

Robust Fault-tolerant Attitude Control of Three-axis Stabilized Satellite

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Abstract: The three-axis stabilized satellite's attitude control under repairable fault conditions is researched in this paper. With the consideration of rotational inertia uncertainty, a control effectiveness factor is used to describe the fault magnitude of satellite actuators. Moreover, the uncertainty of fault diagnosis results is also considered in the design of controller. And a feedforward control unit is introduced to decrease the influence of fault diagnosis uncertainty, Therefore the attitude controller's absolute dependence on fault diagnosis results is avoided ultimately. The H_∞ and H_2 norms are used to signify the performance indexes of attitude information and output torques. The robust fault-tolerant attitude control is achieved by combining the state feedback with a pole assignment method. Finally, the simulation results reveal that this controller can stabilize a three-axis stabilized satellite quickly with a high precision of stabilization and pointing.

Key Words: Fault-tolerant control; Pole assignment; Attitude Control

1 Introduction

The space environment is so harsh that there are many different kinds of satellite instruments fault^[1,2]. For example, the satellite's attitude control system can be out of control with actuator fault. Thus it's absolutely necessary to research on the fault-tolerant control of satellite's attitude control system.

Nowadays, several different control methods, such as sliding-mode control^[3], self-adaptive control^[3,4], PD control^[5], fuzzy control^[3], backstepping control^[4], H_∞ control^[6,7], and H_2/H_∞ control^[2,8,9], are already applied in the satellite attitude control area. However, these control methods usually don't consider the satellite rotational inertia uncertainty or the actuator fault. Moreover, the satellite attitude control accuracies before and after the fault have not been analyzed, so does the recovery ability of these controllers.

In this paper, a robust fault-tolerant controller is introduced based on the works of reference [6]. And this controller involves a control effectiveness factor which enables it to avoid the absolute dependence on fault diagnosis results. This controller can tolerate the system uncertainties and actuator fault and compensate timely fault diagnosis uncertainty. Ultimately, the simulation results demonstrate that this robust fault-tolerant attitude controller of three-axis stabilized satellite is both effective and efficient to deal with the actuator fault and rotational inertia uncertainty problems.

2 Problem Description

2.1 Attitude Control System without Rotational Inertia Uncertainty

The controlled object in this paper is a three-axis stabilized satellite. Its actuators are three orthogonally installed reaction flywheels, and its attitude sensors are gyros and star sensors. Without the consideration of rotational inertias' coupling, the attitude control model of this rigid satellite is below.

$$\begin{aligned} I_x \ddot{\phi} - \omega_0 (I_x - I_y + I_z) \dot{\psi} + 4\omega_0^2 (I_y - I_z) \phi &= u_x + T_{dx} \\ I_y \ddot{\theta} + 3\omega_0^2 (I_x - I_z) \theta &= u_y + T_{dy} \\ I_z \ddot{\psi} + \omega_0 (I_x - I_y + I_z) \dot{\phi} + \omega_0^2 (I_y - I_x) \psi &= u_z + T_{dz} \end{aligned} \quad (1)$$

The dynamical model above can also be transformed into the below matrix differential equation.

$$A_2 \ddot{\alpha} + A_1 \dot{\alpha} + A_0 \alpha = G_d T_d + G_u u \quad (2)$$

in which $T_d = [T_{dx}, T_{dy}, T_{dz}]^T$ is the disturbance torque, and it's mainly composed of aerodynamic torque and geomagnetic torque with the assumption of orbital altitude being approximately 500km. $\alpha = [\phi, \theta, \psi]^T$ is the Euler angle, $u = [u_x, u_y, u_z]^T$ is the control torque. And $I_b = \text{diag}([I_x, I_y, I_z])$ is the satellite's rotational inertia, in which

$$\begin{aligned} A_2 &= I_b, \quad G_d = G_u = I_{3 \times 3} \\ A_0 &= \omega_0^2 \begin{bmatrix} 4(I_y - I_z) & & \\ & 3(I_x - I_z) & \\ & & I_y - I_x \end{bmatrix} \end{aligned}$$

