any 3- or 5-limit sonorities. Because 20 has little to offer in low-limit JI approximations (even compared with 15-EDO), most microtonalists avoid it without ever even having heard it. Of the few compositions that do exist in 20-EDO, most seem to have been written in ignorance of Blackwood tonality and obscure the true strengths of this metatuning. This is a great shame, because 20-EDO *vast* improvements over 15-EDO in melody. With a near-Just 10/9 whole-tone (of which its semitone is 1/3 the size), melodies sound very pleasant and somewhat brighter than in 15 (and more *serious* as well). As a result, the more out-of-tune harmonies can be forgiven, especially with appropriate chord voicings. In the author's experience, 20-EDO is the "sweet-spot" of 5*n*-EDOs, where harmonic quality and melodic quality are most balanced. Note that though 20 is significantly more notes to have per octave than 12, the extra notes are easy to incorporate because of the simple structure of Blackwood tonality. Consequently, 20 may be easier to navigate than either 19 or 22, and its xentonal and xenharmonic aspects are more accessible than those in either of the more popular tunings. A 20-EDO guitar will be only marginally more difficult to adapt to than one in 15-EDO. Also, for those that enjoy the mood of 10-EDO but want more versatility, 20 makes an excellent choice.

25-EDO: where 20-EDO looks bad on paper but sounds good in practice, 25-EDO looks good on paper but sounds unpleasant in practice. Harmonically, it is very good, but in the framework of Blackwood tonality harmonic improvement always entails melodic deterioration. So even though 25-EDO offers excellent 4:5:7:8 chords in the Fair Blackwood scale, melodies played *over* those chords sound very dissonant. In the Unfair Blackwood scale, chords are more dissonant—or at least more xenharmonic—yet melodies sound acceptable. Compared to 20-EDO and even 15-EDO, 25 is weaker overall, because it compromises melody more severely: either the whole tone is too small and is strongly dissonant (in Fair Blackwood), or else the semitone is not large enough and is actually less than a quarter-tone (in Unfair Blackwood). Those looking for low-limit JI approximations will have better results with 31- or 22-EDO; those looking to explore quarter-tonal melodies, however, may have better luck with 25 than 24-EDO, as 25 integrates quarter-tones into a tonal structure where 24 does not. Alternatively, 25-EDO may actually be good for hyperchromatic jazz-type music, where a rigid tonal structure is unnecessary...though such hyperchromaticism may be difficult to apply in practice.

30-EDO: 30 can be looked at as being a more xenharmonic version of 25-EDO, since it mimics 25's melodic properties but offers 9-, 11-, and 13-limit harmonies instead of 5-limit ones. It also has all the tonal possibilities of 10-EDO and 15-EDO combined, for it contains both. The drawback is that with the steps so close together, it is a bit difficult to navigate. It technically has more scalar possibilities than 25, though it is questionable whether the 40¢ interval—1/5 of a whole tone—is large enough to function melodically. Note that while 30-EDO is close to 31-EDO (a meantone-like metatuning popular for its many near-Just intervals), it has little in common with it and would not work well at all for purposes that 31 is used for. However, on a guitar (or other fretted stringed instrument), it will be a bit easier to navigate than 31 because of the symmetric scalar structures and uniform tuning.

Conclusions

Though 5*n*-EDO metatunings perform poorly in approximating 3-limit JI, this should not be a deterrent to exploring them. As a family, 5*n*-EDO metatunings have many unique advantages that cannot be found in any other metatunings. They are easy to explore: their uniform scale shapes and guitar tunings make for less of a learning curve than other metatunings, and their intervals are represented in whole numbers of cents, which makes them easy to approximate on synthesizers with limited retuning accuracy. All 5*n*-EDOs offer an excellent approximation of the 7th Harmonic, and most of them offer excellent approximations of Just intervals up to the 13-limit (excluding the 3-limit). The fascinating Blackwood scale, found only in 5*n*-EDOs, maximizes both the number of possible tonal centers in a given key and the number of degrees from which root motion by a fifth is possible. All things considered, 5*n*-EDOs are excellent metatunings for those looking for practical and tonal alternatives to common practice music and so merit further exploration.

