Monte Carlo Portfolio Optimization

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Introduction

This report presents a Monte Carlo simulation study on portfolio optimization using three well-known assets: Apple Inc. (AAPL), Coca-Cola Co. (KO), and Tesla Inc. (TSLA). The primary objective is to examine how allowing or disallowing short selling affects the distribution of portfolio returns, volatility, and Sharpe Ratios. By simulating thousands of randomly weighted portfolios under both constraints, the study aims to visualize the efficient frontier and provide insights into the practical implications of investment strategies.

Data Summary

The historical data used for this analysis covers approximately one year of daily returns for the three selected assets. From the annualized statistics, TSLA clearly emerges as the most volatile and highest-returning asset, with an expected return of about 57.4% and a standard deviation exceeding 0.5. KO, on the other hand, provides relatively low but stable returns of 7.9%, while AAPL sits between the two with a 3.4% expected return and moderate risk. The covariance matrix reveals meaningful relationships across the assets. AAPL and TSLA share a strong positive covariance, indicating synchronized movements, whereas KO shows weak or even slightly negative correlations with TSLA, creating opportunities for diversification and risk mitigation.

Simulation Design

To evaluate the effect of different investment constraints, two Monte Carlo simulations were conducted, each generating 10,000 random portfolios. In the **long-only scenario**, each asset's weight was sampled from a uniform distribution between 0 and 1 and normalized so that the total portfolio allocation summed to one. This mirrors a traditional investment strategy in which only positive positions are allowed.

In the **short-selling scenario with constraints**, weights were drawn from a uniform distribution between -1 and 1, allowing for negative weights that represent short positions. After normalization to ensure the sum of weights equals one, each portfolio was tested to confirm that all individual weights remained within the [-1, 1] bound post-normalization. If a weight vector violated this constraint, it was discarded and resampled. This additional step prevented unrealistic leverage or exaggerated short exposure, maintaining the financial feasibility of the simulated portfolios.

Analysis of Simulation Results

The **long-only simulation** resulted in a clean and well-defined efficient frontier. The scatter plot displays a convex structure in the return-volatility space, showing that most feasible portfolios lie within a volatility range of 0.2 to 0.6 and achieve returns up to approximately 0.5. The gradient of Sharpe Ratios suggests that the most efficient portfolios cluster toward the upper-left frontier, where higher returns are achieved with relatively lower volatility.

By contrast, the **short-selling simulation with constraints** presents a more complex picture. Because of the imposed bounds on individual weights, the frontier remains largely stable and interpretable. Portfolios span a wider return range compared to the long-only case, but extreme outliers have been eliminated. Notably, high-return, high-risk portfolios driven by shorting volatile assets like TSLA remain possible, but are now tempered by the [-1, 1] weight limitation. The result is a balanced compromise between model flexibility and practical realism. The efficient frontier retains some curvature and depth, capturing the expanded opportunity set introduced by short selling, yet without devolving into distortion.

Interpretation and Implications

The findings from this simulation underscore the critical role of investment constraints in portfolio design. Allowing short sales increases the theoretical variety of portfolio outcomes but can quickly lead to infeasible or highly risky combinations if left unconstrained. The introduction of practical bounds on short positions proves effective in retaining the benefits of short selling while avoiding financial absurdities like infinite leverage or unstable return profiles.

Meanwhile, the long-only approach, while more conservative, offers a cleaner visualization of the efficient frontier and may better suit risk-averse investors or institutional stakeholders bound by regulatory frameworks. This dichotomy between flexibility and feasibility illustrates the importance of tailoring optimization models to investor profiles, regulatory contexts, and real-world implementability.

Conclusion

Monte Carlo simulation proves to be a versatile tool for exploring the landscape of portfolio returns and risks under varying assumptions. Through this analysis, it becomes clear that short selling, when managed with reasonable constraints, can enhance the opportunity set without introducing undue volatility. At the same time, the long-only approach remains a robust baseline for comparison, offering stability and simplicity. Ultimately, successful portfolio optimization demands not just mathematical rigor but also financial realism—and this balance is best achieved by thoughtfully applying constraints that mirror market realities and investor objectives.