2.2 Attitude Control System with Rotational Inertia Uncertainty

The satellite parameter uncertainty in this paper is the rotational inertia, and this uncertainty is described as

$$(A_2 + E_{A_2} \Delta_{A_2} F_{A_2}) \ddot{\alpha} + (A_1 + E_{A_1} \Delta_{A_1} F_{A_1}) \dot{\alpha} + (A_0 + E_{A_0} \Delta_{A_0} F_{A_0}) \alpha = G_d T_d + G_u u \quad (4)$$

in which

$$E_{A_0} = \omega_0^2 \begin{bmatrix} \Delta I_y & 0 & 0 & 0 & -4\Delta I_z & 0 \\ 0 & 0 & 3\Delta I_x & 3\Delta I_z & 0 & 0 \\ 0 & \Delta I_y & 0 & 0 & 0 & -\Delta I_x \end{bmatrix}$$

$$E_{A_1} = \omega_0 \begin{bmatrix} 0 & \Delta I_x & 0 & \Delta I_z & 0 & \Delta I_y \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta I_x & 0 & \Delta I_z & 0 & \Delta I_y & 0 \end{bmatrix}$$

$$F_{A_0} = \begin{bmatrix} 4 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$F_{A_1} = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \end{bmatrix}^T$$

$$E_{A_2} = \text{diag}([\Delta I_x \quad \Delta I_y \quad \Delta I_z]), F_{A_2} = I_{3 \times 3}$$

$$\Delta_{A_2} = \text{diag}\{\delta_x, \delta_y, \delta_z\}$$

$$\Delta_{A_1} = \text{diag}\{\delta_x, \delta_x, \delta_z, \delta_z, \delta_y, \delta_y\}$$

$$\Delta_{A_0} = \text{diag}\{\delta_y, \delta_y, \delta_x, \delta_z, \delta_z, \delta_x\}$$

Suppose

$$G_d = \begin{bmatrix} E_{A_2} & E_{A_1} & E_{A_0} \end{bmatrix} \quad (5)$$

$$\tilde{d} = - \begin{bmatrix} \Delta_{A_2} F_{A_2} \ddot{\alpha} \\ \Delta_{A_1} F_{A_1} \dot{\alpha} \\ \Delta_{A_0} F_{A_0} \alpha \end{bmatrix} \quad (6)$$

Then we can transform the equation (4) into a matrix differential equation below.

$$A_2 \ddot{\alpha} + A_1 \dot{\alpha} + A_0 \alpha = G_d \tilde{d} + G_u u \quad (7)$$

The H_∞ norm is used to evaluate the influences of rotational inertia uncertainty and disturbance on attitude angle, attitude angular velocity and attitude accelerated angular velocity. A general block diagram of mixed H_2/H_∞ can be deduced as Fig.1.

below.

$$I_i = \bar{I}_i + \Delta I_i \delta_i, \|\delta_i\| < 1 \quad (3)$$

With the uncertainty the attitude control model equation can be expressed as below.

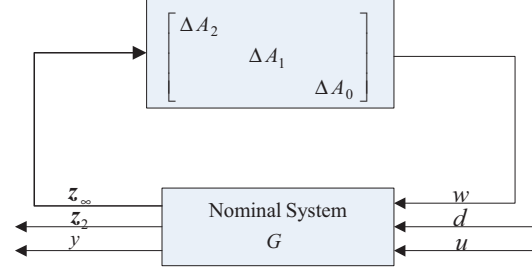


Fig. 1: A general block diagram of mixed H_2/H_∞

On the contrary, the H_2 norm is used to limit the output torque for the reason that the actuator has a saturation degree.

The model with rotational inertia uncertainty has a state space expression as below.