Appendix 1: Table of Just Intervals

Ratio	Cents Value	Name
15/14	119.44	Septimal Minor Second
11/10	165.00	Undecimal Neutral Second
10/9	182.40	Minor Whole Tone
9/8	203.91	Major Whole Tone
8/7	231.17	Septimal Major Second
13/11	289.20	Tridecimal Minor Third
6/5	315.64	Just Minor Third
16/13	359.47	Tridecimal Neutral Third
5/4	386.31	Just Major Third, 5th Harmonic
14/11	417.51	Undecimal Major Third
7/9	435.08	Septimal Major Third
4/3	498.04	Just Perfect Fourth
11/8	551.32	Undecimal Augmented Fourth, 11th Harmonic
7/5	582.51	Septimal Tritone
10/7	617.49	Euler's Tritone
16/11	648.68	Undecimal Diminished Fifth
3/2	701.96	Just Perfect Fifth
14/9	764.92	Septimal Minor Sixth
11/7	782.49	Undecimal Minor Sixth
8/5	813.69	Just Minor Sixth
13/8	840.53	Tridecimal Neutral Sixth, 13th Harmonic
5/3	884.36	Just Major Sixth
22/13	910.79	Tridecimal Major Sixth
7/4	968.83	Septimal Minor Seventh, 7th Harmonic
16/9	996.09	Pythagorean Minor Seventh
9/5	1017.60	Just Minor Seventh
20/11	1035.00	Undecimal Neutral Seventh
28/15	1080.55	Septimal Minor Seventh

Author's Note on Just Intervals: I don't tend to evaluate metatunings on how well they approximate particular JI intervals, but rather based on how they sound when I play in them. If I'd never written music in 15, 20, or 10, or 25-EDO, I'd never have written this paper, because from a JI standpoint they don't seem very appealing. To me, the "stability" or harmonic simplicity of a metatuning's intervals is only of importance when, say, playing those intervals on a heavily-distorted guitar, when the clashing of mismatched partials can become problematic. It will be interesting to see these metatunings perform under such circumstances.

Appendix 2: Harmonic Lattices of Selected Scales

Harmonic lattices are a useful way of representing harmonic relationships between degrees of a scale. Typically, they are two-dimensional, with the horizontal axis representing movement by fifths and the vertical axis representing movement by thirds. Consider the familiar Diatonic scale (numbers represent degrees in the Major scale):

```
...-2-6-3-7
    \/\/\/\
     4-1-5-2-6-3-7
           \/\/\/\
            4-1-5-2-...
```

Note that a backslash ("\") represents movement by a minor third and a forward slash ("/") represents movement by a major third. So we can see that the VII degree is a diminished triad, because it has no fifth but can have two minor thirds stacked on top of it.

Now, in contrast, take a look at how the Diminished/Octatonic scale is represented:

```
...
 \/\
  7-4
   \/\
    5-2
     \/\
      3-8
       \/\
        1-6
         \/\
          7-4
           \/\
            ...
```

Notice how there are no chains of fifths, and that there are basically two repeating chains of minor thirds offset from each other by a fifth.

Next, consider the antimajor:

```
            4-1-5-2-...
           /\/\/\/
     4-1-5-2-6-3-7
    /\/\/\/
...-2-6-3-7
```

Notice that it is exactly a mirror image of the diatonic lattice, so that every major degree in the diatonic is minor in the antimajor (and vice-versa); the VII degree here is thus augmented instead of diminished, as well.

Lastly, examine the Blackwood scale (10th degree is represented by "0"):

```
 ...-1-7-3-9-5-1-7-3-9-5-...
    /\/\/\/\/\/\/\/\/\/\
...-8-4-0-6-2-8-4-0-6-2-8-...
```

Note that the top and bottom chains of fifths repeat indefinitely, and that every degree can be the tonic of a consonant triad.

Appendix 3: Glossary of Selected Terms

Cent (¢): a unit of measurement of pitch-distance, defined as one twelve-hundredth of an octave. Each semitone in 12-EDO is 100¢.

Consonant: a slippery adjective that generally refers to "a harmony which is pleasant to the ear." In this paper, when used to describe a triad, it means that the triad sounds close to the familiar "root-third-fifth" of common-practice music, or more specifically that the triad has a reasonable approximation of the 3/2; *see* "Dissonant". Not to be confused with the type of letter, i.e. a non-vowel.

Diatonic: strictly-speaking, "Diatonic" refers to a genus of Greek tetrachords, wherein the 4/3 "perfect fourth" is divided into four notes and the largest interval between notes is close to a "major second" (usually a 9/8 or an 8/7). In common practice, it refers to a 7-note scale of five large steps (L) and two small steps (s), wherein the small steps are maximally separated from each other.
In this paper, it means something between the two definitions: it refers to any scale composed of x large steps and y small steps, such that $L(x)+s(y)$=1200¢, and 150¢<L≤250¢ & 40¢<s≤150¢. Typically, 1.5s≤L≤4s, but there may be exceptions.

