

Given:

$$S = A_1 \dots A_n \quad (\text{a sequence of consecutive steps in cents})$$

$$\forall x: A_x > 0$$

$$P = \sum_{m=1}^n A_m$$

$$G(x) = x - n \left\lfloor \frac{x-1}{n} \right\rfloor$$

The sum of deviations (measured as fractions of a period) between the multiset of intervals and their optimum equal spacing can be written as the following:

$$D(S) = \sum_{i=1}^n \sum_{j=1}^n \left| \frac{\sum_{k=0}^{j-1} (A_{G(i+k)})}{P} - \frac{j}{n} \right|$$

$$D(S) = \frac{1}{nP} \sum_{i=1}^n \sum_{j=1}^n \left| n \sum_{k=0}^{j-1} (A_{G(i+k)}) - jP \right|$$

Since S cannot contain any negative entries, the most unequal arrangement of steps requires all but one of the steps in the sequence to be infinitesimal. For the sake of demonstration, we can let these steps become 0. The remaining step can be any arbitrary positive quantity.

We can then determine the maximum value of the function $D(S)$ as a function of n . To keep things concise, I will skip the steps required to calculate $\max(D(S))$. (Feel free to derive it on your own!)

$$\max(D(S)) = \frac{n^2 - 1}{3}$$

Now if we want to express the degree of "unequalness" of a scale as a percent, we can divide $D(S)$ by its maximum attainable value and multiply by 100.

$$\%UE = \frac{100 D(S)}{\max(D(S))}$$

The degree of "equalness" of a scale as a percent then follows trivially.

$$\%E = 100 - \frac{100 D(S)}{\max(D(S))}$$

$$\%E = 100 - \frac{300}{n(n^2 - 1)P} \sum_{i=1}^n \sum_{j=1}^n \left| n \sum_{k=0}^{j-1} (A_{G(i+k)}) - jP \right|$$