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_u u \\ z_\infty &= C_1 x + D_{11} w + D_{zu} u \\ z_2 &= u \end{aligned} \quad (8)$$

$$\text{in which } A = \begin{bmatrix} A_2^{-1} A_1 & A_2^{-1} A_0 \\ I & 0 \end{bmatrix}, B_1 = \begin{bmatrix} A_2^{-1} G_d & A_2^{-1} G_d \\ 0 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} F_{A_2} A_2^{-1} A_1 & F_{A_2} A_2^{-1} A_0 \\ F_{A_1} & 0 \\ 0 & F_{A_0} \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} F_{A_2} A_2^{-1} G_d & F_{A_2} A_2^{-1} G_d \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$D_{zu} = \begin{bmatrix} G_u^T & 0 & 0 \end{bmatrix}^T, B_u = G_u, w = \begin{bmatrix} d^T & \tilde{d}^T \end{bmatrix}^T$$

2.3 Attitude Control System with Rotational Inertia Uncertainty and Actuator Fault

The actuators in this paper are zero-momentum reaction flywheels. And the zero-momentum flywheel can fail due to its friction, saturation, dead zone, drift and the external disturbance. A control effectiveness factor is introduced in this paper to describe the fault magnitude of actuators. With the system fault and the consideration of fault diagnosis uncertainty, the relationship between order input u and actual output u_f is as below.

$$u_f = \sum_a (I + \Gamma_a A_a) u \quad (9)$$

Σ_a is the control effectiveness factor. Use $\Sigma_a = \text{diag}([\eta_x, \eta_y, \eta_z])$, $\eta_i (i=x, y, z) \in [0, 1]$ to signify the actuator conditions in three axes. The numbers between 0 and 1 signify the intactness degree of actuator, the bigger the number, the better the actuator. $\eta_i = 1$ means the actuator is in good condition, and $\eta_i = 0$ means the actuator fails totally. Γ_a is the weighting function of uncertainty scope, and Δ_a is the standardized uncertainty.

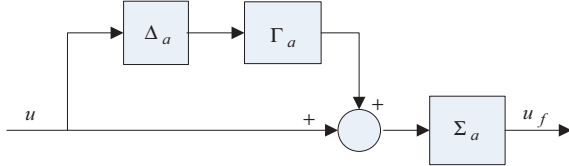


Fig. 2: Block Diagram of Actuator Fault

Under the condition of actuator fault, the state space expression of satellite attitude control system can be evinced as below.

$$\begin{aligned}\dot{x} &= Ax + B_1 w + B_u \Sigma_a (I + \Gamma_a \Delta_a) u \\ z_\infty &= C_1 x + D_{11} w + D_{zu} \Sigma_a (I + \Gamma_a \Delta_a) u \\ z_2 &= u\end{aligned}\quad (10)$$

Suppose $B_2 = B_u \Sigma_a$, $B_f = B_u \Sigma_a \Gamma_a$, $D_{12} = D_{zu} \Sigma_a$, $D_d = D_{zu} \Sigma_a \Gamma_a$, then the state space expression can be simplified as

$$\begin{aligned}\dot{x} &= Ax + B_1 w + B_2 u + B_d f \\ z_\infty &= C_1 x + D_{11} w + D_{12} u + D_d f \\ z_2 &= u\end{aligned}\quad (11)$$

in which $f = \Delta_a u$.

At present, the modeling of satellite attitude control system with actuator fault is completed. Both the rotational inertia uncertainty and the actuator fault are considered in this process. And the fault diagnosis result's uncertainty is also considered so the attitude controller's absolute dependence on fault diagnosis results is avoided.

3 Design of Robust Fault-tolerant Controller

3.1 Dynamic Fault-tolerant Control Law

With the control effectiveness factor Σ_a , an optimal control law is designed as below to stabilize the closed-loop control system.

$$u = K_1 x(t) + K_2 f \quad (12)$$

in which K_1 and K_2 are respectively the controller parameters.