Dissonant: another slippery term that generally refers to "a harmony which is unpleasant to the ear." In this paper, it typically refers to any chord that lacks a reasonable approximation to the 3/2; as an example, a chord composed of two consecutive approximate-minor-thirds would be considered "dissonant". However, this is a very loose definition, as it is certainly possible (at least in the author's experience) for a chord composed in such a way to be consonant, especially if it approximates a low-limit JI relationship such as 5:6:7. It is used here merely to conveniently differentiate the two types of triad typically found in diatonic-like scales.

EDO: short for "Equal Division of the Octave", meaning that an n-EDO divides the octave into n equal parts.

Enharmonic: short for "Enharmonic Equivalent", meaning a note that sounds the same pitch as another note, even though it is designated with a different name or "spelling". An example from 12-EDO would be "C#" and "Db", or "B#" and "C".

Just Intonation (JI): "Just Intonation" refers to a family of metatunings wherein all pitch relationships are described by integer frequency ratios.

Metatuning: a "metatuning" refers to any set or subset of pitch relationships related to some arbitrary fundamental pitch. This subset can be defined arbitrarily, ranging from including ONLY the fundamental pitch to containing ALL possible pitches. 12-EDO is a metatuning defined as "all pitches that are multiples of onetwelfth of an octave from the fundamental." Typically in common practice, the fundamental pitch of 12-EDO is defined as "A=440 Hz", but the metatuning does not define the fundamental, only the relationships other pitches should have to it.

***n*-Limit:** an "*n*-Limit" refers to the largest prime factor found in either of the integers that make up the frequency ratio of a Just Intonation interval. Consider an interval such as 32/25: this is a "5-Limit" interval, because 32 has only the number 2 as its prime factor but 25 has the number 5 (32=2*2*2*2*2, 25=5*5), so 5 is the highest prime factor found in the interval. Similarly, 15/14 would be a "7-Limit" interval because 15=3*5 but 14=7*2, so 7 is the highest prime factor found in the interval.

Neutral: "neutral" refers to any musical interval that is both too sharp to be heard as a "minor" interval and too flat to be heard as a "major" interval. It is typically used to designate pitches close to either 150¢, 350¢, 850¢, or 1050¢.

Scale: simply, a "scale" is any ordered subset of a given metatuning or tuning, according to which sounds can be organized to produce music.

Tuning: a "tuning" refers to a subset of pitches from a metatuning, as related to a specified fundamental pitch, which are chosen to be applied to a musical instrument. In the case of some instruments, such as a piano or a marimba, the subset may consist of the entire set of pitches in the metatuning. In the case of instruments such as a guitar or a violin, a "tuning" refers to the pitches that should be sounded by the open strings; on such instruments, it is the *intonation* that defines any other pitches that will be sounded. Consider that a standard 12-EDO guitar may be *tuned* according to any metatuning, but will always intonate relative to 12-EDO on each string. Also, the term "alternate tuning", though often taken to mean "alternate metatuning" by those in the microtonal community, is commonly used by guitarists to refer to any tuning of a 12-EDO guitar *other than* EADGBE. Hence, "alternate tuning" is not a single-denotative term that refers to non-12-EDO systems of pitch relationships, which is why I insist on the term "alternate metatuning".

Xenharmonic: coined by Ivor Darreg, this word is a portmanteaux of "xenos" (Greek for "foreign" or "alien") and "harmonic". It is used to describe any harmony which sounds different from common-practice 12-EDO.

Xenmelodic: coined by the author (so far as he is aware), after the fashion of Darreg's use of "xenharmonic". It is used to describe any *melody* which sounds different than any melody possible in common-practice 12-EDO.

