A proof for concrete bumps

In the paper, we say a component is included by another component if the set of vertices of this component is covered by another component. We further define a *concrete bump* as such a component that 1) the quality of this component is positive; 2) the quality of this component is not lower than the quality of any component that is included by this component; and 3) merging any adjacent component(s) with this component induces a component that has a lower quality than that of this component or a merged component. Here, we prove that concrete bumps do not overlap with each other.

THEOREM 1. *Concrete bumps do not overlap with each other.*

PROOF. Suppose that there are two concrete bumps C_A and C_B that overlap with each other. Without loss of generality, suppose that $w(C_A) \leq w(C_B)$. Let $V_{A \cap B}$ be the set of vertices that are shared by C_A and C_B. Let $\sum_{i=1}^{u} C_{A \setminus B\_i}$ be u non-overlapping components that are induced by removing $V_{A \cap B}$ from C_A. The second condition above guarantees that $w(C_{A \setminus B\_i}) \leq w(C_A) \mid \forall\ 1 \leq i \leq u$. Let $\sum_{i=1}^{u} E_i$ be u sets of edges that respectively connect $\sum_{i=1}^{u} C_{A \setminus B\_i}$ to $V_{A \cap B}$ in C_A. The second condition above further guarantees that $c(E_i) \leq w(C_{A \setminus B\_i}) \mid \forall\ 1 \leq i \leq u$, as otherwise there is a component that is included by C_A and has a higher quality than C_A. Therefore, by using $\sum_{i=1}^{u} E_i$ to connect $\sum_{i=1}^{u} C_{A \setminus B\_i}$ to $V_{A \cap B}$ in C_B (i.e., to merge $\sum_{i=1}^{u} C_{A \setminus B\_i}$ with C_B), we can construct a larger component $C_{A \cup B}$ such that $w(C_{A \cup B}) \geq w(C_B) \geq w(C_A) \geq w(C_{A \setminus B\_i}) \mid \forall\ 1 \leq i \leq u$, which is not possible (by the third condition above). Hence, this theorem holds. $\square$