The component $K_2 f$ has a function of compensating output torque errors which are caused by the performance decrease of actuators. And this compensation of output torque, which is totally determined by the fault diagnosis uncertainty, is equal to a feedforward control unit. The

selection of K_2 has a limitation as below.

$$B_2 K_2 + B_d f = 0 \quad (13)$$

It can be got from the equation (11) that $K_2 = \Gamma_a$. Therefore K_2 is only related to the scope of fault uncertainty and has no relationship with the system state and the fault magnitude.

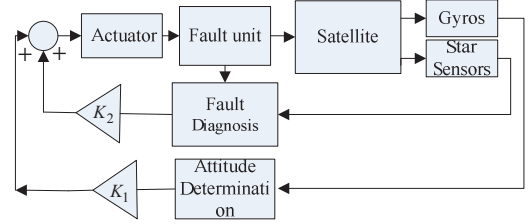


Fig. 3: Block Diagram of Fault-tolerant Attitude Control System

With equation (9) and (10), closed-loop control system can be got.

$$\begin{aligned}\dot{x} &= (A + B_2 \Sigma_a K_1) x + B_1 w \\ z_\infty &= (C_1 + D_{12} \Sigma_a K_1) x + D_{11} w \\ z_2 &= K_1 x\end{aligned}\quad (14)$$

in which

$$A_{cl} = A + B_2 \Sigma_a K_1, B_{cl} = B_1, C_{cl2} = K_1, C_{cl\infty} = C_1 + D_{12} \Sigma_a K_1, D_{cl\infty} = D_{11}.$$

3.2 Fault-tolerant Control with Pole Assignment

The objective of the robust fault-tolerant controller in this paper is to achieve an asymptotic stable condition of the attitude control system with rotational inertia uncertainty and flywheel fault by meeting the following design objectives.

- 1 The w to z_∞ closed-loop transfer function $\|T_{z_\infty w}\|_\infty < \gamma_\infty$, where γ_∞ is a scalar bigger than zero.
- 2 The w to z_2 closed-loop transfer function $\|T_{z_2 w}\|_\infty < \gamma_2$, where γ_2 is a scalar bigger than zero.
- 3 The closed-loop system's pole locates in a district of LMI.

To address the above problems, three lemmas are needed.

Lemma 1 Bounded Real Lemma^[10]: The closed-loop gain from w to z_∞ dose not exceed γ_∞ , namely $\|T_{z_\infty w}\|_\infty < \gamma_\infty$. Suppose $T_{z_\infty w} = (A_{cl}, B_{cl}, C_{cl\infty}, D_{cl\infty})$, if and only if a positive definite matrix X_∞ exists can the following inequality be satisfied.

$$\begin{bmatrix} A_{cl} X_\infty + X_\infty A_{cl}^T & B_{cl} & X_\infty C_{cl\infty}^T \\ * & -I & D_{cl\infty}^T \\ * & * & -\gamma_\infty^2 I \end{bmatrix} < 0 \quad (15)$$

Lemma 2 H_2 Performance^[10]: The closed-loop gain from w to z_2 dose not exceed γ_2 , namely, $\|T_{z_2 w}\|_2 < \gamma_2$.

Suppose $T_{z_w} = (A_{cl}, B_{cl}, C_{cl2}, D_{cl2})$, if and only if $D_{cl2} = 0$, two positive definite matrix X_2 , Q exists to satisfy the conditions below.

$$\begin{bmatrix} A_{cl}X_2 + X_2A_{cl}^T & B_{cl} \\ * & -I \end{bmatrix} < 0 \quad (16)$$

$$\begin{bmatrix} Q & C_{cl2}X_2 \\ * & X_2 \end{bmatrix} > 0 \quad (17)$$

$$R_{11} \otimes X_{opt} + R_{12} \otimes (A_{cl}X_{opt}) + R_{12}^T \otimes (A_{cl}X_{opt})^* + R_{22} \otimes (A_{cl}X_{opt}A_{cl}^T) < 0 \quad (20)$$