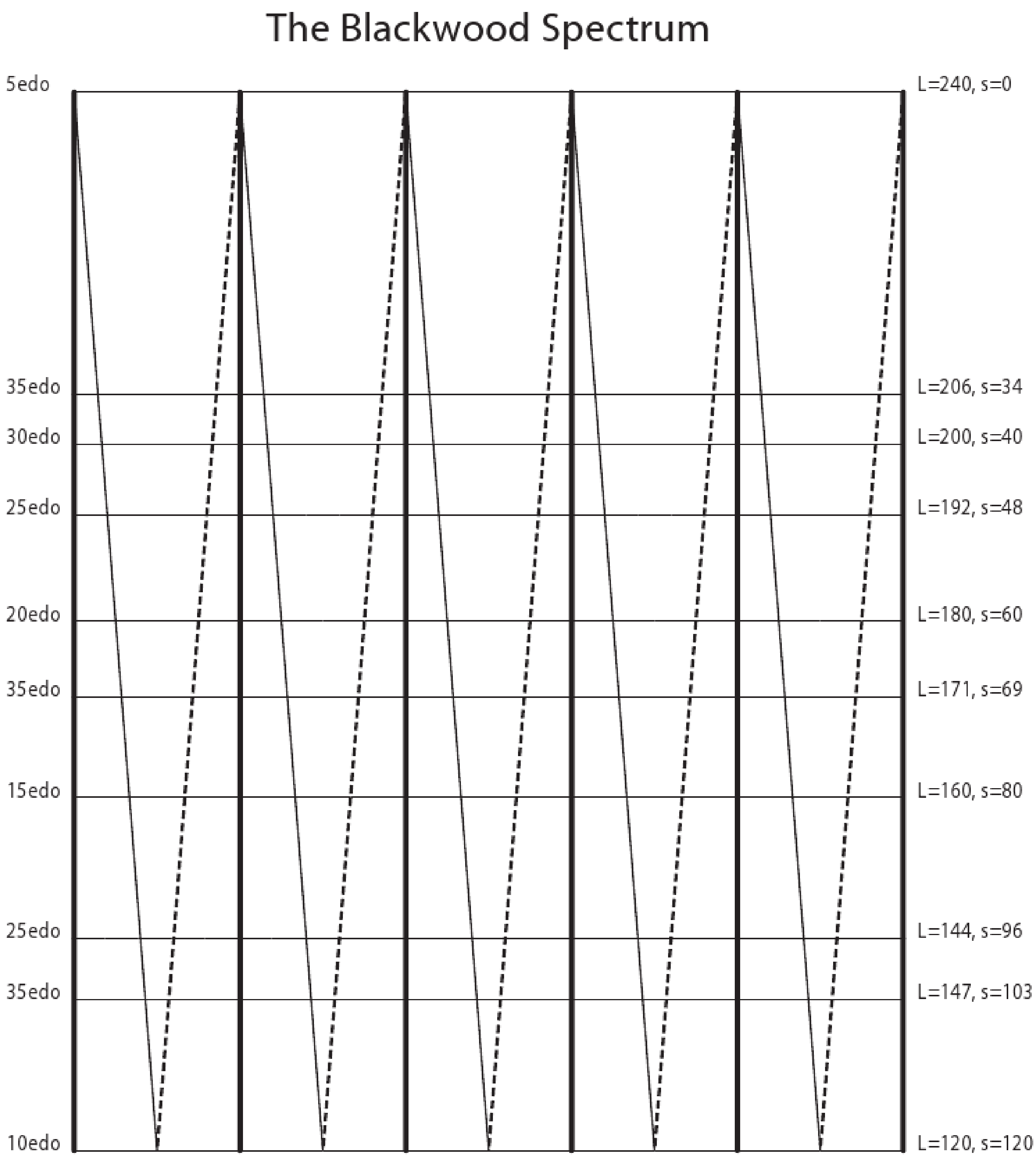
Xentonal: also coined by the author (so far as he is aware). Used to describe any system of pitch organization that sounds distinctly *tonal* (i.e. has a strong feeling of tonal center) but also produces xenharmony and/or xenmelody.

Appendix 4: List of Intervals in 35-, 40-, 45-, and 50-EDO

35-EDO (7x5)	40-EDO (8x5) (10x4) (20x2)	45-EDO (9x5) (3x15)	50-EDO (10x5) (25x2)
34.286	30	26.667	24
68.571	60	53.333	48
102.857	90	80	72
137.143	120	106.667	96
171.429	150	133.333	120
205.714	180	160	144
240	210	186.667	168
274.286	240	213.333	192
308.571	270	240	216
342.87	300	266.667	240
377.143	330	293.333	264
411.429	360	320	288
445.714	390	346.666	312
480	420	373.333	336
514.286	450	400	360
548.571	480	426.667	384
582.857	510	453.333	408
617.143	540	480	432
651.429	570	506.667	456
685.714	600	533.333	480
720	630	560	504
754.286	660	586.667	528
788.571	690	613.333	552
822.857	720	640	576
857.143	750	666.667	600
891.429	780	693.333	624
925.714	810	720	648
960	840	746.667	672
994.286	870	773.333	696
1028.571	900	800	720
1062.857	930	826.667	744
1097.143	960	853.333	768
1131.429	990	880	792
1165.714	1020	906.667	816
1200	1050	933.333	840
	1080	960	864
	1110	986.667	888
	1140	1013.333	912
	1170	1040	936
	1200	1066.667	960
		1093.333	984
		1120	1008
		1146.667	1032
		1173.333	1056
		1200	1080
			1104
			1128
			1152
			1176
			1200

Appendix 5: The Blackwood Spectrum

(Courtesy of Andrew Heathwaite)



Thick solid lines show intervals of 5edo.
Dotted lines indicate Blackwood "major".
Thin solid lines indicated Blackwood "minor".