The poles in this paper are assigned into the elliptical district below.

$$D = \left\{ s = x + jy \mid \frac{(x+q)^2}{a^2} + \frac{y^2}{b^2} < 1 \right\} \quad (21)$$

And this district can be transformed into a LMI form with the equations below.

$$R_{11} = \begin{bmatrix} -1 & B_0 \\ * & -1 \end{bmatrix}; R_{12} = \begin{bmatrix} 0 & B_1 \\ B_2 & 0 \end{bmatrix}; R_{22} = 0$$

in which $B_0 = \frac{q}{a}$, $B_1 = \frac{1}{2a} + \frac{1}{2b}$, $B_2 = \frac{1}{2a} - \frac{1}{2b}$.

It can be got from Lemma 3 that all the eigenvalues of A_{cl} locate in the above elliptical district when a positive definite matrix X_{opt} exists to satisfy the linear matrix inequality below.

$$\begin{bmatrix} (AX + XA^T) + (B_2\Sigma_a Y + Y^T(B_2\Sigma_a)^T) & B_1 & XC_1^T + Y^T(D_{12}\Sigma_a)^T \\ * & -I & D_{11}^T \\ * & * & -\gamma_\infty^2 I \end{bmatrix} < 0 \quad (23)$$

$$\begin{bmatrix} -X & * \\ B_0 X + B_2(AX + B_2\Sigma_a Y) + B_1(AX + B_2\Sigma_a Y)^T & -X \end{bmatrix} < 0 \quad (24)$$

$$\begin{bmatrix} Q & K_1 X \\ * & X \end{bmatrix} > 0 \quad (25)$$

$$Trace(Q) < \gamma_2^2 \quad (26)$$

Finally, we obtain an optimum solution X^* , Y^* by solving LMIs (23),(24),(25),(26) which minimized $\gamma_2 + \gamma_\infty$, The control factor is

$$K_1 = Y^*(X^*)^{-1} \quad (27)$$

4 Simulation

The three principal axes of inertia's rotational inertias are $I_b = \text{diag}([12.49 \ 13.85 \ 15.75])$. The flywheels' maximum angular moment $H_{\max} = \pm 1.25 \text{ N.m.s}$. And the flywheels' maximum output torque $T_{\max} = 0.1 \text{ N.m}$. Suppose there is no fault of the satellite attitude control system, the three-axis stabilized satellite's pointing accuracy is 0.05deg, and its stabilization accuracy is 0.001deg/s. The initial

$$Trace(Q) < \gamma_2^2 \quad (18)$$

Lemma 3 Pole placement^[11]: Suppose the closed-loop system's pole locates in the LMI district below

$$D = \{z \in \mathbb{C} : R_{11} + R_{12}s + R_{12}^T s^* + R_{22}ss^* < 0\} \quad (19)$$

Then the condition of locating all the eigenvalues of A_{cl} into D is that a symmetrical positive definite matrix X_{opt} exists to satisfy the linear matrix inequation (LMI)(20).

$$\begin{bmatrix} -X_{opt} & * \\ B_0 X_{opt} + B_2 A_{cl} X_{opt} + B_1 X_{opt} A_{cl}^T & -X_{opt} \end{bmatrix} < 0 \quad (22)$$

To solve the linear matrix inequalities (15), (16), (17), (18) and (22), an appropriate Lyapunov matrix $X = X_\infty = X_2 = X_{opt}$ is needed.

3.3 Inference

With given scalars γ_2 and γ_∞ , a state feedback controller (12) exists only if positive definite matrixes $X \in \mathbb{R}^{6 \times 6}$, $Q \in \mathbb{R}^{3 \times 3}$, and $Y \in \mathbb{R}^{3 \times 6}$ satisfy the LMIs below.

attitude angles $\alpha = [-0.0017 \ -0.0026 \ 0.0013]$.

The disturbance torques in three axes are respectively $T_{dx} = A_0(3 \cos \omega_0 t + 1)$, $T_{dy} = A_0(1.5 \sin \omega_0 t + 3 \cos \omega_0 t)$, $T_{dz} = A_0(3 \sin \omega_0 t + 1)$, where A_0 is the amplitude of disturbance torque and $A_0 = 1.5 \times 10^{-5} \text{ N.m}$. ω_0 is the orbital angular velocity and $\omega_0 = 0.001 \text{ rad/s}$.

4.1 Attitude Control Simulation without Flywheel Fault

At first, the above controller is used to simulate the attitude control system performance without flywheel fault. Below are the optimal control parameters and the simulation results.

Table 1: Optimal Control Parameters

without Pole Assignment	$\gamma_\infty = 0.153$ $\gamma_2 = 1.733$
with Pole Assignment	$\gamma_\infty = 0.166$ $\gamma_2 = 1.733$ $a=1$ $b=2$ $q=1.3$

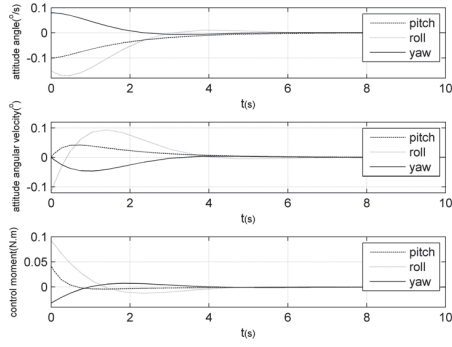


Fig. 4: H_2/H_∞ Control with Pole Assignment

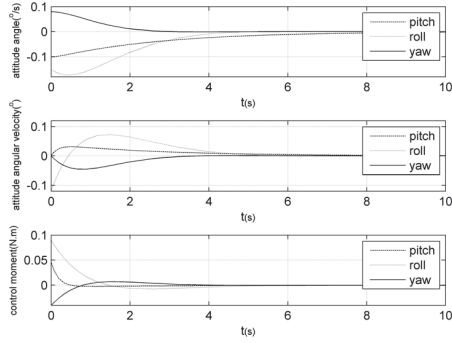


Fig. 5: H_2/H_∞ Control without Pole Assignment

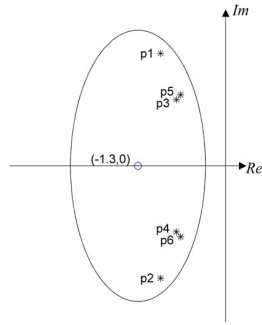


Fig. 6: Pole Location of closed-loop system

The details of poles' locations are that $P_{1,2} = -0.9659 \pm 1.6564i$; $P_{3,4} = -0.7307 \pm 0.9744i$; $P_{5,6} = -0.6686 \pm 1.0462i$.

It can be seen from Fig. 5 and Fig. 6 that the two control methods are both effective to stabilize the satellite when there is no fault, and the stabilization accuracy is 0.0005deg/s, the pointing accuracy is 0.0005deg, which are both far above the required accuracy. However, the control method with pole assignment's dynamic performance is much better, such as the pitch axis's overshoot and oscillation numbers, and its output is also more efficient.

4.2 Attitude Control Simulation with Flywheel Fault

There are three kinds of fault in this paper, namely, the one-axis flywheel fault, two-axis flywheel fault and three-axis flywheel fault. The fault magnitude of each axis is 0.09N.M, and the fault duration is 30-50s. It can be seen from the simulation above that the controller with pole assignment can have a better dynamic performance. Therefore the pole assignment process is used in this paper to perform the fault-tolerant H_2/H_∞ control. Below are the simulation results.

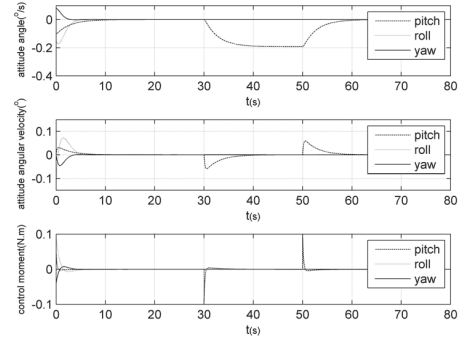


Fig. 7: Fault of Pitch Axis

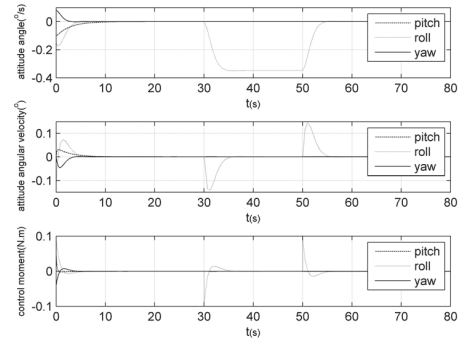


Fig. 8: Fault of Roll Axis

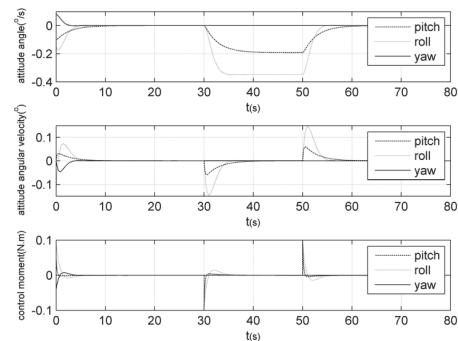


Fig. 9: Fault of Pitch and Roll Axes

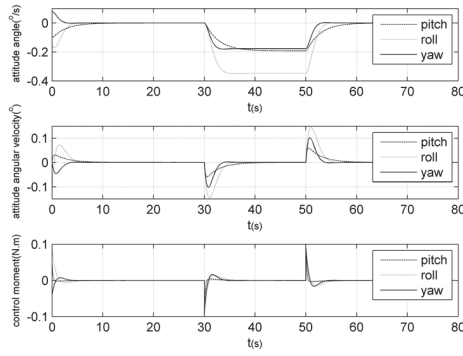


Fig. 10: Fault of Three Axes

The simulation results reveal that this controller is effective to deal with the satellite's actuator fault. And a quick attitude stabilization can be achieved even with a limited output torque after the fault. Moreover, the attitude control system's stabilization accuracy and pointing accuracy can also be maintained after the fault. Additionally, the performance decrease of roll axis is markedly bigger than the decrease of the other two axes under the same fault due to the coupling of pitch axis and yaw axis. Table 2 is the comparison of control performance between the pitch axis and roll axis.

Table 2: Comparison of Different Axes' Control Performance

Fault Axis	stabilization accuracy	pointing accuracy
Pitch	0.06deg/s	0.192deg
Roll	0.15deg/s	0.35deg

A single fault axis almost cannot influence the faultless axes, and the control performance can be well maintained even with two or three fault axes. The above simulation results demonstrate that the controller in this paper can control the attitude effectively with rotational inertia uncertainty and actuator fault. Therefore this control has a good robustness.

5 Conclusion

A robust fault-tolerant attitude controller is designed in this paper by combining H_2/H_∞ norms and the pole assignment method to address the actuator fault problem of three-axis stabilized satellite. This controller has a good robustness for the reason that the fault diagnosis uncertainty is considered in the design process of controller to avoid its absolute dependence on fault diagnosis result. The fault diagnosis uncertainty is compensated by

separating it as a fault signal instead of a noise, and this compensation obviously decrease the calculated amount of attitude controller. Therefore this robust fault-tolerant attitude controller of three-axis stabilized satellite is both effective and efficient.

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